ANALYTICAL DESIGN OF CONTROL STRATEGIES FOR UNSTABLE TIME DELAY SYSTEMS

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CERTIFICATE

This is to certify that the thesis entitled **"Analytical Design of Control Strategies for Unstable Time Delay Systems"** being submitted by **Mr. Purushottama Rao Dasari** for the award of the degree of Doctor of Philosophy (Ph.D) in Chemical Engineering, National Institute of Technology, Warangal, India, is a record of the bonafide research work carried out by him under my supervision. The thesis has fulfilled the requirements according to the regulations of this Institute and in my opinion has reached the standards for submission. The results embodied in the thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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DECLARATION

This is to certify that the work presented in the thesis entitled "Analytical Design of Control Strategies for Unstable Time Delay Systems" is a bonafide work done by me under the supervision of Dr. A. Seshagiri Rao and is not submitted elsewhere for award of any degree.

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ABSTRACT

For a wider range of stable processes, many analytical PID controller tuning rules are available. However, for unstable processes, the availability of analytical tuning rules is limited. In this thesis, H_2 minimization theory in combination with internal model control (IMC) is used to analytically derive PID controller settings which can be used as a ready reference like look-up tables. These analytical settings are developed for a defined range of time delay to time constant ratio. Maximum sensitivity (M_s) is used for evaluating the robustness of the control system. Case studies of unstable systems are considered to evaluate the closed loop performances for set point variations and load disturbance variations. Robustness is evaluated for uncertainties in the process model as well as for the sensor noise. Recently published methods in the literature are considered for the performance comparison with the proposed method. Based on simulation results, it is observed that the current methodology provides significantly enhanced performances when compared with those techniques available in the recent literature. Experimental implementation is carried out on an inverted pendulum for demonstrating the practical applicability of the present method.

Optimal H_2 internal model controller (IMC) is designed for control of unstable cascade processes with time delays. The proposed control structure consists of two controllers in which inner loop controller (secondary controller) is designed using IMC principles. The primary controller (master controller) is designed as a proportional-integral-derivative (PID) in series with a lead-lag filter based on IMC scheme using optimal H_2 minimization. Selection of tuning parameter is important in any IMC based design and in the present work, maximum sensitivity is used for systematic selection of the primary loop tuning parameter. Simulation studies have been carried out on various unstable cascade systems. The present method provides significant improvement when compared to the recently reported methods in the literature particularly for the disturbance rejection. The present method also provides robust closed loop performances for large uncertainties in the process parameters. Quantitative comparison has been carried out by considering integral of absolute error (IAE) and total variation (TV) as performance indices.

Controller design for unstable processes is relatively difficult when compared to stable processes. The complexity increases further for multivariable unstable processes. In this work,

simplified tuning rules are proposed to design PID controller for unstable multivariable processes. Decouplers are applied to make the loops independent and diagonal elements of equivalent transfer function are used to design controllers. Two examples of TITO (two input two outputs) unstable system with time delays are considered for simulation. Comparative analysis has been carried out with the recently reported methods in the literature and observed that the proposed method provides improved closed loop performances. Robustness studies are also carried out with perturbations in the model parameters.

Control of unstable processes with time delays usually result in large overshoots in the closed loop responses. In industry, set-point weighting is one of the recommended methods to minimize the overshoot. In this work, a method is proposed to design the set-point weighting parameters which is relatively simple. Weighting is considered for both proportional (β) and derivative (γ) terms in the PID control law. In the closed loop transfer function for the servo problem, the coefficients of 's' and separately that of 's^3' both in the numerator and denominator are set equal in order to find β and γ. The obtained expressions for β and γ are simple and depends on the controller parameters. The method is carried out first for single input single output (SISO) unstable first order and second order processes with time delays and then for the multi input multi output (MIMO) unstable systems. In control of MIMO systems, decouplers are considered to ensure that the loops have minimum interactions. With the designed values, the closed loop performance is evaluated for different SISO and MIMO unstable systems with time delay. The present method is also compared with the recent methods proposed in the literature and it is observed that enhanced closed loop performances are achieved with the proposed method.

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ABBREVIATIONS

CSTR	Continuous Stirred Tank Reactor
CI	Closed loop Index
DC	Direct Current
DCTF	Desired Closed Loop Transfer Function
DoF	Degrees of Freedom
DS	Direct Synthesis
DS-IAE	Direct Synthesis- Integral Absolute Error
DTC	Dead Time Compensators
ETFs	Equivalent transfer functions
EOTFs	Effective open-loop transfer functions
FOPTD	First Order Plus Time Delay
GM	Gain Margin
IAE	Integral Absolute Error
IMC	Internal Model Control
ISE	Integral Square Error
IP	Inverted Pendulum
IPDT	Integrating Plus Time Delay
ITAE	Integral Time Absolute Error
IUFOPTD	Integrating Unstable First Order Plus Time Delay
LQG	Linear Quadratic Gaussian
LHP	Left Half Plane
MP	Minimum Phase
MIMO	Multi input and Multi output
MVC	Minimum Variance Control
NMP	Non Minimum Phase
PI	Proportional Integral
PD	Proportional Derivative
PID	Proportional Integral Derivative

PM	Phase Margin
RHP	Right Half Plane
SI	Sigma Index
SIMC	Skogestad Internal Model Control
SISO	Single Input and Single Output
SIMO	Single Input and Multi Output
SOPTD	Second Order Plus Time Delay
SP	Smith Predictor
TV	Total Variation
TITO	Two input and Two output
UFOPTD	Unstable First Order Plus Time Delay
USOPTD	Unstable Second Order Plus Time Delay

NOMENCLATURE

Symbol	Meanings
b	Cart Friction Coefficient
b_m, b_v	Blaschke products
d	Disturbance input
g	Acceleration due to gravity
k	Slope of ramp signal
μ	Time delay
n	Time constant
k_c	Proportional gain
k_m	Model gain
k_p	Process gain
k_{p1}	Primary process gain
k_{p2}	Secondary process gain
l	Pole Length
m	Pole Mass
p	RHP zero
r	Reference or set point
r_1	Primary loop set point
r_2	Secondary loop set point
u	Manipulated variable
v	Specific input
x	Cart position
y	Process output
y_1	Primary loop output
y_2	Secondary loop output
C	Exit concentration
C_f	Inlet concentration of reactor

D	Dilution rate
F_r	Set point filter
G_c	Conventional feedback controller
G_m	Model transfer function
G_p	Process transfer function
G_{p1}	Primary process
G_{p2}	Secondary process
H	Closed loop transfer function
I	Pole Moment of Inertia
M	Cart Mass
Ms	Peak value of sensitivity function
Q	Reactor inlet flow rate
Q_c	IMC controller
$\tilde{Q}_c$	H_2 optimal controller
S	Sensitivity function
T	Time delay to Time constant ratio
T_o	Normalized dead time
V	Reactor volume
α	Lead filter parameter
β_1 β_2	Set point weighting parameters for proportional mode
Υ_1 Υ_2	Set point weighting parameters for derivative mode
θ	Process time delay
θ_m	Model time delay
θ_{p1}	Primary time delay
θ_{p2}	Secondary time delay
Λ	Tuning parameter
μ	Filter constant
μ_{IAE}	DS-IAE based index
ς	IMC filter parameter
τ, τ_1, τ_2	Process time constants

τ_d	Derivative time
τ_f	Derivative filter coefficient
τ_i	Integral time
τ_m	Model time constant
τ_{p1}	Primary time constant
τ_{p2}	Secondary time constant
ϕ	Pendulum angle

Chapter 1

Introduction

Chapter 1

Introduction

1.1 General

A system whose transfer function has at least one pole lies in the right half plane (RHP) is known as an unstable system. This open loop instability characteristic moves the system away from the steady state even for small perturbations of process dynamics or operating conditions. The controller design for open-loop unstable systems is fundamentally difficult than that of the stable processes and the difficulty increases when there exists a time delay. The performance specifications that are usually achieved for stable systems are difficult to achieve for unstable systems. Because, for unstable systems, the performance specifications such as overshoot, settling time are larger and there exists a minimum and maximum value of controller gain below which and above which the closed loop system cannot be stabilized. These two values of maximum and minimum controller gain narrow down as the time delay increases thereby restricting the performance of the closed-loop system. The dynamics of many processes can be represented by first or second order system with time delay. For unstable first order plus time delay (UFOPTD) processes, the existence of a right half plane pole and time delay makes it difficult to stabilize the system, particularly with the conventional proportional integral/proportional integral derivative (PI/PID) controllers.

Several methods are proposed for the design of controllers for single-input-single-output (SISO) and multi-input-multi-output (MIMO) unstable systems. However, the interactions among the control loops and the existence of undesirable overshoots and settling times in the closed loop response raise the difficulty bar for MIMO systems as compared to SISO unstable systems. For MIMO systems, decouplers can be used to reduce the process interactions to strengthen system performance. Unstable systems result in larger overshoots in the closed loop set-point tracking responses. In order to minimize the overshoot, either a set-point filter or set-point weighting is recommended. In order to properly address the design of set-point weighting parameters, knowledge about the PID controller parameters (k_c, τ_i, τ_d) is necessary. It is challenging to achieve desired closed-loop performance with the PID controllers for these processes, particularly for set point tracking and load disturbance rejection in the presence of process uncertainties.

1.2 Motivation

Tuning of different types of controllers for right half plane pole processes, which are classified as unstable, has been addressed by many people in the open literature. IMC based design using the Laurent series expansion is developed by Panda (2009) and by using Maclaurin series approximation is developed by Nasution et al. (2011). Based on the need of an operation, i.e., either in a set-point tracking mode or load disturbance rejection mode, Arrieta et al. (2011) developed formulae for a controller for unstable and integrating processes. Summary of different design methodologies for controllers for such processes is addressed by Rao and Chidambaram (2012). To enhance the tracking capability smoothly, either placing a filter for the set point signal or using a weight for set point signal is preferred. The methods developed by Nasution et al. (2011) and Panda (2009) are applicable for a wider range of time delay to time constant ratios [($\mu/n = T_0 = 0.1$ to 1.2) where μ = time delay and n = time constant] and have become popular methods since then. However, the equations are not simple and are tedious to utilize in practice. Also, the method Wang et al. (2015) is not applicable when the time delay to time constant ratio is greater than 0.9. The important points about the methods discussed so far are (1) Analytical tuning rules are not available for many methods and (2) some methods cannot be applied when $\mu/n \geq 1$. Recently, Sree and Chidambaram (2017) discussed the importance of unstable systems and their occurrence in practice.

Many researchers (Tan et al., 2000; Lee and Oh, 2002; Liu et al., 2005a) addressed the design and analysis of cascade control strategies for stable processes. However, limited research work has been carried out for the design of cascade control strategies for unstable processes. Liu et al. (2005b) proposed IMC based cascade control scheme for unstable processes with four controllers. Kaya and Atherton (2008) proposed a cascade control structure for controlling unstable and integrating processes with four controllers. Uma et al. (2009) proposed an improved cascade control scheme for unstable processes using a modified Smith predictor with three controllers and one filter in the outer loop. Padhan and Majhi (2012) proposed a modified Smith predictor based cascade control structure for unstable processes where they used three controllers. Most of the existing methods use more number of controllers and also the design of these controllers is not simple. In practice, a cascade controller structure with only two controllers (one for secondary loop and another for primary loop) is desirable.

Few methods are reported to design controllers for unstable multivariable systems. Over the extensive literature available for multivariable unstable systems, few works (Georgiou et al., 1989; Park, 1991; Flesch et al., 2011) proposed a decentralized PID controller design for unstable multivariable systems using optimization method. Hazarika and Chidambaram (2014) proposed a double loop control structure to decrease the overshoot such as proportional controller followed by PI controller in the outer loop based on the equivalent transfer function. They have shown that by a single loop PI control with a set point filter, the overshoot is reduced significantly and a good servo response is obtained. However, this will not improve the regulatory responses. It may be desirable to use better settings, particularly with a PID controller to improve the performances of both the servo and regulatory performances. All of these methods follow a complex procedure to design the controller.

To get an improved transient response due to the set point change, several researchers proposed either set-point filtering or set-point weighting methods. Prashanti and Chidambaram (2000) developed formulae to calculate the set point weighting parameters for UFOPTD systems for different ratio of time delay to dominant time constant. Chen et al. (2008) recommended the tuning rules for set-point weighting based on a three-element control structure. Begum et al. (2016), Wang et al. (2016) and Begum et al. (2017) proposed controller tuning rules for stable, unstable and integrating processes with time delay based on internal model control (IMC) technique. Nasution et al. (2011) designed controller for time-delayed unstable processes with set point weighting making use of an optimal H_2 IMC-PID control strategy. It is possible to demonstrate that tracking performance of the set-point will be improved if suitable weighting for the derivative mode is determined. This is determined by Nasution et al. (2011), but in a complex way.

Based on the review of literature and identified research gaps, the following objectives are framed.

1.3 Objectives

1. To design optimal H_2 PID controllers for SISO unstable time-delay systems for enhanced closed-loop performances.
2. To experimentally implement the designed controller on an inverted pendulum.
3. To design optimal H_2 PID controllers for unstable cascade time-delay systems for enhanced closed-loop performance.
4. To design multi variable optimal H_2 PID controllers for MIMO unstable time-delay systems.
5. To design set-point weighted PID controllers for SISO and MIMO unstable time-delay systems.

1.4 Organization of the thesis

The organization of the thesis is as follows:

Chapter 2 presents a literature overview for the design of optimal H_2 PID controllers, cascade control, multivariable control and set point weighted PID controllers for SISO and MIMO unstable time delay systems.

Chapter 3 provides an overview of the reported work on the design of optimal H_2 PID controllers for SISO unstable time delay systems for enhanced closed loop performance. H_2 minimization theory in combination with the internal model control (IMC) is used to analytically derive PID controller settings which can be used as a ready reference like look-up tables. These analytical settings are developed for a defined range of time delay to time constant ratio. Maximum sensitivity (M_s) is used for evaluating the robustness of the closed loop system. This chapter highlights the first objective.

Chapter 4 presents the experimental implementation of PID control proposed in chapter 3 to an inverted pendulum for the control of pendulum rod angle by manipulating the cart position. This chapter focused on the second objective.

Chapter 5 proposes an optimal H_2 internal model controller (IMC) designed for the cascade control of unstable processes with time delays which is the third objective. The proposed control structure consists of two controllers in which inner loop controller (secondary controller) is designed using IMC principles. The primary controller (master controller) is designed as a proportional-integral-derivative (PID) in series with a lead-lag filter based on IMC scheme using optimal H_2 minimization. Simulation studies have been executed to show the advantages of the proposed method.

Chapter 6 presents a simplified tuning rules to design optimal H_2 PID controller for unstable multivariable processes. Decouplers are applied to make the loops independent and diagonal elements of equivalent transfer function are used to design the controllers. This chapter deals with the fourth objective.

Chapter 7 proposes a method to design the set-point weighting parameters for UFOPTD and USOPTD processes which is relatively simple and also reduces the overshoot. Weighting is considered for both proportional (β) and derivative (γ) terms in the PID control law. In the closed loop relation for set-point tracking, the coefficients of 's' and separately 's^3' both in the numerator and denominator are made equal in order to find β and γ. The obtained expressions for β and γ are simple and depends on the controller parameters. The design is carried out first for single input single output (SISO) unstable first order and second order processes with time delays and then for the multi input multi output (MIMO) unstable systems. For control of MIMO systems, decouplers are considered to ensure that the loops have minimum interactions. With the designed values, the closed loop performance is evaluated for different SISO and MIMO unstable processes with time delay. This chapter highlights the fifth objective.

Chapter 8 gives the summary and conclusions of the present work along with suggestions for the future work.

Appendix B gives the MATLAB program developed for simulating PID controllers for SISO unstable time delay systems and the corresponding Simulink block diagram for examples considered in Chapter 3.

Appendix C presents the MATLAB program developed for simulating unstable cascade processes with time delays and the corresponding Simulink diagram for examples considered in Chapter 5.

Appendix D gives the MATLAB program for Simplified tuning rules for PID controller for unstable multivariable processes considered in Chapter 6.

Appendix E presents the MATLAB program developed for simulating PID controllers for SISO and MIMO unstable time delay systems with set point weighting and the corresponding Simulink block diagram for examples considered in Chapter 7.

The main contributions of the thesis are

(i) Enhanced design of PID controllers for unstable time delay systems using optimal H_2 framework (objectives - 1, 2)

(ii) Design of controllers for unstable cascade time-delay systems to achieve enhanced closed-loop performance (objective – 3)

(iii) Design of multi variable optimal H_2 PID controllers for MIMO unstable time-delay systems for improved performance (objective – 4).

(iv) Systematic design of set-point weighted PID controllers for both SISO and MIMO unstable time-delay systems for enhanced tracking of set-points (objective – 5).

Chapter 2

Literature Review

Chapter 2

Literature Review

In this chapter, the literature are reviewed on occurrence of unstable process and the design of PID controllers for unstable systems. The methods available for the design of PID controllers for SISO and MIMO unstable time delay systems are presented. The review is given on the control of unstable cascade processes with time delays and on the design of the set-point weighting parameters for UFOPTD and USOPTD processes.

2.1 Existence of unstable processes

Numerous researchers have been focusing on the systems that exhibit unstable behavior and the related literature on the subsistence of unstable processes are listed below. An unstable system is the one which has at least one RHP pole in the complex plane.

Van Heerden (1953) has shown that chemical reactors might exhibit multiple steady states and oscillatory solutions, depending on particular operating conditions. Real-time systems reveal several steady states owing to the certain nonlinearity of the processes. Some of the steady states may be unstable and it is essential to operate the system at unstable steady state for economic and/or safety measures.

Marlin (1995) presented a jacketed continuous stirred tank reactor (CSTR) that is used to perform a simple reaction approximated as an USOPTD model consisting of two unstable complex conjugate poles and a negative zero when the model equations are linearized around an unstable operating point.

Chidambaram (1997) has given the review of the work done on the control of unstable systems. A detailed survey has been stated in the previous works which are related to the control of unstable processes.

Jacobsen (1999) has studied the dynamics of reactor separator networks and has shown that the transfer function model between the composition of the distillate and the recycle ratio of the distillation column results in unstable second order model with one unstable pole and unstable zero.

Bequette (2003) has considered a CSTR with simple reaction; linearization around the unstable operating point gives an USOPTD model with a negative zero with two unstable poles.

Stein (2003) has described the practical, physical consequences of unstable process control. Extensive information on the physical significance of unstable systems is presented.

Sree and Chidambaram (2006) presented an excellent overview of the physical occurrence of unstable processes. Many physical examples are stated as unstable transfer function models. The problems in the control of unstable systems are given in detail.

A summary of the reported work on the existence of unstable processes is given in Table 2.1.

2.2 Design of PID controllers for an unstable process with time delay

This section gives the overview of the literature on PID controllers with and without lead lag filters for unstable processes.

Yang et al. (2002) have proposed IMC based single loop controller design method in which the feedback controller is either PID or higher order form. This can be made automatic for on-line tuning of the first order, second order and higher order unstable processes with time delays.

Tan et al. (2003) have presented a modified IMC structure with three controllers. The set point tracking controller is a lead lag filter and a proportional (P) controller is used for stabilizing the original unstable plant ignoring the time delay, a proportional derivative (PD) controller is used for stabilizing the unstable processes with time delay. Good nominal and control action responses are achieved.

Sree et al. (2004) have designed PI/PID controllers for stable first order plus time delay (FOPTD) systems based on equating the coefficients of corresponding powers of s in the numerator and that in the denominator of the closed loop transfer function for a servo problem. They also extended this method to design PI/PID controllers for an UFOPTD system which gave improved performance and was robust for uncertainty in the model parameters.

Table 2.1 Literature review on existence of unstable processes

S.No.	Author	Description	Remarks
1	**van Heerden** (1953)	Real systems show multiple steady states because of certain nonlinearity of the systems in which some of the steady states may be unstable.	Chemical reactors exhibit multiple steady states and it becomes necessary to operate the system at unstable steady state for economic and/or safety measures.
2	**Marlin** (1995)	Presented jacketed CSTR, when the model equations are linearized around an unstable operating point, resulting in an unstable system.	USOPTD model that contains two unstable complex conjugate poles and a negative zero.
3	**Chidambaram** (1997)	A detailed review of the previous work carried out on unstable processes.	Review of unstable systems.
4	**Jacobsen** (1999)	Transfer function model between the composition of the distillate and the recycle ratio of the distillation column as an unstable system.	An unstable second order model with one unstable pole and one unstable zero
5	**Bequette** (2003)	CSTR performing a simple reaction; linearized around the unstable operating point gives USOPTD model.	Unstable model with two unstable poles and a negative zero.
6	**Stein** (2003)	Described the practical, physical consequences of unstable process control.	The physical significance of unstable systems in the context of airplanes.
7	**Sree and Chidambaram** (2006)	Several examples have been stated for the existence of unstable behavior in the processes.	Gas phase polyolefin reactor, Jacketed CSTR, Isothermal reactor, cart and pole problem, helicopter and airplanes.

Rao and Chidambaram (2005) have proposed a modified form of smith predictor (SP) for unstable processes with time delay for servo and regulatory problems with three controller's viz., the set point PI controller designed based on synthesis method, a P controller for disturbance rejection and a PD stabilizing controller. Better control performances were obtained for unstable processes with a time delay.

Liu et al. (2005a) have proposed an analytical 2 DoF control scheme for open loop unstable first, second order and integrating unstable first order processes with time delay. They have designed three controllers in which proportional or plus derivative controller is employed to stabilize the set point response; a H_2 optimal set point tracking controller is designed based on integral square error (ISE) performance specification. The desired disturbance transfer function is used to design the disturbance estimator in the inner closed loop to obtain the ISE performance objective. Here the servo and regulatory responses can be tuned easily by a single tuning parameter.

Shamsuzzoha and Lee (2007) have elaborated a PID controller using a finest IMC filter structure that produces an enhanced disturbance rejection response for stable, integrating, unstable processes with time delays. The controllers are all tuned to have the same level of robustness in terms of maximum sensitivity (*Ms*). The tuning parameter (λ) guidelines were also proposed for numerous process models over a wide range of θ/τ ratios.

Rao et al. (2007a) have designed the modified smith predictor (SP) with improved closed loop responses with two controllers for open loop UFOPTD processes using direct synthesis method to design the set point tracking controller and simple analytical tuning rules for the load disturbance controller.

Normey-Rico and Camacho (2008) have presented a simple modified smith predictor (SP) structure to improve closed loop characteristics. The proposed structure is simple to analyze and gives totally decoupled disturbance rejection and set point responses for UFOPTD processes.

Shamsuzzoha and Lee (2008a) have designed a PID controller cascaded with a first order lead/lag filter for integrating and first order unstable processes with time delay based on the IMC criterion, which has a single tuning parameter to regulate the performance and robustness of the controller. A set point filter is used to reduce the overshoot in the servo response.

Shamsuzzoha and Lee (2008b) have discussed the design of IMC based PID controller cascaded with a first order lead/lag compensator for a class of second order stable and unstable processes with time delay. A set point filter is used to reduce the overshoot in the set point response.

Panda (2009) have designed IMC controller equivalent PID tuning rules using Laurent series approximation for unstable first order processes, second order processes with and without left half plane (LHP) zeros and integrating processes with time delays. The controller designed is robust, stable and can be implemented easily in real-time process.

Nasution et al. (2011) have presented the synthesis of an optimal IMC based design of H_2-PID controller for unstable processes with single RHP pole and time delay. He compared the control performance and robustness resulting from the five desired closed loop transfer functions (DCLTF) and recommended to use two from the overall consideration. To reduce overshoots, the set point weighting parameters for both proportional and derivative modes have been derived.

Shamsuzzoha et al. (2012) have proposed a modified underdamped IMC filter and derived a PID controller based on the new filter for unstable processes with time delays which provided the desired integral action and improved closed loop performance of the system.

Cho et al. (2014) have presented PI/PID tuning rules based on direct synthesis method by utilizing simple desired closed loop transfer functions and simple approximations of the process time delay. It has one design parameter and a set point filter for the 2 Degree of freedom controller.

Vanavil et al. (2013) have designed a PID with lead lag filter based on direct synthesis method. Set point weight is considered to reduce the overshoot. Systematic guidelines have been provided for selection of the tuning parameter based on the peak value of the sensitivity function and improved closed loop performances are obtained.

Vanavil et al. (2014) have proposed an IMC-PID controller with a lead lag filter based on H_2 minimization concept for integrating and UFOPTD processes and provided good closed loop performance for normalized dead time (θ/τ) up to 1.8.

Table 2.2 Literature review on design of PID controllers for an unstable process with time delay

S.No.	Author	Description	Remarks
1	**Yang et al.** (2002)	IMC based single loop controller design method in which the feedback controller is either PID or high order form.	Applicable for on-line tuning of the first order, second order and higher order unstable processes with time delays and gave satisfactory performance.
2	**Tan et al.** (2003)	Modified IMC structure with three controllers.	For first and second order unstable processes and achieved a good compromise between time domain performance and robustness.
3	**Sree et al.** (2004)	PI/PID controllers based on equating coefficient method.	For stable and unstable first order plus time delay (FOPTD) systems.
4	**Rao and Chidambaram** (2005)	A modified form of smith predictor (SP) with three controllers.	Applicable for UFOPTD systems. Better responses were obtained.
5	**Liu et al.** (2005a)	2 DoF control structure with three controllers, H_2 optimal controller for set point tracking and a load disturbance estimator.	Nominal set point response is decoupled from the load disturbance response for unstable first order, second order and integrating unstable first order processes.
6	**Shamsuzzoha and Lee** (2007)	IMC based PID controller.	Produced improved disturbance rejection response for stable, integrating, unstable processes with time delays.
7	**Rao et al.** (2007a)	Modified smith predictor (SP) with two controllers.	Obtained improved responses for open loop unstable first order plus time delay.
8	**Normey-Rico and Camacho** (2008)	Design of dead time compensators (DTC)	To control unstable FOPTD systems.
9	**Shamsuzzoha and Lee** (2008a)	PID controller cascaded with a first order lead/lag filter based on IMC criterion.	Provided improvement in both set point and disturbance rejection for integrating and first order unstable processes with time delays.

10	**Shamsuzzoha and Lee** (2008b)	IMC based PID controller cascaded with a first order lead/lag compensator.	For a class of second order stable and unstable processes with time delay.
11	**Panda** (2009)	A robust internal model controller equivalent PID tuning rules using Laurent series approximation.	For unstable first order processes, second order processes with and without LHP zero, integrating processes with time delays.
12	**Nasution et al.** (2011)	IMC based design of H_2 optimal PID controller using Maclaurin series approximation.	For unstable processes with single RHP pole and time delays. Due to performance limitation of a 1 DoF controller, 2 DoF controller is derived.
13	**Shamsuzzoha et al.** (2012)	Used a modified underdamped IMC filter and derived a PID controller.	Provided better integral action and improved closed loop performance.
14	**Cho et al.** (2014)	PI/PID tuning rules based on direct synthesis method.	Yielded similar or even improved performance over previous complicated PID tuning methods.
15	**Vanavil et al.** (2013)	PID with lead lag filter based on direct synthesis method for unstable processes.	Set point weight is used and applied to an inverted pendulum experiment.
16	**Vanavil et al.** (2014)	PID with lead lag filter based on H_2 minimization concept.	For controlling integrating and UFOPTD processes and provided good closed loop performance up to $\theta/\tau = 1.8$.
17	**Wang et al.** (2016)	IMC-PID tuning method based on pole zero conversion design.	Provided improved performance than other PID tuning method for stable and unstable processes with time delay.
18	**Begum et al.** (2018)	Developed Optimal controller synthesis for second order time delay systems with at least one RHP pole	The proposed controller design solves the trade-off issue between the robustness and performance with the help of an adjustable parameter.

Wang et al. (2016) have proposed a new IMC-PID tuning method based on pole zero conversion design and PID with a lead lag compensator is designed for first order plus integrating and second order unstable processes with time delay. The method demonstrated better performance than other PID tuning methods. A set point weighing is used to reduce overshoot.

Begum et al. (2018) have proposed IMC-PID tuning method based on H_2 minimization. The controller design is addressed for the second order processes with dead time which has at least one pole in the right half of the s plane.

A summary of the reported work on the tuning of unstable processes with time delays is given in Table 2.2.

2.3 Design of PID controllers based on maximum sensitivity

In any closed loop system, to guarantee a minimum robustness level, there is always a robustness performance trade-off to obtain a smooth response to both step set point and disturbance changes. Analytical tuning rules have been developed for stable processes based on maximum sensitivity values and the literature review of such tuning rules developed is given below.

Alfaro et al. (2010) developed tuning rules for 2 DoF PI controllers for stable FOPTD models. They dealt with the closed loop control system performance robustness trade-off by selecting M_s in the 1.4 to 2.0 range and designed control systems with low, minimum, medium or high robustness levels. Controller tuning rules were provided for FOPTD models with normalized dead times from 0.1 to 2.0.

Padula and Visioli (2011) presented a set of tuning rules for integer and fractional-order PID controllers and analytical expressions for performance assessment. They applied for stable FOPTD models, which minimized integral absolute error (IAE) with a constraint on the Ms and proved that the use of a fractional-order PID controller is more advantageous with respect to improvement in performance.

Vilanova et al. (2012) provided tuning rules for 2 DoF PI controller using M_s as the design parameter for the desired robustness level for stable FOPTD dynamics in terms of normalized dead time allowing the user to select a low/medium/high robust closed loop control system. The proposed auto tuning expressions, when compared with other methods, guaranteed the same robustness level and provided improved performances.

Alfaro and Vilanova (2012) presented a robust tuning method for 2 DoF PI controllers based on the use of a model reference optimization procedure for first order plus time delay (FOPTD) models and second order plus time delay (SOPTD) model process models which allowed the designer to deal with the performance/robustness trade-off of the closed loop control system by selecting M_s in the range from 1.4 to 2.0. They derived a set of controller tuning equations for FOPTD and SOPTD models with normalized dead times from 0.1 to 2.0 that guarantees the achievement of the design robustness level.

Table 2.3 Literature review on design of PID controllers based on maximum sensitivity

S.No.	Author	Description	Remarks
1	**Alfaro et al.** (2010)	Tuning rules for 2 DoF PI controllers. Solved robustness and performance trade-off.	For stable FOPDT models. Selected M_s in 1.4-2.0 range and designed the control system with low, minimum, medium or high robustness levels.
2	**Padula and Visioli** (2011)	A set of tuning rules for integer and fractional-order PID controllers and analytical expressions for performance assessment.	Applied for stable FOPTD models and minimized IAE with a constraint on the M_s.
3	**Vilanova et al.** (2012)	Tuning rules for a 2 DoF PI controller using M_s value as the design parameter.	The user can select a high/medium/low robust closed loop control system for stable first order plus time delay dynamics.
4	**Alfaro and Vilanova** (2012)	A robust tuning method for 2 DoF PI controllers based on the use of a model reference optimization procedure.	For FOPTD and SOPTD controlled process models by selecting Ms in the range from 1.4 to 2.0.
5	**Alfaro and Vilanova** (2013)	A model reference robust tuning method for 2 DoF PID controllers based on the use of an optimization procedure.	For inverse response controlled process modeled by a second order plus a right half plane zero transfer function by selecting robustness levels between 1.6 and 2.0.
6	**Begum et al.** (2016)	Developed PID tuning rules based on maximum sensitivity (M_s) for unstable dead time processes	These tuning rules allow the designer to design closed loop control system with a specified low, medium, or high robustness level by selecting the corresponding value of M_s.

Alfaro and Vilanova (2013) presented a model reference robust tuning method for 2 DoF PID controllers based on the use of a model reference optimization procedure for inverse response controlled process modelled by a second order plus a right half plane zero transfer function which allowed the designer to deal with the performance/robustness trade-off of the closed loop control system by selecting robustness levels between 1.6 and 2.0.

Begum et al. (2016) Developed PID tuning rules based on maximum sensitivity (M_s) for unstable dead time processes. These tuning rules allow the designer to design closed loop control system with a specified low, medium, or high robustness level by selecting the corresponding value of M_s.

A summary of the reported work regarding the maximum sensitivity based analytical tuning of stable processes are given in Table 2.3.

2.4 Design of series cascade controllers for an unstable processes

A cascade control can be used to obtain better disturbance rejection existing in the inner loop and is used to improve single loop control performance when the disturbances are associated with the manipulated variable or when the final control element exhibits nonlinear behavior. The desired performance can be obtained with simple PID controllers, IMC controllers and by using SP structures. This section gives the overview of the literature on controllers for tuning of series cascade processes.

Lee et al. (2002) proposed a general IMC-PID control structure to handle stable, integrating, and unstable processes for cascade control systems. The new structure was more robust to measurement noises with improved performance than a conventional cascade control structure.

Kaya and Atherton (2008) designed an improved cascade control structure and controller design method for controlling unstable and integrating processes where PI-PD-SP scheme was used in the outer loop and IMC controller in the inner loop. Simulation results were provided to illustrate the proposed structure's superiority.

Table 2.4 Literature review on design of series cascade controllers for an unstable process.

S. No.	Author	Description	Remarks
1	**Lee et al.** (2002)	A robust IMC-PID control structure.	For stable, integrating, and unstable processes for cascade control systems with improved performance.
2	**Kaya and Atherton** (2008)	An improved cascade control structure with PI-PD-SP scheme used in the outer loop and IMC controller in the inner loop.	Provided improved performance for unstable and integrating processes.
3	**Uma et al.** (2009)	Modified SP structure with three controllers: IMC controller for inner loop and two PID controllers with a filter for the outer loop.	Applied for unstable processes with and without zero. The primary loop controllers designed based on direct synthesis method.
4	**Uma et al.** (2010)	Modified SP structure with three controllers: IMC controller for the inner loop and PID and PD controller with filters for the outer loop.	Applied for integrating processes with and without zero.
5	**Padhan and Majhi** (2012)	Modified SP with three controllers. Direct synthesis based method for set point tracking and 2 PID with second order lead lag filter for disturbance rejection.	Provided improved disturbance rejection capability.
6	**Santosh and Chidambaram** (2013)	P/PI controllers based on equating coefficient method.	For a series cascade control system with UFOPTD models. The degree of robustness for uncertainty in the model parameters was studied.
7	**Raja and Ali** (2017)	Various series cascade control strategies are briefly reviewed and their advantages and disadvantages are discussed	Suitable tuning strategies for a class of stable, unstable and integrating process models are recommended in order to help the user in selecting the appropriate control strategy.
8	**Yin et al.** (2019)	Improved Cascade Control System for a Class of Unstable Processes with Time Delay	proposed cascade control scheme based on modified Smith predictor for controlling a class of unstable processes with time delay

Uma et al. (2009) presented enhanced modified SP structure for the control of open loop unstable cascade processes with/without zero using three controllers in which the secondary loop has one IMC controller and the primary loop has two controllers designed based on direct synthesis method viz., PID with lag filter for set point tracking and PID with lead lag filter for disturbance rejection which provided significant improvement in disturbance rejection characteristics.

Uma et al. (2010) proposed modified SP structure for control of integrating processes with and without zero using three controllers in which the secondary loop has one IMC controller and the primary loop has two controllers designed based on direct synthesis method viz., PID with lag filter for set point tracking and PD with lead lag filter for disturbance rejection.

Padhan and Majhi (2012) presented a modified SP for controlling open loop unstable time delay processes. The proposed structure has three controllers of which one is meant for servo response designed based on direct synthesis method and the other two are PID controllers cascaded with a second order lead/lag filter for regulatory responses with improved disturbance rejection capability. Kharitonov's theorem is used for the robustness analysis.

Santosh and Chidambaram (2013) designed P/PI controllers for a series cascade control system for UFOPTD systems based on equating the coefficients of corresponding powers of s and s^2 in the numerator to α_1 and α_2 times those of the denominator of the closed loop transfer function for a servo problem with only two tuning parameters. The robustness for uncertainty in the model parameters was studied and the performances were found to be better.

Raja and Ali (2017) discussed various series cascade control strategies and reviewed their advantages and disadvantages. Suitable tuning strategies for a class of stable, unstable and integrating process models are recommended in order to help the user in selecting the appropriate control strategy.

Yin et al. (2019) proposed cascade control scheme based on modified Smith predictor for controlling a class of unstable processes with time delay. The proposed control structure consist three controllers of which the secondary loop has one controller and the primary loop has two controllers.

A summary of the reported work on the tuning of series cascade unstable processes is given in Table 2.4.

2.5 Design of controllers for multi-input-multi-output (MIMO) unstable systems

This section gives the literature review on design of PID controller for unstable multivariable processes.

Georgiou et al. (1989) presented a multivariable controller design technique for open loop unstable systems and the controller is tuned in four stages step optimization procedure. The closed loop performance and robustness of the multi loop SISO controllers are established by an effective damping coefficient and its corresponding effective closed loop time constant. They considered a complex system of two reactors in series, where exothermic second order reaction occurs. However, the system does not take into account large values of time delay.

García and Albertos (2010) designed a new dead-time compensator to deal with unstable multivariable systems with multiple time delays and a MIMO dead time compensator is suitable for any linear plants has been presented which is both applicable for stable and unstable plants.

Rajapandiyan and Chidambaram (2012) a decoupler with a decentralized control system is designed based on ETF models and the proposed method has shown the better performance of compared with the centralized control system, ideal, inverted, and normalized decoupling methods.

Hazarika and Chidambaram (2014) designed multivariable proportional integral controllers for unstable multivariable systems and used equivalent transfer function model to design multivariable PI controllers for diagonal elements and simplified decouplers are used to decompose the unstable multi loop systems into independent loops and the double loop control structure is used to reduce the overshoot for unstable systems.

Chandrasekhar et al. (2016) Proposed simple method of designing decentralised PID controllers for stable systems by synthesis method and extended the method to unstable systems. The method gives improved responses and decreased interactions when compared to that of the Govindhakannan decentralized PID control system design method.

Table 2.5 Literature review on design of controllers for multi-input-multi-output (MIMO) unstable systems

S. No.	Author	Description	Remarks
1	**García and Albertos** (2010)	Designed a new dead-time compensator to deal with unstable multivariable systems with multiple time delays.	Here a MIMO dead time compensator suitable for any linear plants has been presented which is both applicable for stable and unstable plants.
2	**Rajapandiyan and Chidambaram** (2012)	Controller Design for MIMO Processes Based on Simple Decoupled Equivalent Transfer Functions and Simplified Decoupler.	A decoupler with a decentralized control system is designed based on ETF models and the proposed method has shown the better performance of compared with the centralized control system, ideal, inverted, and normalized decoupling methods.
3	**Hazarika and Chidambaram** (2014)	Designed multivariable proportional integral controllers for unstable multivariable systems	Used equivalent transfer function model to design multivariable PI controllers for diagonal elements and simplified decouplers are used to decompose the unstable multi loop systems into independent loops and double loop control structure is used to reduce the overshoot for unstable systems
4	**Chandrasekhar et al.** (2016)	Proposed simple method of designing decentralized PID controllers for stable systems by synthesis method is extended to unstable systems.	Decentralized control system is designed using maclaurin Series, gives improved responses and decreased interactions when compared to that of the Govindhakannan decentralized PID control system design method.

A summary of the reported work regarding the design of PID controller for unstable multivariable processes are given in Table 2.5.

2.6 Design of set-point weighting parameters for unstable systems

This section gives the overview of the literature on set-point weighted PID controllers for SISO and MIMO unstable time-delay systems.

Sree and Chidambaram (2003a) designed set point weighted PI controller for UFOPTD process with a zero based on equating coefficient method by matching the corresponding first power of *s* in the numerator and that in the denominator of the closed loop transfer function. Simulation results were provided to illustrate the robust performance of the controller which is evaluated by simulation on a CSTR with non-ideal mixing carrying out an enzymatic reaction.

Sree and Chidambaram (2003b) proposed a set point weighted PI controller for stabilizing an unstable bioreactor with a dominant unstable zero based on direct synthesis method. The controller design proved to be robust for perturbations in the controller settings. A set point weighting has been considered for the controller to reduce the initial jump and undershoot of the servo response.

Liu et al. (2005a) have proposed an analytical 2 DoF control scheme for open loop unstable first, second order and integrating unstable first order processes with time delay. They have designed three controllers in which proportional or plus derivative controller is employed to stabilize the set point response; a H_2 optimal set point tracking controller is designed based on ISE performance specification. The desired disturbance transfer function is proposed to design the disturbance estimator in the inner closed loop to obtain the ISE performance objective. Here the set point and load disturbance responses can be tuned easily by a single tuning parameter.

Shamsuzzoha and Lee (2008c) designed a simple 2 DoF IMC based PID controller for integrating processes with positive and negative zeros focusing on disturbance rejection. Comparisons with the other tuning methods have been carried out for the same level of robustness. Guidelines are provided for the selection of tuning parameter.

Uma et al. (2010) proposed a modified smith predictor (SP) design for controlling the non minimum phase integrating processes with/without a zero. They used two controllers based on direct synthesis approach. The set point tracking controller is PID with lag filter and PD with lead lag structure is the disturbance rejection controller. The method provided good disturbance rejection response with significant improvement in servo responses by using a set point weight.

Chen et al. (2008) developed set point weighted PID controller tuning for time delayed unstable systems. Based on the set point weighting parameter, they used a simple PID-PD controller to achieve basic and modified PID structures

Ali and Majhi (2010) presented a tuning method of minimizing the ISE criterion with the constraint that the slope of the Nyquist curve has a user specified value at the gain crossover frequency, to get the optimal controller parameters for integrating processes providing satisfactory set point tracking and load disturbance rejection responses. Guidelines have been provided for selecting the gain crossover frequency and the slope of the Nyquist curve.

Jin and Liu (2014) addressed the analytical tuning method of 1 DoF PID controller and 2 DoF PID controller with set point weights for integrating processes with and without RHP zero using IMC technique and achieved a good performance /robustness trade-off, by specifying the desired robustness.

Lee et al. (2014) proposed simple analytical tuning rules for PI/PID controller based on Skogestad IMC (SIMC) method. They also designed a set point filter to reduce the overshoot for stable, integrating and double integrating processes.

Anil and Sree (2015) designed direct synthesis based PID controller with lead lag filter for integrating time delay systems. The effectiveness of the proposed method was shown by providing the simulation results on various models and on nonlinear model equations of CSTR.

Padma Sree and Chidambaram (2017) presented an excellent overview of controlling unstable single and multi-variable systems. Many physical examples are stated as unstable transfer function models. The problems in the control of unstable systems are given in detail.

Table 2.6 Literature review on set-point weighting parameters for UFOPTD and USOPTD processes.

S. No.	Author	Description	Remarks
1	**Sree and Chidambaram** (2003a)	Set point weighted PI controller based on equating coefficient method.	Applied for UFOPTD systems with a zero and provided simulation results on a CSTR process.
2	**Sree and Chidambaram** (2003b)	PI controller based on direct synthesis method.	For stabilizing an unstable bioreactor with a dominant unstable zero, provided robust performance.
3	**Liu et al.** (2005a)	2 DoF control scheme with three controllers.	For UFOPTD, USOPTD and integrating UFOPTD.
4	**Shamsuzzoha and Lee** (2008c)	2 DoF IMC based PID controller.	For integrating processes with positive and negative zeros focussing on disturbance rejection.
5	**Uma et al.** (2010)	Modified SP design with two controllers designed based on direct synthesis method.	Provided good disturbance rejection for non minimum phase integrating processes with/without a zero.
6	**Chen et al.** (2008)	Developed set point weighted PID controller tuning for time delayed unstable systems.	Based on the set point weighting parameter, used a simple PID-PD controller to achieve basic and modified PID structures
7.	**Ali and Majhi** (2010)	PID controller based on minimizing ISE and gain crossover frequency in the Nyquist curve.	Provided good servo/regulatory responses for integrating processes.
8	**Jin and Liu** (2014)	IMC-PID from 2 DoF to 1 DoF.	For integrating processes with and without RHP zero.
9	**Lee et al.** (2014)	PI/PID controller based on SIMC method	For stable and integrating processes.
10	**Anil and Sree** (2015)	Improved PID controller with lead lag filter using direct synthesis method	For unstable integrating processes with time delays.
11	**Padma Sree and Chidambaram** (2017)	Presents the equating coefficient method for the design of PI and PID controllers for stable and unstable systems. Several examples have been stated for the existence of unstable behavior in the processes.	CSTR, bioreactor, fluidization reactor, fluid catalytic reactor, Jacketed CSTR, Isothermal reactor, cart and pole problem, helicopter and airplanes.

A summary of the reported work for set-point weighted PID controllers for SISO and MIMO unstable time-delay systems are given in Table 2.6.

Based on literature survey the following important research problems are noted:

- Analytical design rules may be developed like look up tables
- Design of H_2 optimal IMC based controllers to unstable systems both theoretically and experimentally
- Enhanced design of H_2 optimal PID controllers for cascade and MIMO unstable systems
- Propose simple set-point weighted PID controllers design for SISO and MIMO unstable systems to further reduce the overshoot

Chapter 3

Analytical PID Tuning Rules for Unstable Processes

Chapter 3

Analytical PID Tuning Rules for Unstable Processes

For a wider range of stable processes, many analytical PID controller tuning rules are available. However, for unstable processes, the availability of analytical tuning rules is limited. In this chapter, H_2 minimization theory in combination with internal model control (IMC) is used to analytically derive PID controller settings which can be used as a ready reference like look-up tables. These analytical settings are developed for a defined range of time delay to time constant ratio. Robustness of the control system is evaluated by Maximum sensitivity (M_s).

3.1 Introduction

Tuning of different types of controllers for right half plane pole processes, which are classified as unstable, has been addressed by in literature. IMC based design using the Laurent series expansion is developed by Panda (2009) and by using Maclaurin series approximation is developed by Nasution et al. (2011). Based on the need of an operation, i.e., either in a set-point tracking mode or load disturbance rejection mode, Arrieta et al. (2011) developed formulae for a controller for unstable and integrating processes. IMC based design using the Laurent series expansion is developed by Panda (2009) and by using Maclaurin series approximation is developed by Nasution et al. (2011).

To enhance the tracking capability smoothly, either placing a filter for the set point signal or using a weight for set point signal is preferred. The methods developed by Nasution et al. (2011) and Panda (2009) are applicable for wider range of time delay to time constant ratios ($\mu/n = T_0 = 0.1$ to 1.2). However, the equations are not simple and are tedious to utilize in practice. Also, the method Wang et al. (2015) is not applicable when the time delay to time constant ratio is greater than 0.9. A method to design PI controller is proposed by Cho et al. (2014) but the major limitation of the method is that it does not work when the time delay to time constant ratio is equal to 1. A method to design PID controller is addressed by Anusha and Rao (2012) for second order unstable processes and IMC based controller with lead-lag filter is developed by Vanavil et al. (2014). The important points about the methods discussed so far are (1) Analytical tuning rules are not available for many methods and (2) some methods cannot be applied when $\mu/n \geq 1$.

Recently, Sree and Chidambaram (2017) discuss the importance of unstable systems and their occurrence in practice. Begum et al. (2016) proposed analytical relations for controller for the desired level of robustness. The present work extends the idea used by Nasution et al. (2011) to develop robust PID tuning rules for unstable processes using IMC method with optimal H_2 minimization theory. The current design methodology evaluates the performance and robustness trade-offs which aim to achieve a smoother response to both servo and regulatory changes while at the same time guaranteeing desired robustness. Also, the present method works even when μ/n ≥ 1. Further, with the provided examples, it is shown that the proposed method is much superior to the existing methods regarding performance, particularly for higher values of μ/n.

3.2 Controller design

IMC control is shown in Figure 3.1. Analytical relations for PID controller are reported for unstable first order plus time delay (UFOPTD) processes by Nasution et al. (2011), however, the developed formulae are not easy to use in practice. In the present method, analytical tuning rules are provided which are relatively easy for practical use. Let us consider the UFOPTD process as

$$P(s) = \frac{me^{-\mu s}}{ns-1} \tag{3.1}$$

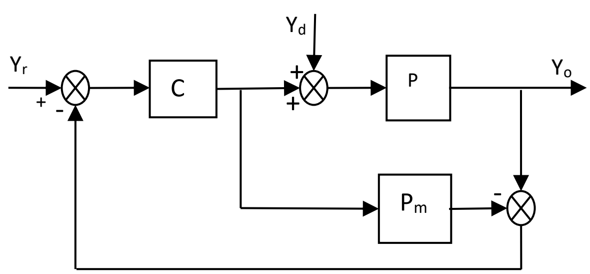

Figure 3.1 Internal Model Control

Based on IMC philosophy, considering perfect model ($P = P_m$) and optimal H_2 theory, the IMC controller is derived and is given as Nasution et al. (2011)

$$C(s) = \frac{(ns-1)}{m}\{(e^{\mu/n} - 1)ns + 1\}F \tag{3.2}$$

Here, F is the filter, which is selected as

$$F = (\gamma s + 1)/(\lambda s + 1)^3 \quad (3.3)$$

here, λ need to be selected properly. Also, the analytical expression for γ is derived based on the internal stability conditions for any IMC based structure (Nasution et al., 2011) and is obtained as

$$\gamma = \{(\lambda/n)^2 + 3(\lambda/n) + 3\}\lambda \quad (3.4)$$

The equivalent controller G_C in single loop control is obtained as

$$G_C = \frac{C}{1 - CP_m} \quad (3.5)$$

Incorporating the expressions for C and P_m, one can achieve

$$G_C = \frac{\{(e^{\mu/n} - 1)ns + 1\}(\gamma s + 1)(ns - 1)}{m\left[(\lambda s + 1)^3 - \{(e^{\mu/n} - 1)ns + 1\}(\gamma s + 1)e^{-\mu s}\right]} \quad (3.6)$$

the above expression for controller is converted into a PID controller form using Maclaurin series by defining

$$w(s) = sG_c(s) \quad (3.7)$$

Expanding w(s) using Maclaurin series, we get

$$G_c(s) = \frac{1}{s}\left(W(0) + W'(0)s + \frac{W''(0)}{2!}s^2 + \cdots\right) \quad (3.8)$$

Eq. 3.8 is written as equivalent to a conventional PID controller as

$$G_c(s) = k_c\left(1 + \frac{1}{\tau_i s} + \tau_d s\right) \quad (3.9a)$$

From Eq. (3.8) & (3.9a), the controller settings are derived as

$$k_c = W'(0), \quad \tau_i = \frac{W'(0)}{W(0)} and \quad \tau_d = \frac{W''(0)}{2W'(0)} \quad (3.9b)$$

on substitution and simplification, the controller equations can be obtained as a function of μ/n and λ/n and which are complicated. Each expression for k_c, τ_i, τ_d contain expressions that are intricate and not easy to implement. Moreover, user friendly rules in simple analytical form are always recommended in industries. Note that the design equations for the techniques reported by Morari and Zafiriou (1989), Nasution et al. (2011), Panda (2009) and Wang et al. (2015) are more involved. Also, it is always preferred to have robust tuning rules to take care of process uncertainties.

3.3 Robust tuning of controller

For designing a robust controller, it is necessary to select the IMC tuning parameter λ as all the controller parameters (k_c, τ_i, τ_d) are functions of the transfer function model parameters which are known except λ. For robust design, let us again consider the UFOPTD process transfer function given in eq. (3.1).

$$P(s) = \frac{me^{-\mu s}}{ns-1} \tag{3.10}$$

Using the transformation ŝ = ns and $\mu_p = \mu/n$, after normalization, the normalized process model is obtained as

$$C^g(\hat{s}) = \frac{e^{-\mu_p \hat{s}}}{\hat{s}-1} \tag{3.11}$$

similarly, the controller given in eq. (3.9a) is also normalized using the transformations and obtained as

$$G_c^g(\hat{s}) = mk_c\left(1 + \frac{n}{\tau_i \hat{s}} + \frac{\tau_d}{n}\hat{s}\right) \tag{3.12}$$

with the normalized process and controller expressions, the sensitivity function S is defined as

$$S = \frac{1}{1+G_c^g P^g} \tag{3.13}$$

Based on this, M_s is defined as the maximum value of |S| for all of the frequencies. Using the controller expressions, the robustness is evaluated to achieve a minimum possible value for Ms for different values of μ/n. The equivalent values are depicted in Table 3.1. From this table, for a given process, once μ/n is known, one can select λ/n In order to further simplify the selection of λ/n, an analytical relation is determined based on regression analysis. The data is plotted as shown in Figure 3.2. The corresponding equation for selection of λ/n is obtained as

$$\lambda/n = 2.0957\,(\mu/n)^2 + 0.9634\,(\mu/n) - 0.0889 \quad (3.14)$$

Similarly, an equation for M_s is also determined based on λ/n and is given below.

$$M_s = 0.2547\,(\lambda/n)^3 - 1.7432\,(\lambda/n)^2 + 4.7569\,(\lambda/n) + 1.3371 \quad (3.15)$$

Let us assume μ/n =T, λ/n=R

From eq.3.14 and 3.15

$$R = 2.0957\,(T)^2 + 0.9634\,(T) - 0.0889$$

$$M_s = 0.2547\,(T)^3 - 1.7432\,(R)^2 + 4.7569\,(R) + 1.3371$$

In this work, from eq. (3.9b), simple expressions are developed for the controller settings. The expressions for k_c, τ_i, τ_d are developed from eq. (3.9b) after carrying out simple mathematics and are obtained as

$$K_c m = f_1(R,T) \quad (3.16)$$

$$\frac{\tau_i}{n} = f_2(R,T) \quad (3.17)$$

$$\frac{\tau_d}{n} = f_3(R,T) \quad (3.18)$$

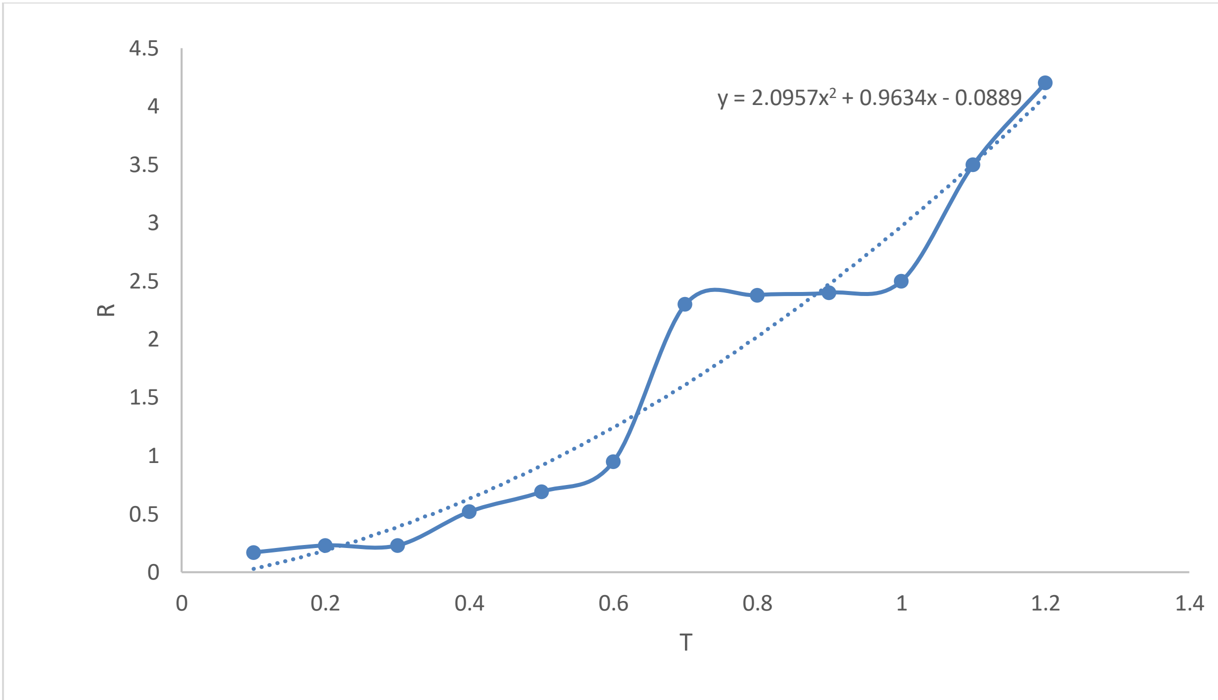

Figure 3.2 R for specified T values.

Selection of R is already discussed and λ can be obtained from eq. (3.14) for a given process. Regression is used to develop simple tuning formulas for controller parameters. For a given process, T is known and based on this, the controller parameters are found out as a function of R based on eq. (3.9b) and obtained the following equations.

$$K_c m = a_1(R)^{b_1} + c_1 \quad (3.19)$$

$$\frac{\tau_i}{n} = a_2(R)^{b_2} + c_2 \quad (3.20)$$

$$\frac{\tau_d}{n} = a_3(R)^{b_3} + c_3 \quad (3.21)$$

Where a_i, b_i, c_i (i = 1, 2, 3) are the coefficients whose values change for each T. In this work, the range of T is considered as 0.1 to 1.2. After carrying out regression, the values of these coefficients are obtained for each T and are shown in the Table 3.1. R can be obtained from eq. (3.14).

Table 3.1 R and M_s Evaluation for defined T range

T	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
R	0.17	0.23	0.23	0.52	0.69	0.95	2.3	2.38	2.4	2.5	3.5	4.2
Minimum possible M_s	1.61	2.17	2.86	3.18	3.96	5.13	5.68	5.05	5.8	7.69	8.12	9.18

While the a_1,b_1,c_1,a_2,b_2,c_2,a_2,b_2,c_2,a_3,b_3,c_3 parameter values for 0.1< T <1.2 are available in the above table, coefficients for intermediate values of *T* can be obtained by interpolation. To further have simplified tuning rules, the coefficients are also evaluated analytically based on the data given in Table 3.1 using curve fitting and the corresponding equations are given in Appendix A.

Now, for any value of *T*, one can find out the controller parameters after appropriately selecting the λ value. Here, for selection of λ, which plays major role in determining the robustness of the closed loop control system, M_s is used. To summarize the present tuning method, for the known T value, use eq. (3.19, 3.20, and 3.21) along with eqs gives in Appendix A and obtain the controller parameters as a function`n of *R*.

To summarize the proposed robust tuning, the systematic steps are given here.

Step 1: For the known unstable transfer function model of the process, calculate *T*.

Step 2: Select λ based on eq. (3.14)

Step 3: Use eqs. 3.19, 3.20 and 3.21 along with eqs gives in Appendix A and find out the controller parameters k_c, τ_i, τ_d

These steps can be used like look-up tables to design the PID controllers by the operators.

3.4 Simulation results

Evaluation of the performances is verified on many unstable processes and is compared with the techniques developed by Wang et al. (2015), Panda (2009) and Nasution et al. (2011). For reasonable evaluation, IAE and TV defined at same M_s are taken into account as metrics.

Controller Performance Metrics:

Integral Absolute Error (IAE), Integral Square Error (ISE) and Total variance (TV) are the criteria used to estimate the performance of the closed-loop process.

$$\mathrm{IAE} = \int_0^{\infty} |\mathrm{E(t)}|\mathrm{dt}$$

Where $\mathrm{E(t)} = \mathrm{Y(t)} - \mathrm{U(t)}$.

Total Variance is to measure the total variance in the controller output $\mathrm{U(t)}$ which provides an acceptable measure of the smoothness of the control.

$$\mathrm{TV} = \sum_{j=1}^{\infty} \left|\mathrm{U_j} - \mathrm{U_{j-1}}\right|$$

Example – 1: Control of a dimerization reactor process is considered here whose dynamics are (Sree, 2017)

$$P(s) = \frac{-0.017e^{-2.4s}}{5.8s-1} \tag{3.23}$$

Based on this model, the controller is designed and the corresponding values are given in Table 3.3. For comparison, other recent methods are also accounted and their controller values are also given in Table 3.2. Based on these controllers, a unit step change is provided to the set point and at a time of 60 sec, a step signal of 0.5 is provided to the disturbance. The simulation graphs are given in Figure 3.3 & 3.4 for nominal and perturbed cases. The metrics IAE and TV for both cases are calculated and provided in Table 3.3. The current approach provides good responses. It can be observed that even with increased perturbations, the TV value remains low for the current method.

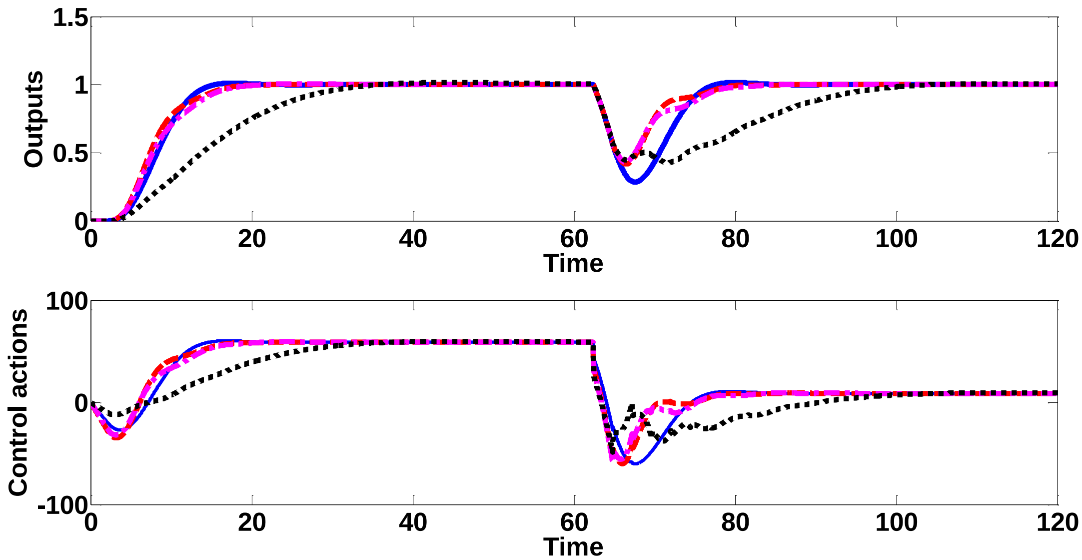

Figure 3.3 Output and control action behavior under exact model for example-1, dash dot – (Panda, 2009), solid – Present work, dash – (Nasution et al., 2011), , dot - (Wang et al., 2015).

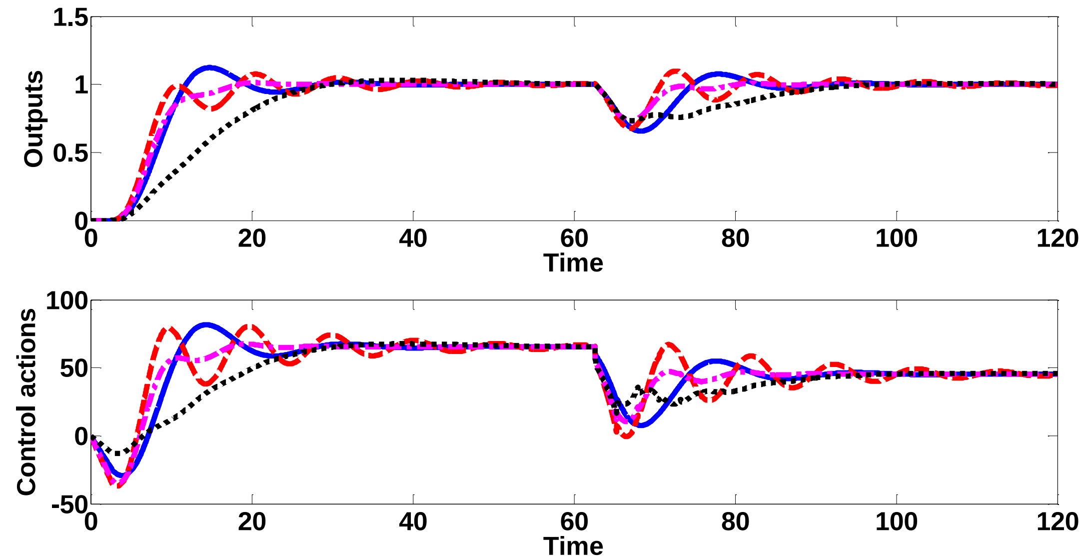

Figure 3.4 Output and control action behavior under mismatch model for example 1, dash – (Nasution et al., 2011), solid – Present work, dash dot – (Panda, 2009), dot - (Wang et al., 2015).

Example – 2: Here, unstable process with more than first order is studied (Liu and Gao, 2012)

$$P(s) = \frac{e^{-0.5s}}{(5s-1)(2s+1)(0.5s+1)} \tag{3.24}$$

λ is obtained as 5.2 according to eq. (3.14) which corresponds to $M_s = 4.6$. For this value of λ, the controller values are provided in Table 3.2. Based on this controller, a unit step change is provided to the set point and at a time of 100 sec, a step signal of 0.5 are provided to the disturbance. The simulation graphs are given in Figure 3.5 for nominal and perturbed cases. The metrics IAE and TV for both cases are calculated and provided in Table 3.2. Notice that the current approach resulted in fair responses and also smooth control action responses.

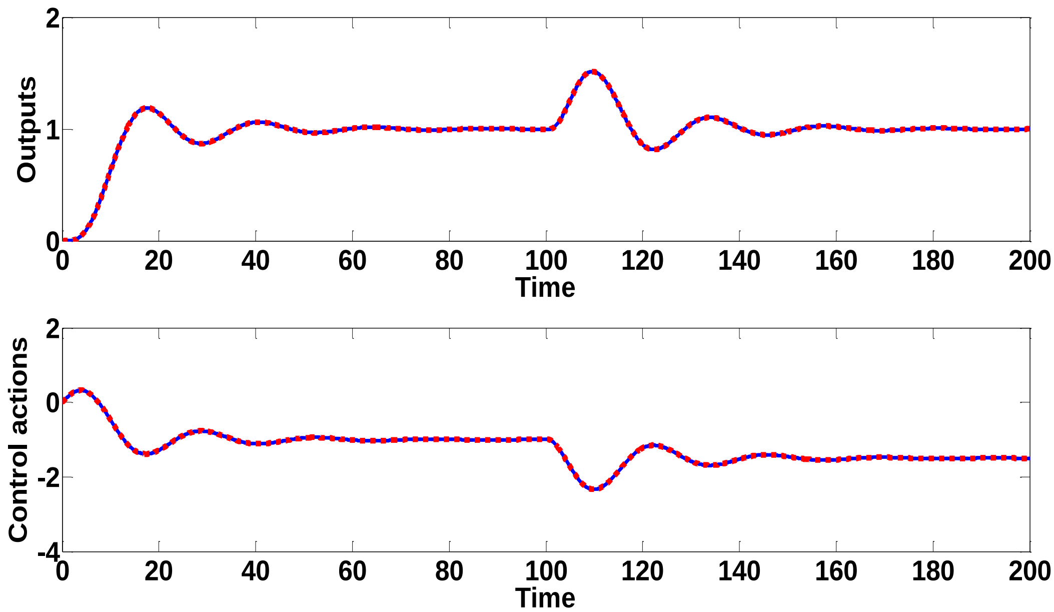

Figure 3.5 Output and control action behavior under exact and mismatch model for example 2, dash – Mismatch model, Solid - Perfect model.

Example – 3: The following unstable process is considered here (Liu and Gao, 2012)

$$P(s) = \frac{e^{-0.5s}}{(2s-1)(0.5s+1)} \tag{3.25}$$

Here, λ is obtained as 1.72 for which M_s is 3.9. For this value of λ, the controller values are provided in Table 3.2. Based on this controller, a unit step change is provided to the set point and at a time of 30 sec, a step signal of 0.5 is provided to the disturbance. The simulation graphs are given in Figure 3.6 for nominal and perturbed cases. The metrics IAE and TV for both cases are calculated and provided in Table 3.2. Notice that the current approach resulted in fair responses and also smooth control action responses.

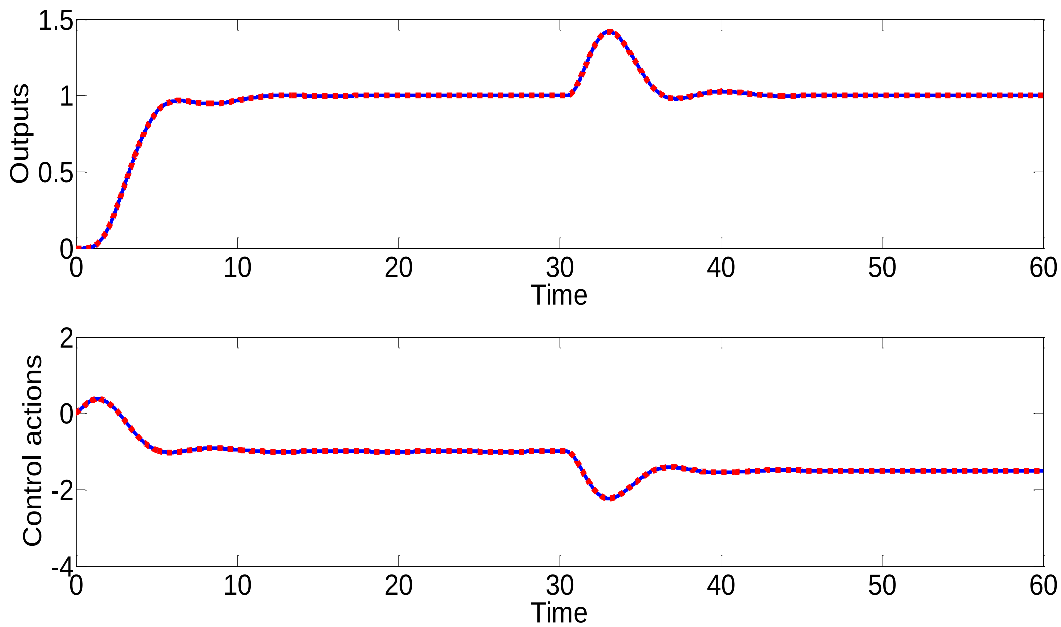

Figure 3.6 Output and control action behavior under exact and mismatch model for example 3, dash – Mismatch model, solid - Perfect model.

Example – 4: Here, an unstable process with significant time delay is taken whose dynamics are

$$P(s) = \frac{e^{-1.2s}}{s-1} \tag{3.26}$$

For this example also, the designed controllers for all considered cases are given in Table 3.3. Based on these controllers, a unit step change is provided to the set point and at a time of 40 sec, a step signal of 0.08 is provided to the disturbance. The simulation graphs are given in Figure 3.7 & 3.8 for nominal and perturbed cases. The metrics IAE and TV for both cases are calculated and provided in Table 3.2. From the table, it can be observed that even with increased time delay, the metrics remain low for the current method than previous techniques. It should be worth mentioning here that Wang et al. (2015) technique provides unbounded responses for significant delay process for any λ and thus those graphs are kept in Figure 3.7 & 3.8.

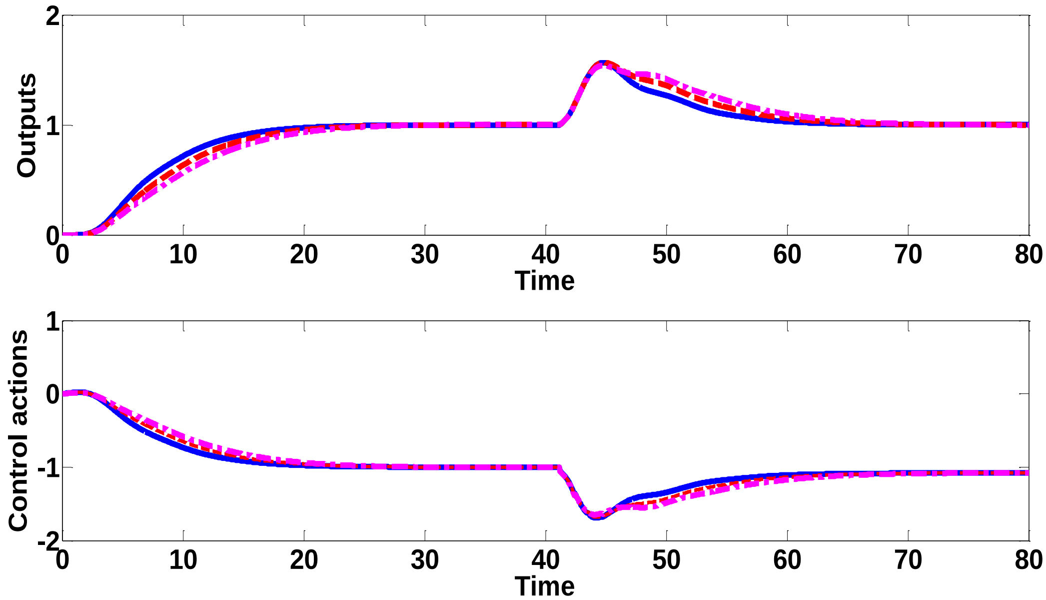

Figure 3.7 Output and control action behavior under exact model for example 4, dash – (Nasution et al., 2011), dash dot – (Panda, 2009), solid - Present work.

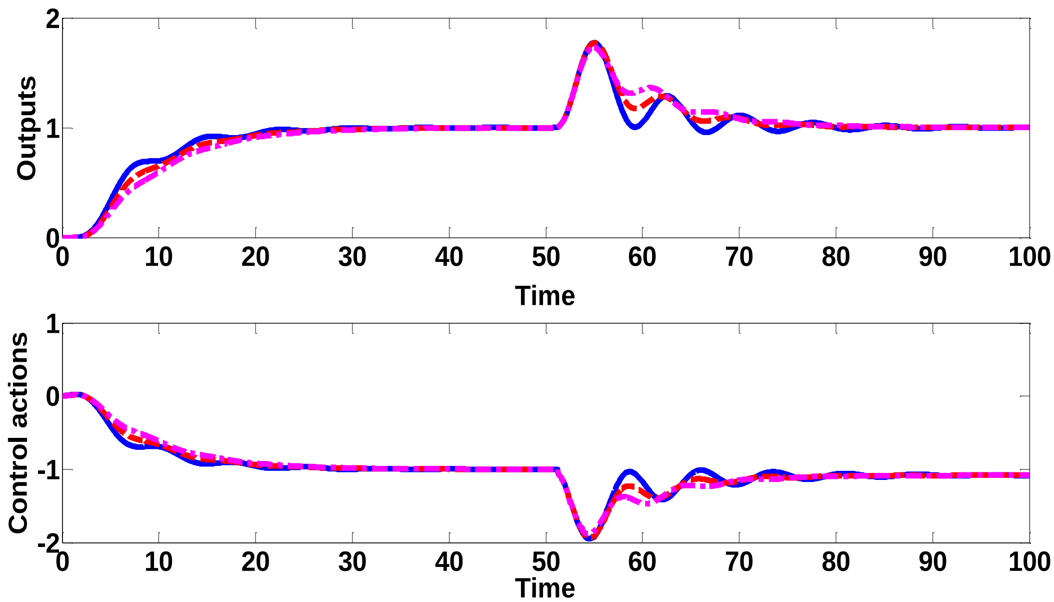

Figure 3.8 Output and control action behavior under mismatch model for example 4, dash – (Nasution et al., 2011), solid - Present work ,dash dot – (Panda, 2009).

Table 3.2 Comparative evaluation in terms of IAE and TV for all methods.

Example	Method	λ	k_c	τ_i	τ_d	Ms	Perfect model		Perturbations of -10% in k_p +10% in θ and -10% in τ	
							IAE	TV	IAE	TV
Example 1 $P(s)=\frac{-0.017e^{-2.4s}}{5.8s-1}$	**Proposed**	3.87	-137	14.73	0.7	3.81	14	314	12.1	304
	Nasution et al.	2.56	-166	12.6	0.9	3.81	12	351	11.7	636
	Panda	2.51	-164	13.38	1.1	3.81	13	357	9.8	271
	Wang et al.	4.12	-128	27.2	1.7	3.81	26	372	18.93	221
Example 2 $P(s)=\frac{0.9492e^{-2.774s}}{5.2644s-1}$									**-30% in k_p +30% in θ and -30% in τ**	
	Proposed	5.2	2	16.3	0.84	4.6	19.5	6.96	19.58	6.96
Example 3 $P(s)=\frac{0.9657e^{-1.0416s}}{2.4278s-1}$									**-10% in k_p +10% in θ and -10% in τ**	
	Proposed	1.72	2.34	6.29	0.3	3.9	5	4.3	5.01	4.31
Example 4 $P(s)=\frac{e^{-1.2s}}{s-1}$									**+5% in θ and -5% in τ**	
	Proposed	4	1.14	63.79	0.53	9	12.6	2.3	13	4.5
	Nasution et al.	3.28	1.13	77.9	0.55	9	14.7	2.37	14.7	3.37
	Panda	4.19	1.12	89	0.58	9	16.53	2.35	16.46	3
	Wang et al.	Unstable								

Example – 5: Control of a Chemical Reactor:

Mathematical model of isothermal chemical reactor is considered here (Vanavil et al., 2014).

$$\frac{dC_A}{dt} = \frac{F}{V}\left(C_{A,f} - C_A\right) - \frac{k_1 C_A}{(k_2 C_A + 1)^2} \qquad (3.27a)$$

Where F is the flow rate (0.0333 L/s) and $C_{A,f}$ is the inlet concentration. V = 1 L, k_1 = 10 L/s, and k_2 = 10 L/mol. The reactor is operated with an inlet concentration of 3.288 mol/L. Corresponding to this, the multiple steady states for the exit concentration are C_A = 1.7673, 0.01424 and 1.316 mol/L. Of these three, C_A = 1.316 is an unstable steady state. Inlet concentration is considered as the manipulated variable. The above nonlinear model is linearized around this operating condition and obtained the Unstable transfer function model as 3.433/ (103.1s-1). For this particular case, a time delay of 20 sec is considered. With that, the model is obtained as

$$P(s) = \frac{3.433 \ e^{-20\ s}}{103.1\ s-1} \tag{3.27b}$$

By considering this transfer function model, from eq. (3.14), λ = 18 is obtained to achieve M_s = 2.6. For this value ofλ, the controller values are provided in Table 3.3. Based on this controller, a unit step change is provided to the set point and at a time of 400 sec, a step signal of 0.5 is provided to the disturbance. The simulation results are given in Figure 3.9 for perfect model and in Figure 3.10 for uncertainties. The metrics IAE and TV for both cases are calculated and provided in Table 3.3. The current method is comparatively better.

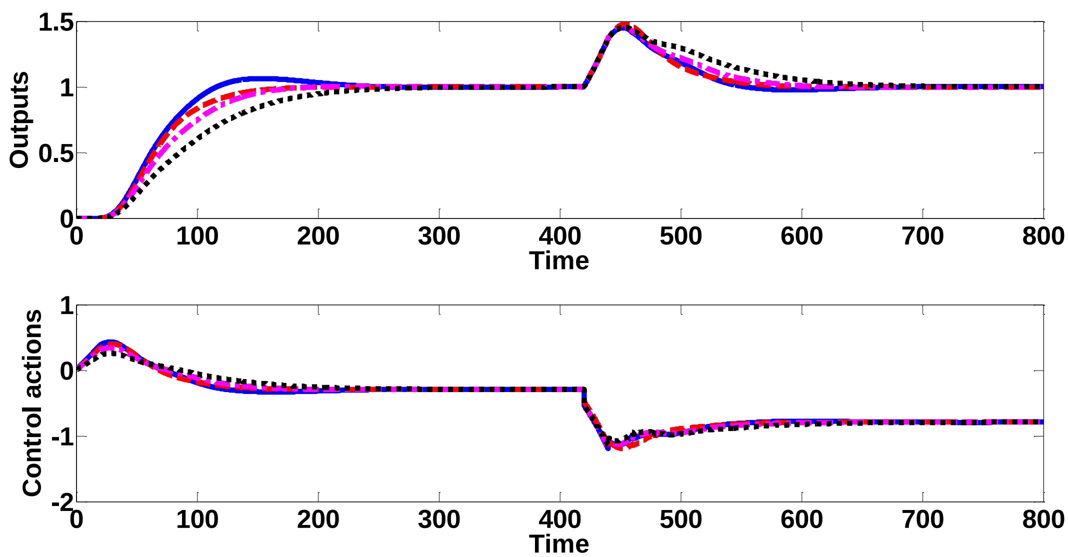

Figure 3.9 Output and control action behavior under exact model conditions for example 5, dash – (Nasution et al., 2011), Solid - Present work, dash dot – (Panda, 2009), dot - (Wang et al., 2015).

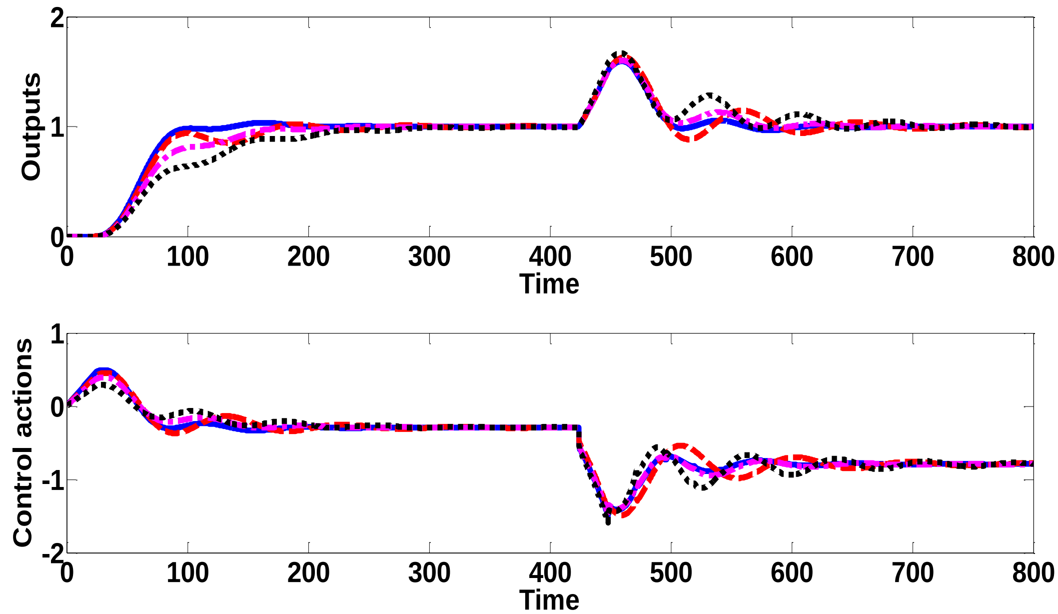

Figure 3.10 Output and control action behaviour under mismatch model conditions for example 5, dash – (Nasution et al., 2011), solid - Present work, dash dot – (Panda, 2009), dot - (Wang et al., 2015).

Non – linear Simulation:

To analyze the performance of the controllers in more realistic manner, closed loop simulations are carried out on the original nonlinear model by giving a step change in the set point from 1.316 to 1.34 at time t = 0 and a step disturbance of magnitude of 0.034 at t = 400 sec. The corresponding results are shown in Figure 3.11 for perfect model and in Figure 3.12 for perturbations. From the figure, one can see that the present work shows fair closed loop tracing of set point with smooth less oscillatory controller output. The IAE and TV values are given in Table 3.3. Further, both increased (positive) and decreased (negative) step inputs of different magnitudes are considered with a noise of power 0.0001 in the measurement and the simulation results are shown in Figure 3.13. The present method tracks the set point well.

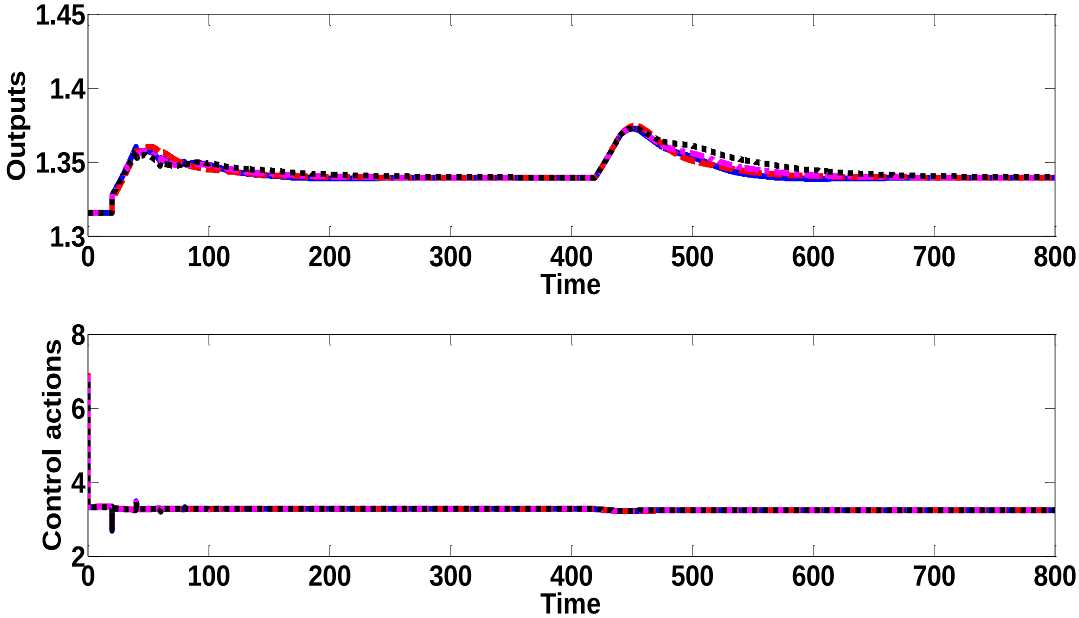

Figure 3.11 Output and control action behaviour based on Non Linear true model for example 5, dash – (Nasution et al., 2011), solid - Present work, dash dot – (Panda, 2009), dot - (Wang et al., 2015).

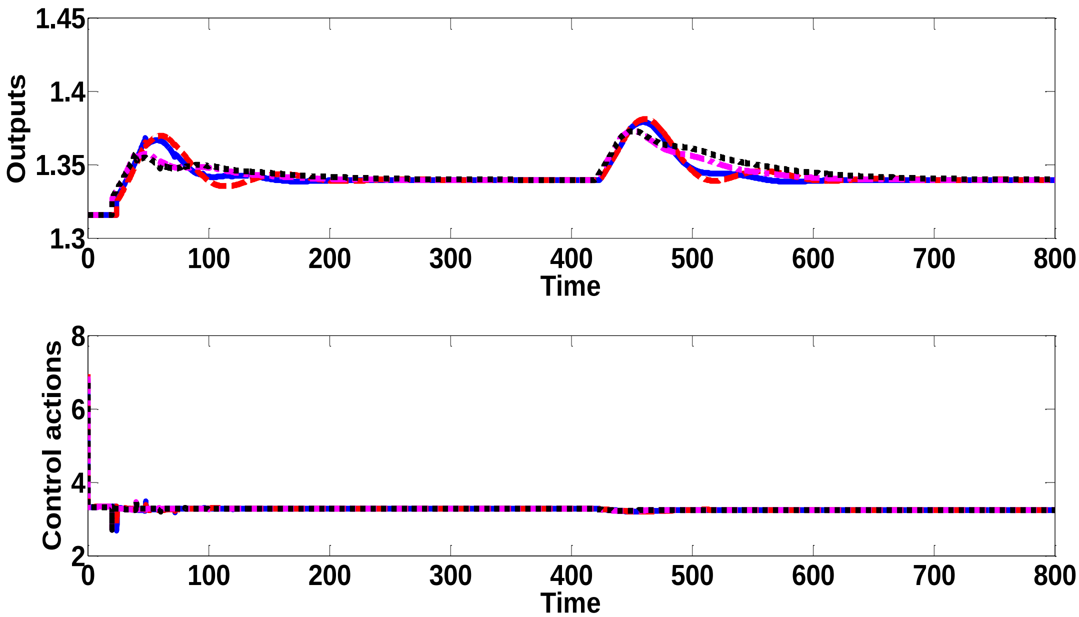

Figure 3.12 Output and control action behaviour based on Non Linear mismatch model for example 5. dash – (Nasution et al., 2011), dash dot – (Panda, 2009), solid - Present work, dot - (Wang et al., 2015).

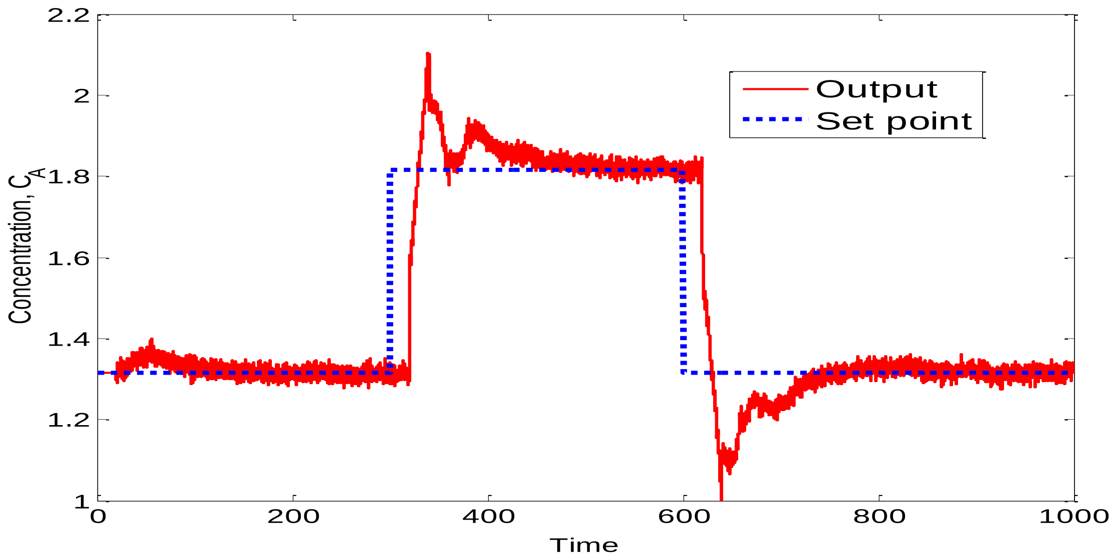

Figure 3.13 Closed loop response for different set points example 5 with Noise.

Example – 6: Control of a Bio-reactor

A nonlinear continuous bioreactor exhibiting output multiplicity behavior is considered whose model is given by (Sree and Chidambaram, 2017)

$$\frac{dX}{dt} = (\mu - D)X \tag{3.28}$$

$$\frac{dS}{dt} = \left(S_f - S\right)D - \frac{\mu X}{\gamma} \tag{3.29}$$

Where $\mu = \mu_m S/(K_m + S + K_l S^2)$, γ = 0.4 g/g, S_f = 4g/l, m = 0.53 h^{-1}, D = 0.3 h^{-1}, K_m = 0.12 g/l, K_l = 0.45451 l/g. X and S are the cell and substrate concentrations. The reactor exhibits multiple steady states at (X = 0, S = 4), (0.9951, 1.5122) and (1.5301, 0.1746). It is desired to operate the reactor at the intermediate unstable steady state (X = 0.9951, S = 1.5122). The dilution rate, D is used as a manipulated variable. Measurement delay of 2.4 hours is assumed for X. Linearizing the nonlinear model equations around the unstable operating point, the transfer function model is obtained as.

$$\mathrm{P(s)} = \frac{-5.89\, e^{-2.4s}}{5.86s-1} \tag{3.30}$$

For this process, λ is obtained as 3.85 as per eq. (3.14) for which M_S is obtained as 3.2. Design methods are developed in references (Wang et al., 2015) (Panda, 2009) (Nasution et al., 2011) are considered for comparison. The controller values for all cases are provided in Table 3.3. Based on these controllers, a unit step change is provided to the set point and at a time of 50 sec, a step signal of 0.5 is provided to the disturbance. The simulation graphs are given in Figure 3.14 & 3.15 for nominal and perturbed cases. The metrics IAE and TV for both cases are provided in Table 3.3. It can be seen that the IAE and TV are close to Nasution et al. (2011) and better than Wang et al. (2015) and Panda (2009).

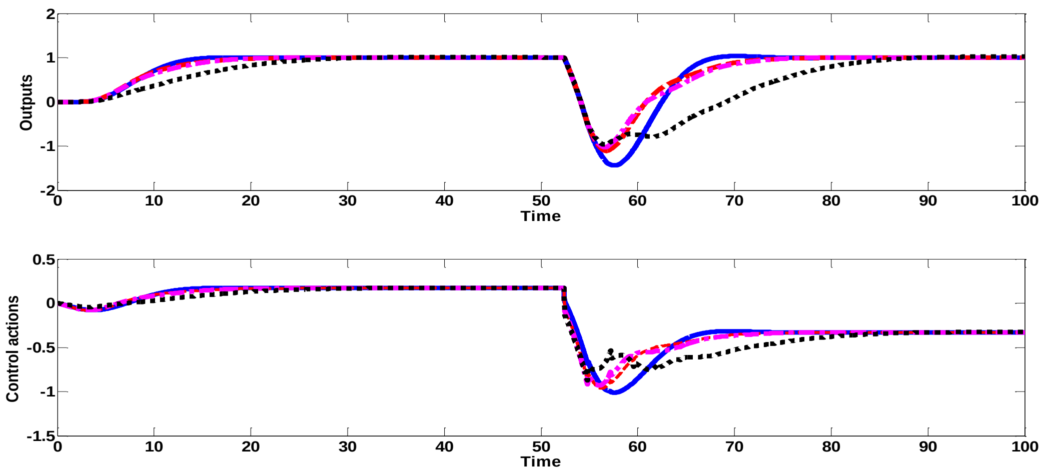

Figure 3.14 Output and control action behavior under exact model for example 6, dash – (Nasution et al., 2011), dash dot – (Panda, 2009), solid - Present work.

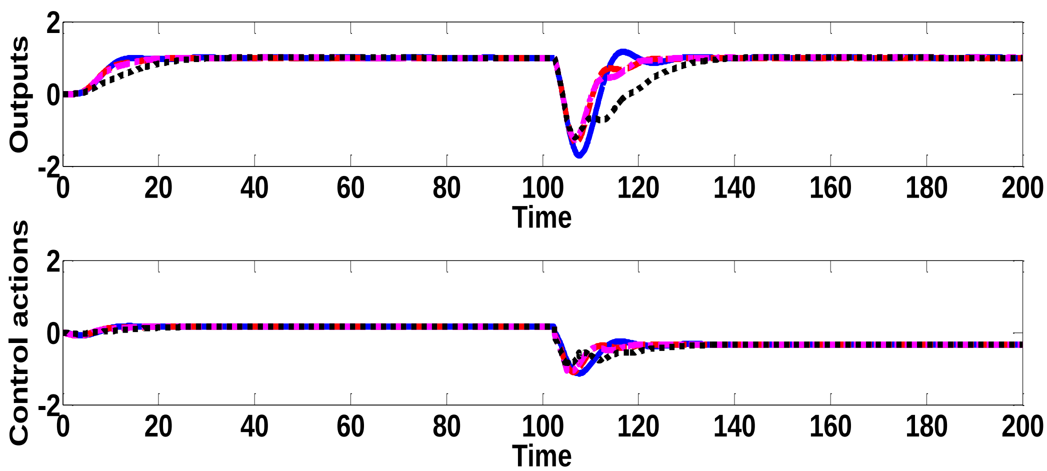

Figure 3.15 Output and control action behavior under mismatch model conditions for example 6, dash – (Nasution et al., 2011), Solid - Present work ,dash dot – (Panda, 2009)

Non – linear Simulation:

Closed loop simulation is carried out by giving a step change in the set point from 0.9951 to 1.2 at time t = 0 and a step disturbance of magnitude of 0.35 at t = 75 sec for the original nonlinear model. The corresponding closed loop and control action responses are shown in Figure 3.16 for perfect model and in Figure 3.17 for perturbations. One can see that the present work shows fair closed loop tracing of set point with smooth less oscillatory controller output. IAE and TV values are given in Table 3.2. Evaluation is also carried out in the presence of Noise with power = 0.0001 for both positive and negative step inputs of different magnitudes and shown in Figure 3.18. The present method tracks the set point well even in the presence of noise.

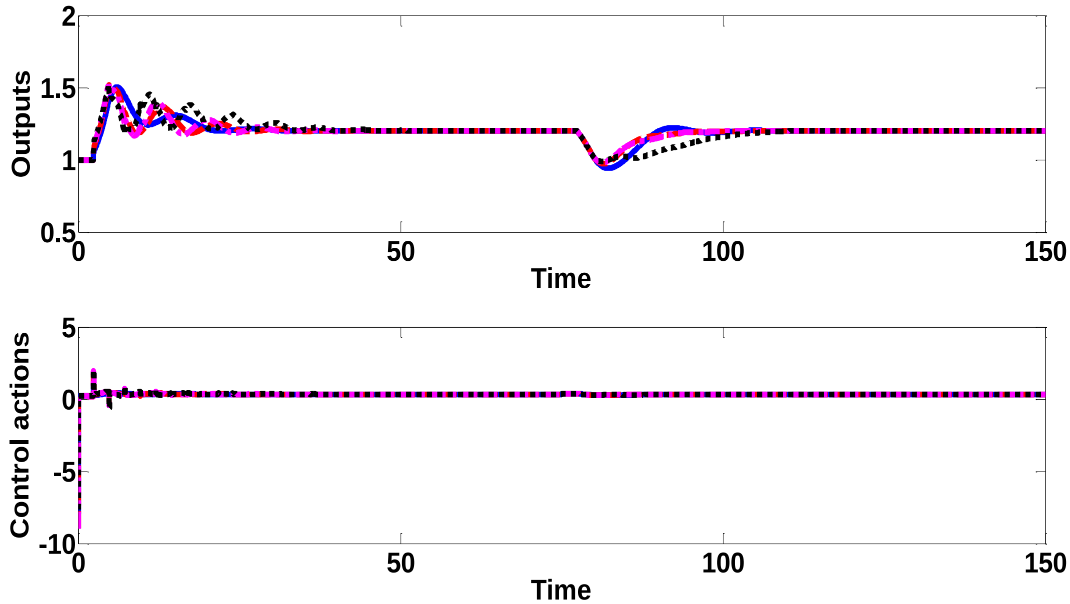

Figure 3.16 Performance evaluation for Non Linear true model for example 6. dash –(Nasution et al., 2011), solid - Present work, dash dot –(Panda, 2009), dot - (Wang et al., 2015).

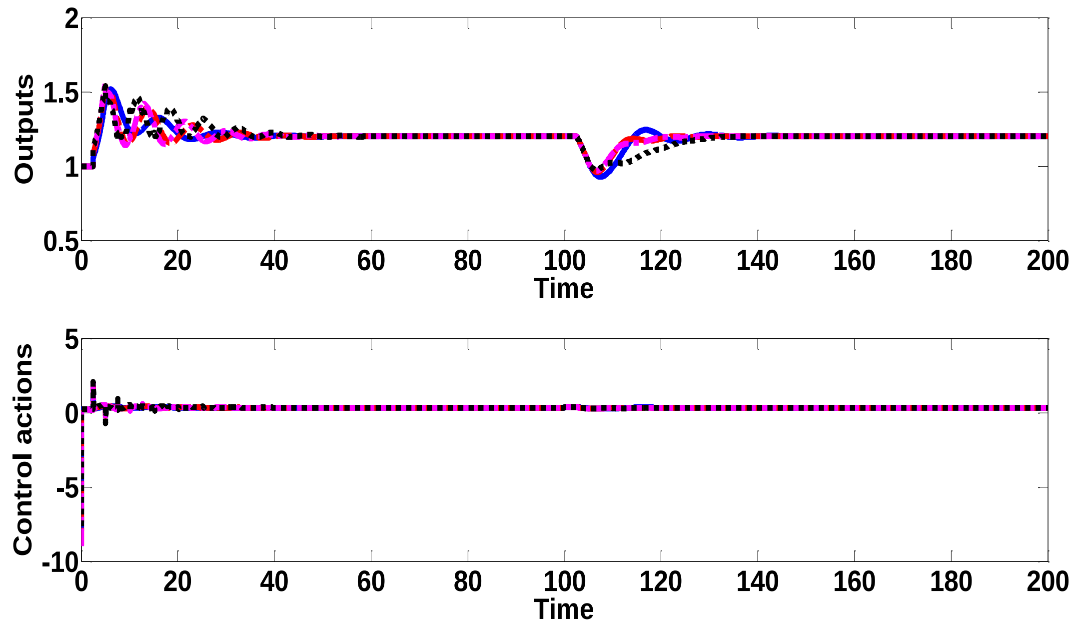

Figure 3.17 Performance evaluation for Non Linear mismatch model for example 6. dash – (Nasution et al., 2011), solid - Present work, dash dot – (Panda, 2009) , dot - (Wang et al., 2015)**.**

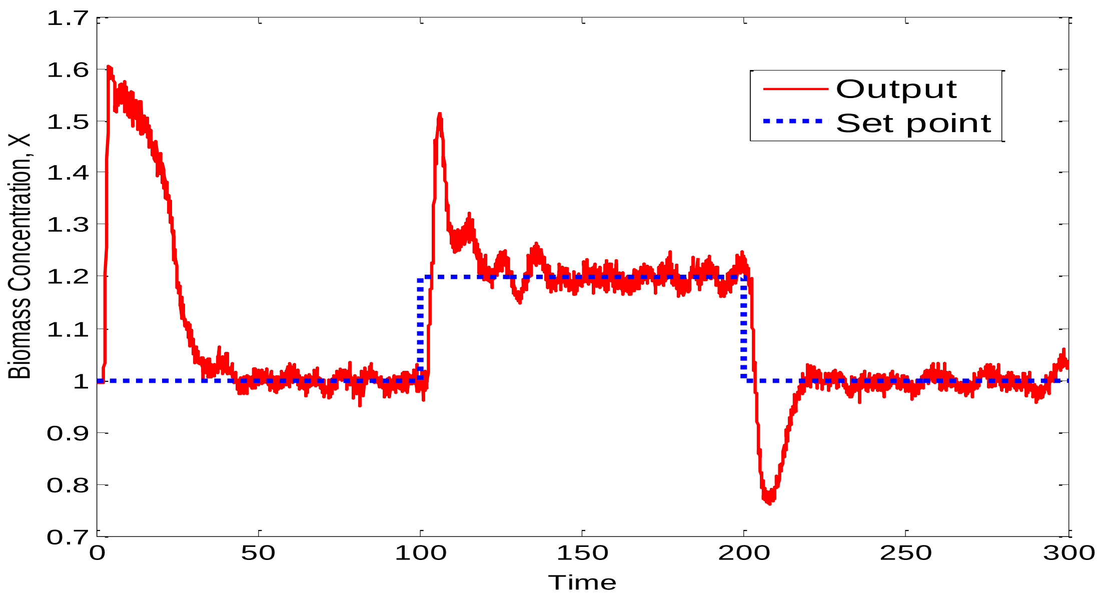

Figure 3.18 Closed loop response for different set points example 6 with Noise for perfect condition. Dot – input, solid – output.

Table 3.3 Comparative evaluation in terms of IAE and TV for all methods

Example 5	**Method**	**λ**	**k_c**	**τ_i**	**τ_d**	**Ms**	**Perfect model**		**+20% in θ and -10% in τ**	
							IAE	**TV**	**IAE**	**TV**
$P(s)=\frac{3.433\ e^{-20s}}{103.1s-1}$	**Proposed**	18	1.45	75.9	10.1	2.6	99.9	2.85	92.3	4.31
	Nasution et al.	22	1.51	88.9	7.2	2.6	101.2	2.54	112.7	5.33
	Panda	24	1.48	97.3	9.4	2.6	112.2	2.51	111.8	4.14
	Wang et al.	26	1.41	123.4	10.2	2.6	141.7	2.34	143.2	5.83
Example 5 Non-linear Simulation	**Proposed**	18	1.45	75.9	10.1	2.6	3.76	5.8	3.92	5.86
	Nasution et al.	22	1.51	88.9	7.2	2.6	3.9	5.18	4.39	5.3
	Panda	24	1.48	97.3	9.4	2.6	4.18	5.71	4.1	5.71
	Wang et al.	26	1.41	123.4	10.2	2.6	5.2	5.57	5.2	5.5
Example 6									**+5% in θ and -5% in τ**	
$P(s)=\frac{-5.89e^{-2.4s}}{5.86s-1}$	**Proposed**	3.85	-0.39	14.8	0.7	3.2	27.38	2.28	28.2	2.82
	Nasution et al.	2.9	-0.44	14.7	0.9	3.2	25.5	2.3	25.5	2.76
	Panda	2.9	-0.44	15	1.07	3.2	26.2	2.35	26	2.8
	Wang et al.	3.6	-0.38	23.6	1.52	3.2	44.7	2.6	44.4	3
Example 6 Non-linear Simulation	**Proposed**	3.85	-0.39	14.8	0.7	3.2	4.5	11.2	5	11.3
	Nasution et al.	2.9	-0.44	14.7	0.9	3.2	4.04	15.6	4.55	15.8
	Panda	2.9	-0.44	15	1.07	3.2	4.16	17.6	4.66	17.7
	Wang et al.	3.6	-0.38	23.6	1.52	3.2	6.43	20.24	6.53	20.27

3.5 Summary

Robust analytical relations for PID controller are developed for time delayed unstable systems. These rules can be used like look-up tables by the operators for tuning of PID controllers. For unstable systems, it is very crucial to select the tuning parameters to acquire stable responses. Robustness always requires lower Ms values which is usually not easy to achieve for such systems. The tuning parameter is selected to achieve minimum possible Ms value and analytical formula is given to calculate λ. Further, the developed simple tuning formulae provide fair and enhanced performances. The present methods can be utilized as look up tables for selection of the PID controller tuning parameters. Six case studies are considered to evaluate the applicability of the current method. Analytical formula is provided to determine λ based on μ/n. The evaluated responses of the current design are superior when compared with existing techniques, especially when μ/n is significant. The current methodology is relatively simple and can be applied for any system with a right half plane pole. Comparative analysis has also been done using IAE and TV.

Chapter 4

Experimental Studies on an Inverted Pendulum

Chapter 4

Experimental Studies on an Inverted Pendulum

In the present chapter, an experimental assessment of the developed method in chapter 3 is verified by testing it on an Inverted Pendulum (IP). The performance of controlled system is compared with the methods proposed by Begum et al. (2018) and Cho et al. (2014).

4.1 Introduction

Inverted Pendulum (IP) is a platform for study of control theories. There are two separate control problems in IP. First is the crane control problem, in which the goal is to move the cart to a desired position with as little oscillation of the load (pendulum arms) as possible. The other is to stabilize the IP in an upright position. The IP task can be seen as a self-erecting control problem, which is present in missile launching and control applications. Furthermore, the pendulum application involves a swing up control aspect if initially the pendulum hangs freely in the vertical position. These two control problems (inverted pendulum and crane control) have one very important difference, which is the stability. The pendulum serving as a crane is stable without a working controller. Due to energy loss through friction and air resistance it will always end up at an equilibrium point. The inverted pendulum is inherently unstable. Left without a stabilizing controller, it will not be able to remain in an upright position when disturbed.

4.2 System description of inverted pendulum

An automated digital pendulum is considered as shown in figure 4.1 which consists of a cart moving along a one meter track length. The cart is associated with a shaft attached with two freely rotating pendulums. The present study aims in maintaining a vertical upright position of the pendulum by cart's action. When the cart's movement is towards the extreme end (beyond the limit switches), the power supply is interrupted with the help of sensors seated on either sides of the rail. The belt and DC motor assembly helps the cart to move freely in horizontal directions. The belt and DC motor assembly helps the cart to move freely in horizontal directions.

This digital IP is a Single Input Multiple Output (SIMO) system. By applying a voltage to the motor the force is controlled with which the cart is pulled. The value of the force depends on the value of the control voltage. The input signal to system is the control voltage (u) and the output are, the cart position coordinate (x) and pendulum angle (ϕ), which can be read using optical encoders. The controller's task will be to change the DC motor voltage depending on these two variables, in such a way that the desired control task is fulfilled (stabilizing in an upright position, swinging or crane control). Here, our main objective is to control the pendulum angle. The system can be thought of an inverted pendulum when ϕ = 0 at vertical position and crane control when ϕ = π i.e., when it is suspended freely. Both the cart position and the control signal are bounded in a real time application. The bound for the control signal is set to [–2.5 V to +2.5 V] and the generated force magnitude of around [–20.0 N to +20.0 N]. The cart position is physically bounded by the rail length and is equal to [-0.5 m to +0.5 m]. The model parameters of the above mentioned system are given in Table 4.1.

Table 4.1 Model parameters of Inverted Pendulum

Parameters	Notations	Values
Cart Mass	M	2.4 Kg
Pole Mass	m	0.23 Kg
Pole Length	l	0.4 m
Pole Moment of Inertia	I	0.099 $Kg.m^2$
Cart Friction Coefficient	b	0.05 $N.s/s^2$
Acceleration due to gravity	g	9.81 m/s^2

4.3 Inverted pendulum modelling

The phenomenological model of the pendulum is nonlinear, meaning that at least one of the states (x and its derivative or ϕ and its derivative) is an argument of a nonlinear function. For such a model to be presented as a transfer function (a form of linear plant dynamics representation used in control engineering), it has to be linearized. Without a properly designed controller it is tough to control an IP to reach its objective, as it is unstable in upright vertical position. The objective is to control the pendulum angle. An appropriate mathematical modelling is necessary for the design of a controller. The nonlinear differential equations derived through the modelling of the IP system are (Rao and Chidambaram, 2015) :

$$(m+M)\ddot{x} + b\dot{x} + ml\ddot{\phi}\cos(\phi) - ml\dot{\phi}^2\sin(\phi) = u \qquad (4.1)$$

$$(I+ml^2)\ddot{\phi} + ml\ddot{x}\cos(\phi) + mgl\sin(\phi) = 0 \qquad (4.2)$$

When the force is applied to the cart (u), the cart position changes (x) and hence the pendulum angle with respect to vertical position (ϕ). These nonlinear equations, Eq. 4.1 and Eq. 4.2 needs to be linearized around steady state condition to design a suitable controller. The following equation shows the conversion of nonlinear model in to linearized one based on which the pendulum angle transfer function is derived.

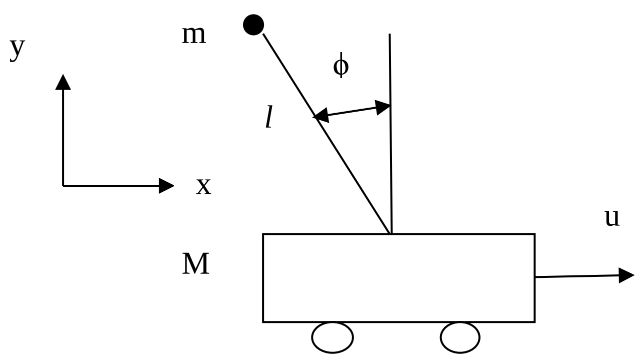

Figure 4.1 Schematic diagram of Inverted pendulum (IP)

$$G_p(s) = \frac{\phi(s)}{u(s)} = \frac{mls/q}{\left(s^3 + \frac{b(I+ml^2)}{q}s^2 - \frac{(M+m)mgl}{q}s - \frac{bmgl}{q}\right)} \qquad (4.3)$$

$$G_p(s) = \frac{mls}{(qs^3 + b(I+ml^2)s^2 - (M+m)mgls - bmgl)} \qquad (4.4)$$

Where $q = (M+m)(I+ml^2) - (ml)^2$

After substituting all the model parameters in Eq. 4.4, the following transfer function is obtained.

$$G_p(s) = \frac{0.01119s}{(0.0424s^3 + 0.0008256s^2 - 0.2886s - 0.005487)} \tag{4.5}$$

Assuming cart friction coefficient b = 0, we get

$$G_p(s) = \frac{0.01119s}{(0.0424s^3 - 0.2886s)} \tag{4.6}$$

Cancelling out the s terms and on factorization of the denominator in Eq. 4.6, we obtain

$$G_p(s) = \frac{0.0388}{(0.3833s - 1)(0.3833s + 1)} \tag{4.7}$$

$$G_p(s) = \frac{-0.0388}{(0.3833s - 1)(-0.3833s - 1)} \tag{4.8}$$

4.4 Controller design

The PID controller is the most commonly used controller in industry. The controller design for unstable process discussed in Chapter 3, section 3.2 is considered here. The structure of the controller is considered as PID controller derived based on H_2 minimization. The methods proposed by Cho et al. (2014) and Begum et al. (2018) are considered here for comparison with the above method without using a pre-filter. They have used a simple desired closed loop transfer function and the first order Taylor series approximation of process time delay ($e^{-\theta s} = 1 - \theta s$).

4.5 Experimental results

An automated digital pendulum is considered as shown in Figure 4.1 which consists of a cart moving along a one meter track length. The cart is associated with a shaft attached with two freely rotating pendulums. The present study aims in maintaining a vertical upright position of the pendulum by cart's action. When the cart's movement is towards the extreme end (beyond the limit switches), the power supply is interrupted with the help of sensors seated on either sides of the rail. The belt and DC motor assembly helps the cart to move freely in horizontal directions. The design and methodology used for the experimentation are briefly elucidated in Begum et al. (2018).

The present work is tested on the process with k_p = -0.0388, τ_1 = 0.3833 and τ_2 = -0.3833. For M_s value of 8.6, λ is taken as 0.88 and the controller parameters are k_c = 34.5, τ_i = 6.42, τ_d = 0.16. For a balanced evaluation, the present work, Begum et al. (2018) and Cho et al. (2014) methods are tuned for same M_s value of 8.6 and the controller settings for Begum et al. (2018) and Cho et al. (2014) are k_c = 31.0453, τ_i = 14.8144, τ_d = 0.1721 for λ =0.81 and k_c = 29.2793, τ_i = 45.0954, τ_d = 0.2155 for λ =1.8 respectively.

The tuning parameters obtained by the proposed method, are used for real time simulation on a pendulum setup, where $\phi = \pi$ is stable position and ϕ = 0 or 2π implies unstable critical position. The aim of controller is to maintain pendulum vertical position shown in Figure 4.2. An initial control voltage of 0.18V and the proposed controller parameters being k_c = 34.5, τ_i = 6.42, τ_d = 0.16 are provided to the system. The experiment is performed with a notion of controlling the angle of the pendulum by balancing the cart acceleration. The set point for the angle of the pendulum is fixed at ϕ = 0 i.e. upright vertical position and the cart position is maintained at zero. The controller tracks the required angular set-point as depicted in Figure 4.3 for which the controller settings helps the pendulum to reach the upper position at ϕ = 0.

The response of the recommended method is displayed in Figure 4.3. To ensure that performance of the present method is improved, the responses achieved using the controller settings of Begum et al. (2018) and Cho et al. (2014) method as presented in Figure 4.4 and Figure 4.5. It is noticed from Figure 4.3 that the pendulum reaches the upright position for a faster settling time of 10 sec while it takes 24 sec and 28 sec for the other methods (Figure 4.4 and Figure 4.5). This implies that the proposed method gives a smoother response in comparison to the methods available in literature.

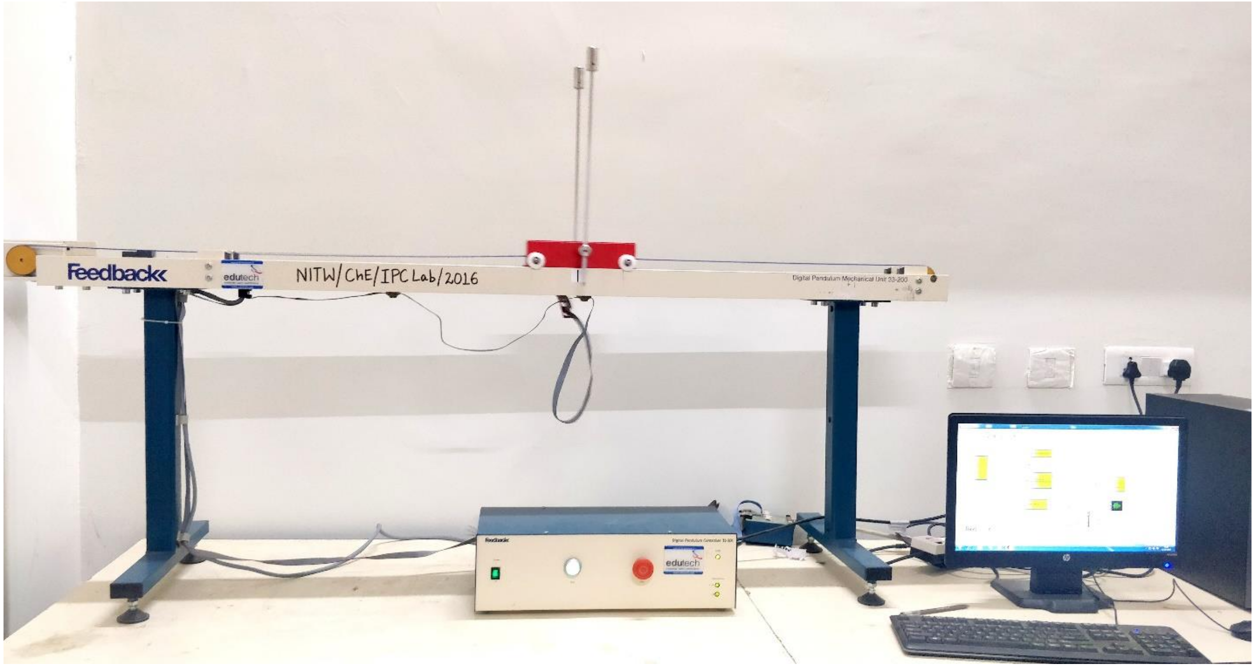

Figure 4.2 Pendulum in the final balanced vertical position.

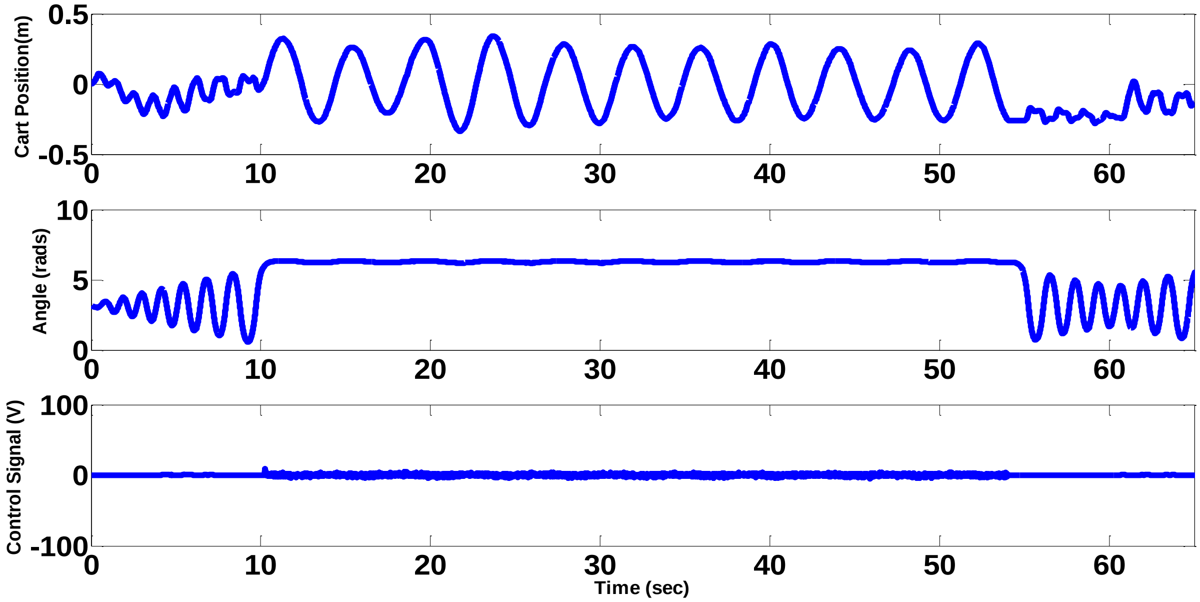

Figure 4.3 Response of the Inverted Pendulum from the experiment for the proposed method.

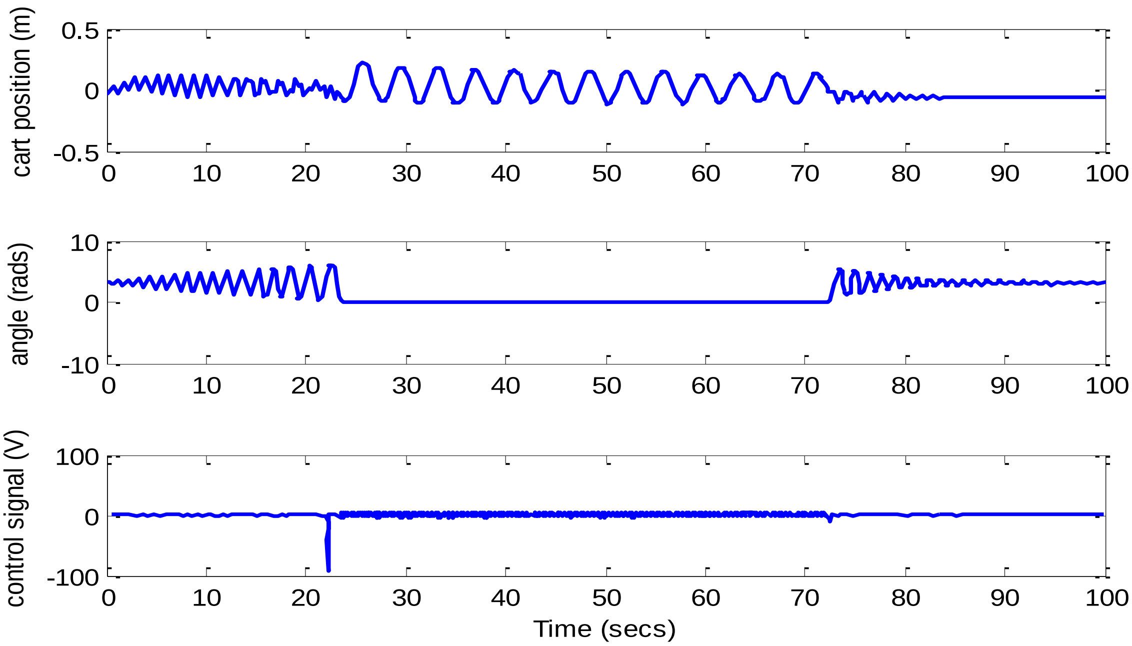

Figure 4.4 Response of the Inverted Pendulum from the experiment for Begum et al. Method.

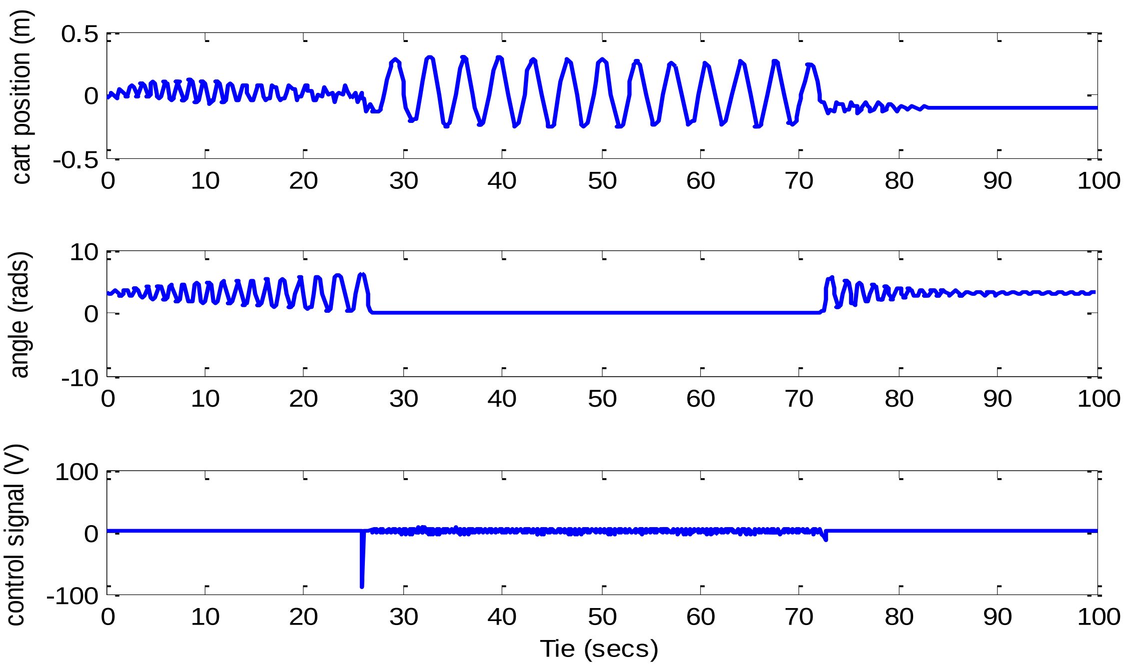

Figure 4.5 Response of the Inverted Pendulum from the experiment for Cho et al. method.

4.6 Summary

The majority of control loops are of PID type for set point tracking and disturbance rejection. In this work, a H_2 minimization based IMC-PID controller is developed for controlling the angle of an inverted pendulum. The transfer function model identification of the process has been carried out based on available parameters of the inverted pendulum system. Experimental implementation of the developed PID controller, shows a good response in maintaining the set point angle. The performances of the present method is better than that of Begum et al. (2018) and Cho et al. (2014).

Chapter 5

IMC-PID Design for Series Cascade Systems

Chapter 5

IMC-PID Design for Series Cascade Systems

Optimal H_2 internal model controller (IMC) is designed for control of unstable cascade processes with time delays. The proposed control structure consists of two controllers in which inner loop controller (secondary controller) is designed using IMC principles. The primary controller (master controller) is designed as a proportional-integral-derivative (PID) in series with a lead-lag filter based on IMC scheme using optimal H_2 minimization. Selection of tuning parameter is important in any IMC based design and in the present work, maximum sensitivity is used for systematic selection of the primary loop tuning parameter.

5.1 Introduction

Unstable processes are comparatively difficult to control than that of stable processes. The desired performance for unstable systems accompanying large time delays cannot be achieved with simple PID controllers. Despite the fact that Smith delay compensation proved to be a powerful tool to deal with time delay systems, it is inapplicable to unstable systems (Camacho, 2007). It is a well-known fact that cascade control scheme drastically improves the closed loop performance with disturbance rejection. A cascade control structure comprises of two control loops, a secondary intermediate loop (slave loop) and a primary outer loop (master loop). In typical cascade control structure, the secondary loop process dynamics are faster when compared to the primary loop. This provides faster disturbance attenuation and minimizes the possible effect of the disturbances before they affect the primary output.

Kaya (2001) proposed a cascade control scheme combined with Smith predictor for stable processes with dominant time delay and achieved improved control performances. Many researchers (Huang et al., 1998; Lee et al., 1998; Lee and Oh, 2002; Liu et al., 2005a; Tan et al., 2000) worked on the design and analysis of cascade control strategies for stable processes. But, limited research work has been carried out for the design of cascade control strategies for unstable processes. Liu et al. (2005) suggested IMC based cascade control scheme for unstable processes with four controllers. Kaya and Atherton (2008) designed a cascade control structure for controlling unstable and integrating processes with four controllers. Uma et al. (2009) proposed an improved cascade control scheme for unstable processes with a modified Smith predictor with three controllers and one filter in the outer loop. Garcia et al. (2010) developed filtered Smith predictor cascade control and generalized predictor cascade control, in which they proposed the design in discrete domain. Their method is applicable for stable, integrating and unstable time delay processes.

Padhan and Majhi (2012) proposed a modified Smith predictor based cascade control structure for unstable processes where they used three controllers. Recently, Nandong and Zang (2014) proposed a multi scale control scheme for cascade processes. In the works of (Kaya and Atherton, 2008; Liu et al., 2005b; Padhan and Majhi, 2012; Uma et al., 2009) more than three controllers and/or filters were used in the cascade control architecture to improve the performance of the unstable time delay processes. Most of the existing methods use more controllers and also the design of these controllers is not simple. In practice, a cascade controller structure with only two controllers (one for secondary loop and another for primary loop) is desirable.

In this chapter, a cascade control scheme is proposed with only one primary loop controller and one secondary loop controller. Tuning rules are derived for the controllers for effective control of open-loop unstable plants.

5.2 Proposed cascade control scheme

The cascade control structure used in the proposed method for the control of open-loop unstable processes is shown in Figure 5.1 where G_{c1} is the primary loop controller, G_{c2} is the secondary loop controller. G_{p1} and G_{p2} are the primary and the secondary loop processes. Simple IMC control scheme is used in the secondary loop. G_{m2} is the secondary loop process model and F_R is the set point filter. Usually, the dynamics of the secondary process are stable in nature and dynamics of the primary process are unstable in nature. Hence, control of secondary process is simple as compared to primary process.

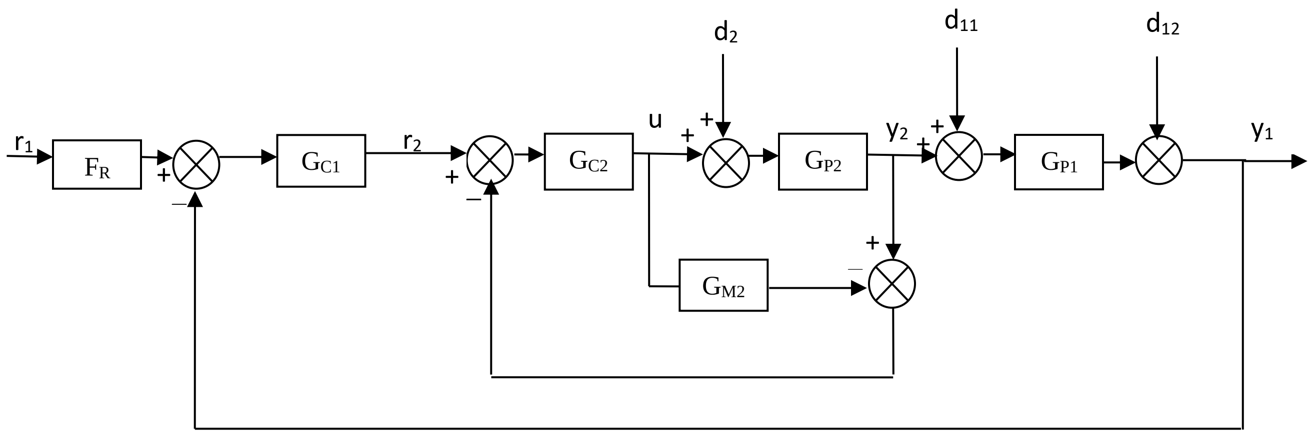

Figure 5.1 Proposed cascade control structure.

5.3 Controller design

The design of controllers in cascade loops depends on the dynamics of the secondary and primary processes. If the dynamics of secondary loop are fast compared to that of primary loop, the secondary controller needs to be designed first followed by the primary controller. If the dynamics of both secondary as well as primary processes are similar, then simultaneous design of controllers in both the loops is more appropriate and need to be carried out. In the present effort, the dynamics of secondary loop is considered to be fast and hence the secondary controller is designed first followed by the primary controller. Once the secondary controller is designed, an overall primary loop process model is obtained. Based on the overall primary process model, the primary controller is designed using H_2 norm minimization. In the following sections, design of secondary controller is discussed first following by design of primary controller.

5.3.1 Design of secondary loop controller

The secondary controller is designed as a simple IMC controller. The controller in the secondary loop represented by G_{c2} is based on IMC principle and it stabilizes the process through good disturbance rejection in the secondary loop. The closed loop transfer function of the secondary loop is given by

$$\frac{y_2}{r_2} = \frac{G_{c2}\, G_{p2}}{1\text{-}G_{c2}\, G_{m2} + G_{c2}\, G_{p2}} \tag{5.1}$$

As mentioned earlier, the secondary process dynamics are stable in nature and hence the secondary loop process is considered as a first order plus time delay (FOPTD) process as

$$G_{p2} = \frac{k_{p2} e^{-\theta_{p2} s}}{(\tau_{p2} s+1)} \tag{5.2a}$$

G_{m2} is the model of the secondary process and is considered as

$$G_{m2} = \frac{k_{m2} e^{-\theta_{m2} s}}{(\tau_{m2} s+1)} \tag{5.2b}$$

As per the IMC strategy, the secondary controller is obtained as

$$G_{c2}(s) = \frac{(\tau_{m2} s+1)}{k_{m2}(\lambda_2 s+1)} \tag{5.3}$$

Assuming that the model exactly corresponds to the process ($G_{m2} = G_{p2}$) and substituting G_{c2},G_{p2},G_{m2}, the closed loop transfer function of the secondary loop is obtained as

$$\frac{y_2}{r_2} = \frac{e^{-\theta m_2 s}}{(\lambda_2 s+1)} \tag{5.4}$$

Where λ_2 is the secondary loop tuning parameter.

5.3.2 Design of primary loop controller

The primary loop controller is designed using H_2 minimization. To design G_{c1}, the overall primary process model, G_m (relation between y_1 and r_2) is required and assuming a perfect secondary loop process model ($G_{m2} = G_{p2}$), we get

$$G_m = \frac{y_1}{r_2} = G_{c2} G_{p2} G_{p1} \tag{5.5}$$

In this work, the primary loop process is a considered as an unstable FOPTD process as given in Eq. 5.6a

$$G_{p1} = \frac{k_{p1} e^{-\theta_{p1} s}}{(\tau_{p1} s - 1)} \tag{5.6a}$$

The corresponding primary loop process model is considered as

$$G_{m1} = \frac{k_{m1} e^{-\theta_{m1} s}}{(\tau_{m1} s - 1)} \tag{5.6b}$$

Upon substitution in Eq. 5.5, we get

$$G_p = \frac{y_1}{r_2} = \frac{k_{p1} e^{-(\theta_{p1} + \theta_{p2}) s}}{(\lambda_2 s + 1)(\tau_{p1} s - 1)} \tag{5.7a}$$

Where G_p is the overall primary loop process. Assuming perfect primary process model, (G_{m1}= G_{p1}), the overall primary process model (G_m) is obtained as

$$G_m = \frac{k_{m1} e^{-\theta_m s}}{(\lambda_2 s + 1)(\tau_{m1} s - 1)} \tag{5.7b}$$

Where $\theta_m = \theta_{m1} + \theta_{m2}$.

As a generalization, Eq. 5.7b is rewritten as

$$G_m = \frac{k e^{-\theta_m s}}{(\tau_1 s - 1)(\tau_2 s - 1)} \tag{5.8}$$

Where $\tau_1 = \tau_{m1}$, $\tau_2 = -\lambda_2$, $k = -k_{m1}$

Based on this model (Eq. 5.8), the primary loop process controller (G_{c1}) is designed based on H_2 minimization theory and IMC principles.

Note: The present method addresses the design only for first order unstable time delay processes. However, if the primary loop process has two unstable poles, then Eq. (5.8) will have one more pole and becomes third order. In such cases, suitable identification techniques can be applied to reduce the third order unstable process into a second order unstable process and still the present method can be applied.

According to IMC principles, the primary loop IMC controller $Q_{C,p}$ is equivalent to

$$Q_{c,p} = \widetilde{Q}_{c,p}\, F \tag{5.9}$$

Where F is a filter which is used for altering the robustness of the controller.

The filter structure should be selected such that the IMC controller $Q_{c,p}$ is proper and realizable and also the control structure is internally stable. In addition to these requirements, it should be selected such that the resulting controller provides improved closed loop performances. In this work, $\widetilde{Q}_{c,p}$ is designed for a specific type of step input disturbance (v) to obtain H_2 optimal performance by Nasution et al. (2011) and is based on the invertible portion of the process model. The process model and the input are divided as

$$G_m = G_{m-}G_{m+} \quad and \quad v = v_- v_+ \tag{5.10}$$

Where the subscript " – " refers to minimum phase part and " + " refers to non-minimum phase part. The Blaschke product of RHP poles of G_m and v are defined as

$$b_m = \prod_{i=1}^{k} \frac{-s + p_i}{s + \overline{p}_i} \quad and \quad b_v = \prod_{i=1}^{\tilde{k}} \frac{-s + p_i}{s + \overline{p}_i} \tag{5.11}$$

Where p_i and $\overline{p_i}$ are the ith RHP pole and its conjugate respectively. Based on this, the H_2 optimal controller is derived by using the following formula (Morari and Zafiriou, 1989)

$$\tilde{Q}_C = b_m (G_{m-} b_v v_-)^{-1} \{(b_m G_{m+})^{-1} b_v v_-\}|_* \tag{5.12}$$

Where $\{.....\}|^*$ is defined as the operator that operates by omitting all terms involving the poles of $(G_{m+})^{-1}$ after taking the partial fraction expansion.

In the present endeavour, The quantities required for the operator are obtained based on the overall primary loop process model (G_m) as (Anusha and Rao, 2012)

$$\tilde{G}_{m-}(s) = \frac{k}{\tau_1 \tau_2 \left(-s + \frac{1}{\tau_1}\right)\left(-s + \frac{1}{\tau_2}\right)}; \; \tilde{G}_{m+}(s) = e^{-\theta_m s} \tag{5.13}$$

$$v_-(s) = \frac{k}{\tau_1 \tau_2 \left(-s + \frac{1}{\tau_1}\right)\left(-s + \frac{1}{\tau_2}\right) s}; \; v_+(s) = 1 \tag{5.14}$$

$$b_p(s) = \left(-s + \frac{1}{\tau_1}\right)\left(-s + \frac{1}{\tau_2}\right) / \left(s + \frac{1}{\tau_1}\right)\left(s + \frac{1}{\tau_2}\right) \tag{5.15}$$

$$b_v(s) = \left(-s + \frac{1}{\tau_1}\right)\left(-s + \frac{1}{\tau_2}\right) / \left(s + \frac{1}{\tau_1}\right)\left(s + \frac{1}{\tau_2}\right) \quad (5.16)$$

Substituting in eq. 5.9, the IMC controller is obtained as

$$\tilde{Q}_{c,p} = \frac{(\tau_1 s - 1)(\tau_2 s - 1)}{k}\left[\frac{\tau_1\tau_2(\tau_1 - \tau_2 - \tau_1 e^{\theta_m/\tau_1} + \tau_2 e^{\theta_m/\tau_2})s^2 + (\tau_1^2 e^{\theta_m/\tau_1} - \tau_2^2 e^{\theta_m/\tau_2} + \tau_2^2 - \tau_1^2)s + (\tau_1 - \tau_2)}{(\tau_1 - \tau_2)}\right] \quad (5.17)$$

Considering the filter as $F(s) = (\gamma s + 1)/(\lambda_1 s + 1)^4$, the IMC controller is obtained as

$$Q_{c,p}(s) = \tilde{Q}_{c,p}(s)F(s) \quad (5.18a)$$

Where λ_1 is the primary loop tuning parameter which is to be carefully selected so that good nominal and robust closed loop performances are obtained.

The desired closed loop transfer function for set point changes is then obtained as

$$\mathrm{H(s)} = \mathrm{Q_{c,p}(s)G_m(s)}$$

The equivalent controller in a conventional feedback form is obtained from IMC structure as

$$G_{c1} = Q_{c,p}/(1 - Q_{c,p}G_m) \quad (5.18b)$$

The conditions to be followed for internal stability of the above controller are

Condition 1: $Q_{C,p}$ must be stable and should cancel the right half plane poles of G_m

Condition 2: $Q_{C,p}G_m$ should be stable

Condition 3: $(1-G_m Q_{C,p})$ at the RHP poles of the process should be zero

Eq. (5.18b) can also be written as

$$\mathrm{G_{c1}} = \frac{\mathrm{H(s)}}{[1-\mathrm{H(s)}]\mathrm{G_m(s)}} \quad (5.19)$$

It can be seen that the controller, G_{c1}, has the zeros and poles at the location of RHP poles of $G_m(s)$. The RHP zeros of G_{c1} will be canceled out by the zeros of [1 - H(s)] as we satisfy the internal stability according to IMC rules i.e.

$Q_{C,\,p}$ must be stable and should cancel the right half plane poles of G_m and should contain only the minimum phase elements of G_m.

Therefore there is no RHP poles-zeros cancellation in the outer loop in conventional feedback control structure (Figure 5.1).After substituting $Q_{c,\,p}$ from eq. 5.18a and G_m from eq. 5.8, the primary loop controller $G_{c1}$is obtained as

$$Gc_1 = \frac{(\tau_1 s-1)(\tau_2 s-1)(z_1 s^2+z_2 s+z_3)(\gamma s+1)}{[z_3(\lambda_1 s+1)^4-(z_1 s^2+z_2 s+z_3)e^{-\theta_m s}(\gamma s+1)]} \tag{5.20}$$

To bring the structure of G_{c1} to a conventional PID controller format, approximations for the time delay are required unless one uses maclaurin series or Laurent series. Here, we did not use those approximations instead used first order pade's approximation so that the final controller is in PID format.

Considering pade's first order approximation for the time delay term as

$$e^{-\theta_m s} = \frac{1-0.5\theta_m s}{1+0.5\theta_m s} \tag{5.21}$$

The resulting controller is obtained as

$$G_{c1} = \frac{(1+0.5\theta_m s)(\tau_1 s-1)(\tau_2 s-1)(z_1 s^2+z_2 s+z_3)(\gamma s+1)}{k[(1+0.5\theta_m s)z_3(\lambda_1 s+1)^4-(z_1 s^2+z_2 s+z_3)(1-0.5\theta_m s)(\gamma s+1)]} \tag{5.22}$$

Where,

$$z_1 = \tau_1\tau_2(\tau_1 - \tau_2 - \tau_1 e^{\theta/\tau_1} + \tau_2 e^{\theta/\tau_2}) \tag{5.23}$$

$$z_2=\tau_1{}^2 e^{\theta/\tau_1} - \tau_2{}^2 e^{\theta/\tau_2} + \tau_2{}^2 - \tau_1{}^2 \tag{5.24}$$

$$z_3=\tau_1 - \tau_2$$

The denominator in Eq. (5.22) can be simplified as

$$x_0 s + x_1 s^2 + x_2 s^3 + x_3 s^4 + x_4 s^5 \tag{5.25}$$

Where

$$x_0 = 4\lambda_1 z_3 + \theta z_3 - \gamma z_3 - z_2 \tag{5.26}$$

$$x_1 = 6z_3\lambda_1^2 + 2\theta z_3\lambda_1 + 0.5\theta\gamma z_3 - z_1 + 0.5\theta z_2 - \gamma z_2 \tag{5.27}$$

$$x_2 = 4z_3\lambda_1^2 + 3\theta\lambda_1^2 z_3 - \gamma z_1 + 0.5\theta\gamma z_2 + 0.5\theta z_1 \tag{5.28}$$

$$x_3 = z_3\lambda_1^4 + 2\theta\lambda_1^3 z_3 + 0.5\theta\gamma z_1 \tag{5.29}$$

$$x_4 = 0.5\theta\lambda_1^4 z_3$$

After simple mathematical algebraic rearrangements, the controller is obtained as

$$G_{c1} = \frac{(z_1 s^2 + z_2 s + z_3)}{k x_0 s}\left[\frac{(\tau_1 s-1)(\tau_2 s-1)(\gamma s+1)(1+0.5\theta s)}{(1+\frac{x_1}{x_0}s+\frac{x_2}{x_0}s^2+\frac{x_3}{x_0}s^3+\frac{x_4}{x_0}s^4)}\right] \tag{5.30}$$

$$X_1 = \frac{x_1}{x_0}, X_2 = \frac{x_2}{x_0}, X_3 = \frac{x_3}{x_0}, X_4 = \frac{x_4}{x_0}$$

This expression should be simplified to a PID controller form. Maclaurin series or Laurent series may be applied here for approximation to a PID controller form. In the present work, this controller is approximated to a PID controller with lead-lag filter as given in eq. 5.31 with simple approximations.

Note that the first term in Eq. (5.23) is in the form of a PID controller. The second term needs to be approximated as a lead-lag filter. After retaining the numerator term $(\gamma s + 1)$, the remaining terms are taken to the denominator and made equal to $(\beta s + 1)$. With that, the controller is obtained as

$$G_{c1} = k_c\left(1 + \frac{1}{\tau_i s} + \tau_d s\right)\frac{(\gamma s+1)}{(\beta s+1)} \tag{5.31}$$

Where the denominator is

$$\beta s + 1 = \frac{(1+X_1 s+X_2 s^2+X_3 s^3+X_4 s^4)}{(1+0.5\theta s)(\tau_1 s-1)(\tau_2 s-1)} \tag{5.32}$$

Taking first derivative for Eq. (5.32) with respect to 's' and by substituting s = 0, the lag filter parameter is obtained as

$$\beta = X_1 + \tau_1 + \tau_2 - 0.5\theta \quad (5.33)$$

With that, the controller parameters of Eq. (5.31) are obtained as

$$k_c = \frac{z_2}{kx_0}, \tau_i = \frac{z_2}{z_3}, \tau_d = \frac{z_1}{z_2} \quad (5.34)$$

The value of γ is obtained from the condition-3 of internal stability for IMC structure

i.e. $(1-G_m Q_{C,p})$ at the RHP poles of the process should be zero.

This condition can be applied as

$$(1 - Q_{C,p}G_m)|_{s=1/\tau_1} = 0 \quad (5.35)$$

Substituting $Q_{C,p}$ from eq. 5.18a, the values of γ is obtained as

$$\gamma = \frac{z_3\left(\frac{\lambda_1}{\tau_1}+1\right)^4 e^{\frac{\theta}{\tau_1}}\tau_1}{\left(\frac{z_1}{\tau_1{}^2}+\frac{z_2}{\tau_1}+z_3\right)} - \tau_1 \quad (5.36)$$

By applying this condition, all the three conditions for internal stability are satisfied. With the above equations, the analytical expressions for all the controller parameters of Eq. (5.31) are available except the tuning parameters λ_1 and λ_2. In the next section, the guidelines for selection of these two tuning parameters are provided. It should be noted that for all practical applications, derivative filtering is required in a PID controller to attenuate noise in the process output. Hence, the designed PID controller (eq. 5.31) is implemented in the form

$$G_{c1} = k_c\left(1 + \frac{1}{\tau_i s} + \frac{\tau_d s}{\tau_f s+1}\right)\frac{(\gamma s+1)}{(\beta s+1)} \quad (5.37)$$

Where τ_f is the derivative filter coefficient. Selection of τ_f depends on both performance, robustness, noise attenuation and hence trade off exits for selection of τ_f. As a compromise between performance, robustness and noise attenuation, in the present work, τ_f is selected as $\tau_d/2$ for all the simulation studies. However, this value can be varied based on the requirement and the presence of noise magnitude in the process output.

5.4 Guidelines for selection of tuning parameters

5.4.1 Selection of tuning parameter λ_2

In synthesis and IMC methods, low values of tuning parameters results in good nominal performance. Large values of these tuning parameters produces robust control performance with compromise on nominal responses. This sets a tradeoff in deciding the tuning parameter values. Over the extensive simulations performed on various processes, the range of tuning parameter is selected as $\lambda_2 = 0.4\theta_{m2} - 2\theta_{m2}$.

5.4.2 Selection of tuning parameter λ_1

To have clear understanding for selection of λ_1, a systematic analysis is carried out using maximum sensitivity (M_s) as the performance index. (Skogestad and Postlethwaite, 2005) M_s is also a robust performance measure like Gain margin (GM) and Phase margin (PM) and is related to these margins as $GM \geq M_s/(M_s - 1),\ PM \geq 2\sin^{-1}(1/2M_s)$. In the present work, M_s values are plotted against the tuning parameter λ_1 and from this plot one can select the tuning parameter and obtain the controller based on the required level of robustness. Note that for the same value of M_s, there exist two values of λ_1 in which the higher value need to be selected to ensure robustness of the closed loop system.

5.4.3 Set point filter

For unstable systems, usually there exist undesirable overshoot in the closed loop response. To avoid this undesirable overshoots, either set-point weighting or set point filters are recommended (Astrom and Hagglund, 1995). In this work, set point filter is considered to eliminate the undesirable overshoots in the servo responses. The closed loop relation between y_1 and r_1 consists of the term $(\gamma s + 1)$ and this term causes overshoot in the closed loop output response. To minimize the undesirable overshot, the set point filter is selected as given in Eq. (5.38) which is first order in nature.

$$F_R = \frac{1}{(\gamma s+1)} \tag{5.38}$$

5.5 Simulation results

Simulation studies have been performed on different unstable cascaded time delay processes and the results are compared with some of the recently reported methods. For quantitative comparison, Integral of Absolute Error (IAE) and Total Variation (TV) are used as performance indices.

Example – 1: An example studied by Uma et al. (2009) is considered here. The secondary and primary processes considered here are $G_{p2}= e^{-0.6s}/ (2.07s+1)$ and $G_{p1}= e^{-0.339s} / (5s-1)$ respectively. For the proposed method, the inner loop controller G_{c2} is an IMC controller as described earlier, and the secondary loop tuning parameter is considered as $\lambda_2 = 0.5\theta_{m2} = 0.3$. With that the secondary loop controller is obtained as $G_{c2} = (2.07s + 1)/(0.3s + 1)$. Based on this controller, the overall primary process model is obtained from eq. 5.8. To select the primary loop tuning parameter (λ_1), an analysis is carried out based on maximum sensitivity. Figure 5.2 shows the variation of Ms with respect to λ_1. It can be observed from the figure that one should not select λ_1 corresponding to the Ms peak value of 52.75. Based on this analysis, $\lambda_1 = 1.2$ is selected and the corresponding primary loop controller settings are obtained as $k_c = 0.478$, $\tau_i = 0.99$, $\tau_d = 0.213$, $\gamma = 6.821$, $\beta = 0.567$. The set point filter constant is selected as $\gamma = 6.821$.

With these controller settings, simulation studies are performed by giving a unit step change in set point at t = 0 sec and a negative disturbance of magnitude 4 at t = 50 sec in the inner loop (d_2) and a unit negative step disturbance at t = 100 sec in the outer loop (d_{11}) respectively. The closed loop performances and the corresponding control action responses are shown in Figure 5.3 for perfect parameters. The proposed method provides good closed loop responses. In order to analyse the robustness, perturbations of +25% in primary time delay and -25% in both time constants are given and the corresponding control action and closed loop responses are presented in Figure 5.3. It can be observed from the responses that the suggested method provides robust closed loop and control action responses.

For a fair comparison, the methods proposed by Uma et al. (2009) and Padhan and Majhi (2012) are also considered. Whenever, a comparison is carried out with any other method, it should be fair and proper. The proposed method is a simple cascade control scheme with only two loops and two controllers whereas the methods proposed by Uma et al. (2009) and Padhan and Majhi (2012) are based on modified Smith predictor and consist of more loops with more controllers. Also, these methods have different tuning parameters for each controller. In order to have fair comparison with these methods, a unit step change is given in the primary loop disturbance (d_{11}) for all the three methods and the corresponding disturbance rejection controllers are tuned to provide same value of IAE. For the method of Uma et al. (2009), G_{cd} is tuned by selecting λ_d = 1.14 and the other two controllers are obtained by selecting λ_2 = 0.6 and λ_s = 0.7042. The corresponding set point tracking controller settings are obtained as k_{is} = 2.8953, k_{cs} = 9.1554, β_s = 0.2373, and k_{ds} = 4.4519. The primary disturbance rejection controller (G_{cd}) parameters are obtained as k_{id} = 0.5712, k_{cd} = 3.8728, k_{dd} = 2.1937, β_d = 0.0322, and α_d = 0.4695.

For the method of Padhan and Majhi (2012), the disturbance rejection controller G_{cd1} is tuned by selecting λ_1 = 1.24 and the corresponding controller parameters are obtained as K_{c1} = 0.3028, T_{i1} = 0.6260 and T_{d1} = 0.2348, c_{f1} = 8.7967, c_{f2} = 13.924, d_{f1} = 0.5535, d_{f2} = 0.1842. The set point tracking controller is obtained as $G_{cs} = (10.35s^2+2.93s)/(0.64s^2+1.6s+1)$ by choosing the tuning parameter as λ_{cs} = 0.8. The parameters of G_{cd2} are obtained as K_{c2} = 0.2752, T_{i2} = 0.4 and T_{d2} = 0.15, a_{f1} = 6.1635, a_{f2} = 8.4736, b_{f1} = 1.8817, b_{f2} = 0.3705, by selecting the tuning parameter as $\lambda_2 = 1.7\theta_{m2} = 1.02$. These controller settings for all the three methods provide an IAE of 2.07 when a unit step change is given in the primary disturbance d_{11}. A fair comparison can be carried out with this approach for all the three methods. Set point responses are not considered for comparative analysis as the shape of the set point responses can be altered by adding either a set point filter or set point weighting.

All the three methods are compared using the suggested controller settings by giving a step change of magnitude 4 in the secondary loop disturbance (d_2) at time t = 0 and unit step change in the primary loop disturbance (d_{11}) at t = 40 respectively. Their respective closed loop responses are presented in Figure 5.4. It can be observed from Figure 5.4 that the proposed method shows improved performances with smooth control action responses. In order to analyse the robustness, perturbations of +25% in primary time delay and -25% in both time constants are given and the corresponding control action and closed loop responses are presented in Figure 5.5. From the responses, it can be noted that the proposed method provides improved closed loop and smooth control action responses when compared to the methods of Uma et al. (2009) and Padhan and Majhi (2012). The corresponding IAE and TV values are shown in Table 5.1. It can be observed from the Table 5.1 that the IAE values for the proposed method are low. From the IAE and TV values, it can be identified that the suggested method is better.

To analyse the effect of the secondary loop tuning parameter (λ_2) on the selection of the primary loop tuning parameter (λ_1), a plot of M_s verses λ_1 is drawn for different values of λ_2 and is shown in Figure 5.6. It can be observed that as λ_2 increases, the number of peaks for Ms Increases which will restrict the selection of λ_1 to limited zones. This kind of phenomenon is peculiar only for unstable processes. To analyse the effect of noise, it is assumed that white noise is present in the measurement device with noise power = 0.0001, sampling time of 0.5 and seed = 0 for perfect model condition. The corresponding closed loop and control action responses are shown in Figure 5.7. Note that there will be variations in the control action responses in the presence of noise. To reduce the effect of noise, one can consider improved derivative filtering in the PID controller with suitable value for the filter coefficient τ_f.

Table 5.1 IAE and TV values for Example 1

Method	**Proposed model**		**Mismatch of -25 % in both time constants and +25 % in primary time delay**	
	IAE	**TV**	**IAE**	**TV**
Present	4.23	16.43	4.26	23.12
Uma et al. (2009)	4.75	54.88	4.35	121.4
Padhan and Majhi (2012)	4.38	15.88	4.35	25.42

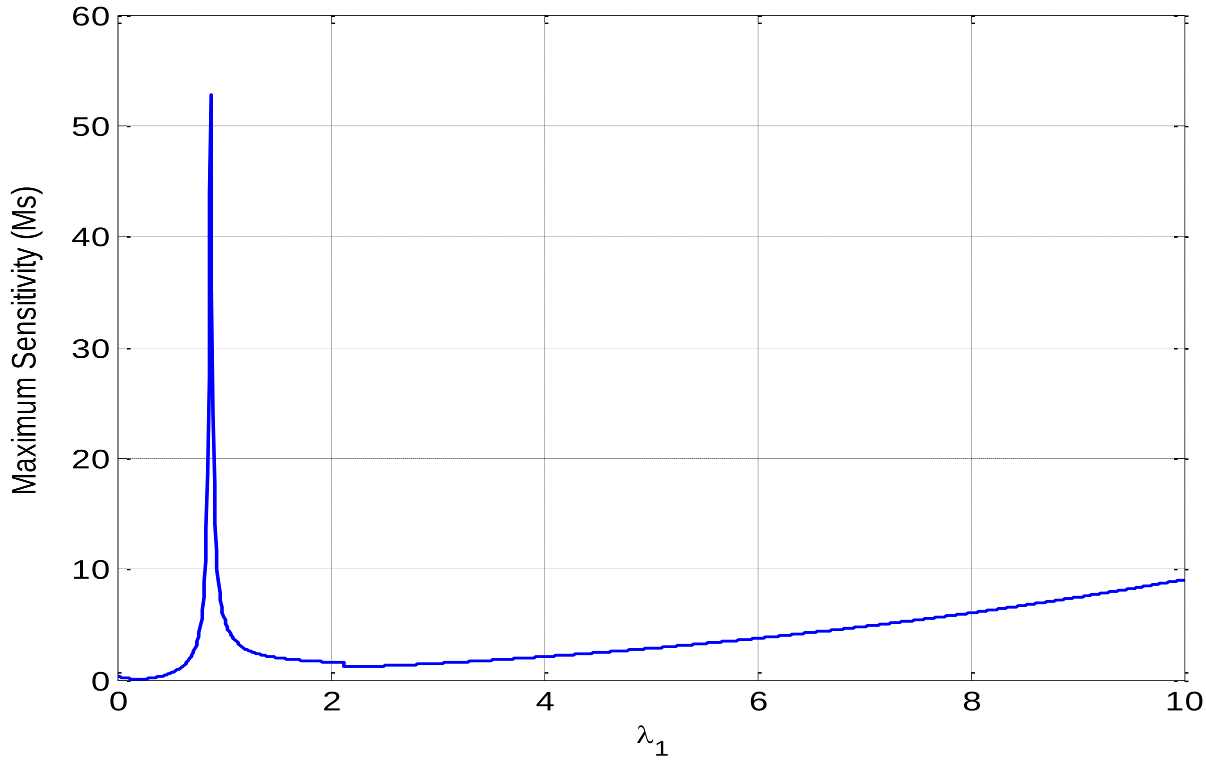

Figure 5.2 Variation of Ms with λ_1 for example-1.

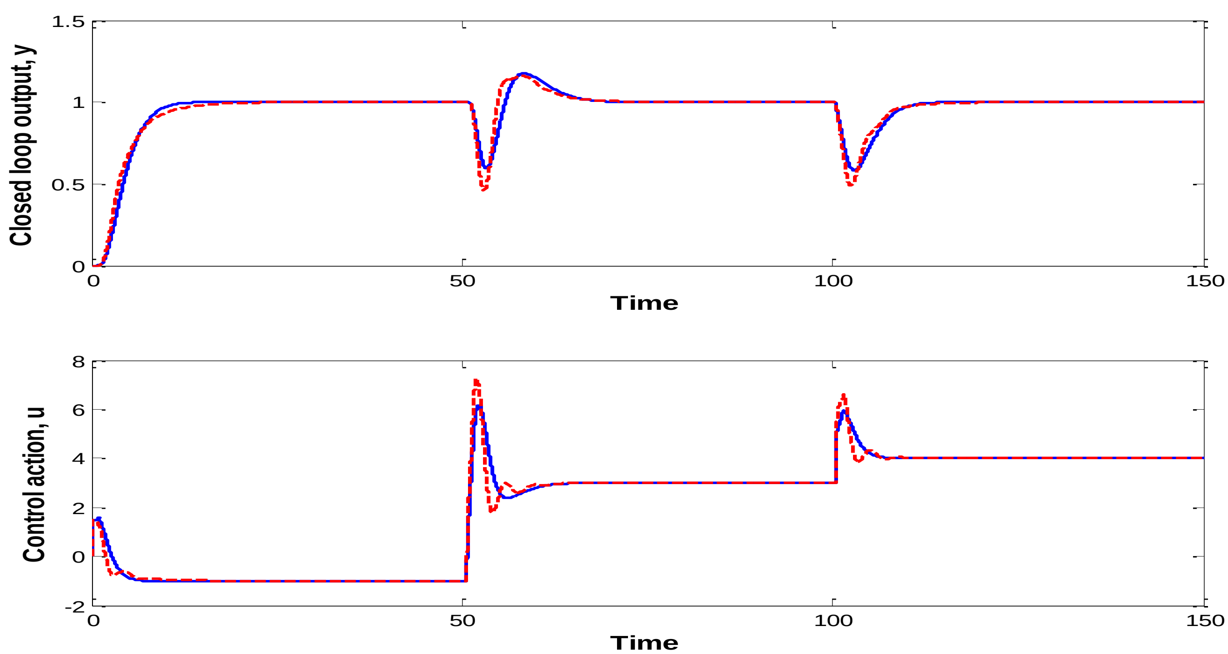

Figure 5.3 Output and control action behavior for example-1, dash – Perturbations, solid – Perfect model.

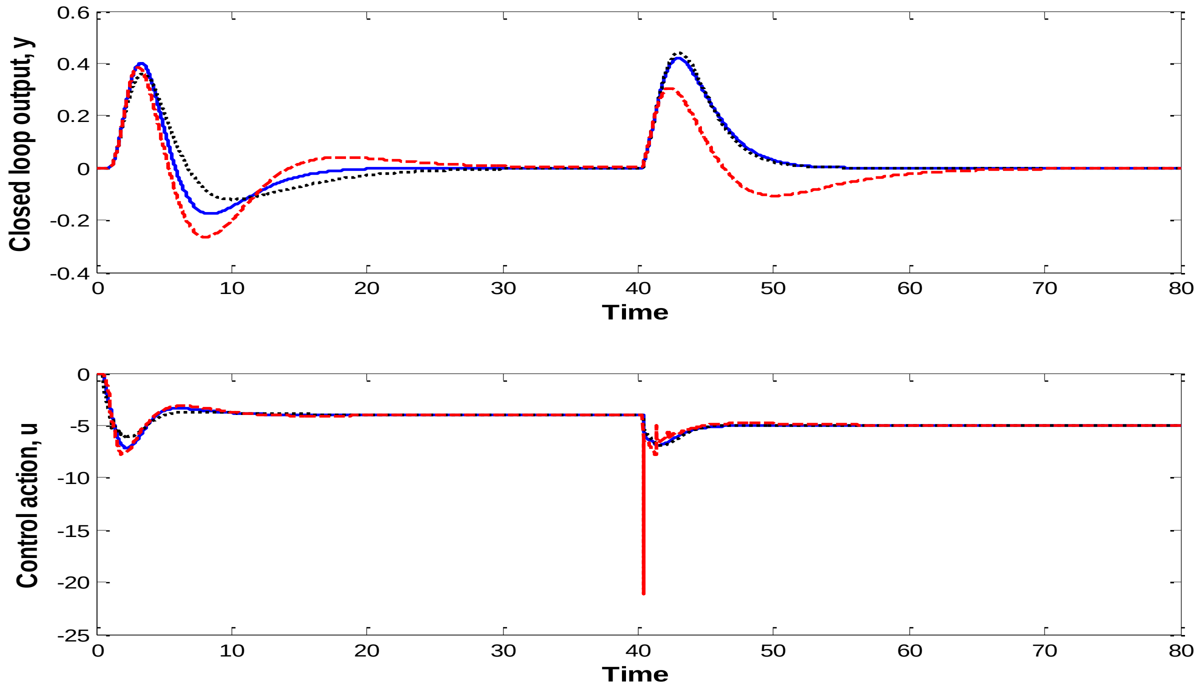

Figure 5.4 Output and control action behavior under exact model for example-1, dot - Padhan and Majhi (2012), solid - Proposed method, dash – Uma et al. (2009).

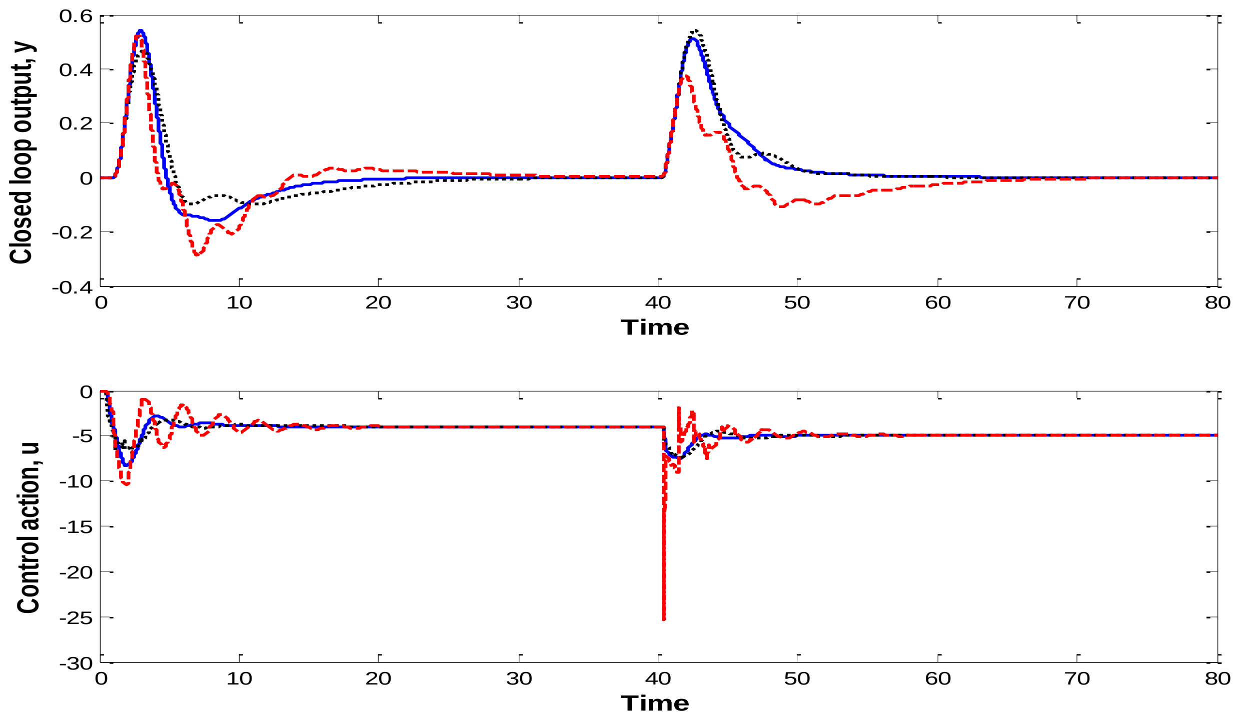

Figure 5.5 Output and control action behavior under mismatch model for example-1, dash – Uma et al. (2009), solid - Proposed method, dot - Padhan and Majhi (2012).

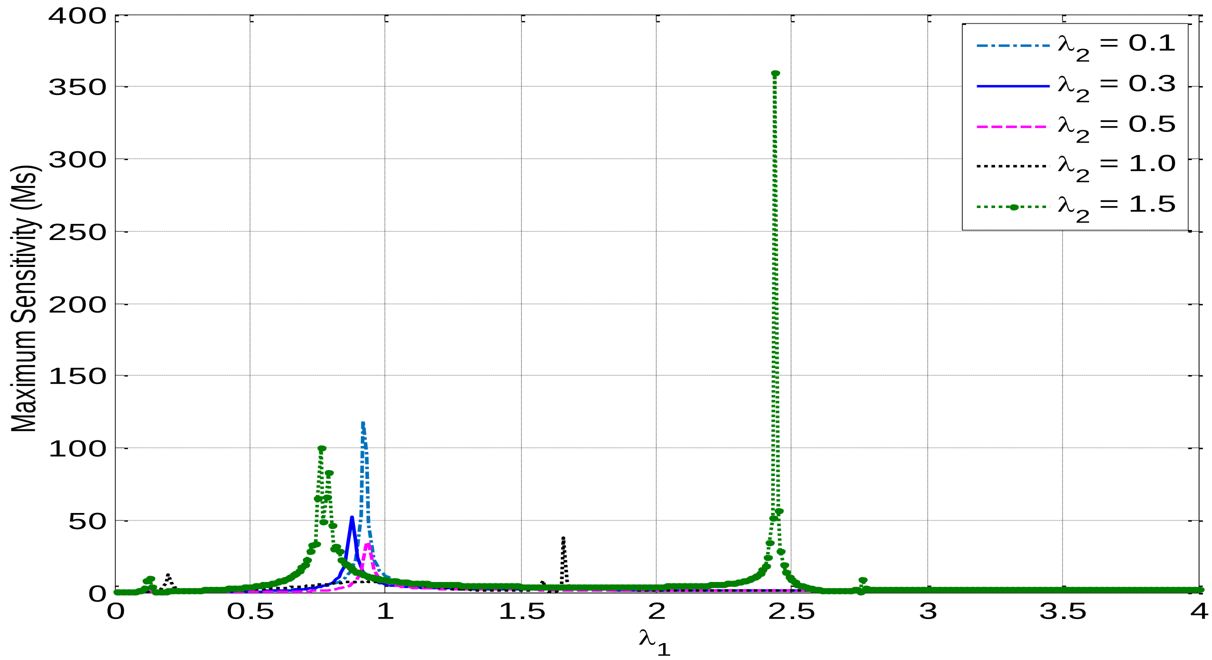

Figure 5.6 Variation of Ms with λ_1 for different values of λ_2 for example-1.

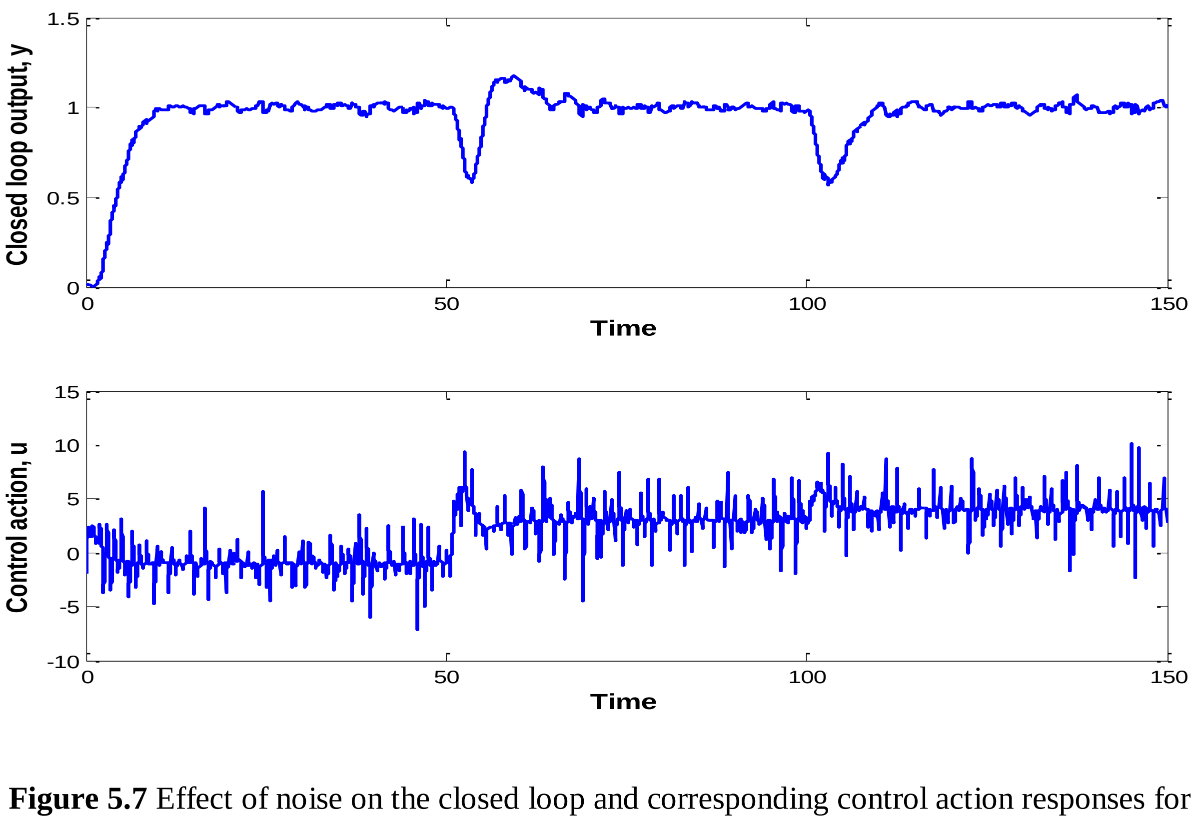

Figure 5.7 Effect of noise on the closed loop and corresponding control action responses for example-1.

Example – 2: An example studied by many researchers (Liu et al., 2005b; Uma et al., 2009) is considered in which the secondary and primary processes are given as $G_{p2} = 2e^{-2s}/(20s+1)$ and $G_{p1} = e^{-4s} / (20s-1)$ respectively. The inner loop controller G_{c2} is designed as $G_{c2} = (20s+1)/2(s+1)$ by choosing the tuning parameter $\lambda_2 = 0.5\theta_{m2} = 1$. Based on this, the overall primary process model (eq. 5.8) is obtained and the primary loop controller is designed by appropriately selecting the tuning parameter. To select the primary loop tuning parameter (λ_1), an analysis is carried out based on maximum sensitivity. Figure 5.8 shows the variation of M_s with respect to λ_1. From the graph, note that λ_1 should not be selected where peaks exist for M_s value of 40.77 to avoid non-robust responses. Here, the tuning parameter is selected as $\lambda_1 = 6.296$. With this value of tuning parameter, the primary loop controller settings are obtained as $k_c = 0.438$, $\tau_i = 6.71$ *and* $\tau_d = 0.85$, $\gamma = 39.76$, $\beta = 4.21$. The set point filter time constant is considered as 39.76.

With these controller settings, simulation studies are performed by giving a unit step change in set point at t = 0 sec and a negative disturbance of magnitude 4 at t = 50 sec in the inner loop (d_2) and a unit negative step disturbance at t = 100 sec in the outer loop (d_{11}) respectively. The closed loop performances and the corresponding control action responses are shown in Figure 5.9 for perfect parameters. The proposed method provides good closed loop responses. In order to analyse the robustness, perturbations of -20 % in secondary time constant, -10% in primary time constant, +20 % in secondary process gain and time delay & +10% in primary process gain and time delay are considered and the corresponding control action and closed loop responses are also shown in Figure 5.9. From the responses, it can be noted that the suggested method provides robust closed loop and control action responses.

For a fair comparison, the methods proposed by Garcia et al. (2010) and Uma et al. (2009) are considered. The methods considered here are also based on modified Smith predictor and they are composed of more loops with more controllers. In order to have all the methods at the same level, a unit step change is given in the primary loop disturbance (d_{11}) for all the three methods and the corresponding disturbance rejection controllers are tuned to provide same value of IAE. Generalized predictor cascade controller (GPCC) is used for the method of Garcia et al. (2010). For the method of Uma et al. (2009), G_{cd} is tuned by selecting $\lambda_d = 5.15$ and the other two controllers are obtained by selecting λ_2 = 2 and λ_s = 5. The corresponding set point tracking controller settings are obtained as k_{is} = 0.1829, k_{cs} = 4.6571, β_s = 2.8571, and k_{ds} = 12.2857. The controller parameters for primary disturbance rejection (G_{cd}) are obtained as k_{id} = 0.0874, k_{cd} = 3.063, k_{dd} = 6.637, α_d = 3, and β_d = 0.1537. The three methods provide an IAE of 15.34 when a unit step change is given in the primary disturbance d_{11}. A fair comparison can be carried out with this approach for all the three methods. As explained in example-1, set point responses are not considered for comparative analysis as the shape of the set point responses can be altered by adding either a set point filter or set point weighting.

All the three methods are compared using the suggested controller settings by giving a step change of magnitude 4 in the secondary loop disturbance (d_2) at time t = 0 and unit step change in the primary loop disturbance (d_{11}) at t = 150 respectively. Their respective closed loop responses are presented in Figure 5.10. It can be noted from Figure 5.10 that the proposed method shows improved performances with smooth control action responses. In order to analyse the robustness, perturbations of -20 % in secondary time constant, -10% in primary time constant, +20 % in secondary process gain and time delay & +10% in primary process gain and time delay are considered and the corresponding control action and closed loop responses are shown in Figure 5.11. From the responses, it can be noted that the suggested method provides enhanced closed loop and smooth control action responses when compared to the methods of Garcia et al. (2010) and Uma et al. (2009). In fact, the oscillations in the control action responses for other two methods are more whereas the proposed method does not provide such oscillatory responses even if there are more perturbations in the process parameters. The corresponding IAE and TV values are shown in Table 5.2. It can be observed from the Table 5.2 that the IAE values for the proposed method are low. From the IAE and TV values, it can be identified that the proposed method is better.

To analyse the effect of noise, it is assumed that white noise is present in the measurement device with noise power = 0.0001, sampling time = 0.5 and seed = 0 for perfect model condition. The corresponding closed loop and control action responses are shown in Figure 5.12. It can be noted that the suggested method is able to provide good set point and disturbance rejection performances in the presence of noise. The proposed method is also compared with the method of Nandong and Zang (2014) and it is observed that the method of Nandong and Zang (2014) is sensitive to model uncertainties (graph not shown here). The proposed method provides significantly improved performances when compared to Nandong and Zang (2014) for model uncertainties.

Table 5.2 IAE and TV values for Example 2

Method	**Proposed model**		**Mismatch of -10% in primary time constant , -20 % in secondary time constant, +20 % in secondary process gain and time delay & +10% in primary gain and time delay**	
	IAE	**TV**	**IAE**	**TV**
Present	32.0	18.59	32.60	40.76
Garcia et al. (2010)	34.78	18.60	37.24	104.39
Uma et al. (2009)	34.57	96.11	Unstable	Unstable

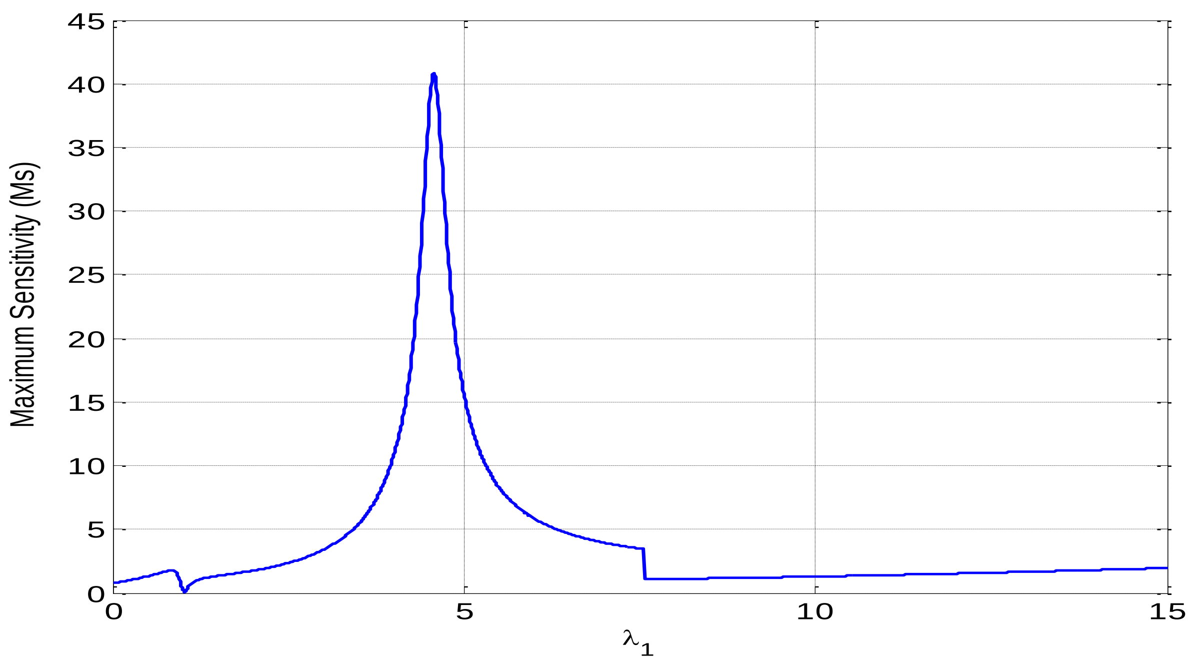

Figure 5.8 M_s versus λ_1 for example-2.

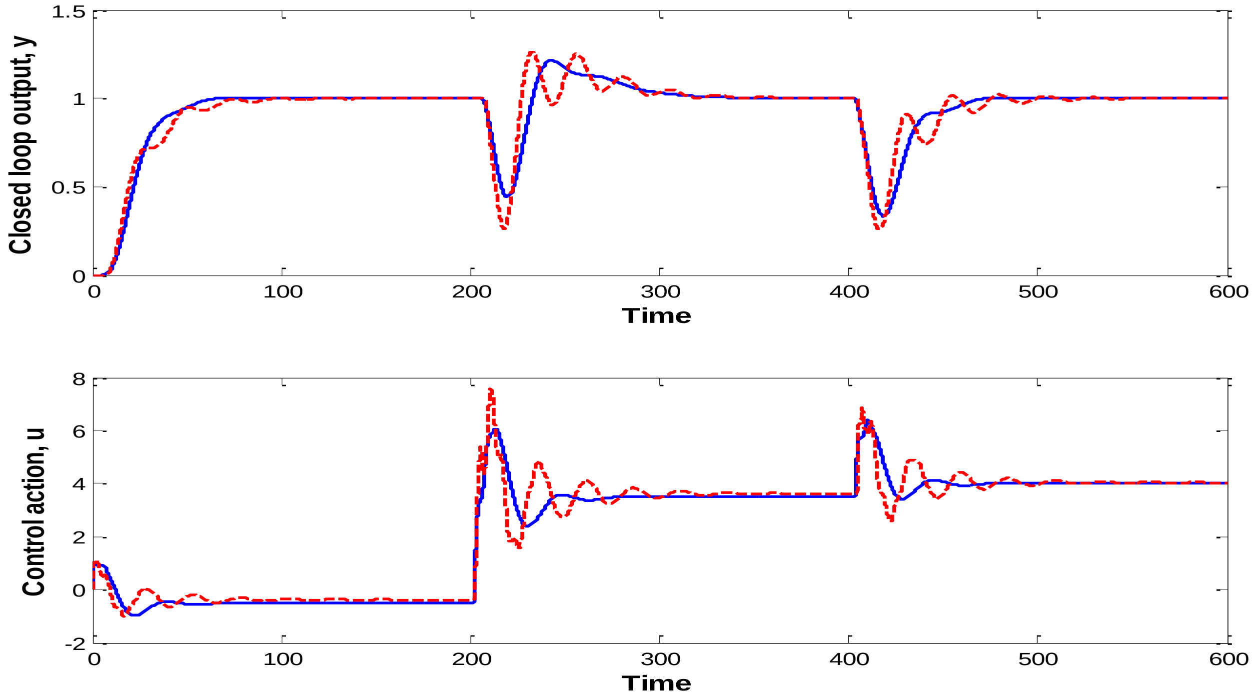

Figure 5.9 Output and control action behavior for example-2, dash – Perturbations, solid – Perfect model.

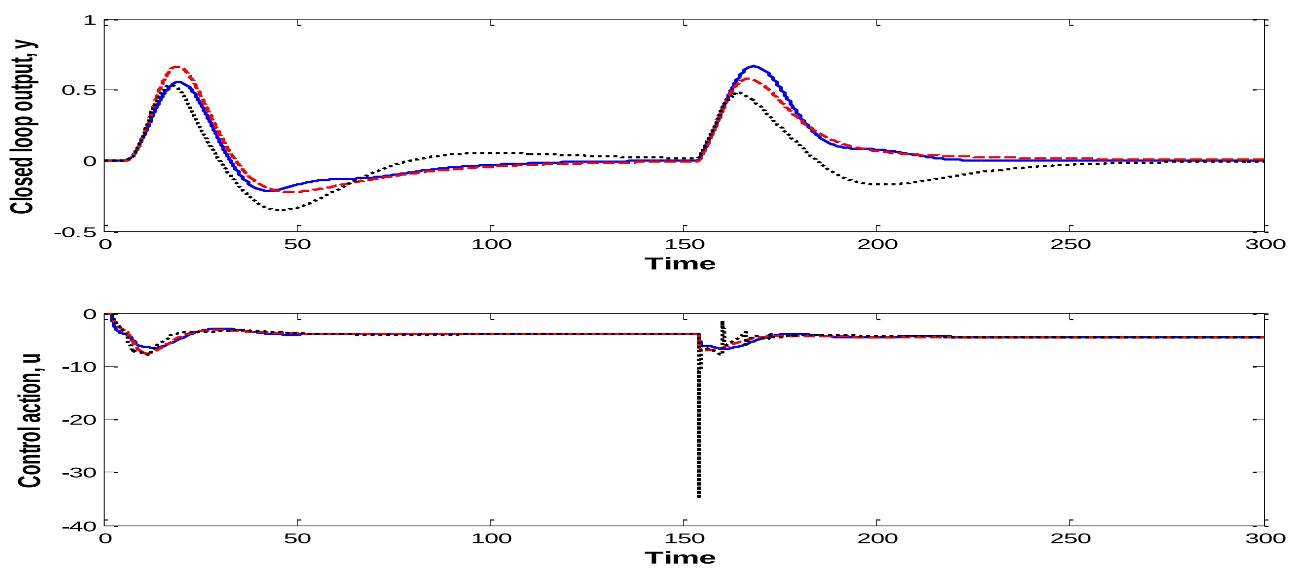

Figure 5.10 Output and control action behavior under exact model for example-2, dash – Garcia et al. (2010), solid - Proposed method, dot – Uma et al. (2009).

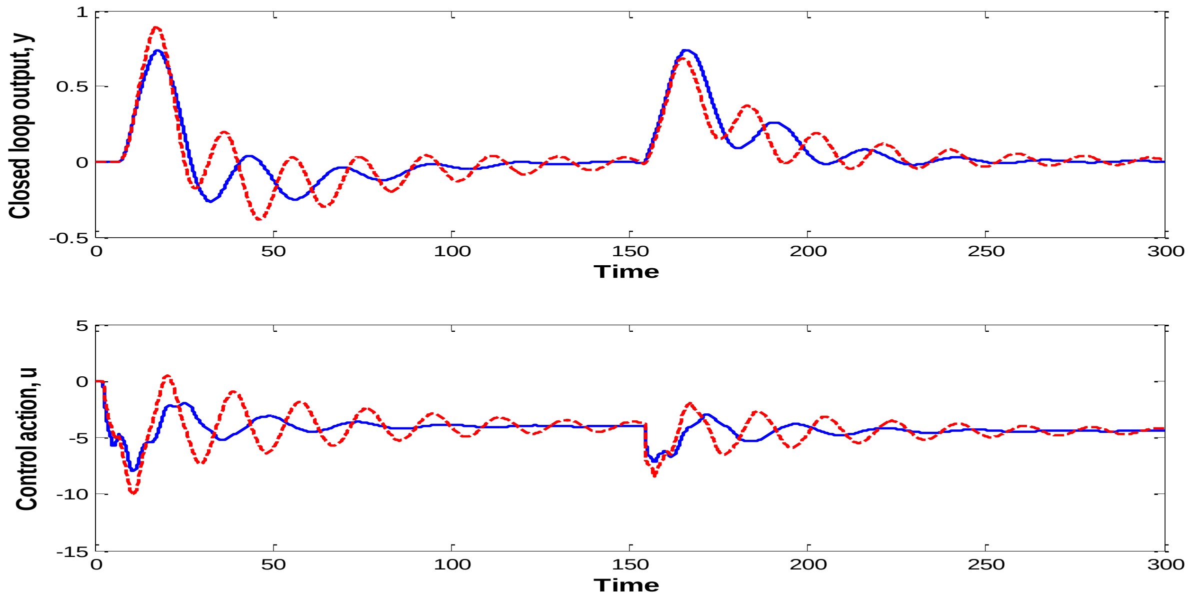

Figure 5.11 Output and control action behavior under mismatch model for example-2, dash – Garcia et al. (2010), solid - Proposed method.

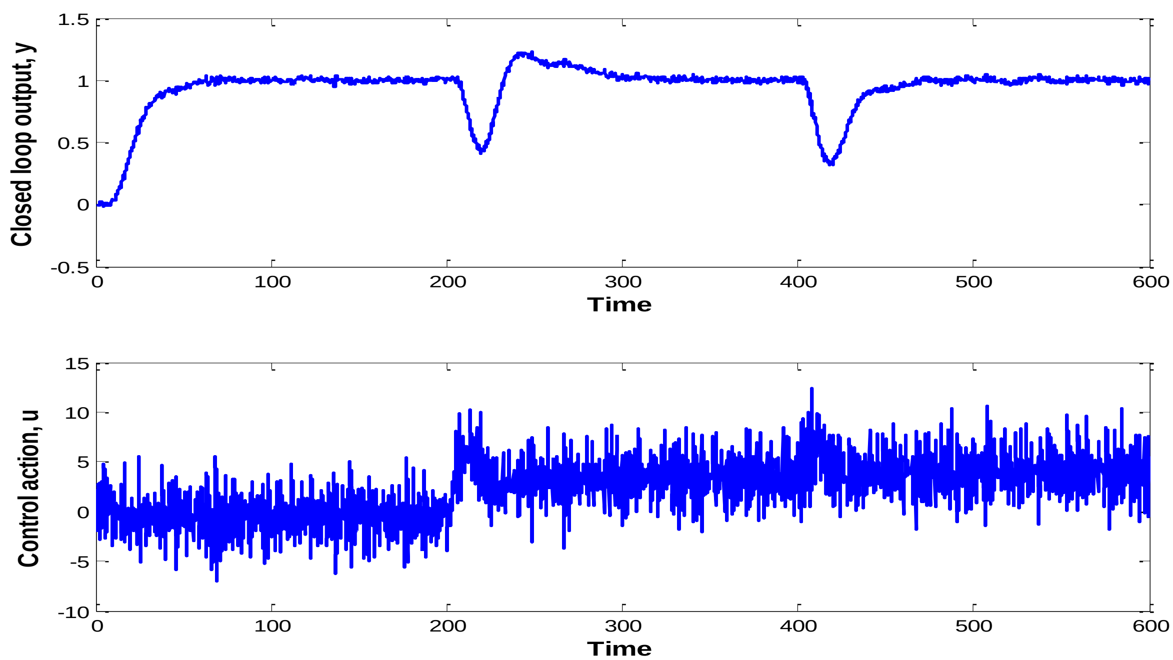

Figure 5.12 Effect of noise on the closed loop and corresponding control action responses for example-2

Example – 3: An example studied by (Liu et al., 2005b; Padhan and Majhi, 2012; Uma et al., 2009) is considered. The secondary and the primary processes are given as $G_{p2}= 2e^{-2s}/(s+1)$ and $G_{p1}= e^{-3s} / (10s-1)$ respectively. For the proposed method, the inner loop controller G_{c2} is designed as $G_{c2} = (s + 1)/2(3s + 1)$ after selecting the tuning parameter as $\lambda_2 = 1.7\theta_{m2} = 3.4$. The overall primary loop model is obtained as per eq. (5.8) and the primary loop controller is designed as per eq. (5.13). To select the primary loop tuning parameter (λ_1), M_s versus λ_1 is plotted and is shown in Figure 5.13. From the graph, it can be observed that there exist three peak values for M_s i.e. 415, 108, 127 and hence M_s value should not be considered as any of these peaks for selection of λ_1. Based on this, the tuning parameter is selected as $\lambda_1 = 6.935$ and the corresponding controller settings are obtained as $k_c = 0.1223$, $\tau_i = 5.5$, $\tau_d = 1.782$, $\gamma = 72.25$ and $\beta = 0.8856$. The set point filter time constant is considered as $\alpha_3 = 72.25$.

With these controller settings, simulation studies are performed by giving a unit step change in the set point at t = 0 sec and a negative step change of magnitude 0.5 in the disturbance at t = 200 in the inner loop (d_2) and a negative step change of magnitude 0.5 in the disturbance at t = 400 sec in the outer loop (d_{11}) respectively. The closed loop performances and the corresponding control action responses are shown in Figure 5.14 for perfect parameters. The proposed method provides good closed loop responses. In order to analyse the robustness, perturbations of -20 % in both primary and secondary time constants, +20 % in both primary and secondary time delays are considered and the corresponding control action and closed loop responses are also shown in Figure 5.14. From the responses, it can be noted that the suggested method provides robust closed loop and control action responses.

For a fair comparison, the methods suggested by Uma et al. (2009) and Padhan and Majhi (2012) are considered. Again, in order to have fair comparison with these methods, a unit step change is given in the primary loop disturbance (d_{11}) for all the three methods and the corresponding disturbance rejection controllers are tuned to provide same value of IAE. For the method of Uma et al. (2009), G_{cd} is tuned by selecting $\lambda_d = 7.5$ and the other two controllers are obtained by selecting $\lambda_2 = 2$ and $\lambda_s = 1.5$. The corresponding set point tracking controller settings are obtained as $k_{is} = 1.7429$, $k_{cs} = 11.4575$, $k_{ds} = 15.97$, and $\beta_s = 0.4412$. The controller parameters for primary disturbance rejection (G_{cd}) are obtained as $k_{id} = 0.0181$, $k_{cd} = 1.5884$, $k_{dd} = 4.945$, α_d

$= 2.5$, and $\beta_d = 0.2864$. For the method of Padhan and Majhi (2012), the disturbance rejection controller G_{cd1} is tuned by selecting $\lambda_1 = 7$ and the corresponding controller parameters are obtained as $K_{c1} = 0.037$, $T_{i1} = 3.333$ and $T_{d1} = 1.25$, $c_{f1} = 72.0$, $c_{f2} = 71.0$, $d_{f1} = 2.455$, $d_{f2} = 4.103$. The set point tracking controller is obtained as $G_{cs} = (10s^2+9s+1)/(4.5s^2+6s+2)$ by choosing the tuning parameter as $\lambda_{cs} = 1.5$. The parameters of G_{cd2} are obtained as $K_{c2} = 0.0015$, $T_{i2} = 1.333$ and $T_{d2} = 0.5$, $a_{f1} = 449.55$, $a_{f2} = 448.55$, $b_{f1} = 0.841$, $b_{f2} = 0.1633$, by selecting the tuning parameter as $\lambda_2 = 6.8$. These controller settings for all the three methods provide an IAE of 44.9 when a unit step change is given in the primary disturbance d_{11}. Set point responses are not considered for comparative analysis as the shape of the set point responses can be altered by adding either a set point filter or set point weighting.

All the three methods are compared using the suggested controller settings by giving a unit step change in the secondary loop disturbance (d_2) at time $t = 0$ and unit step change in the primary loop disturbance (d_{11}) at $t = 250$ respectively. Their respective closed loop responses are presented in Figure 5.15. It can be noted from Figure 5.15 that the proposed method shows improved performances with smooth control action responses. In order to analyse the robustness, perturbations of +20% in both primary and secondary time delays and -20% in both time constants are given and the corresponding control action and closed loop responses are presented in Figure 5.16. From the responses, it can be noted that the suggested method provides enhanced closed loop and smooth control action responses when compared to the method of Uma et al. (2009). The method of and Padhan and Majhi (2012) shows unstable responses and hence are not shown in the figure. The corresponding IAE and TV values are shown in Table 5.3. It can be noted from the Table 5.3 that the IAE values for the proposed method are low. From the IAE and TV values, it can be observed that the proposed method is better. Note that if the perturbations are increased further, the other methods are giving more oscillatory or unstable responses whereas the proposed methods provides stable closed loop responses with comparatively smooth control action responses.

To analyse the effect of noise, it is assumed that white noise is present in the measurement device with noise power = 0.0001, sampling time = 0.5 and seed = 0 for perfect model condition. The corresponding closed loop and control action responses are shown in Figure 5.17. It can be identified that the suggested method is able to provide good set point and disturbance rejection performances in the presence of noise. To reduce the effect of noise, one can alter the derivative filtering coefficient to achieve desirable performances.

Table 5.3 IAE and TV values for Example 3

Method	Proposed model		Mismatch of -20 % in both time constants and +20 % in both time delays	
	IAE	TV	IAE	TV
Present	75.86	6.36	9.68	10.82
Uma et al. (2009)	76.44	8.79	100.98	36.02

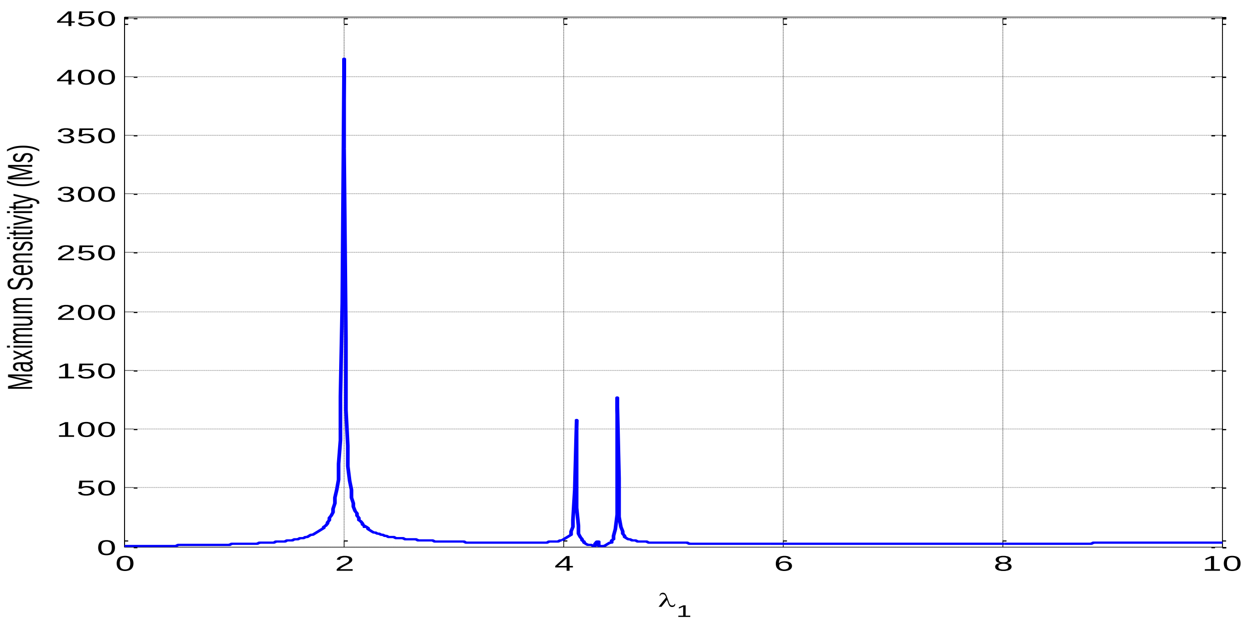

Figure 5.13 M_s versus λ_1 for example-3.

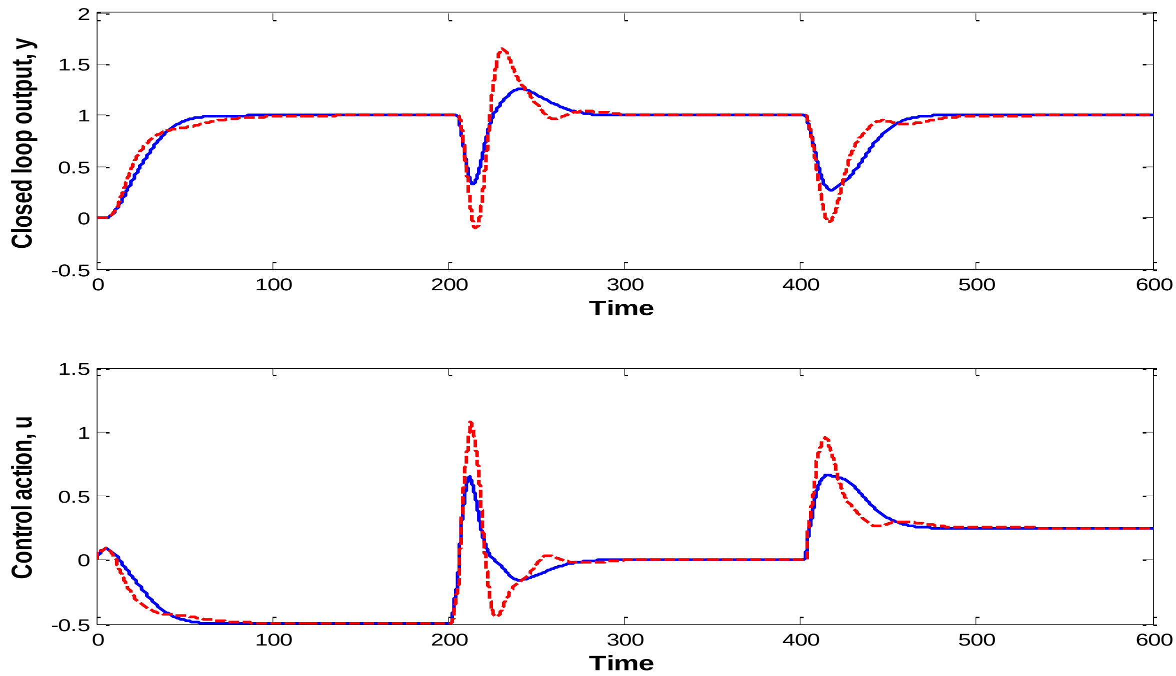

Figure 5.14 Output and control action behavior for example-3, dash – Perturbations, solid – Perfect model.

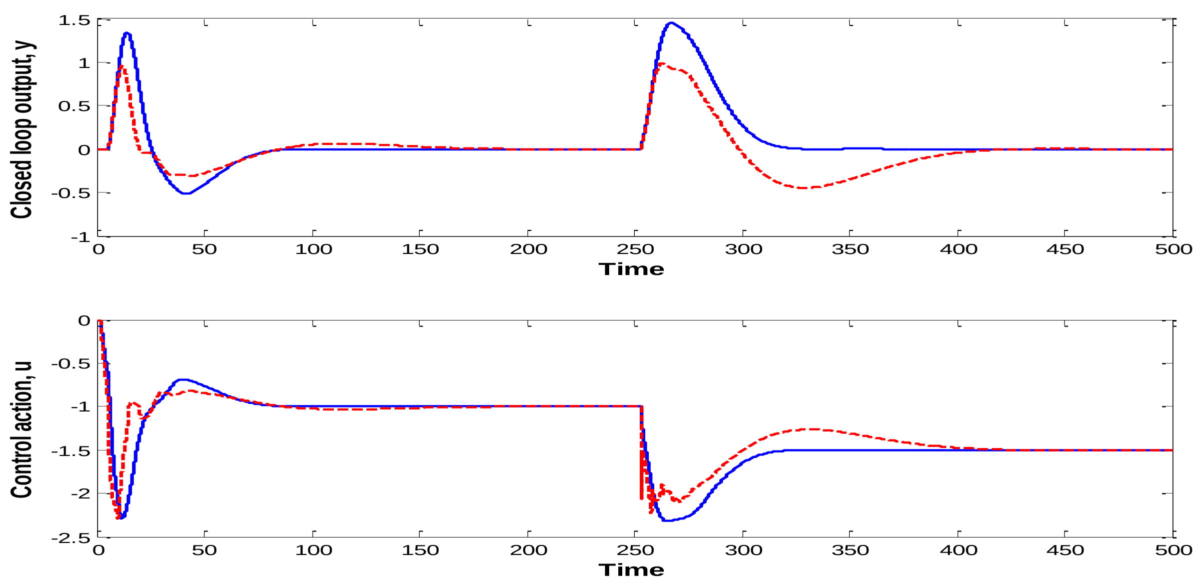

Figure 5.15 Output and control action behavior under exact model for example-3, dash – Uma et al. (2009), solid - Proposed method.

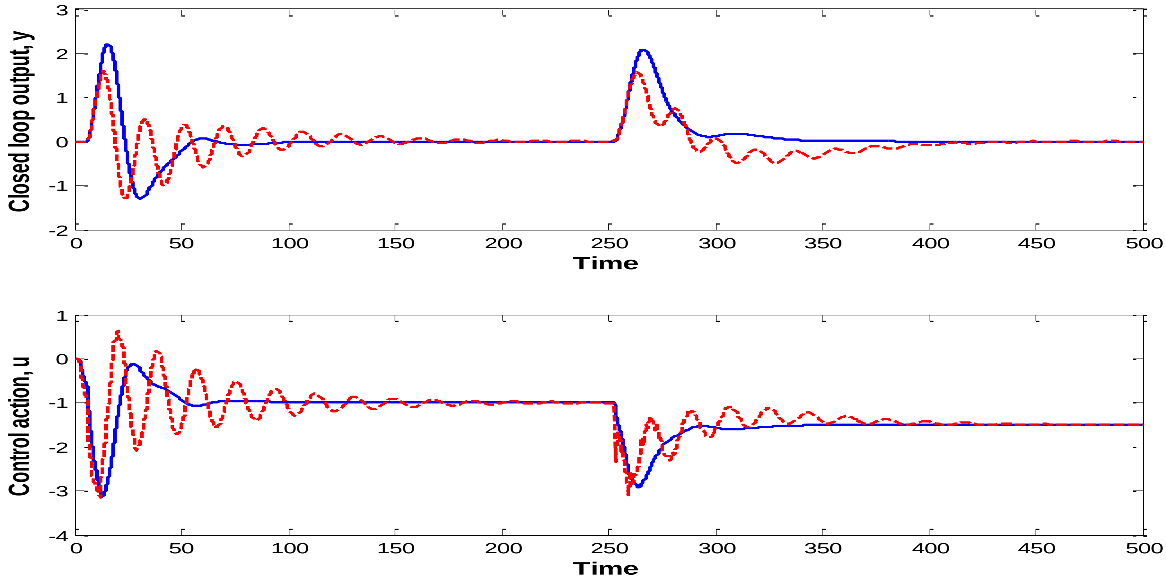

Figure 5.16 Output and control action behavior under mismatch model for example-3, dash – Uma et al. (2009), solid - Proposed method.

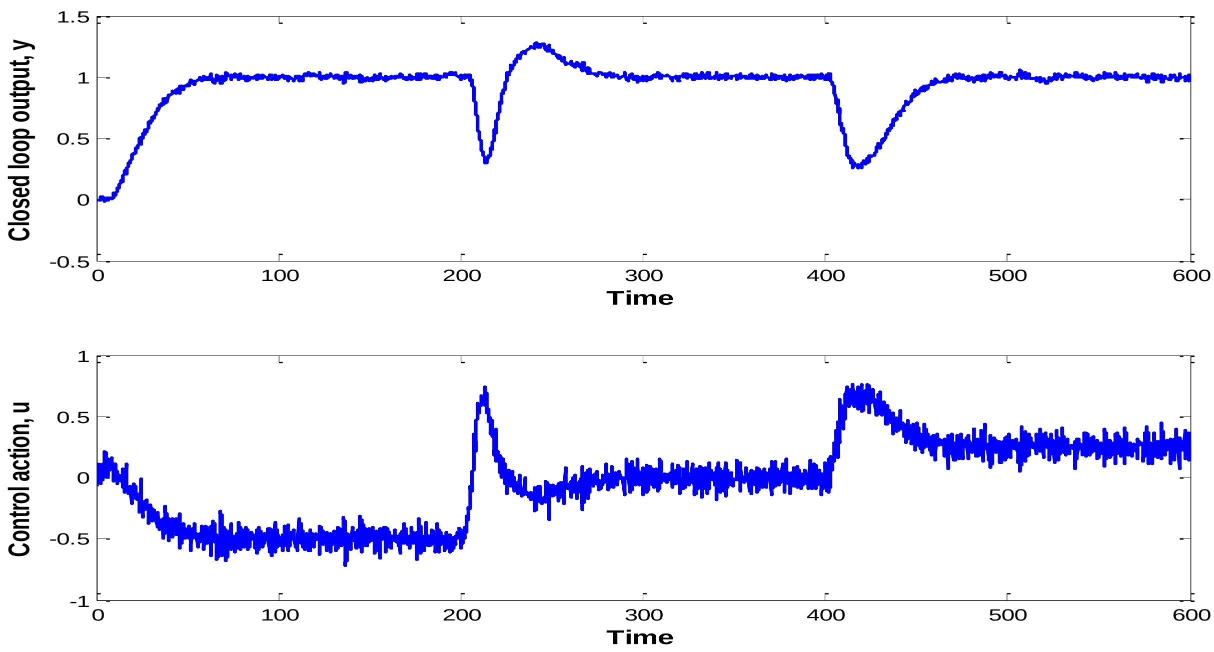

Figure 5.17 Effect of noise on the output and control action responses for example-3.

Example – 4: Control of a chemical reactor

An isothermal continuous stirred tank reactor is considered which exhibits multiple steady state solutions. The mathematical model of the reactor is given as Sree and Chidambaram (2006)

$$\frac{dC}{dt} = \frac{Q}{V}(C_f - C) - \frac{k_1 C}{(k_2 C+1)^2} \tag{5.39}$$

Where C_f is the inlet concentration, Q is the inlet flow rate, V is the volume of the reactor, C is the exit concentration, k_1 & k_2 are the kinetic parameters. The corresponding values of the parameters and constatns are given as Q = 0.0333 L/s, V = 1L, k_1 = 10 L/s, and k_2 = 10 L/mol. By considering C_f = 3.288 mol/L, three steady states are obtained as C = 1.7673, 1.316 and 0.01424 mol/L. Out of the three steady states, there is one unstable steady state at C = 1.316 mol/L. Inlet concetration is considered as the manipulated variable and exit concentration as the controlled variable. Linearization of the manipulated variable around this operating condition C = 1.316 gives the unstable transfer function model as 3.433/(103.1s-1). For this particular case , the time delay is considered as 20 sec . Hence, the primary loop unstable transfer function model is obtained as,

$$G_{p1} = \frac{3.433e^{-20s}}{103.1s-1} \tag{5.40}$$

The inlet flow rate is acting as a disturbance and hence one can implement cascade control scheme. Let us assume that the secondary loop dynamics are given as $G_{p2} = e^{-0.5s}/(3s+1)$.

The inner loop controller G_{c2} is designed as $G_{c2} = (3s+1)/(0.25s+1)$ by choosing the tuning parameter as $\lambda_2 = 0.5\theta_{m2} = 0.25$. The overall primary loop model is obtained as per eq. 5.8 and the primary loop controller is designed as per eq. 5.13. To select the primary loop tuning parameter (λ_1), M_s versus λ_1 is plotted and is shown in Figure 5.18. From the graph, it can be observed that there exist peak value for M_s at 142 and hence M_s value should not be considered at this peak for selection of λ_1. Based on this, the tuning parameter is selected as λ_1 = 22 and the corresponding controller settings are obtained as k_c = 0.19, τ_i= 22.62, τ_d = 0.247, γ = 120.38 and β = 18.93. The set point filter time constant is considered as α_3 = 120.38. With the corresponding controller settings, the proposed method is simulated by giving a unit step change in set point at t = 0 sec and a negative step disturbance of magnitude 0.5 at t = 500 sec in the inner loop (d_2) and a negative step disturbance of magnitude 0.2 at t = 800 sec in the outer loop (d_{11}) respectively. The corresponding closed loop performances and control action responses are shown in Figure 5.19 for perfect model parameters. To analyse the robustness, perturbations of +20% in both primary and secondary loop time delays are given and the corresponding closed loop and control action responses are also shown in Figure 5.19. It can be noted that the suggested method provides good closed loop responses.

To analyse the effect of noise, it is assumed that white noise is present in the measurement device with noise power = 0.0001, sampling time = 0.5 and seed = 0 for perfect model condition. The corresponding closed loop and control action responses are shown in Figure 5.20. It can be noted that the suggested method provides good set point and disturbance rejection performances in the presence of noise. To reduce the effect of noise, one can alter the derivative filtering coefficient to achieve desirable performances. Simulation studies are also carried out for different values of τ_f and the corresponding closed loop and control action responses are shown in Figure 5.20 for $\tau_f = \tau_d/2$, τ_d, $\tau_d/0.1$. It can be observed that the control action responses have less variance for $\tau_f = \tau_d/0.1$ when compared to that of $\tau_f = \tau_d/2$.

Discussion:

The proposed method follows a simple cascade control strategy and comparatively easy for tuning of the controllers. However, the previous approaches make use of more number of controllers and more number of tuning parameters, which is difficult for the operator for tuning. Hence, the main merit of the proposed method is its simple structure and only two controllers. Also, systematic guidelines are provided for the proposed method based on maximum sensitivity (M_s).

For unstable systems, selection of tuning parameters is very important and in the literature, the tuning guidelines are given based on some heuristic rules. Whereas in the present method, M_s is used to select the tuning parameter which ensures desired level of robustness for the closed loop. In modified Smith predictor based cascade control schemes, one has to consider separately Ms values for set point tracking and disturbance rejection which is not easy. Note that the methods of Uma et al. (2009), Garcia et al. (2010), Padhan and Majhi (2012) have already shown superiority over many existing methods in the literature. One more contribution of the proposed method is that it has a traditional cascade structure with only two loops and controllers (one for secondary loop and another for primary loop) which provides smooth control action responses and better disturbance rejection performances when compared to the recently stated methods (Garcia et al. (2010; Nandong and Zang, 2014; Padhan and Majhi, 2012; Uma et al., 2009). Note that the other methods Uma et al. (2009) Padhan and Majhi (2012) Nandong and Zang (2014) have more number of sub loops in the primary loop. Even though Garcia et al. (2010) method is applicable for stable and integrating systems, their method is considered here only for unstable systems.

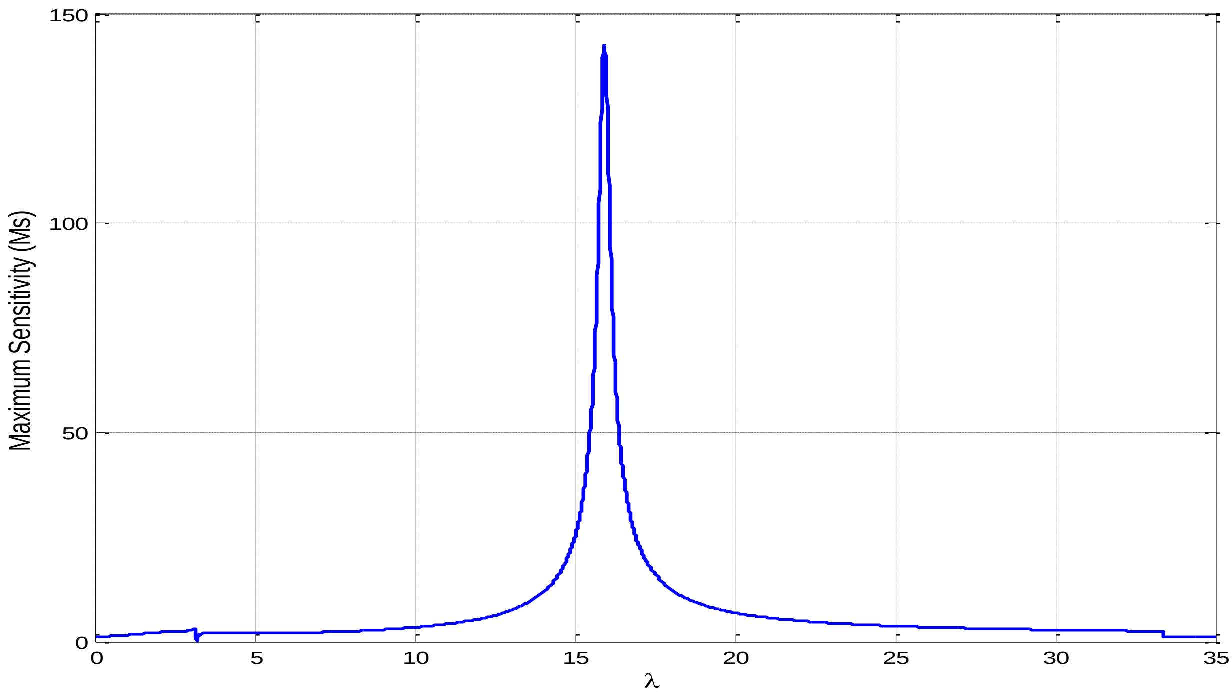

Figure 5.18 M_s versus λ_1 for example-4.

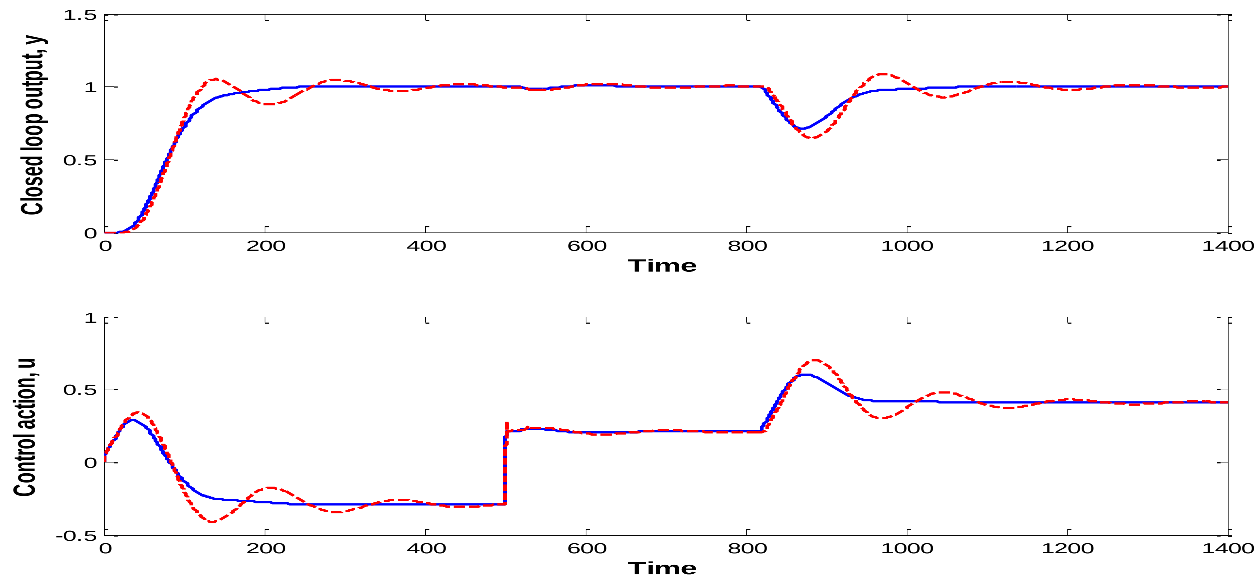

Figure 5.19 Output and control action behavior under mismatch model for example-4, solid – Perfect model, dash – Perturbations.

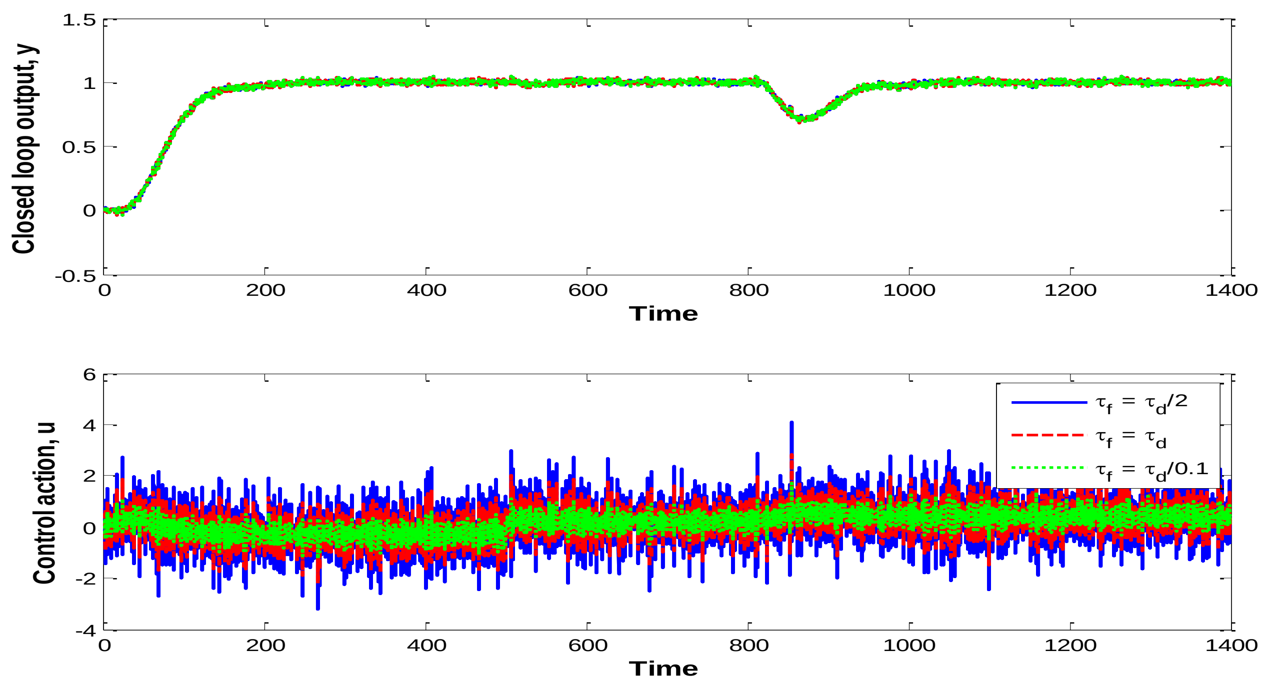

Figure 5.20 Output and control action behavior of the proposed method for different values of the derivative filter constant for perfect model for example-4.

5.6 Summary

In the present work, enhanced design of controllers is proposed for unstable time delayed cascade processes. The performance of the system is analyzed with four different examples. Performance of the system for the proposed method is much better than that of the previously existing methods particularly for disturbance rejection. The proposed method consists of only two controllers whereas in the previous methods, at least two or three controllers were used. The design is comparatively easy and can be implemented for any unstable cascade system. The ability to provide good stable closed loop response even when there are large amount of perturbations in the process parameters is a major advantage of the proposed method over previously existing methods. Quantitative comparison is carried out using IAE and TV values and the proposed method is superior over existing methods. One more main advatange of the proposed method is that the control action respones are smooth in all examples and correspondingly provides low TV values which is recommended for any control system.

Chapter 6

PID Design for Multivariable Unstable Processes

Chapter 6

PID Design for Multivariable Unstable Processes

Controller design for unstable processes is relatively difficult when compared to stable processes. The complexity increases further for multivariable unstable processes. In this work, simplified tuning rules are proposed to design optimal H_2 PID controller for unstable multivariable processes. Decouplers are applied to make the loops independent and diagonal elements of equivalent transfer function are used to design controllers.

6.1 Introduction

Unstable systems are more difficult to control than that of the stable systems. Several methods are proposed for the design of controllers for single-input-single-output (SISO) unstable systems (Rao and Chidambaram, 2012; Sree Chidambaram, 2006). Design of controllers for multivariable systems is difficult than that of SISO systems due to the interactions among the control loops. This difficulty increases for multivariable unstable systems as there exist undesirable overshoots, settling times in the closed loop responses. Several methods (Katebi, 2012; Wang and Nie, 2012) are available in the literature for multivariable stable processes. However, the design methods for multivariable unstable systems are limited. Georgiou et al. (1989) have developed optimization based method. However the system considered does not have a significant time delay. Agamennoni et al. (1992) have proposed a method of designing controllers based on optimization method. In the above two methods, the considered systems have unstable components only in one of the inputs.

Govindhakannan and Chidambaram (1997a) have developed a centralized design of controllers for unstable multivariable processes. However, in their method, the interactions are found to be significant. Georgiou et al. (1989) designed controllers based on four steps optimization approach for multivariable unstable processes. Decentralized PI controllers do not stabilize the system if unstable pole is present in all the transfer functions of multivariable system. Only centralized PI controllers stabilize such systems. Govindhakannan and Chidambaram (2000) have applied a two stage P−PI controllers for the unstable systems. Many works published in the literature introduced the concepts of equivalent transfer functions/effective open-loop transfer functions (ETFs/EOTFs) to take into account the loop interactions in the design of multi-loop stable systems. Rajapandiyan and Chidambaram (2012) recently proposed a method of designing controllers for multi-loop stable systems by combining the simplified decoupler approach with the ETF model approximation. Their method provides less interactions and better performances when compared to the ideal and inverted decoupling methods. Very recently, Hazarika and Chidambaram (2014) proposed a method for unstable two-input-two-output (TITO) systems based on ETF model.

6.2 Theoretical developments

6.2.1 ETF model development

The TITO block diagram with decouplers and controllers is shown in Figure 6.1. If the second feedback controller is in the automatic mode, with $y_{r2} = 0$, then the overall closed-loop transfer function between y_1 and u_1 is

$$\frac{y_1}{u_1} = g_{p,11} - \frac{g_{p,12}g_{p,21}(g_{c,2}g_{p,22})}{(1+g_{c,2}g_{p,22})g_{p,22}} \tag{6.1}$$

And similarly for the second loop, the relation can be written as

$$\frac{y_2}{u_2} = g_{p,22} - \frac{g_{p,21}g_{p,12}g_{c,1}g_{p,11}}{(1+g_{c,1}g_{p,11})g_{p,11}} \tag{6.2}$$

Based on these relations, the ETF is derived as given in Hazarika and Chidambaram (2014).For obtaining ETFs, the controller need not be known apriori. Once the ETFs are obtained, the corresponding controller is designed.

6.2.2 Controller design

The open loop transfer function is

$$Y(s) = G_p(s)D(s)U(s) \tag{6.3}$$

$$G_p(s)D(s) = \begin{bmatrix} g_{p,11} & g_{p,12} \\ g_{p,21} & g_{p,22} \end{bmatrix} \begin{bmatrix} 1 & d_{12} \\ d_{21} & 1 \end{bmatrix} = \begin{bmatrix} g^*_{p,11} & 0 \\ 0 & g^*_{p,22} \end{bmatrix} \tag{6.4}$$

Where the simplified decoupler is designed as

$$d_{12}(s) = -\frac{g_{p,12}(s)}{g_{p,11}(s)}; d_{12}(s) = -\frac{g_{p,21}(s)}{g_{p,22}(s)} \tag{6.5}$$

For these systems, if there exist time delay, it may lead to unrealizable situations. Hence, an extra time delay (θ) is to be incorporated into the decoupler matrix which is further added to the corresponding ETF. In the presence of the decoupler, the TITO system behaves like two independent loops for which the controllers can be designed independently. In the present work, diagonal controllers are designed by optimal H_2 – IMC based method based on the corresponding unstable ETFs. ETFs are developed to take into account the loop interactions in the design of multi-loop control systems.

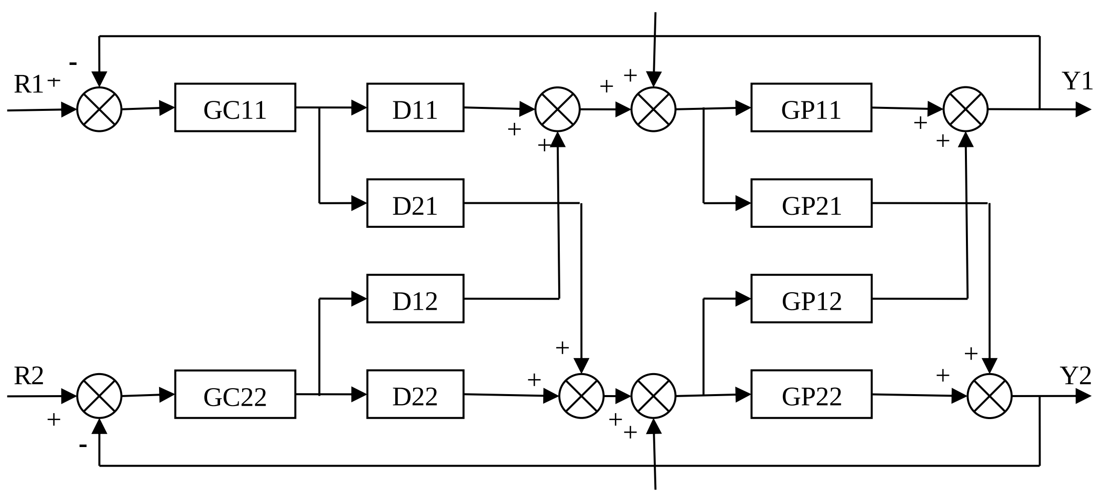

Figure 6.1 Closed loop control for TITO system

Once the ETFs are derived, based on pairing using relative gain array and Neiderlinski Index, the corresponding controllers are designed. The design is based on unstable first order plus time delay system. Anusha and Rao (2012) developed a PID design method based on optimal-H_2 minimization concept for second order unstable processes.

However, in the present work, the controller design is addressed for first order unstable processes. Vanavil et al. (2014) recently proposed design of PID controller in series with lead lag filter and is used here for controlling TITO loops. The method is presented here. Assuming the ETFs are in the form of

$$G_p(s) = \frac{k_p e^{-\theta s}}{\tau_p s - 1} \tag{6.6}$$

According to IMC principle, the IMC controller Q_C is equivalent to

$$Q_C = \tilde{Q}_C F \tag{6.7}$$

Where F is IMC filter which is used for altering the robustness of the controller.

In eq. (6.7), $\tilde{Q}_c$ is designed for a specific type of step input disturbance (v) to obtain H_2 optimal performance and is based on the invertible portion of the process model. The process model and the input are divided as

$$G_m = G_{m-} G_{m+} \quad and \quad v = v_- v_+ \tag{6.8}$$

Where the subscript "– " refers to minimum phase part and " + " refers to non-minimum phase part.

The Blaschke product of RHP poles of G_m and v are defined as

$$b_m = \prod_{i=1}^{k} \frac{-s + p_i}{s + \bar{p}_i} \quad and \quad b_v = \prod_{i=1}^{\tilde{k}} \frac{-s + p_i}{s + \bar{p}_i} \tag{6.9}$$

Where p_i and $\bar{p}_i$ are the ith RHP pole and its conjugate respectively. Based on this, the H_2 optimal controller can be derived by using the following formula (Morari and Zafiriou, 1989).

$$\tilde{Q}_C = b_m (G_{m-} b_v v_-)^{-1} \{(b_m G_{m+})^{-1} b_v v_-\}|_* \tag{6.10}$$

Where {.....}|* is defined as the operator that operates by omitting all terms involving the poles of $(G_{m+})^{-1}$ after taking the partial fraction expansion.

Substituting all expressions, one will get,

$$\tilde{Q}_C = \frac{(\tau_p s - 1)}{k_p} \{(e^{\theta/\tau_p} - 1)\tau_p s + 1\} \tag{6.11}$$

To get the final form of the IMC controller, here, the filter is selected as

$$F = (\alpha s + 1)/(\lambda s + 1)^3 \quad (6.12)$$

Therefore, the IMC controller is obtained as

$$Q_C = \frac{(\tau_P s-1)}{k_P}\{(e^{\theta/\tau_P} - 1)\tau_P s + 1\}\frac{(\alpha s+1)}{(\lambda s+1)^3} \quad (6.13)$$

Here, λ is the closed loop tuning parameter. The value of α is obtained from the conditions of internal stability for IMC structure. The conditions to be followed for internal stability are

Condition 1: Q_C must be stable and should cancel the right half plane poles of G_m

Condition 2: $Q_C G_m$ should be stable

Condition 3: $(1-G_mQ_C)$ at the RHP poles of the process should be zero

The first two conditions are satisfied from the above design procedure and third condition can be applied as

$$(1 - Q_C G_m)|_{s=1/\tau_P} = 0 \quad (6.14)$$

Now, this IMC controller is converted in to a unity feedback control system and the corresponding unity feedback controller G_C is obtained as

$$G_c = Q_c/(1 - Q_c G_m) \quad (6.15)$$

Substituting all the terms, we will get

$$G_C = \frac{\{(e^{\theta/\tau_P}-1)\tau_P s+1\}(\alpha s+1)(\tau_P s-1)}{k_P[(\lambda s+1)^3-\{(e^{\theta/\tau_P}-1)\tau_P s+1\}(\alpha s+1)e^{-\theta s}]} \quad (6.16)$$

This expression is approximated to a PID controller with lead-lag filter as given below with simple approximations.

$$G_c(s) = k_c\left(1 + \frac{1}{\tau_i s} + \tau_d s\right)\frac{(\delta s+1)}{(\gamma s+1)} \tag{6.17}$$

With the controller parameters as

$$k_c = -\frac{4\theta}{k_P[18\lambda+6\theta-6\alpha-6(e^{\theta/\tau_P}-1)\tau_P]} \tag{6.18}$$

$$\tau_i = 2\,\theta/3 \tag{6.19}$$

$$\tau_d = \theta/4 \tag{6.20}$$

$$\delta = \alpha$$

$$\gamma = \frac{[18\lambda^2+12\lambda\theta+\theta^2+2\theta\alpha-6\alpha(e^{\theta/\tau_P}-1)\tau_P+2\theta(e^{\theta/\tau_P}-1)\tau_P]}{[18\lambda+6\theta-6\alpha-6(e^{\theta/\tau_P}-1)\tau_P]} + \tau_P - (e^{\theta/\tau_P}-1)\tau_P \tag{6.21}$$

However, selection of λ is very important for unstable processes and there should be systematic guidelines for selection of λ.The guidelines are based on the maximum sensitivity of the individual closed loops Vanavil et al. (2014).

To reduce the undesirable overshoots, set point weighting is considered with a weighting of 0.3.The usual range of set-point weighting is 0-1. Here, the selection of 0.3 is based on many simulation studies on different types of unstable processes and is not random.

6.3 Simulation results

Two examples are considered to show the effectiveness of the proposed design method. For the purpose of comparison, method proposed by Hazarika and Chidambaram (2014) is considered. For quantitative comparison, Integral value of Absolute Error (IAE) is considered.

Example – 1: Consider an unstable TITO process

$$G_p(s) = \begin{bmatrix} \dfrac{1.6e^{-s}}{-2.6s+1} & \dfrac{0.6e^{-1.5s}}{2.5s+1} \\ \dfrac{0.7e^{-1.5s}}{3s+1} & \dfrac{1.7e^{-s}}{-2.2s+1} \end{bmatrix} \tag{6.22}$$

After pairing, the ETF matrix is obtained as

$$\hat{G}_p = \begin{bmatrix} \dfrac{1.3529e^{-0.9383s}}{-2.4396s+1} & \dfrac{-3.2857e^{-0.9008s}}{1.5013s+1} \\ \dfrac{-3.8333e^{-0.9008s}}{1.8016s+1} & \dfrac{1.4375e^{-0.9383s}}{-2.0643s+1} \end{bmatrix} \tag{6.23}$$

Based on this ETF matrix, the simplified decouplers are obtained as

$$D(s) = \begin{bmatrix} 1 & \dfrac{(7.8s-3)e^{-0.5s}}{20s+8} \\ \dfrac{(15.4s-7)e^{-0.5s}}{51s+17} & 1 \end{bmatrix} \tag{6.24}$$

Now, based on the diagonal elements of the ETF matrix, the corresponding controllers (G_{c11} and G_{c22}) are designed as PID with lead lag controllers.

With these controllers, the two methods are compared by giving unit step change in the set point and load disturbance separately. For Hazarika and Chidambaram (2014) method, stabilization with a P controllers and then outer controller is considered. Figure 6.2 shows the closed loop responses for servo problem. Figure 6.3 shows the closed loop responses for servo problem when there exist +10% perturbations in all time delays of the process. It can be observed that the proposed method performs better for set point changes. Figure 6.4 shows the corresponding control action responses and it can be observed that the proposed method shows comparatively smooth responses. Figure 6.5 shows the responses for load disturbance and Figure 6.6 shows the load disturbance responses for +10% uncertainty in time delays. Again, it can be observed that the proposed method performs better. Figure 6.7 shows the corresponding control action responses and it can be observed that the proposed method shows comparatively smooth responses.

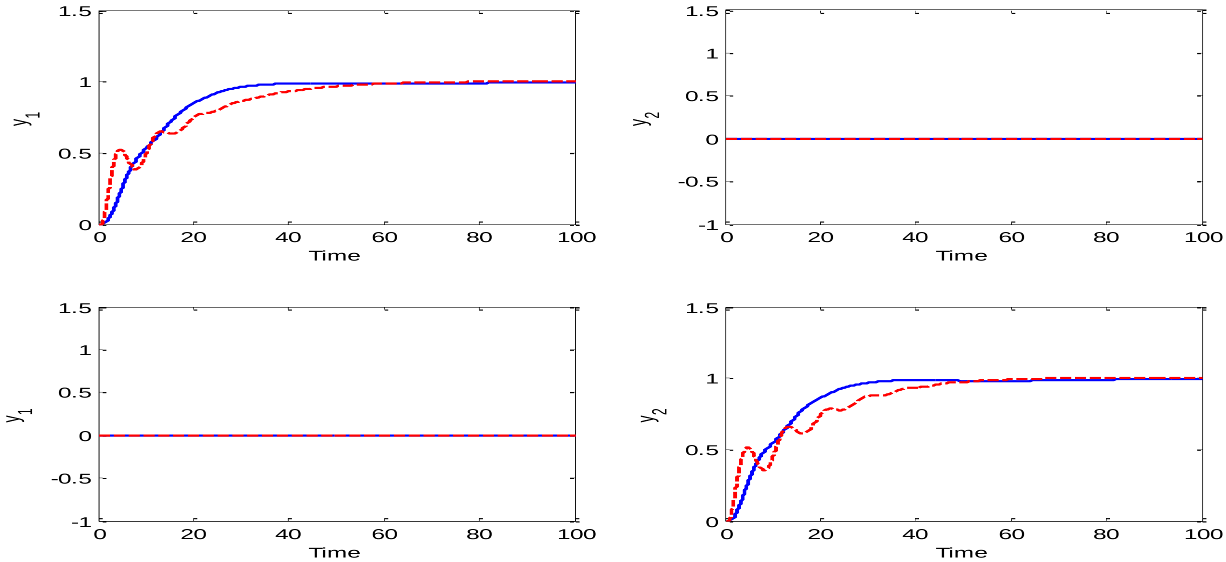

Figure 6.2 Servo response for perfect model for example-1. Top two figures for a set point change in y_{r1} and bottom two figures for a set point change in y_{r2}. Solid – present method, dash - Hazarika and Chidambaram (2014).

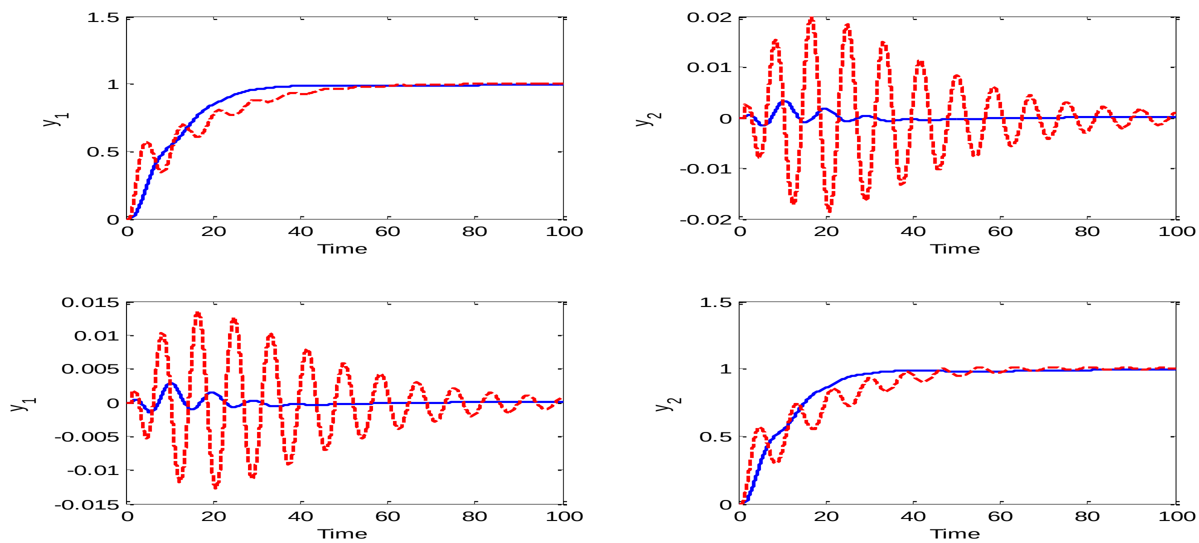

Figure 6.3 Servo response for perturbations of +10% in all process time delays for example-1, legend: as shown in Figure 6.2.

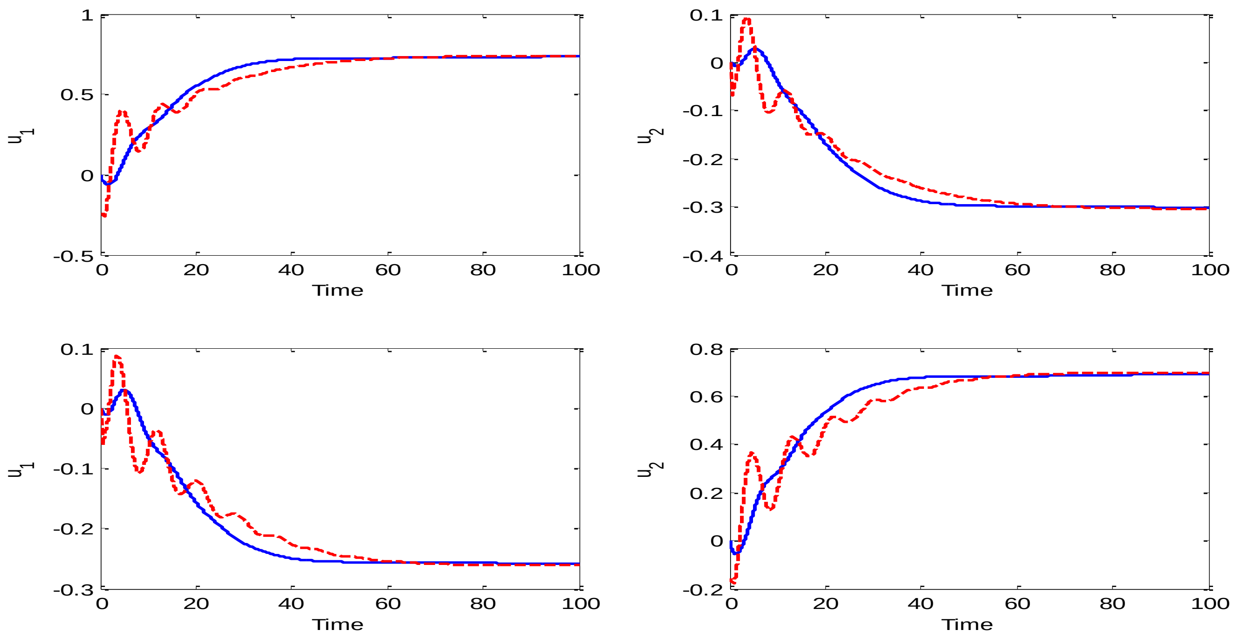

Figure 6.4 Control action responses for perturbations of +10% in all process time delays for example-1, legend: as shown in Figure 6.2.

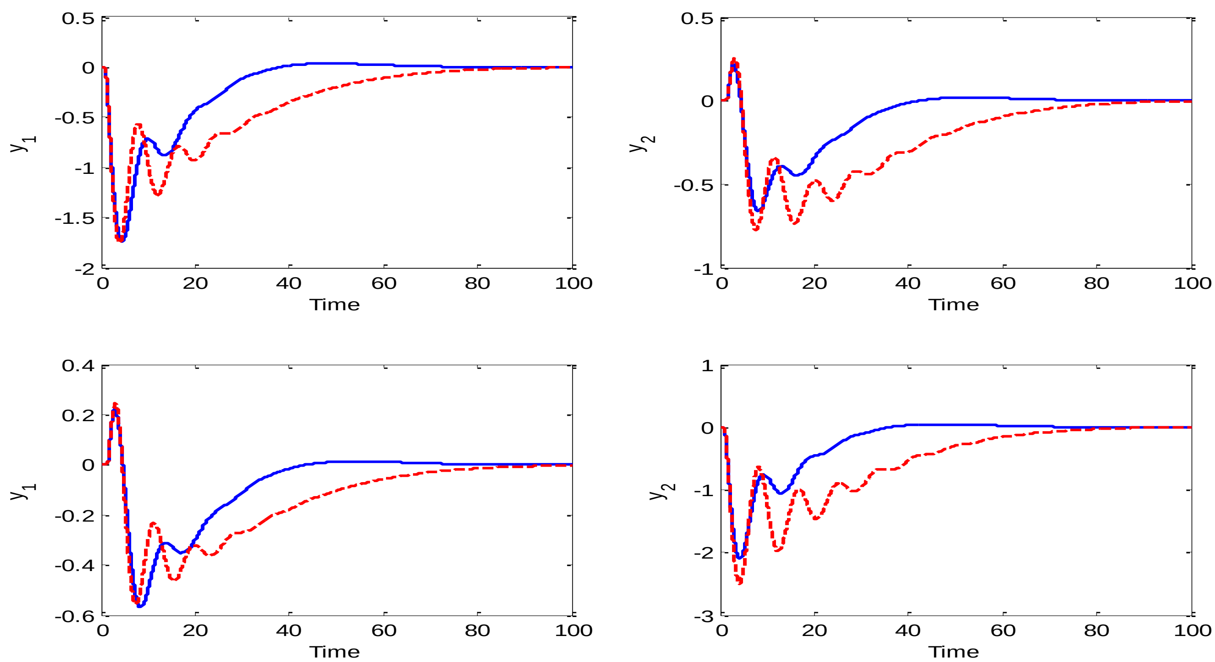

Figure 6.5 Regulatory response for perfect model for example-1, legend: as shown in Figure 6.2.

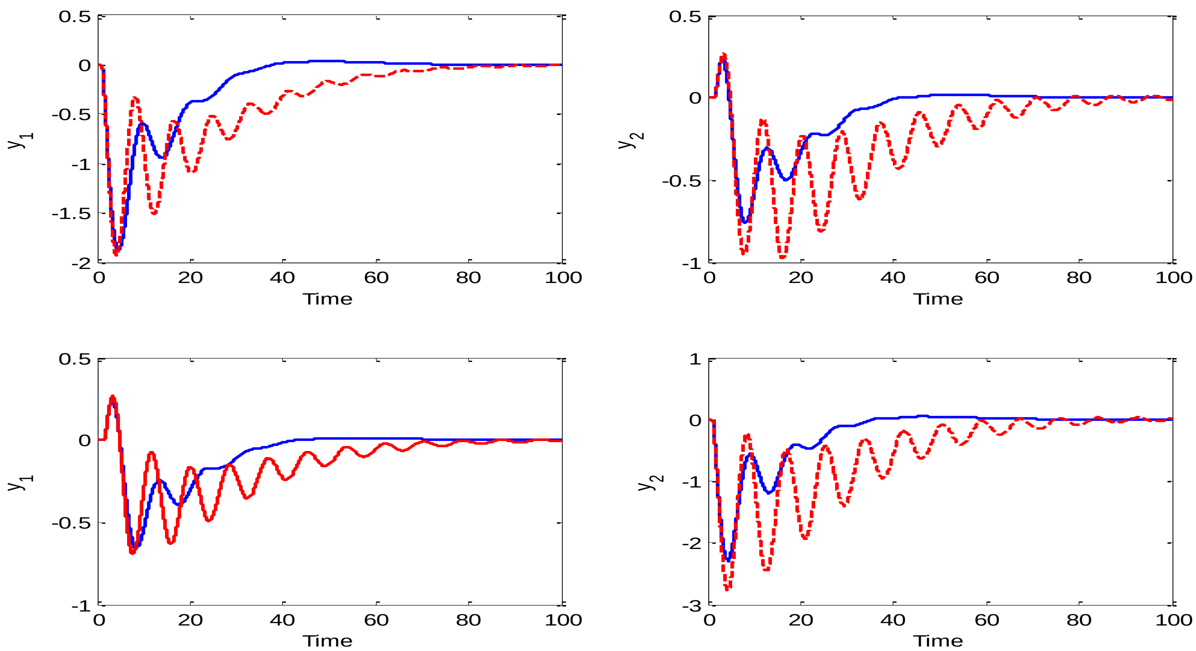

Figure 6.6 Regulatory responses for perturbations of +10% in all process time delays for example-1, legend: as shown in Figure 6.2.

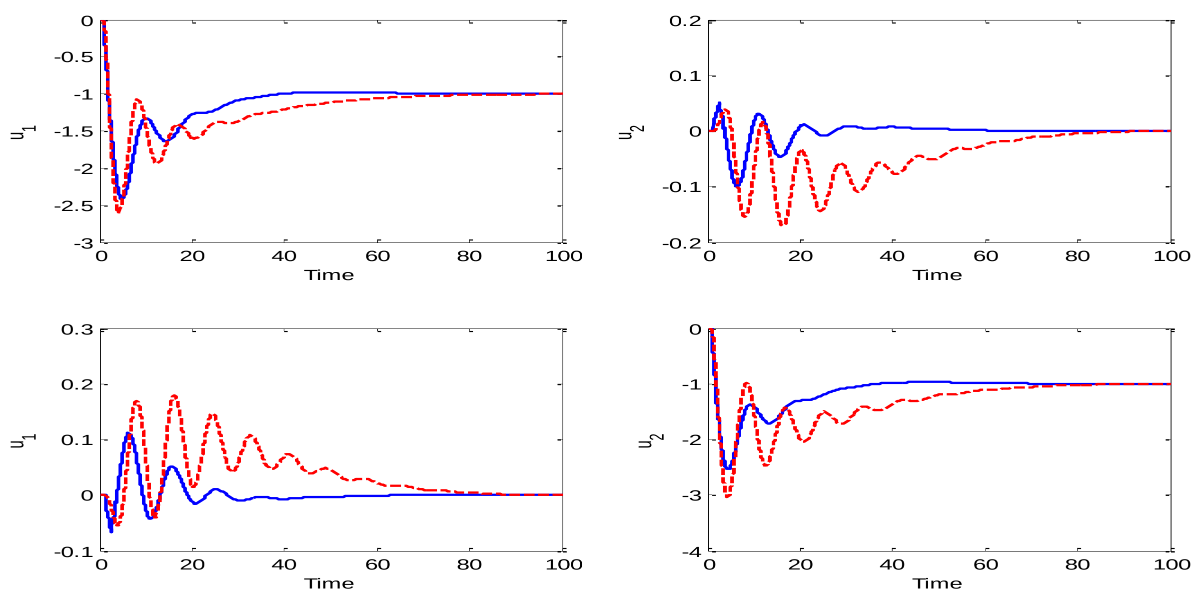

Figure 7.7

Example – 2: Consider another unstable TITO process

$$G_P(S) = \begin{bmatrix} \dfrac{-1.6667e^{-s}}{-2.6s+1} & \dfrac{-1e^{-s}}{-1.6667s+1} \\ \dfrac{-0.8333e^{-s}}{-1.6667s+1} & \dfrac{-1.6667e^{-s}}{-1.6667s+1} \end{bmatrix} \tag{6.25}$$

After pairing, the ETF matrix is obtained as

$$\hat{G}_p = \begin{bmatrix} \dfrac{2.3348e^{-s}}{-1.6667s+1} & \dfrac{-1.16711e^{-s}}{-2.6s+1} \\ \dfrac{-1.16711e^{-s}}{-1.6667s+1} & \dfrac{1.9456e^{-s}}{-1.6667s+1} \end{bmatrix} \tag{6.26}$$

Based on this ETF matrix, the simplified decouplers are obtained as

$$\text{D(s)} = \begin{bmatrix} 1 & -1.6667 \\ -2.001 & 1 \end{bmatrix} \tag{6.27}$$

Now, based on the diagonal elements of the ET1F matrix, the corresponding controllers (Gc_{11} and Gc_{22}) are designed as PID with lead lag controllers. With these controllers, the two methods are compared by giving unit step change in the set point and load disturbance separately. Figure 6.8 shows the closed loop responses and Figure 6.9 shows the corresponding control action response for servo problem. Figure 6.10 shows the closed loop responses for servo problem for +10% perturbations in all time delays of the process. It can be observed that the proposed method performs better for set point changes.

Figure 6.11 shows the responses for load disturbance and Figure 6.12 shows the corresponding control action response. Figure 6.13 shows the load disturbance responses for +10% uncertainty in time delays. Again, it can be observed that the proposed method performs better. The corresponding IAE values for example-1 is shown in Table 6.1 and for example-2 is shown in Table 6.2. It can be observed that the present method shows low IAE values for perfect model and for perturbations. Also shown in the table, the IAE values for different perturbations in the process parameters. Note that the present method provides low IAE values for all cases and hence the present method is better.

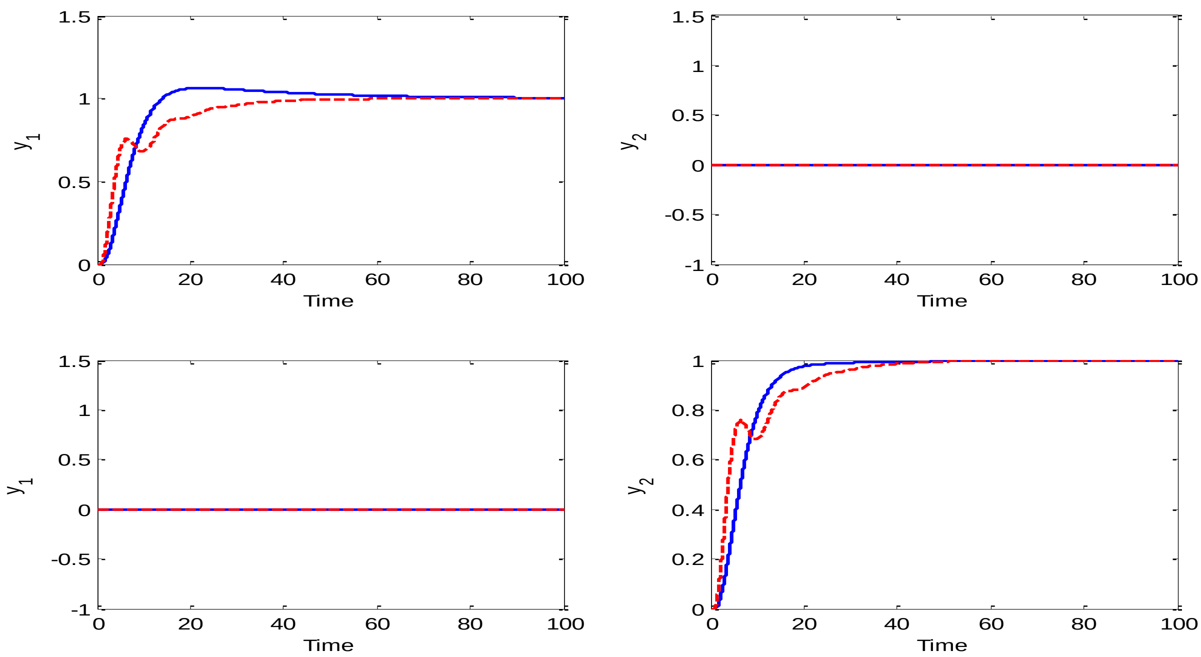

Figure 6.8 Servo response for perfect model for example-2. Top two figures for a set point change in y_{r1} and bottom two figures for a set point change in yr_2. Legend: as shown in Figure 6.2.

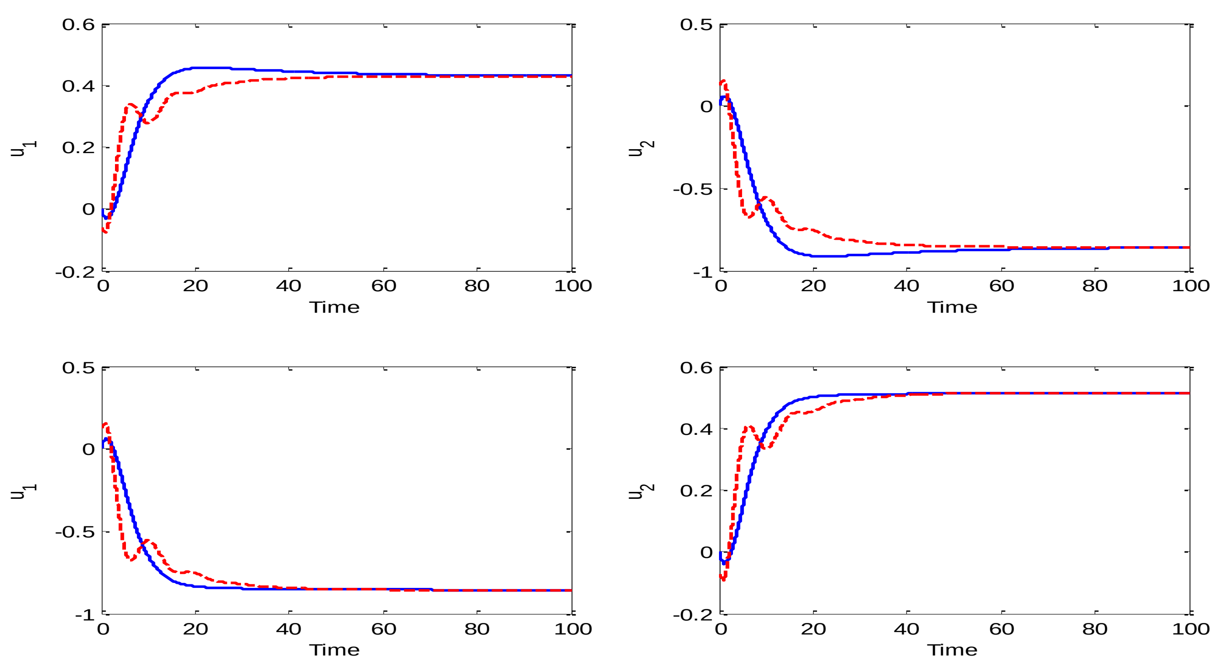

Figure 6.9 Control action responses corresponding to Figure 6.8.

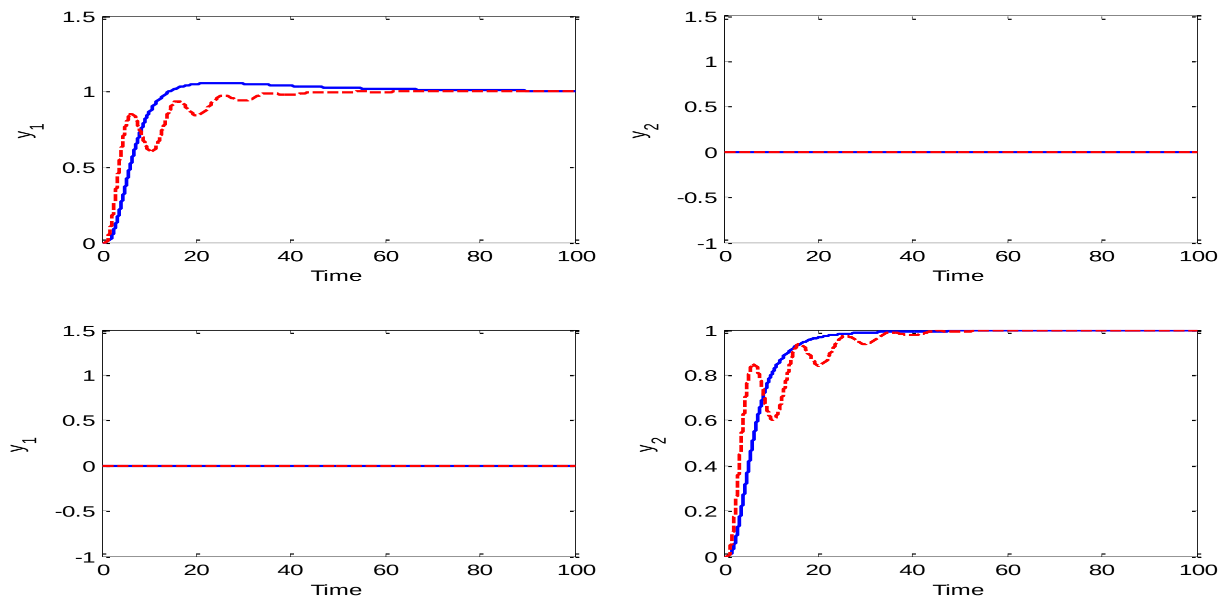

Figure 6.10 Servo response for perturbations of +10% in time delays for example-2, legend: as shown in Figure 6.6.

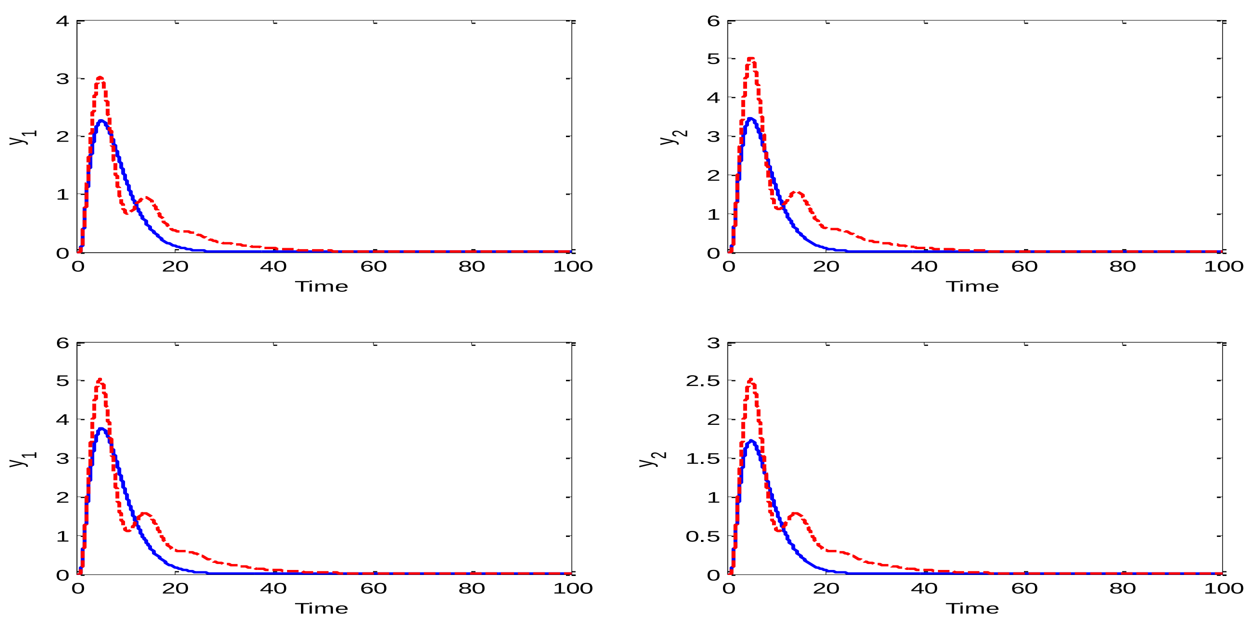

Figure 6.11 Regulatory response for perfect model for example-2, legend: as shown in Figure 6.6.

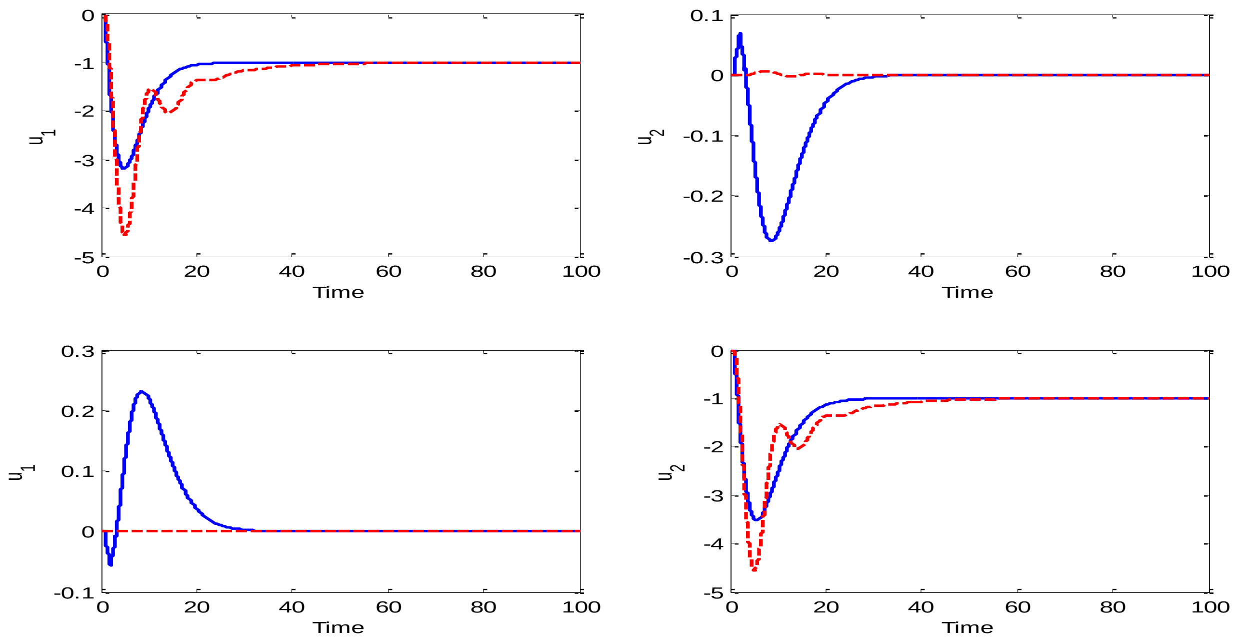

Figure 6.12 Control action responses corresponding to Figure 6.11.

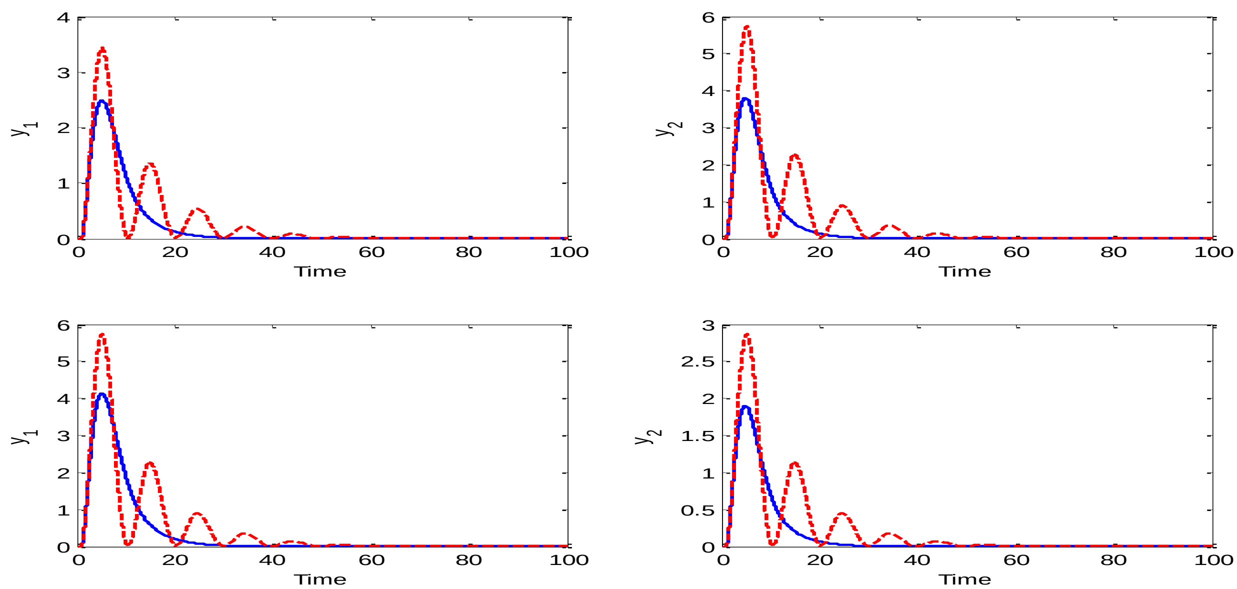

Figure 6.13 Regulatory responses for perturbations of +10% in time delays for example-2, legend: as shown in Figure 6.2.

Table 6.1 IAE value for example-1.

	Servo		Regulatory	
Perturbations	**Proposed**	**H-C method**	**Proposed**	**H-C method**
K_p	12.42	13.97	21.54	36.54
$1.1k_p$	13.54	16.03	20.86	36.35
$0.9k_p$	11.11	11.87	22.78	36.79
θ	12.42	13.97	21.54	36.54
1.1θ	12.42	11.87	21.46	36.53
0.9θ	12.42	13.98	21.62	36.55
τ	12.42	13.97	21.54	36.54
1.1τ	12.42	14.01	21.72	36.56
0.9τ	12.42	13.96	21.33	36.52

Table 6.2 IAE value for example-2.

	Servo		Regulatory	
Perturbations	**Proposed**	**H-C method**	**Proposed**	**H-C method**
K_p	8.8	7.91	20.58	27.81
$1.1k_p$	9.22	10.44	20.56	27.79
$0.9k_p$	9.87	4.82	27.44	27.81
θ	8.8	7.91	20.58	27.81
1.1θ	8.47	7.91	20.58	27.81
0.9θ	9.12	7.91	20.72	27.81
τ	8.8	7.91	20.58	27.81
1.1τ	9.31	7.91	20.86	27.81
0.9τ	8.35	7.91	20.58	27.81

6.4 Summary

Multivariable PID controller in series with lead lag filter is applied based on the equivalent transfer function (ETF) model for unstable multivariable systems with time delay. The method uses simplified decouplers which decompose the unstable multi-loop systems into independent loops with ETFs as the resulting decoupled process model having unstable poles. To reduce the undesirable overshoot, set point weighting is used. Two simulation examples are studied and showed that the present method provides significantly improved closed loop performances for servo responses compared to other methods in the literature.

Chapter 7

Set-Point Weighting Design for Unstable Systems

Chapter 7

Set-Point Weighting Design for Unstable Systems

Control of unstable processes with time delays usually result in large overshoots in the closed loop responses. In industry, set-point weighting is one of the recommended methods to minimize the overshoot. In this chapter, a method is proposed to design the set-point weighting parameters which is relatively simple and also reduces the overshoot significantly. Weighting is considered for both proportional (β) and derivative (γ) terms in the PID control law. In the closed loop relation for the set-point tracking, the coefficients of 's' and 's^3' both in the numerator and denominator are made equal in order to find β and γ. The obtained expressions for β and γ are simple and depends on the controller parameters. The design is carried out first for single input single output (SISO) unstable first order and second order processes with time delays and then for multi input multi output (MIMO) unstable processes. In MIMO process control, decouplers are considered to ensure that the loops have minimum interactions.

7.1 Introduction

The PID controller is universally adopted in chemical processes. This is primarily due to its basic structure. Nonetheless, time delay is unavoidable in majority of the chemical processes due to the presence of recycle loops and transportation delays. It is intractable for conventional PID controllers to ascertain the stability for unstable time delay processes. Additionally, it becomes more problematic to design the PID controller for a process that demonstrates a time delay which is open-loop unstable. Unstable systems result in larger overshoots in the closed loop set-point tracking responses. In order to minimize the overshoot, either a set-point filter or set-point weighting is recommended. In this work, set-point weighting is considered for minimization of the overshoot.

In order to properly address the design of set-point weighting parameters, knowledge about the PID controller parameters (k_c, τ_i, τ_d) is necessary. There exit many methods in the literature for design of PID controllers for unstable first order plus time delay (UFOPTD) and unstable second order plus time delay (USOPTD) processes of which internal model control (IMC) and direct synthesis methods are recommended (Sree and Chidambaram, 2017). In the interest to acquire an improved transient response due to set point change, several researchers proposed either set-point filtering or set-point weighting methods. Prashanti and Chidambaram (2000) developed formulae to calculate the set point weighting parameters for UFOPTD systems for different ratio of time delay to dominant time constant. Sree and Chidambaram (2004) developed a method for unstable second order systems with a positive or negative zero. Later they developed a method by minimizing the integral square error (ISE) for a servo problem (Sree and Chidambaram, 2005). Rao et al. (2007) proposed set-point weighting based modified Smith predictor method for integrating processes. Rao et al. (2009) used set-point weighting to a PID controller integrated with a lead-lag compensator for different types of integrating systems.

Chen et al. (2008) recommended the tuning rules for set-point weighting based on three-element control structure. Vijayan and Panda (2012) designed a set point filter for curtailing the overshoot for low order processes. Based on learning automata, a method is projected to select the set-point weighted parameter for unstable systems by Musmade and Chidambaram (2014). Begum et al. (2016, 2017), Dasari et al. (2017), Rao et al. (2016) and Wang et al. (2016) proposed controller tuning rules for stable, unstable and integrating processes with time delay based on internal model control (IMC) technique. Bingi et al. (2017) proposed a new fractional order filter for the implementation of PID based control strategy for unstable systems.

In all the formerly cited works, the derivative action is set free from set-point weighting by considering the weighting parameter as one. Nasution et al. (2011) designed controller for time delayed unstable processes with set point weighting making use of an optimal H_2 IMC-PID control strategy. It is possible to demonstrate that tracking performance of the set-point will be improved if it is possible to determine the suitable derivative mode weighting. This is determined by Nasution et al. (2011), but in a complex way.

Design of the controller for MIMO unstable systems with interactions between the loops turned to be an interesting research area in the recent years. Decouplers can be used to overrule the process interactions to strengthen the system performance. However, these decouplers are susceptible to process changes and demand extremely rigorous process models, which are tough to search out. Georgiou et al. (1989) worked on unstable multivariable systems with a decentralized PID controller based on optimization method. Govindhakannan, J. and

Chidambaram (1997) designed single stage PI controllers for unstable multivariable systems using Tanttu and Lieslehto (1991) method. Besta, C. S., and Chidambaram (2016) used synthesis method to come up with an effective solution for the design of decentralized PID controller for unstable systems. Dasari et al. (2016), Hazarika and Chidambaram (2014), Rajapandiyan and Chidambaram (2012) and Raviteja et al. (2016) used ETF (equivalent transfer function) procedure for the controller design for time delay unstable systems. Recently, Dasari and Chidambaram (2018) suggested a simple method for calculating dynamic set-point weighting parameters for time delayed unstable processes. In the present chapter, this method is extended for higher order SISO and MIMO unstable systems.

7.2 Set point weighted PID algorithm

Let us consider the well-known ideal form of the PID controller with set-point weighting on both proportional and derivative terms as

$$u(t) = k_c\left[e_p(t) + (1/\tau_I)\int e(t)dt + \tau_D\frac{de_d(t)}{dt}\right] \tag{7.1}$$

Where

$$e_p(t) = \beta y_r - y(t) \qquad e(t) = y_r - y(t) \quad e_d(t) = \gamma y_r - y(t) \tag{7.2}$$

The parameters γ and β are established in order to account for the errors in the derivative and proportional terms, respectively. In order to fully comprehend the function of the set-point weighted PID controller, eq. 7.1 can be re-written as

$$u(t) = k_c\left[\beta e(t) - (1-\beta)\mathrm{y(t)} + (1/\tau_I)\int e(t)dt + \gamma\tau_D\frac{de_d(t)}{dt} - (1-\gamma)\tau_D\frac{\mathrm{dy}(t)}{dt}\right]$$

$$u(t) = \left[k_c\beta e(t) + (k_c/)\int e(t)dt + k_c\gamma\tau_D\frac{de(t)}{dt}\right] - \left[k_c(1-\beta)y(t) + (k_c(1-\gamma)\tau_D)\frac{\mathrm{dy}(t)}{dt}\right]$$

$$u(t) = \left[P_1 e(t) + P_2\int e(t)dt + P_3\frac{de(t)}{dt}\right] - \left[P_4 y(t) + P_5\frac{\mathrm{dy}(t)}{dt}\right] \tag{7.3}$$

Where

$P_1 = K_c\beta$

$P_2 = K_c/\tau_I$

$P_3 = K_c\,\gamma\,\tau_D$

$P_4 = K_c\,(1-\beta)$

$P_5 = K_c\,(1-\gamma)\,\tau_D$

Eq. 7.3 can be represented as shown in Figure 7.1 for any process $G_p(s)$. Set point weighted PID algorithm can be represented as shown in Figure 7.1 which is equivalent to a PID feedback with an inner loop consisting of a PD controller. There is no set-point weighting done for integral action. This is due to the integral error should be true error. It is to prevent any steady-state control error.

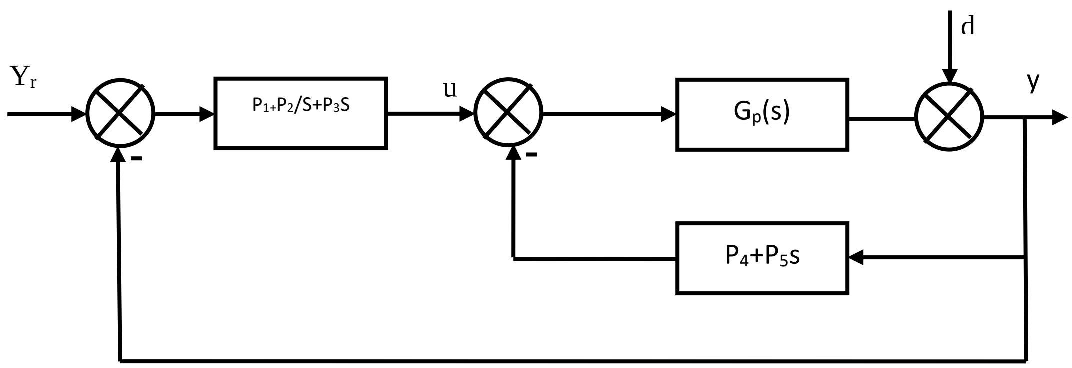

Figure 7.1 Equivalent representation of a set-point weighted PID controller.

7.3 Design of set point weighting parameters for UFOPTD processes

In eq. 7.1, for UFOPTD processes, let us consider

$$e_p = \beta_1 y_r - y \qquad e = y_r - y \quad e_d = \gamma_1 y_r - y \tag{7.4}$$

Here, β_1 and γ_1 are set point weighting parameters.

Representing the UFOPTD process as

$$G_p(s) = \frac{k_p e^{-\theta s}}{(\tau_p s - 1)} \tag{7.5}$$

With the PID controller, the transfer function between y and y_r is

$$\frac{y}{y_r} = \frac{[k_c k_p(\beta_1 s + m + \gamma_1 s^2) e^{-\theta s}]}{[s(\tau_p s - 1) + k_c k_p (s + m + s^2) e^{-\theta s}]} \tag{7.6}$$

Where $m = 1/\tau_i$. The present method requires the numerator and denominator terms in the form of a polynomial in ‘s’. To achieve this, a first order Pade’s Approximation for $e^{-\theta s}$ in the denominator is considered and with that, the above equation becomes

$$\frac{y}{y_r} = \frac{[k_c k_p(\beta_1 s + m + \gamma_1 s^2)(1 + 0.5\theta s) e^{-\theta s}]}{[s(\tau_p s - 1)(1 + 0.5\theta s) + k_c k_p (s + m + s^2)(1 - 0.5\theta s)]} \tag{7.7}$$

Eq. (7.7) can be rewritten as

$$\frac{y}{y_r} = \frac{[k_c k_p (m + c_1 s + c_2 s^2 + c_3 s^3)\exp(-\theta s)]}{[-s + c_4 s^2 + c_5 s^3 + k_c k_p (m + s c_6 + s^2 c_7 + s^3 c_8)]} \tag{7.8}$$

Where

$c_1 = \beta_1 + 0.5m\theta$

$c_2 = 0.5\beta_1\theta + \gamma_1\tau_D$

$c_3 = 0.5\gamma_1\tau_D$

$c_4 = \tau_p - 0.5\theta$

$c_5 = -0.5\theta\tau_p$

$c_6 = 1 - 0.5\theta m$

$c_7 = \tau_D - 0.5\theta$

$c_8 = -0.5\tau_D\theta$

In the present work, based on the coefficients of 's' and 's^3' in the numerator and denominator polynomial, the set point weighting parameters are determined. Let L_1 represent the ratio of coefficients of 's' in the numerator to that in the denominator without any set point weighting i.e. β_1=1

$$L_1 = \frac{[k_c k_p (1 + 0.5m\theta]}{[-1 + k_c k_p (1 - 0.5m\theta)]} \tag{7.9}$$

If $L_1 > 1$, then the corresponding coefficient of s in the numerator is equated to that in the denominator i.e.

$$k_c k_p (\beta_1 + 0.5m\theta) = -1 + k_c k_p (1 - 0.5m\theta) \tag{7.10}$$

From Eq. (7.10), we obtain

$$\beta_1 = 1 - \frac{1}{k_c k_p} - \frac{\theta}{\tau_I} \tag{7.11a}$$

If $L_1 \le 1$, then the corresponding coefficient of s in the numerator is equated to L_1 times that in the denominator i.e.

$$k_c k_p (\beta_1 + 0.5m\theta) = L_1(-1 + k_c k_p (1 - 0.5m\theta)) \tag{7.11b}$$

Based on the simulation performed on various transfer models, it is noticed that a reduced overshoot is obtained for $\beta_1 = 0.7L_1$.

The ratio of corresponding coefficients of s^3 with unit set-point weighting ($\gamma_1 = 1$) is represented as L_2.

$$L_2 = \frac{k_c k_p [0.5\gamma_1\tau_D\theta]}{[0.5\theta\tau_p + k_c k_p (-0.5\tau_D\theta)]} \tag{7.12}$$

The corresponding coefficients can be equated as shown below for $L_2 > 1$.

$$k_c k_p[0.5\gamma_1 \tau_D \theta] = \left[0.5\theta\tau_p + k_c k_p(-0.5\tau_D \theta)\right] \quad (7.13)$$

From Eq. (7.13), we get

$$\gamma_1 = -1 + \frac{\tau_p}{\tau_D k_c k_p} \quad (7.14a)$$

The numerator is equated to L_2 times the denominator for $L_2 \leq 1$

$$k_c k_p[0.5\gamma_1 \tau_D \theta] = L_2\left[0.5\theta\tau_p + k_c k_p(-0.5\tau_D \theta)\right] \quad (7.14b)$$

Based on the simulation performed on various transfer models, it is noticed that a reduced overshoot is obtained for $\gamma = 0.3L_2$.

7.4 Design of set point weighting parameters for USOPTD processes

In eq. 7.1, for USOPTD processes, let us consider

$$e_p = \beta_2 y_r - y \qquad e = y_r - y \quad e_d = \gamma_2 y_r - y \quad (7.15)$$

Here β_2 and γ_2 are set point weighting parameters.

Representing the USOPTD process as

$$G_p(s) = \frac{k_p e^{-\theta s}}{(\tau_1 s - 1)(\tau_2 s - 1)} \quad (7.16)$$

For simplification the above equation

$$G_p(s) = \frac{k_p e^{-\theta s}}{(a_1 s^2 + a_2 s + a_3)} \quad (7.17)$$

Here $a_1 = \tau_1 \tau_2$, $a_2 = -(\tau_1 + \tau_2)$, $a_3 = 1$

With a PID controller in the closed loop, the transfer function relating y to y_r is obtained as

$$\frac{y}{y_r} = \frac{\left[k_c k_p(\beta_2 s + m + \gamma_2 s^2)e^{-\theta s}\right]}{\left[s(a_1 s^2 + a_2 s + a_3) + k_c k_p(s + m + s^2)e^{-\theta s}\right]} \quad (7.18)$$

Considering first order Pade's approximation for $e^{-\theta s}$ in the denominator, Eq. (7.18) becomes

$$\frac{y}{y_r} = \frac{\left[k_c k_p(\beta_2 s + m + \gamma_2 s^2)(1 + 0.5\theta s)e^{-\theta s}\right]}{\left[s(a_1 s^2 + a_2 s + a_3)(1 + 0.5Ls) + k_c k_p(s + m + s^2)(1 - 0.5\theta s)\right]} \quad (7.19)$$

Eq. (7.19) can be rewritten as

$$\frac{y}{y_r} = \frac{\left[k_c k_p(m + v_1 s + v_2 s^2 + v_3 s^3)\exp(-\theta s)\right]}{\left[a_3 s + v_4 s^2 + v_5 s^3 + v_6 s^4 + k_c k_p(m + s v_7 + s^2 v_8 + s^3 v_9)\right]} \quad (7.20)$$

Where

$$v_1 = \beta_2 + 0.5m\theta$$

$v_2 = 0.5\beta_2\theta + \gamma_2\tau_D$

$v_3 = 0.5\gamma_2\tau_D\theta$

$v_4 = a_2 + 0.5a_3\theta$

$v_5 = a_1 + 0.5a_2\theta$

$v_6 = 0.5a_1\theta$

$v_7 = 1 - 0.5m\theta$

$v_8 = \tau_D - 0.5\theta$

$v_9 = -0.5\tau_D\theta$

In the present work, based on the coefficients of 's' and 's^3' in the numerator and denominator polynomial, the set point weighting parameters are determined. Let L_3 represent the ratio of coefficients of '*s*' in the numerator to that in the denominator without any set point weighting i.e. β_2=1

$$L_3 = \frac{[k_c k_p(1+0.5m\theta]}{[a_3+k_c k_p(1-0.5m\theta)]} \tag{7.21}$$

If $L_3 > 1$, then the corresponding coefficient of s in the numerator is equated to that in the denominator i.e.

$$k_c k_p(\beta_2 + 0.5m\theta) = a_3 + k_c k_p(1 - 0.5m\theta) \tag{7.22}$$

From Eq. (7.22), we obtain

$$\beta_2 = 1 + \frac{a_3}{k_c k_p} - m\theta \tag{7.23a}$$

If $L_3 \leq 1$, then the corresponding coefficient of s in the numerator is equated to L_3 times that in the denominator i.e.

$$k_c k_p(\beta_1 + 0.5m\theta) = L_3(-1 + k_c k_p(1 - 0.5m\theta)) \tag{7.23b}$$

Based on the simulation performed on various transfer models, it is noticed that a reduced overshoot is obtained for$\beta_2 = 0.7L_3$.

The ratio of corresponding coefficients of s^3 with unit set-point weighting ($\gamma_2 = 1$) is represented as L_4.

$$L_4 = \frac{k_c k_p[0.5\gamma_2\tau_D\theta]}{[a_1+0.5\theta a_2+k_c k_p(-0.5\tau_D\theta)]} \tag{7.24}$$

The corresponding coefficients can be equated as shown below for$L_4 > 1$.

$$.k_c k_p[0.5\gamma_2\tau_D\theta] = \left[a_1 + 0.5\theta a_2 + k_c k_p(-0.5\tau_D\theta)\right] \tag{7.25}$$

From Eq. (7.25), we get

$$\gamma_2 = \frac{a_1}{0.5\theta\tau_D k_c k_p} + \frac{a_2}{\tau_D k_c k_p} - 1 \quad (7.26a)$$

The numerator is equated to L_4 times the denominator for $L_4 \leq 1$

$$k_c k_p[0.5\gamma_2\tau_D\theta] = L_4\left[a_1 + 0.5\theta a_2 + k_c k_p(-0.5\tau_D\theta)\right] \quad (7.26b)$$

Based on the simulation performed on various transfer models, it is noticed that a reduced overshoot is obtained for $\gamma_2 = 0.3L_4$.

7.5 Controller saturation

It is more practical to manage the control saturations for unstable processes in order to protect from reset windup. A set-point weighted PID loop along with the reset-feedback form is depicted in Figure 2 (Nasution et al., 2011).

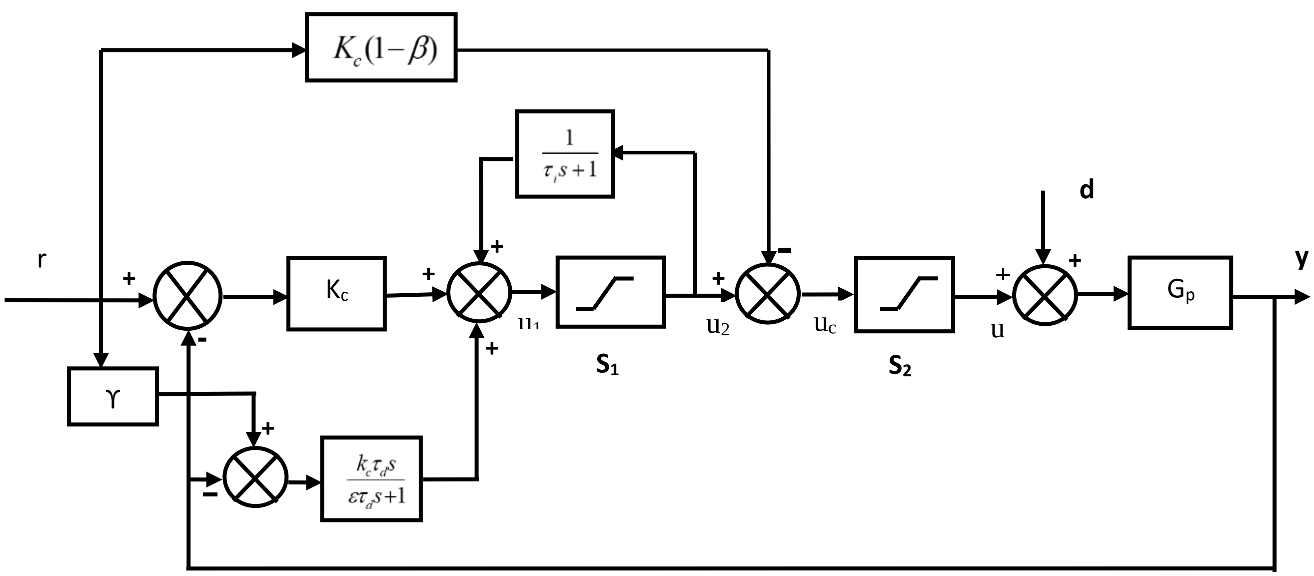

Figure 7.2 Reset feedback scheme of set point weighted PID control system

The two saturation functions, S_1 and S_2 in figure 7.2 are defined as

$$u_2(t) = \begin{cases} u_{\max} + k_c(1-\beta)r & for \quad u_1(t) > u_{\max} + k_c(1-\beta)r \\ u_1(t) & for \quad u_{\min} - k_c(1-\beta)r \leq u_1(t) \leq u_{\max} + k_c(1-\beta)r \\ u_{\min} - k_c(1-\beta)r & for \quad u_1(t) < u_{\min} - k_c(1-\beta)r \end{cases} \quad (7.27)$$

$$u_2(t) = \begin{cases} u_{\max} & for \quad u_c(t) > u_{\max} \\ u_c(t) & for \quad u_{\min} \le u_c(t) \le u_{\max} \\ u_{\min} & for \quad u_c(t) < u_{\min} \end{cases} \tag{7.28}$$

Where u_{max} and u_{min} correspond to the maximum and minimum physical saturation limitations of the control signal. The signals u_1 and u_2 are given as

$$u_1(s) = k_c[r(s) - y(s) + (1/\tau_I s + 1)u_2(s) + k_c\tau_D s/(\varepsilon\tau_D s + 1)(\gamma r(s) - y(s))] \tag{7.29}$$

$$u_c(t) = u_2(t) - k_c(1 - \beta)r \tag{7.30}$$

Here, ε is 0.1 which accounts for the derivative filter parameter. The saturation function S_1 is useful to take care of the anti-reset windup by considering the set-point weighting on proportional mode. Additionally, control limitations are taken care by the saturation function S_2. By all means, the control performance would diminish once the constraints of control saturations are activated. The saturation limits are set as u_{max} = +2.5 and u_{min} = -2.5. A point worth noting is that regardless of the constraints being met by the controller output, the proposed method resulted in a satisfactory performance.

7.6 Design of set point weighting parameters for MIMO processes

Several MIMO systems can be altered using a more feasible choice of using a decentralized control strategy as recommended by Hazarika and Chidambaram (2014). Figure 7.3 illustrates the process under study with U and Y as input and output respectively. The set point weighting technique is induced into the decoupler control loops and studied. The proposed set-point weighting design (Figure 7.2) is incorporated independently for the two control loops in Figure 7.3 and simulation studies are carried out for different examples.

Note that Dasari et al. (2016) method is the technique proposed in Chapter 6. This varies from the method proposed in Chapter 7 in the view of fixed set-point weighting of 0.3 and in Chapter 7, the set-point weighting varies according to the design technique.

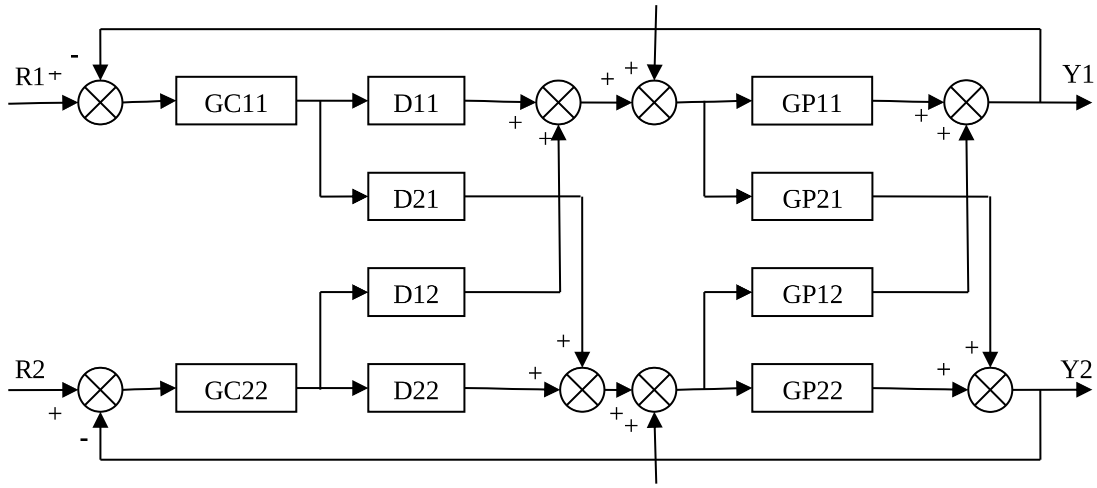

Figure 7.3 Decoupled control scheme

7.7 Simulation results

PID tuning method proposed by Nasution et al. (2011), Wang et al. (2016) and Dasari et al. (2016) are used for calculating the PID settings for SISO and TITO processes. Simulation studies were carried out on various unstable processes and the closed loop set-point performances are compared with the method proposed by Nasution et al. (2011) and Wang et al. (2016) for SISO processes and Hazarika and Chidambaram (2014), Dasari et al. (2016) for TITO processes.The controller performance is assessed based on integral of absolute error (IAE) and total variation of the manipulated variable at a value of maximum sensitivity. The set-point weighting parameters (β_1 and γ_1) for UFOPTD process are determined from eq. 7.11 & 7.14. Similarly, the set-point weighting parameters (β_2 and γ_2) for USOPTD processes are calculated from eq. 7.23 & 7.26. All the simulations for SISO and TITO processes are carried out based on Figure 7.2 and Figure 7.3.

Example – 1: A UFOPTD process is considered as shown below

$$G_p(s) = \frac{e^{-1.2s}}{s-1} \tag{7.31}$$

For this process, different λ are selected and the corresponding M_s values are obtained based on Nasution et al. (2011). Simulations are performed by giving a step change of unity to the set point. Figure 7.4 represents the closed loop and control responses. The designed controller settings, set point weighting parameters and the controller performance for the conditions specified are illustrated in Table 7.1. It can be observed that the M_s values are higher as the time delay to time constant ratio is greater than 1. M_s value of 9 yields good responses as compared to other M_s values.

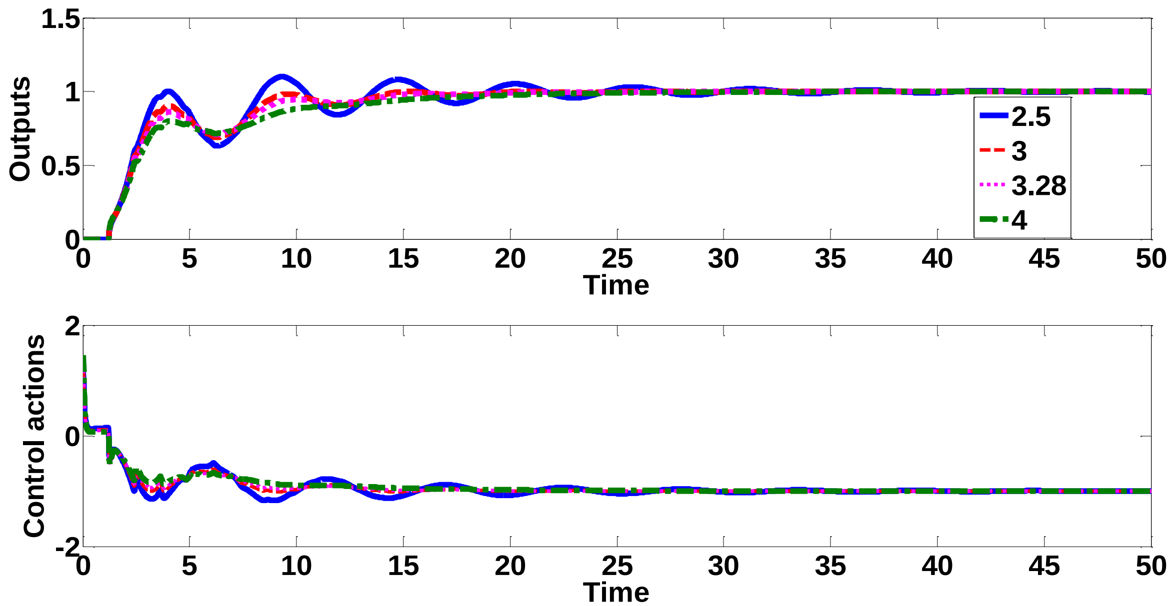

Figure 7.4 Output and control action behavior under exact model conditions for example 1 for different values of λ.

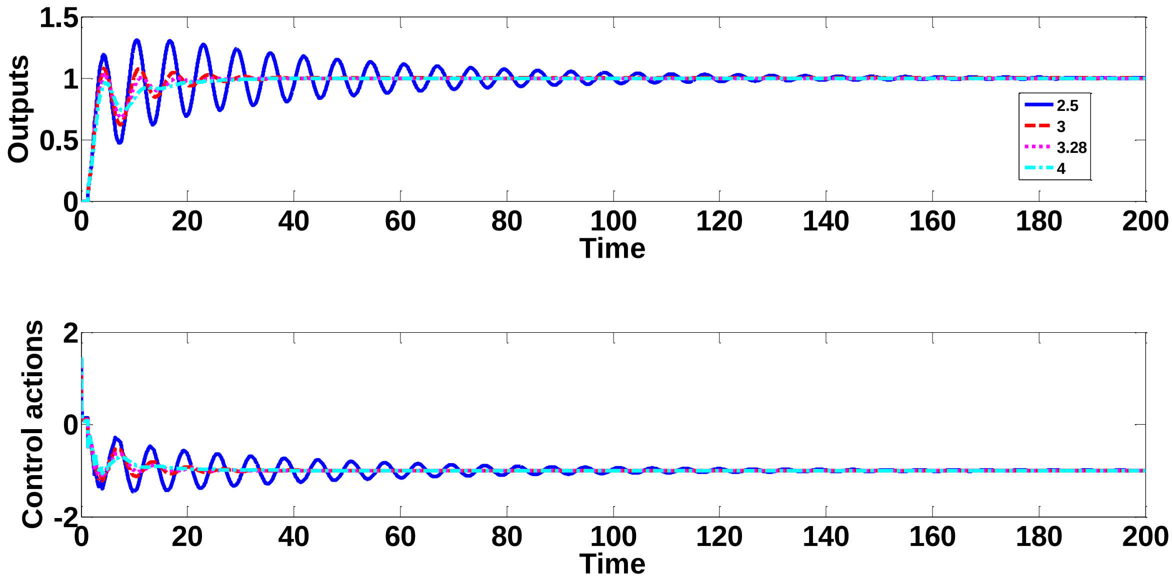

Figure 7.5 Output and control action behavior under uncertainties model conditions for example 1 for different values of λ.

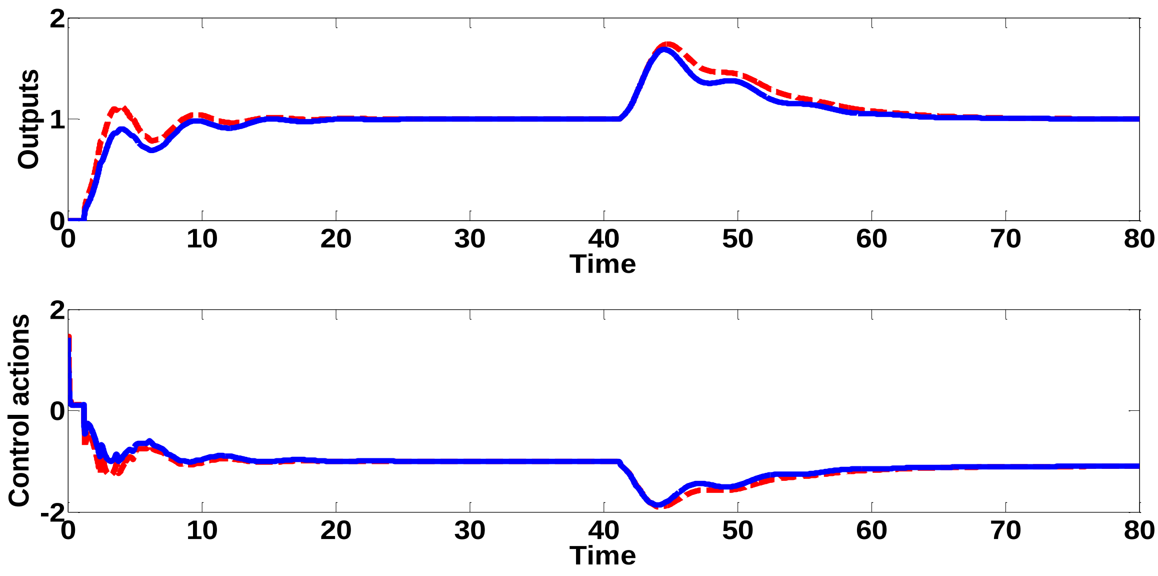

Figure 7.6 Output and control action behaviour under exact model conditions for example 1, solid – Proposed work, dash – (Nasution et al., 2011).

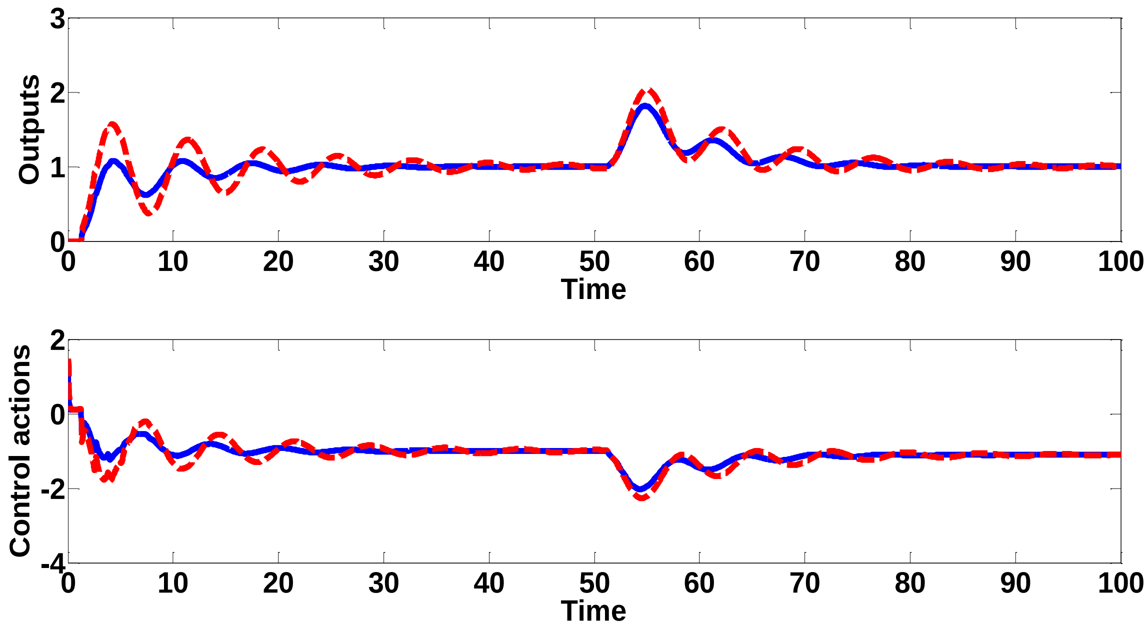

Figure 7.7 Output and control action behaviour under uncertainties model conditions for example 1, solid – Proposed work, dash – (Nasution et al., 2011).

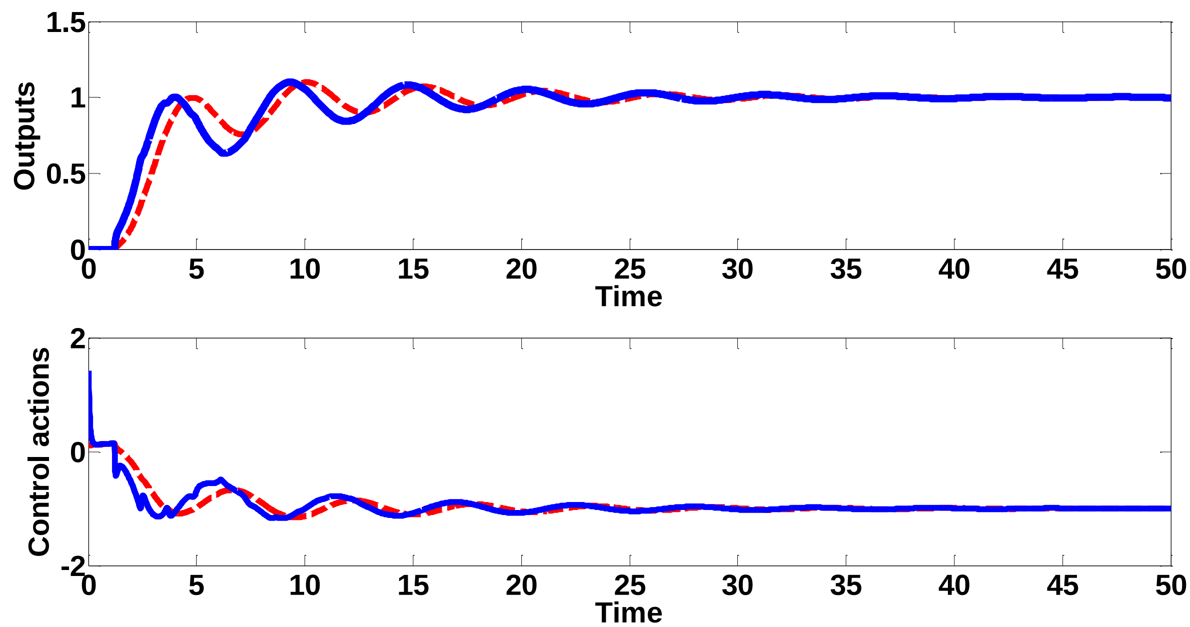

Figure 7.8 Output and control action behaviour under exact model conditions for example 1, solid – Proposed work with gama, dash – Present work without gama.

Example – 2: A higher-order unstable process studied in (Nasution et al., 2011) is considered

$$G_p(s) = \frac{e^{-0.5s}}{(5s-1)(2s+1)(0.5s+1)} \tag{7.32}$$

Table 7.1 shows the corresponding controller settings. The simulation responses values are shown in Table 7.1 for perfect model and perturbations of +10% in time delay and -10% in time constant. It is observed that the controller performance is improved using this method.

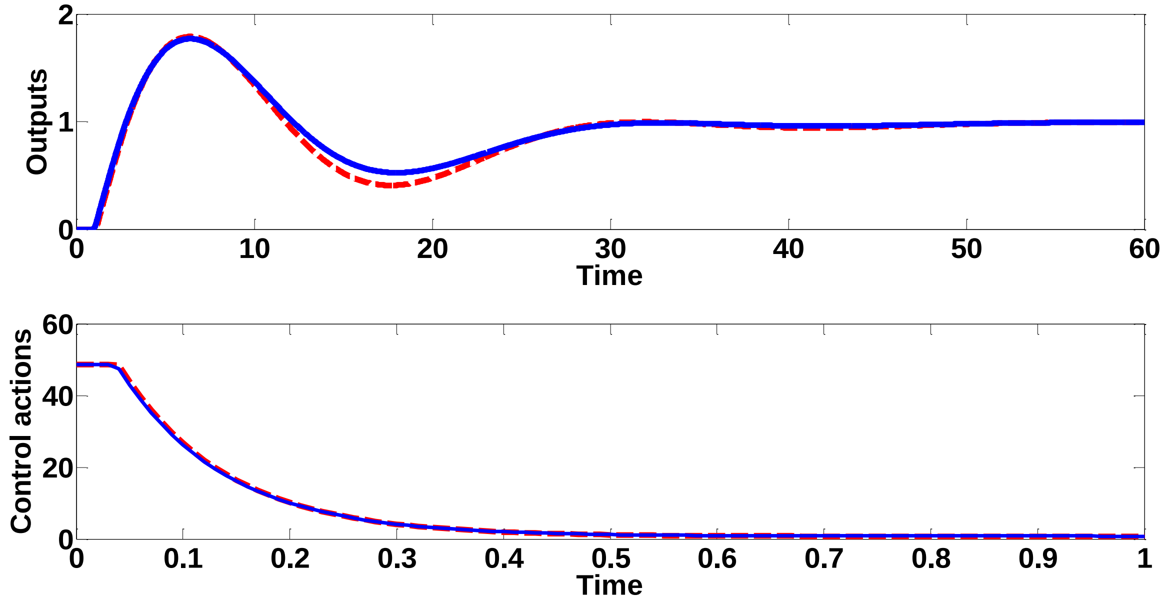

Figure 7.9 Output and control action behaviour under exact and uncertainties model conditions for example 2, solid – Proposed, dash – uncertainties.

Table 7.1 Tuning Parameters and the performance indices for Example 1 and 2.

	Method	λ	k_c	τ_i	τ_d	M_s	β	γ	Proposed model		+5% in θ	
									IAE	TV	IAE	TV
Example 1	Proposed	2.5	1.19	42.4	0.57	12	0.09	0.14	4.3	6.8	13	20
		3	1.15	63.5	0.56	9.5	0.07	0.16	3.8	4.9	4.4	6.2
		3.28	1.13	77.9	0.55	9	0.06	0.18	3.9	4.5	4	5
		4	1.1	124	0.53	11.6	0.05	0.21	4.63	4.2	4.6	4.4

									Proposed model		+10% in θ and -10% in τ	
Example 2	**Proposed**	5	1.86	34.7	0.99	2.8	0.3	3.7	11.9	54	13.1	55.3

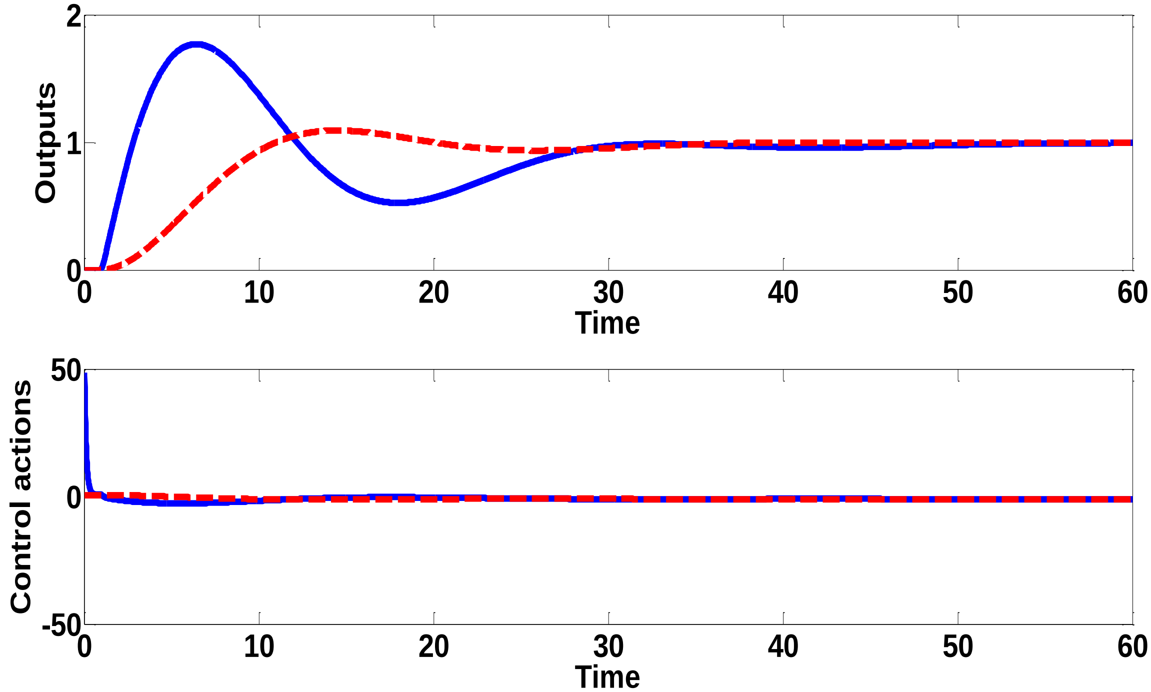

Figure 7.10 Output and control action behaviour under exact model conditions for example 2, solid – Present work with gama, dash – Proposed work without gama.

Example – 3: Consider an UFOPTD process as (Sree and Chidambaram, 2017)

$$G_p(s) = \frac{4e^{-2s}}{4s-1} \tag{7.33}$$

For this process, λ = 2.1 was selected as per the Nasution et al. (2011) method. Similarly, when λ is considered as 3, Ms is obtained as 1.6 for both the proposed and Wang et al. (2016) methods. Simulation studies are carried out separately by giving a step change of unity to the set points for all the methods. Figure 7.11 represent the closed loop and control responses. The designed controller settings, set point weighting parameters and the controller performance for the conditions specified are illustrated in Table 7.2. It is observed that the proposed technique provides good satisfactory performances with smoother control action compared to remaining methods.

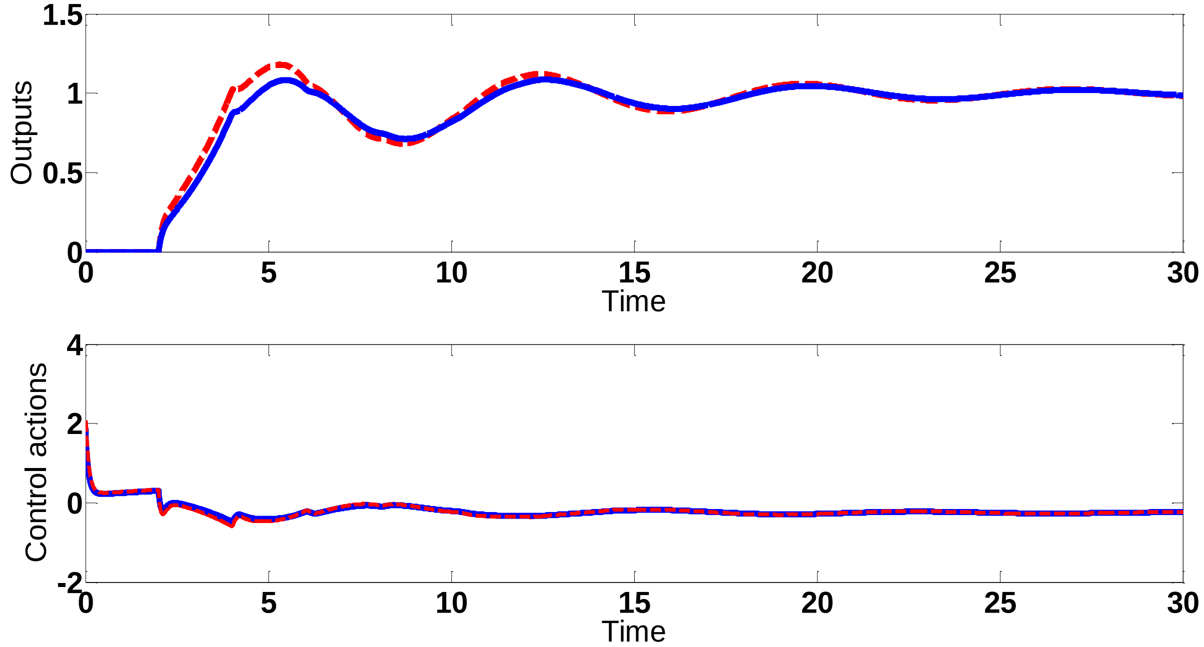

Figure 7.11 Output and control action behaviour under exact model conditions for example 3, solid – Proposed work, dash – (Nasution et al., 2011).

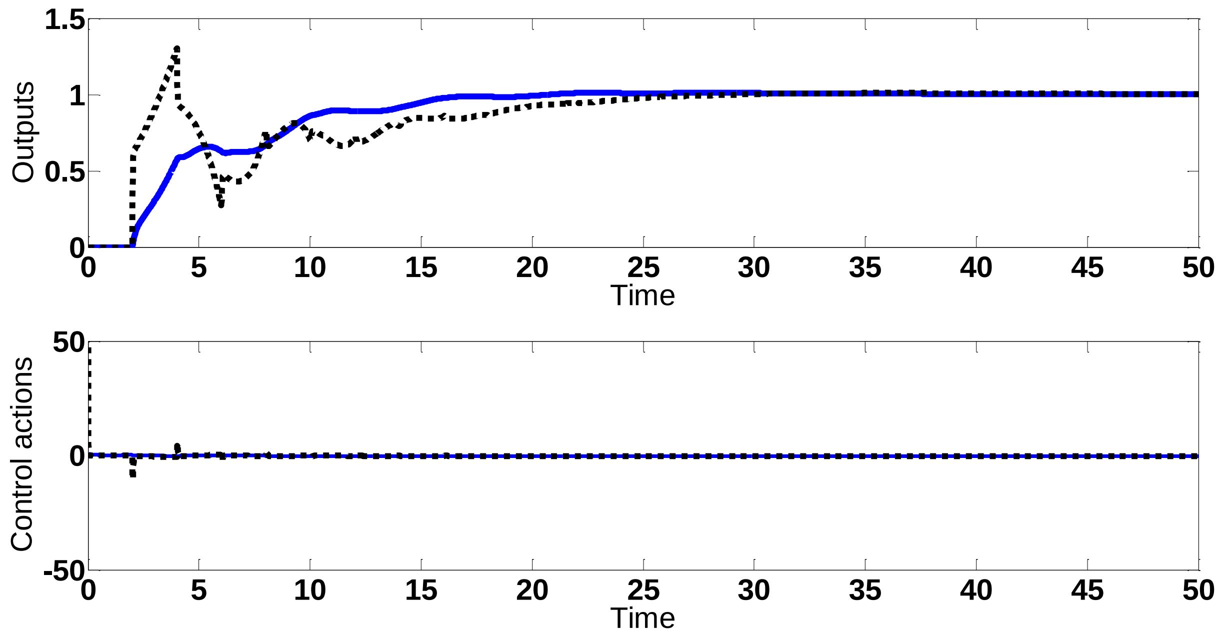

Figure 7.12 Output and control action behaviour under exact model conditions for example 3, solid – Proposed work, dash –(Wang et al., 2016).

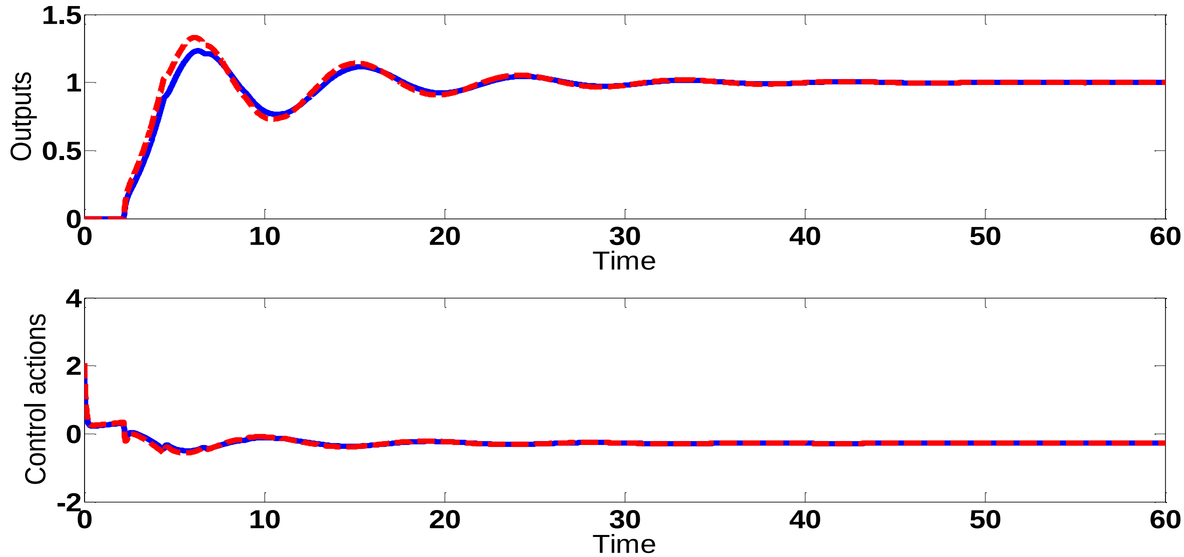

Figure 7.13 Output and control action behaviour under uncertainties model conditions for example 3, solid – Proposed work, dash – (Nasution et al., 2011).

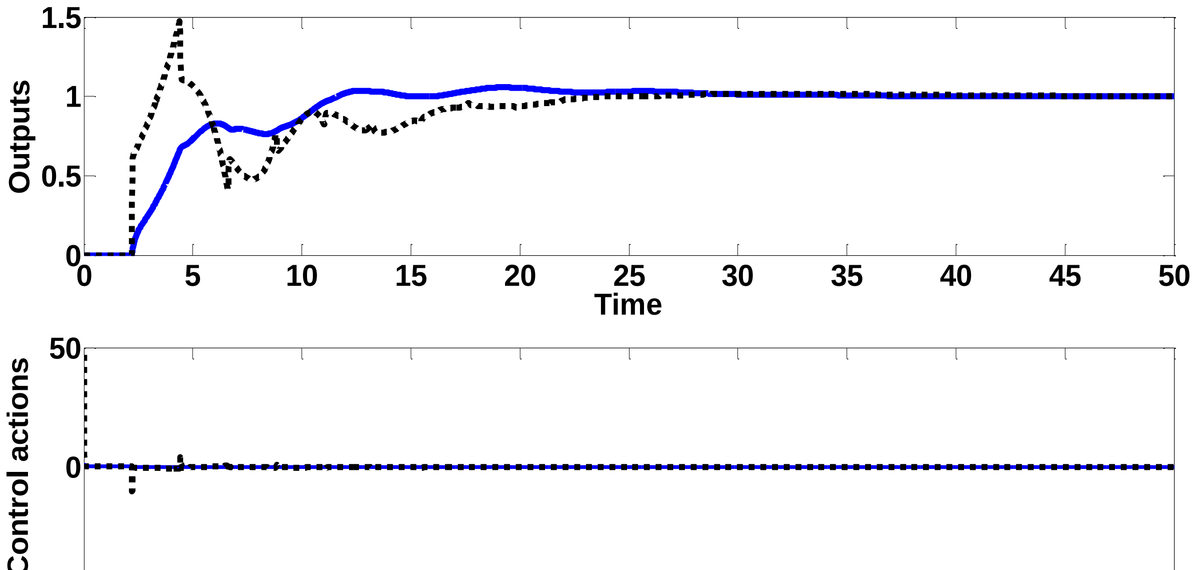

Figure 7.14 Output and control action behaviour under uncertainties model conditions for example 3, solid – Proposed work, dash – (Wang et al., 2016).

Example – 4: Another UFOPTD process is considered as below

$$G_p(s) = \frac{e^{-0.5s}}{s-1} \tag{7.34}$$

For this process, λ value of 0.8 is chosen which complements to Ms value of 2.8. The designed controller settings, set point weighting parameters and the controller performance for the conditions specified are illustrated in Table 7.2. In the same way, when λ is taken as 0.9, the obtained Ms value is 1.5 for both the proposed and Wang et al. methods. Simulation studies are carried out separately by giving a step change of unity to the set points for both methods. In addition to the perturbations given in example 3, time constant is also subjected to -10% change to test the robustness; the consequent responses are illustrated in Table 7.2.

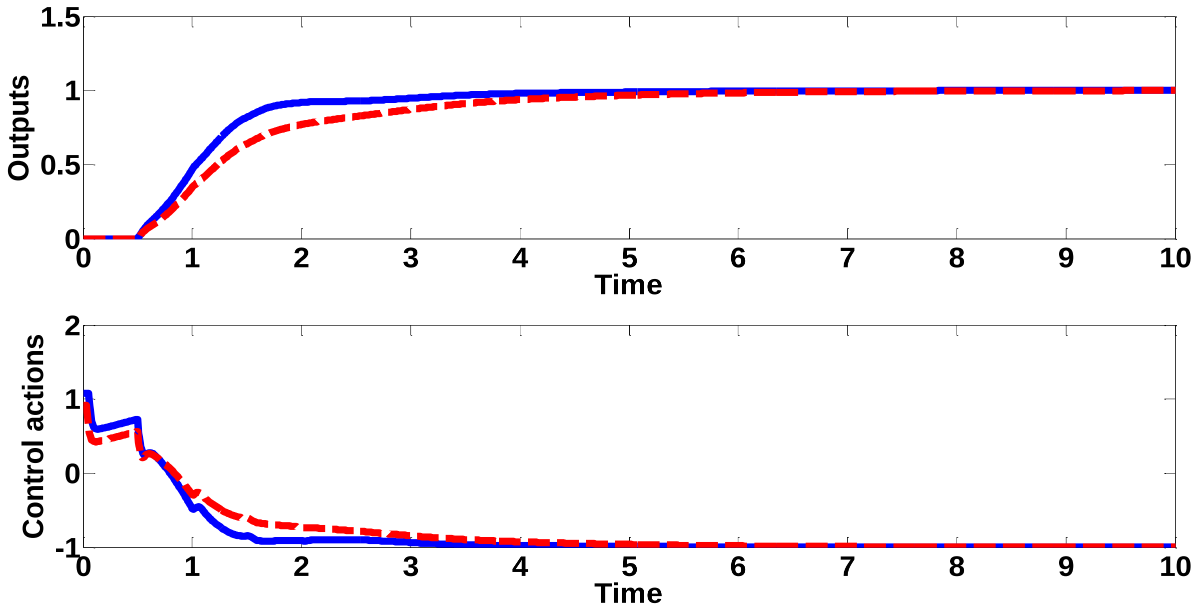

Figure 7.15 Output and control action behaviour under exact model conditions for example 4, solid – Present work, dash – (Nasution et al., 2011).

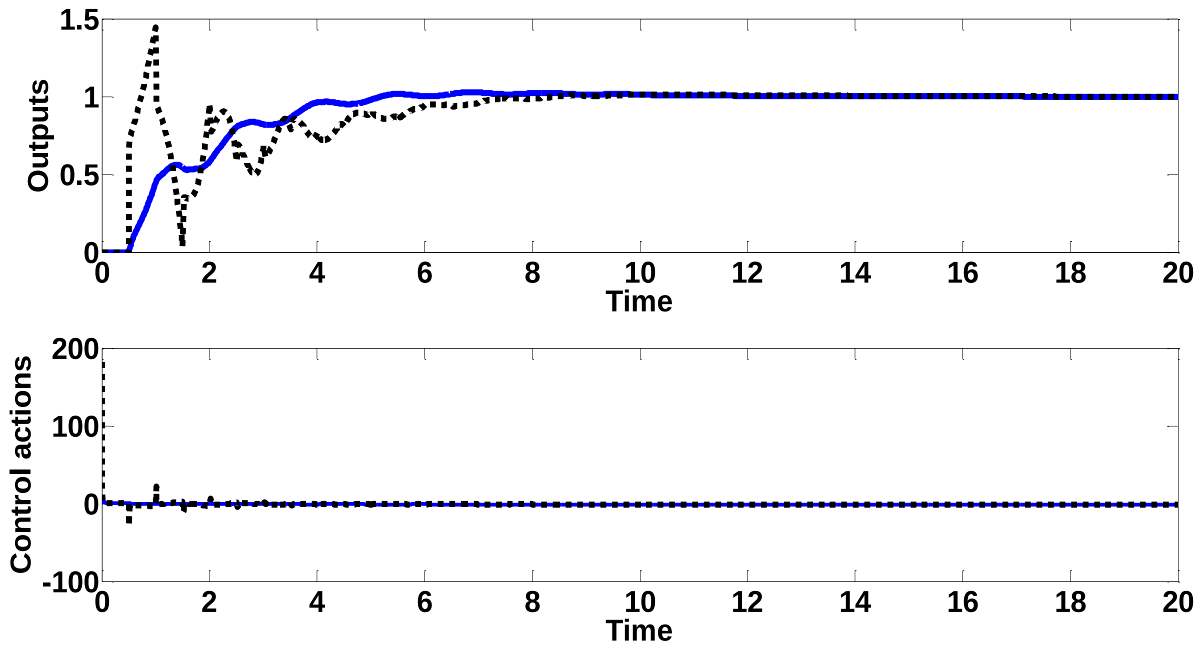

Figure 7.16 Output and control action behaviour under exact model conditions for example 4, solid – Present work, dash – (Wang et al., 2016).

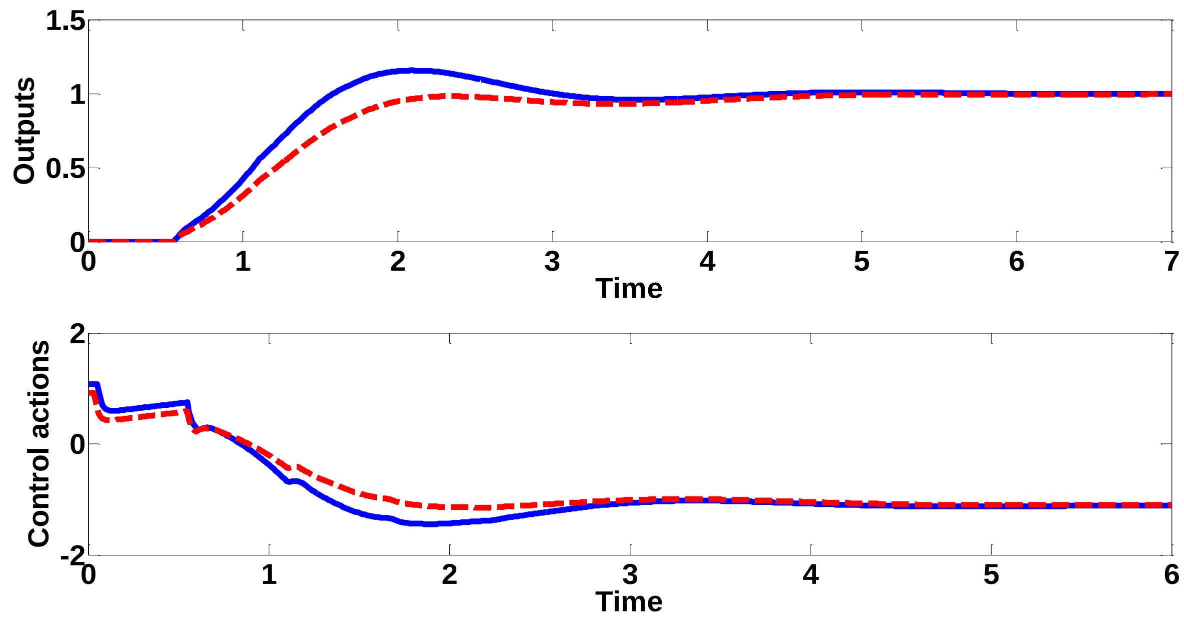

Figure 7.17 Output and control action behaviour under uncertainties model conditions for example 4, solid – Present work, dash – (Nasution et al., 2011).

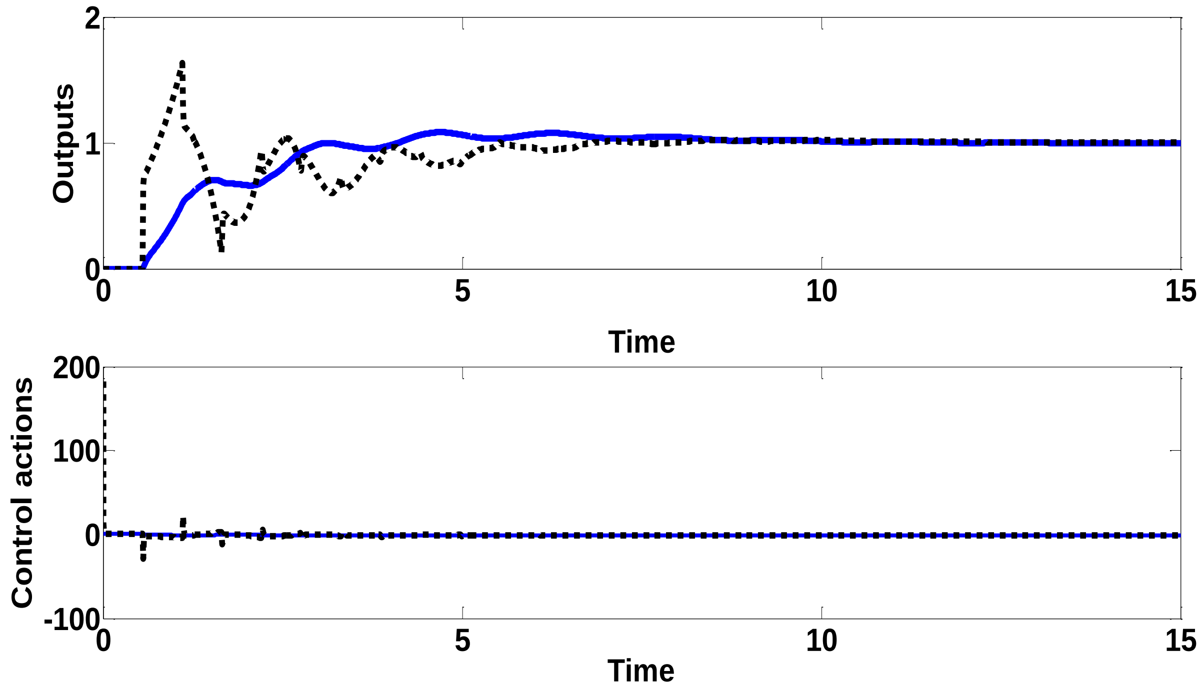

Figure 7.18 Output and control action behaviour under uncertainties model conditions for example 4, solid – Present work, dash – (Wang et al., 2016).

Table 7.2 Tuning parameters and the performance indices for Example 3 and 4.

	Method	λ	k_c	τ_i	τ_d	M_s	β	γ	Proposed model		-10% in K_p and +10% in θ	
									IAE	TV	IAE	TV
Example 3	Proposed	2.1	0.61	11	0.84	4.6	0.29	0.27	4.83	4.35	5.5	4.1
	Nasution et al.	2.1	0.61	11	0.84	4.6	0.32	0.37	5	5.22	5.9	4.9
	Proposed	3	0.47	23.5	1.3	1.6	0.26	0.19	6.1	2.2	5.52	2.33
	Wang et al.	3	0.47	23.5	1.3	1.6	0.2	0.16	6.81	90.7	5.92	90.9
											-10% in K_p +10% in θ and -10% in τ	
Example 4	Proposed	0.8	1.94	5.02	0.18	2.8	0.27	0.53	1.20	2.5	1.25	3.5
	Nasution et al.	0.8	1.94	5.02	0.18	2.8	0.18	0.19	1.67	2.4	1.38	2.8
	Proposed	0.9	1.81	7.12	0.39	1.5	0.26	0.12	1.86	5.37	1.76	5.9
	Wang et al.	0.9	1.81	7.12	0.39	1.5	0.2		2	380	1.73	382

Example – 5: A second order unstable process (Nasution et al., 2011) is considered

$$G_p(s) = \frac{e^{-0.5s}}{(2s-1)(0.5s+1)} \tag{7.35}$$

The corresponding controller values are given in Table 7.3. The time constant and delay time are subjected to -10% and +10% perturbations; the consequent responses are illustrated in Table 7.3. The results comprehend to better robust performances which reveals that the present method is superior.

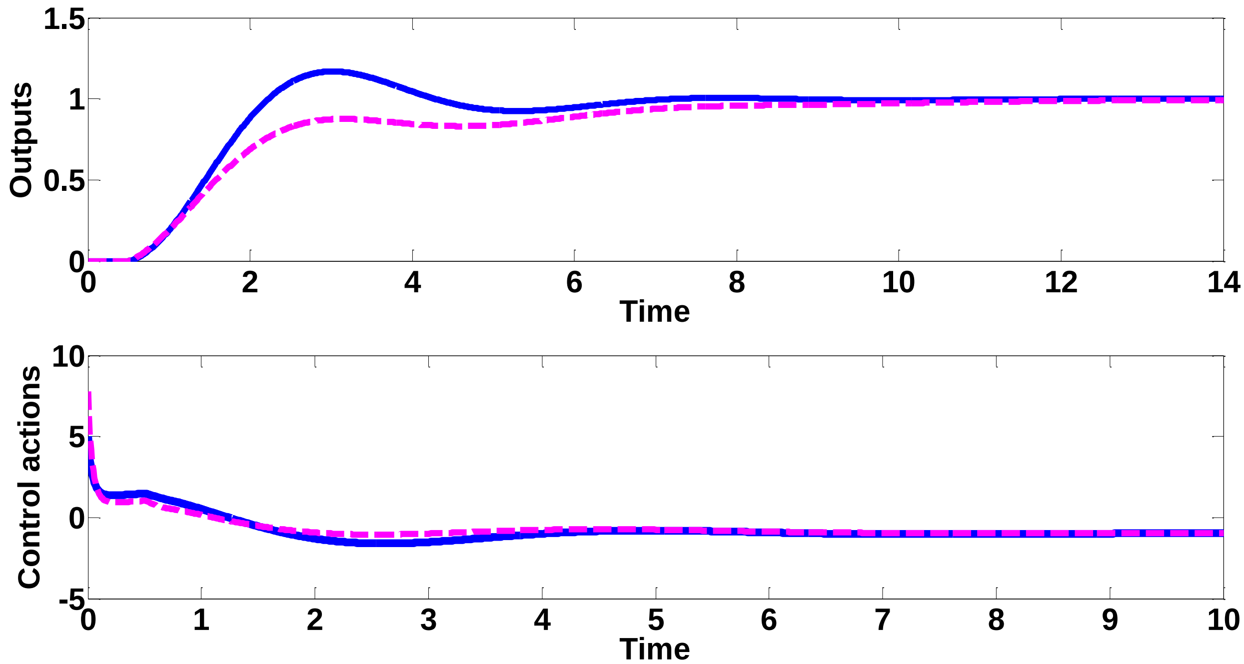

Figure 7.19 Output and control action behaviour under exact model conditions for example 5, solid – Present work, dash – (Nasution et al., 2011).

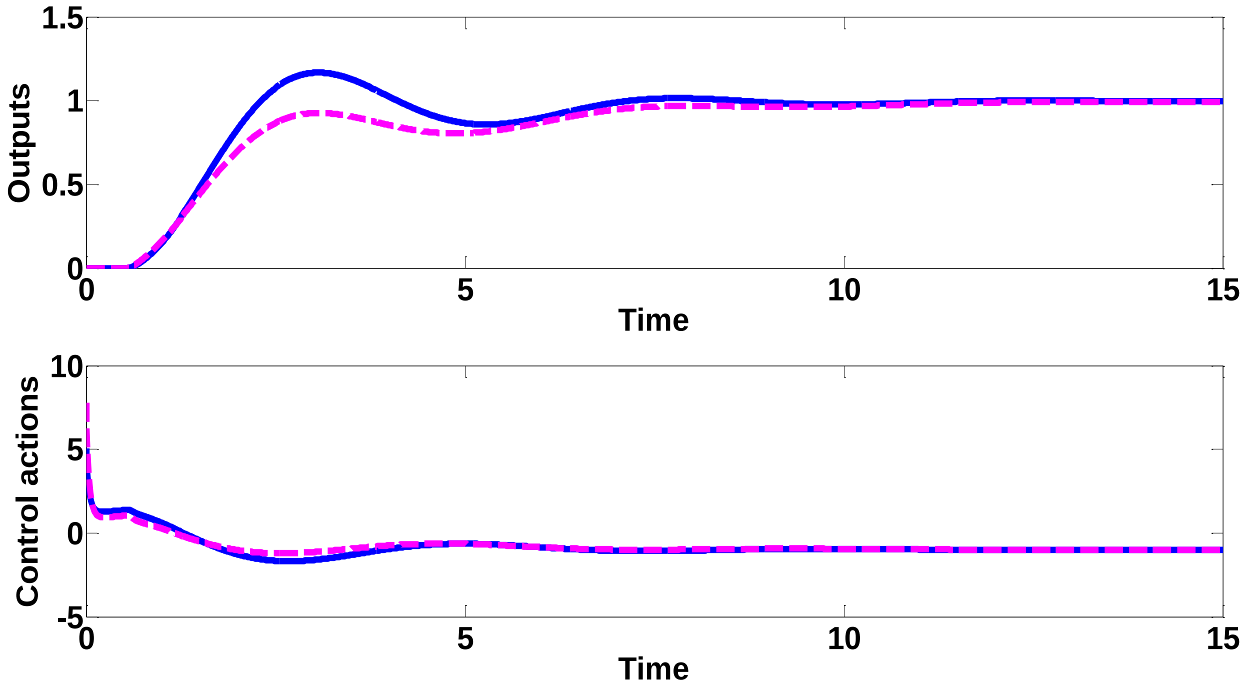

Figure 7.20 Output and control action behaviour under uncertainties model conditions for example 5, solid – Present work, dash – (Nasution et al., 2011).

Example – 6: **Control of a Bio-reactor**:

A nonlinear continuous bioreactor exhibiting multiplicity behavior is modelled (Sree and Chidambaram, 2017) by the following equations:

$$\frac{dX}{dt} = (\mu - D)X \tag{7.36}$$

$$\frac{dS}{dt} = \left(S_f - S\right)D - \frac{\mu X}{\gamma} \tag{7.37}$$

Where$\mu = \mu_m S/(K_m + S + K_l S^2)$. The parameters are taken as γ = 0.4 g/g, S_f = 4g/l, m=0.53 h^{-1}, D =0.3 h^{-1}, K_m = 0.12 g/l, K_l = 0.45451 l/g. X and S represent the cell and substrate concentrations respectively. An unstable linearization point (X = 0.9951, S = 1.5122) is obtained for which a transfer function model is developed as followed.

$$G_p(s) = \frac{-5.89\, e^{-2.4s}}{5.86s - 1} \tag{7.38}$$

For this process, λ=2.56 was selected as per the Nasution et al. (2011) method, which corresponds to an M_s Value of 3.78. Simulation studies are carried out separately by giving a step change of unity to the set points for all the methods. The designed controller settings, set point weighting parameters and the controller performance for the conditions specified are illustrated in Table 7.3. The results comprehend to better robust performances which reveals that the present method is superior.

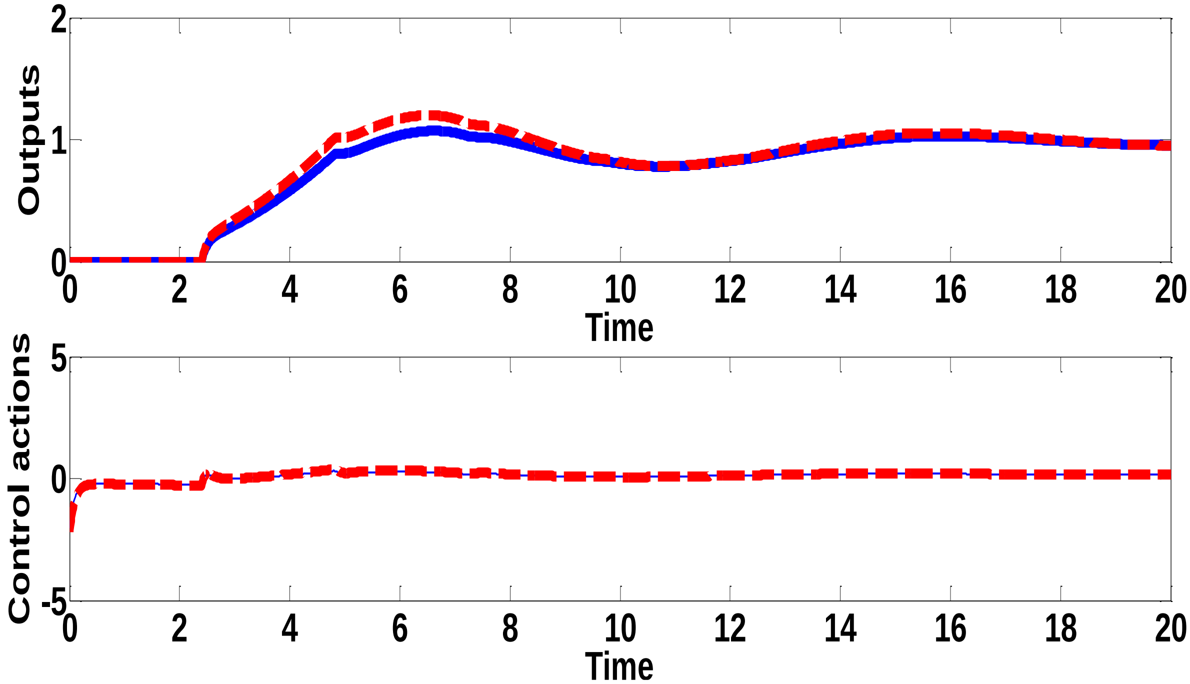

Figure 7.21 Output and control action behaviour under exact model conditions for example 6, solid – Present work, dash – (Nasution et al., 2011).

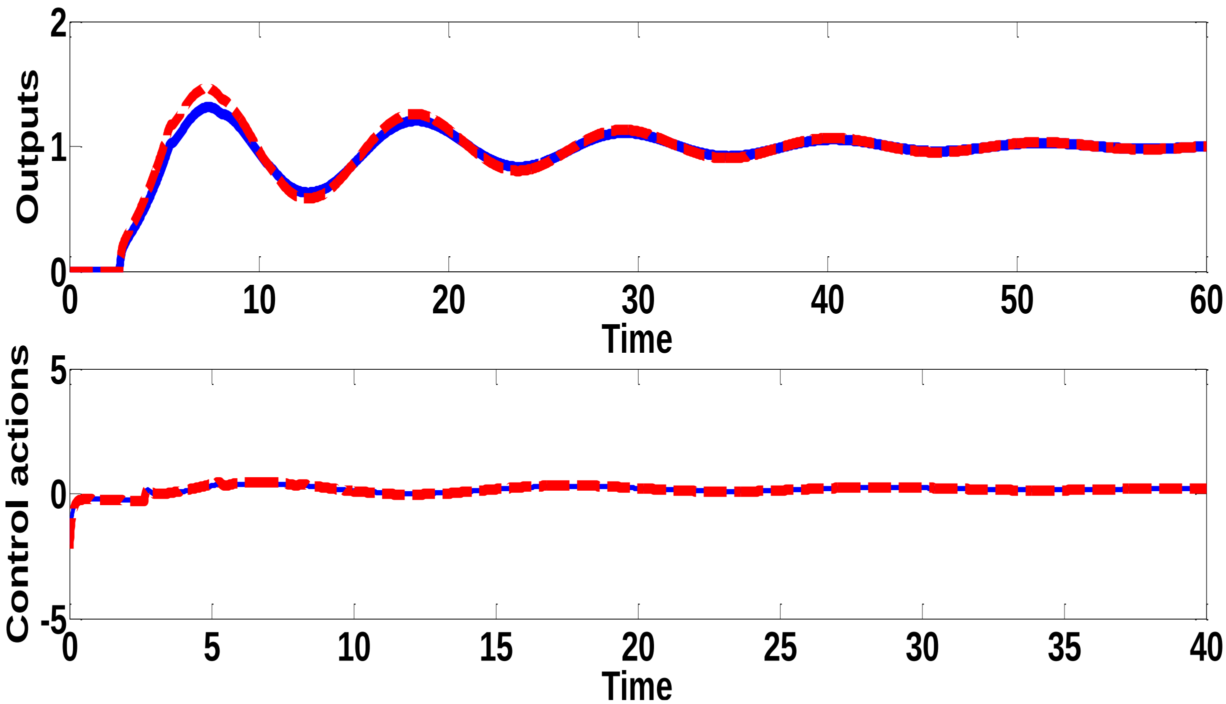

Figure 7.22 Output and control action behaviour under uncertainties model conditions for example 6, solid – Present work, dash – (Nasution et al., 2011).

Non – linear Simulation:

It is more appropriate to perform simulation directly on the original nonlinear model instead of transfer function model. To serve this purpose, λ is selected 2.56 which complements to M_s value of 3.78. The designed controller settings, set point weighting parameters and the controller performance for the conditions specified are illustrated in Table 7.3. Simulation studies are carried out separately by giving a step change from 0.9951 to 1.1 at time t = 0 sec to the set point. Figure 7.23 represent the closed loop and the corresponding control responses respectively. The delay time is subjected to +5% perturbations to test the robustness; the consequent responses are illustrated in Table 7.3. The results again comprehend to better robust performances which reveals that the present method is superior with smooth control action.

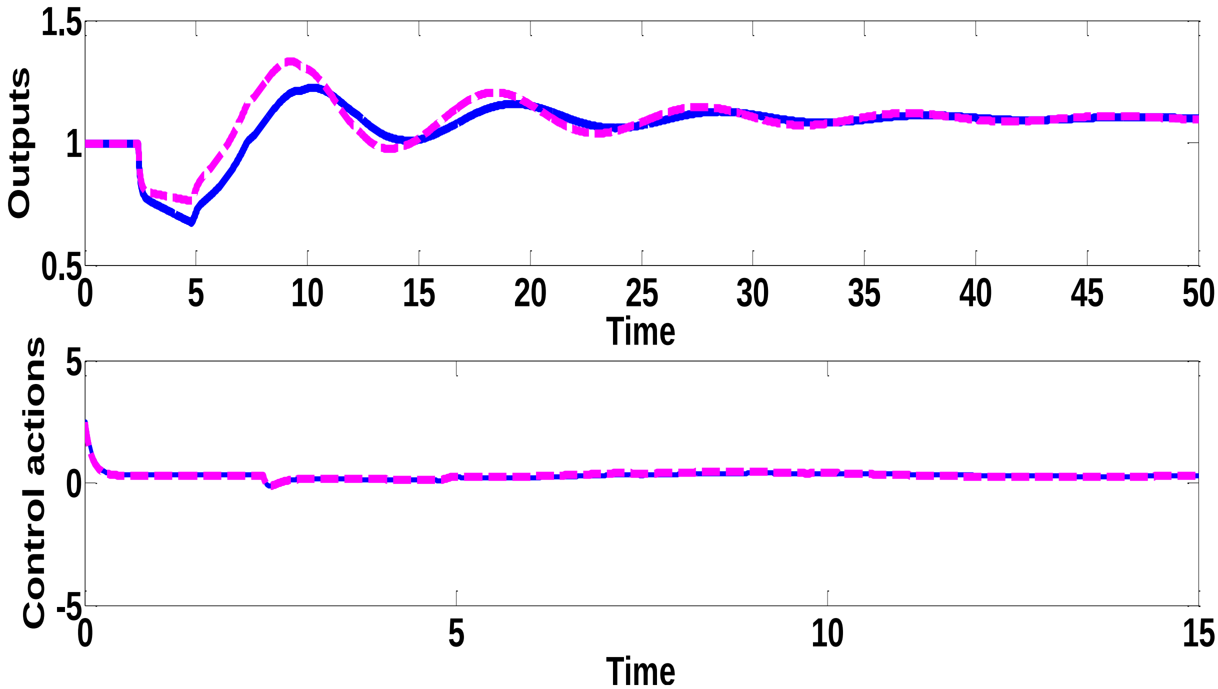

Figure 7.23 Output and control action behaviour for Non Linear exact model conditions for example 6, solid – Present work, dash – (Nasution et al., 2011).

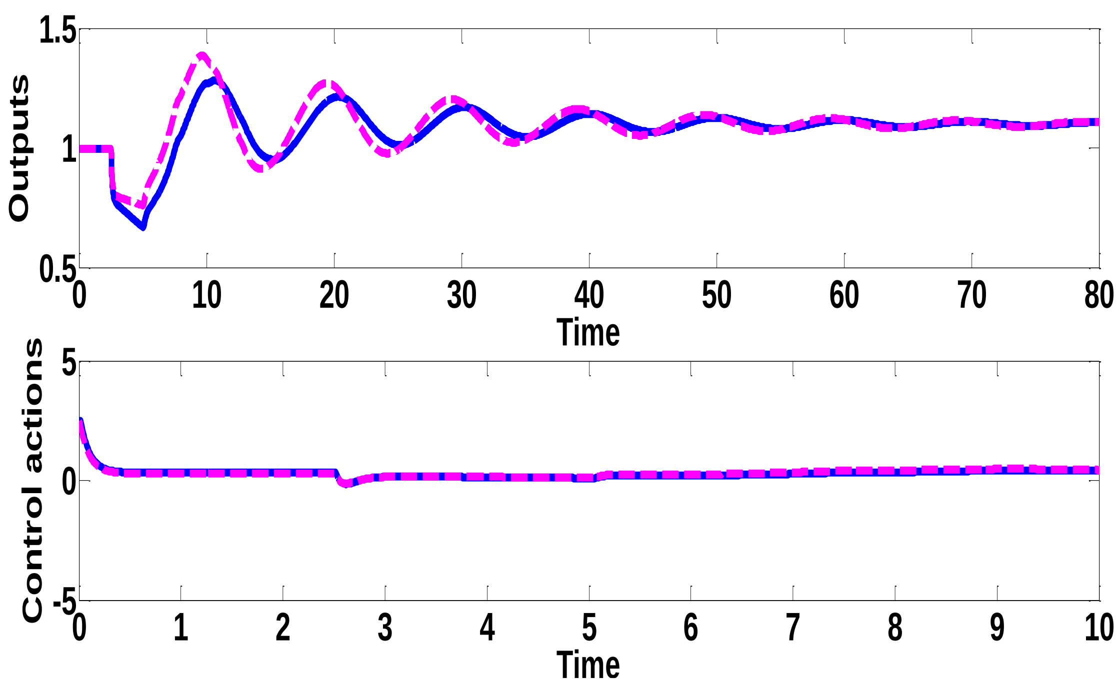

Figure 7.24 Output and control action behaviour for Non Linear uncertainties model conditions for example 6, solid – Present work, dash – (Nasution et al., 2011).

Tracking of Different Set-points:

Both increase (positive) and decrease (negative) step inputs of different magnitudes are considered. Step magnitudes of 1, 0.2 and -0.2 at times t = 0, 60 and 250 are considered and the corresponding closed loop responses for perfect conditions without and with noise are shown in Figure 7.25,7.26 and Figure 7.27,7.28. Hence, it is inferred that the present method has the ability to track multi set-points without much overshoot.

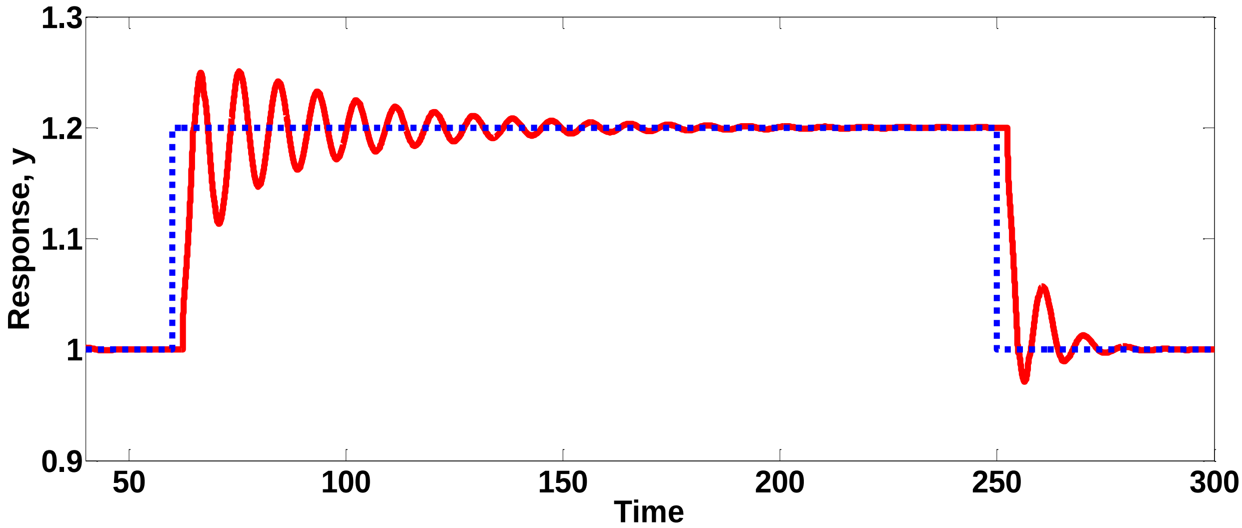

Figure 7.25 Closed loop response for different setpoints responses for example 6 for exact condition. Dot – set point, Solid – output.

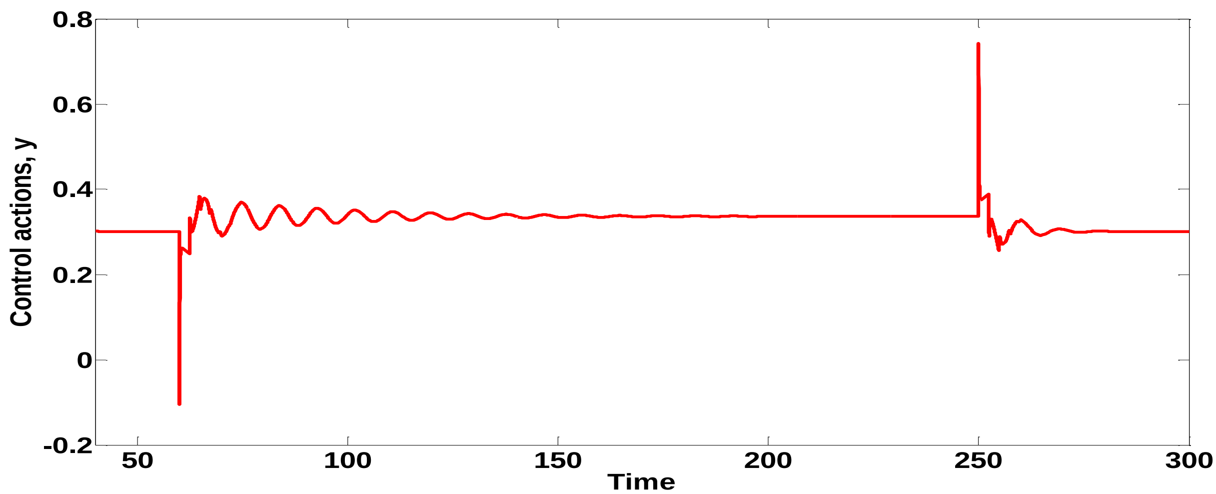

Figure 7.26 Closed loop response for different setpoints control actons for example 6, for perfect condition.Solid – output.

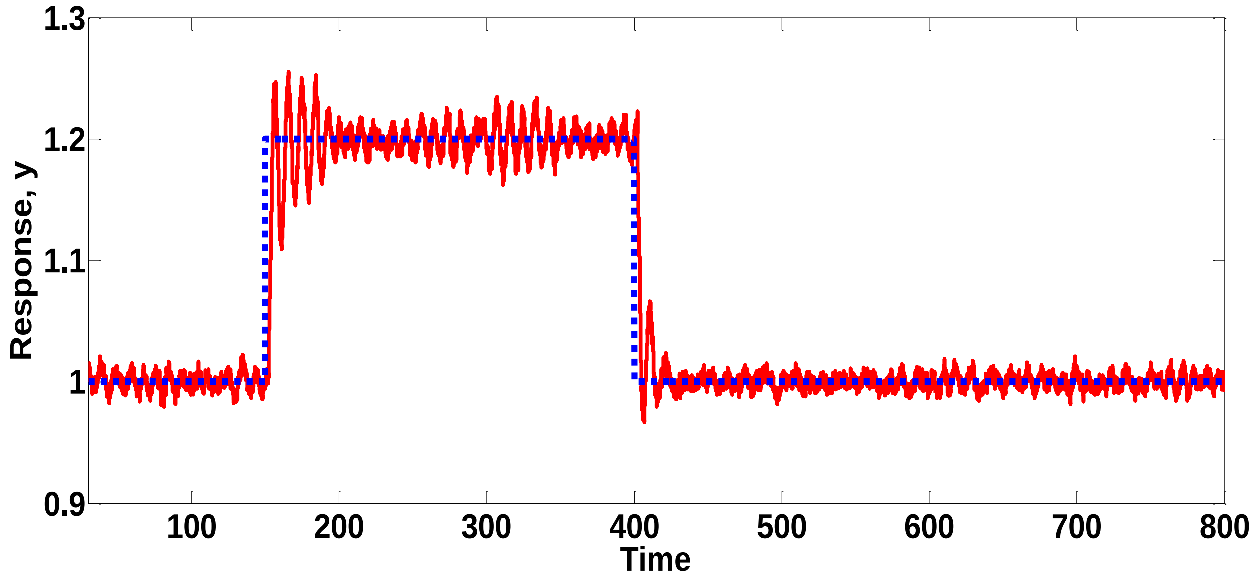

Figure 7.27 Closed loop response for different setpoints for example 6 with Noise for perfect condition. Dot – set point, Solid – output.

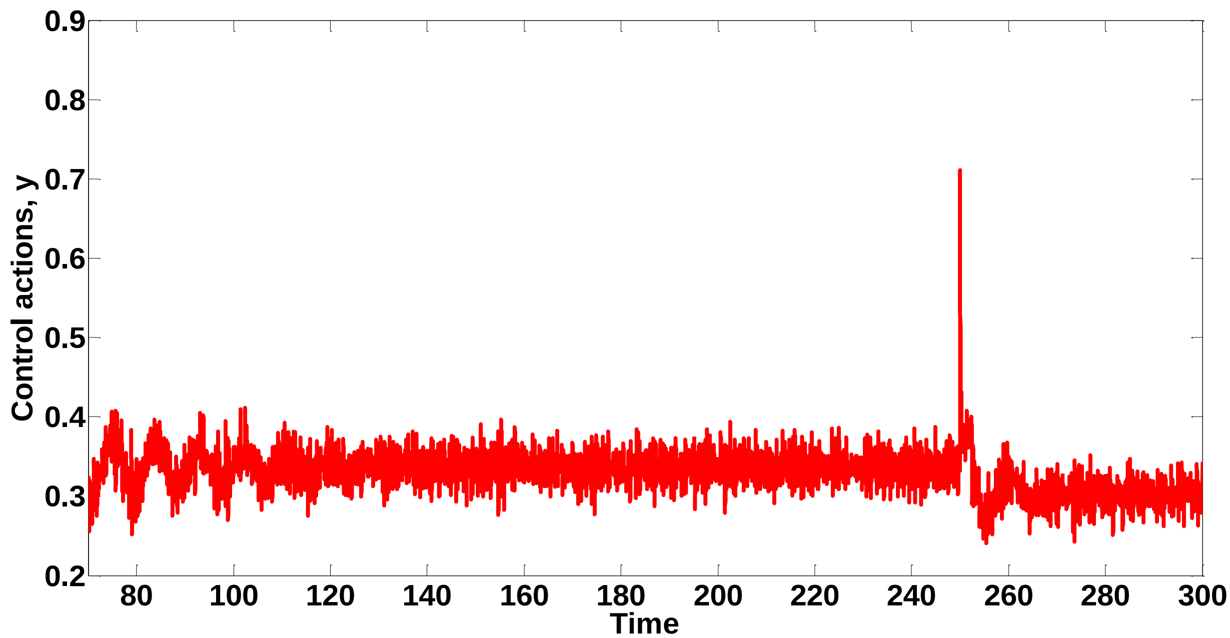

Figure 7.28 Closed loop response for different setpoints control actions for example 6 with Noise for perfect condition. Dot – set point, Solid – output.

Example - 7: Control of a Chemical Reactor:

An isothermal chemical reactor exhibiting multiple steady-state solutions is modelled as (Musmade and Chidambaram, 2014):

$$\frac{dc}{dt} = \frac{Q}{V}\left(C_f - C\right) - \frac{k_1 C}{(k_2 C + 1)^2} \qquad (7.39)$$

Where C_f and Q are the inlet concentration and flow rate respectively. The values of the operating parameters are Q=0.0333 L/s, V=1L, k_1=10 L/s, and K_2 =10 L/mol. Two steady state concentrations (C=1.7673 and 0.01424 mol/L) and one unstable steady state (C=1.316mol/L) are obtained around a nominal value of C_f =3.288 mol/L. An unstable transfer function model is developed by linearizing the manipulated variable i.e., the feed concentration at C=1.316. A time delay of 20s is considered which yield an unstable transfer function model as

$$G_p(s) = \frac{3.433e^{-20s}}{103.1s-1} \qquad (7.40)$$

For this process, λ is selected as 26 which complements to M_s value of 2.6. Simulation studies are carried out by giving a step change of unity to the set point. Figure 7.29 represent the closed loop and the corresponding control responses. The designed controller settings, set point weighting parameters and the controller performance for the conditions specified are illustrated in Table 7.3.

Example - 8: Consider a TITO process (Hazarika and Chidambaram, 2014)

$$G_p(s) = \begin{bmatrix} \frac{1.6e^{-s}}{-2.6s+1} & \frac{0.6e^{-1.5s}}{2.5s+1} \\ \frac{0.7e^{-1.5s}}{3s+1} & \frac{1.7e^{-s}}{-2.2s+1} \end{bmatrix} \qquad (7.41)$$

RGA and NI are calculated to estimate the pairing selection for the above mentioned system.

$$K = \begin{bmatrix} 1.6 & 0.6 \\ 0.7 & 1.7 \end{bmatrix} \text{RGA} = \begin{bmatrix} 1.1826 & -0.1826 \\ -0.1826 & 1.1826 \end{bmatrix} \qquad (7.42)$$

NI=0.6969

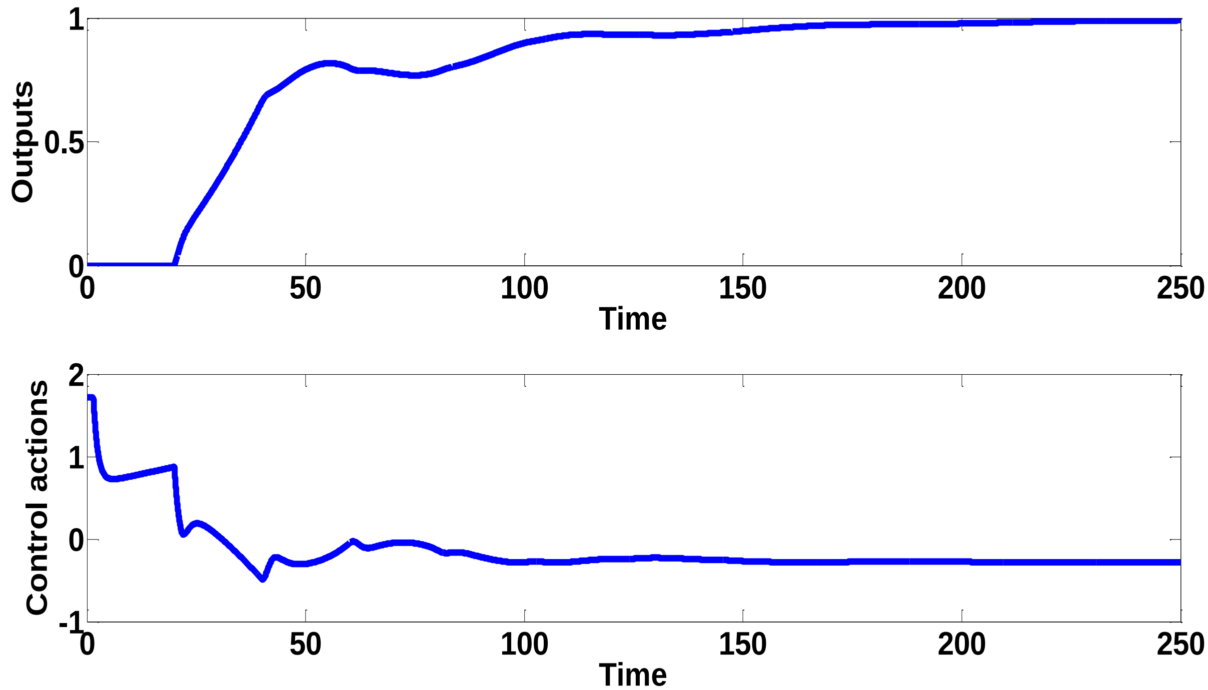

Figure 7.29 Output and control action behaviour under exact model conditions for example 7, solid – Present work.

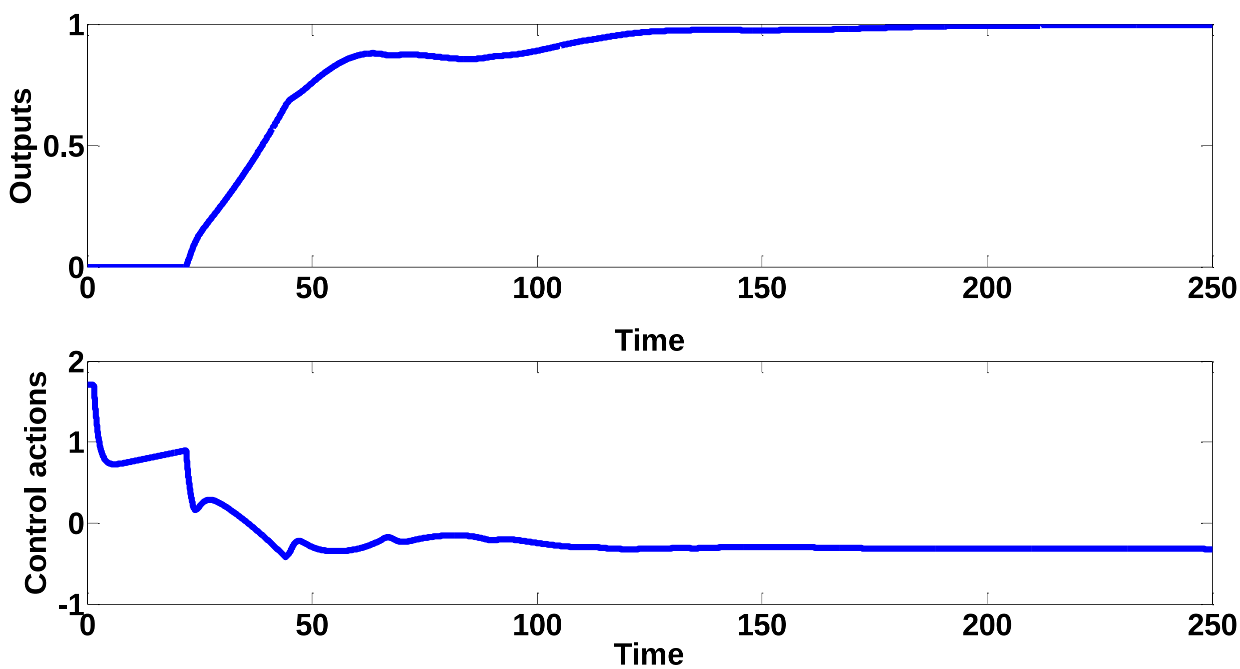

Figure 7.30 Output and control action behaviour under uncertainties model conditions for example 7, solid – Present work.

Table 7.3 Tuning parameters and the performance indices for Example 5, 6 and 7.

	Method	λ	k_c	τ_I	τ_D	M_s	β	γ	Proposed model		+10% in θ and -10% in τ	
									IAE	TV	IAE	TV
Example 5	**Proposed**	1.23	2.67	6.31	0.41	3.4	0.46	0.14	1.81	7.91	2	8.71
	Nasution et al.	1.23	2.67	6.31	0.41	3.4	0.28	0.26	2.43	9.61	2.42	10.3
											-10% in K_p +10% in θ and -10% in τ	
Example 6	**Proposed**	2.56	-0.48	12.57	0.9	3.78	0.31	0.39	5	3.5	8.7	4.6
	Nasution et al.	2.56	-0.48	12.57	0.9	3.78	0.37	0.45	5.06	4.02	9.84	5.29
Example 6 Non-linear simulation	**Proposed**	2.56	-0.48	12.57	0.9	3.78	0.31	0.39	2.97	3.85	4.58	4.37
	Nasution et al.	2.56	-0.48	12.57	0.9	3.78	0.37	0.45	3.29	4.05	5.26	4.69
Example 7	**Proposed**	26	1.41	123	10.2	2.6	0.44	0.32	51	3.9	48	3.6

The tuning parameters are derived from (Hazarika and Chidambaram, 2014) which are found to be 1.4434, 1.6142. The pairing criteria is to get a positive value for NI as a same number of open loop unstable poles is obtained for both Gp(s) and $\overline{G}$p(s) = diag[gp,ii(s)].

$$D(s)=\begin{bmatrix} 1 & \dfrac{(7.8s-3)e^{-0.5s}}{20s+8} \\ \dfrac{(15.4s-7)e^{-0.5s}}{51s+17} & 1 \end{bmatrix} \tag{7.43}$$

The controller parameters – normalized gain array, relative average residence time array (RARTA), average residence time array, and relative normalized gain array (RNGA) which are mentioned below are used for the evaluation of equivalent transfer function

$$T_{ar}=\begin{bmatrix} 3.6 & 4 \\ 4.5 & 3.2 \end{bmatrix} K_N=\begin{bmatrix} 0.444 & 0.15 \\ 0.1556 & 0.5313 \end{bmatrix} \tag{7.44}$$

$$\phi=\begin{bmatrix} 1.1097 & -0.1097 \\ -0.1097 & 1.1097 \end{bmatrix} \Gamma=\begin{bmatrix} 0.9383 & 0.6005 \\ 0.6005 & 0.9383 \end{bmatrix} \tag{7.45}$$

Hence, the developed equivalent transfer function is

$$\hat{G}_p = \begin{bmatrix} \dfrac{1.3529e^{-0.9383s}}{-2.4396s+1} & \dfrac{-3.2857e^{-0.9008s}}{1.5013s+1} \\ \dfrac{-3.8333e^{-0.9008s}}{-1.8016s+1} & \dfrac{1.4375e^{-0.9383s}}{-2.0643s+1} \end{bmatrix} \tag{7.46}$$

A lead lag filter based method is used as in Dasari et al. (2016) to design the controller which is as represented below

$$G_c = \begin{bmatrix} -0.0371(1+\dfrac{1}{0.6255s}+0.2346) & 0 \\ 0 & -0.0335(1+\dfrac{1}{0.6255s}+0.2346) \end{bmatrix} \tag{7.47}$$

$$F^{'}(s) = \begin{bmatrix} \dfrac{20.3717s+1}{0.4537s+1} & 0 \\ 0 & \dfrac{20.2497s+1}{0.3965s+1} \end{bmatrix} \tag{7.48}$$

This designed controller is applied with set point weighting of magnitudes that is for first loop β_1=0.3, γ_1=0.79 and for the second loop β_1=0.25, γ_1=0.74 present in the process. A step change of unity is given to the set point and disturbance and their corresponding closed loop responses are plotted in comparison to Hazarika and Chidambaram (2014) and Dasari et al. (2016). The closed loop servo responses are illustrated in Figure 7.31. Other parameters are also subjected to perturbations and the respective controller performances values are presented in Tables 7.4 & 7.5. It is noticed that the proposed method gives better performance in comparison to the other methods and also is stable in nature.

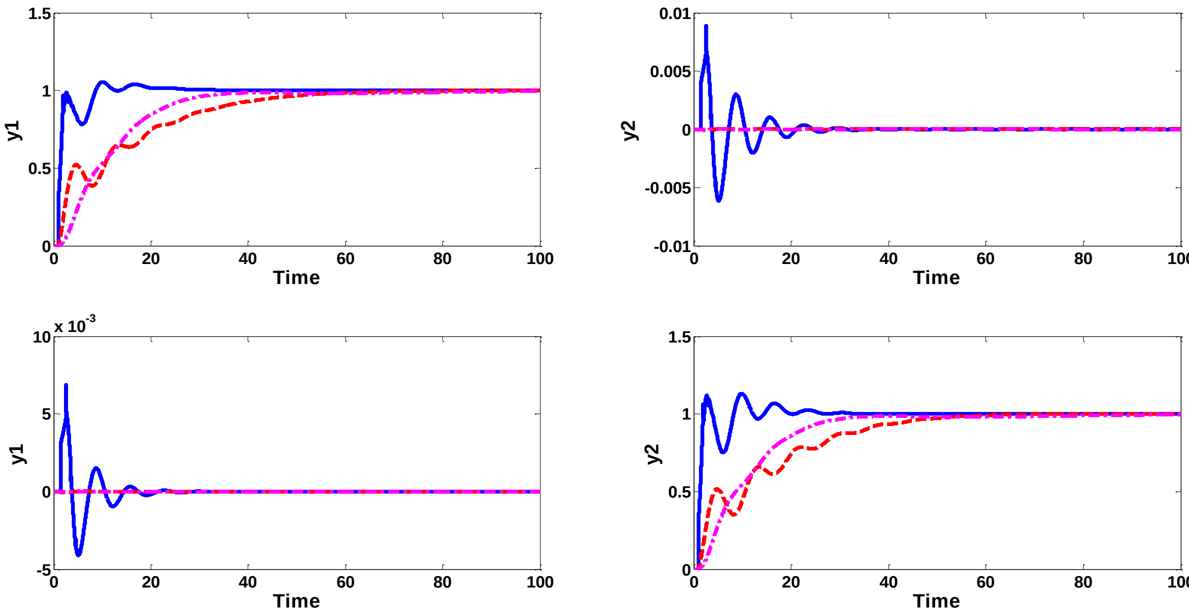

Figure 7.31 Output and control action behaviour for servo under exact model conditions for y_1 and y_2 for example 8, solid – Present work, dash – (Hazarika and Chidambaram, 2014), dash dot– (Dasari et al., 2016)

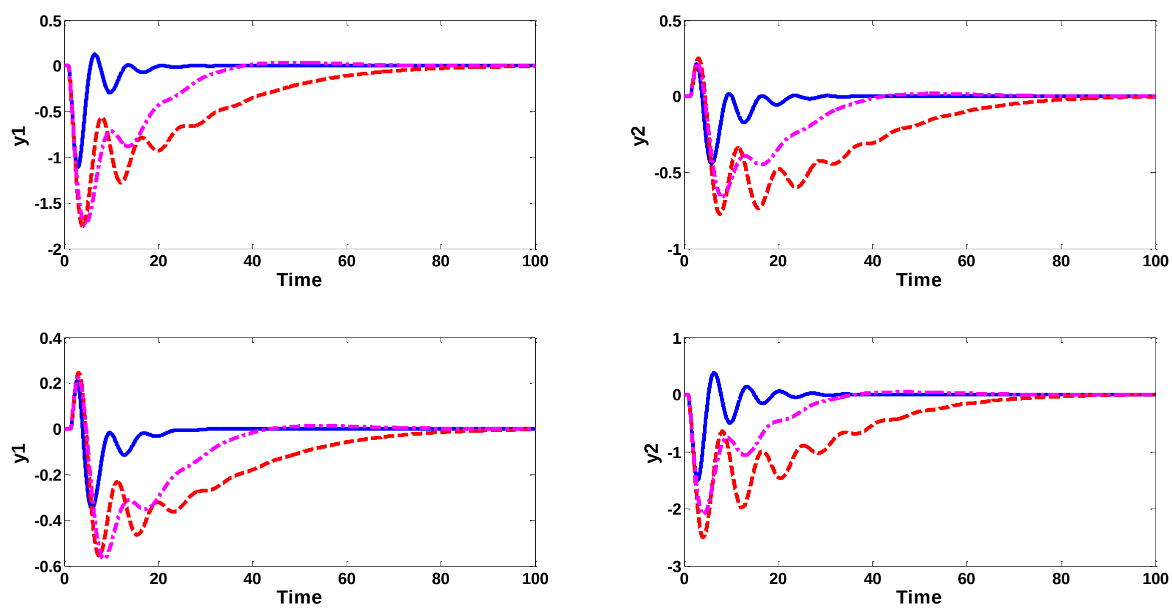

Figure 7.32 Output and control action behaviour for regulatory under exact model conditions for y_1 and y_2 for example 8, solid – Present work, dash – (Hazarika and Chidambaram, 2014), dash dot– (Dasari et al., 2016)

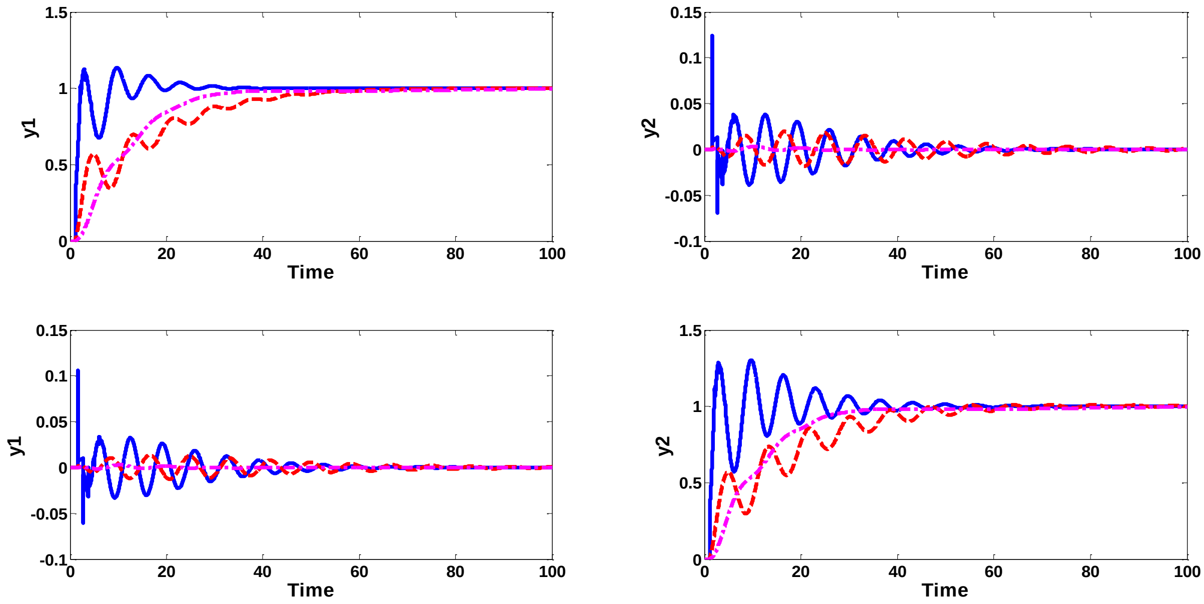

Figure 7.33 Output and control action behaviour for servo under 10% uncertainties to time delay for y_1 and y_2 for example 8, solid – Present work, dash – (Hazarika and Chidambaram, 2014), dash dot– (Dasari et al., 2016)

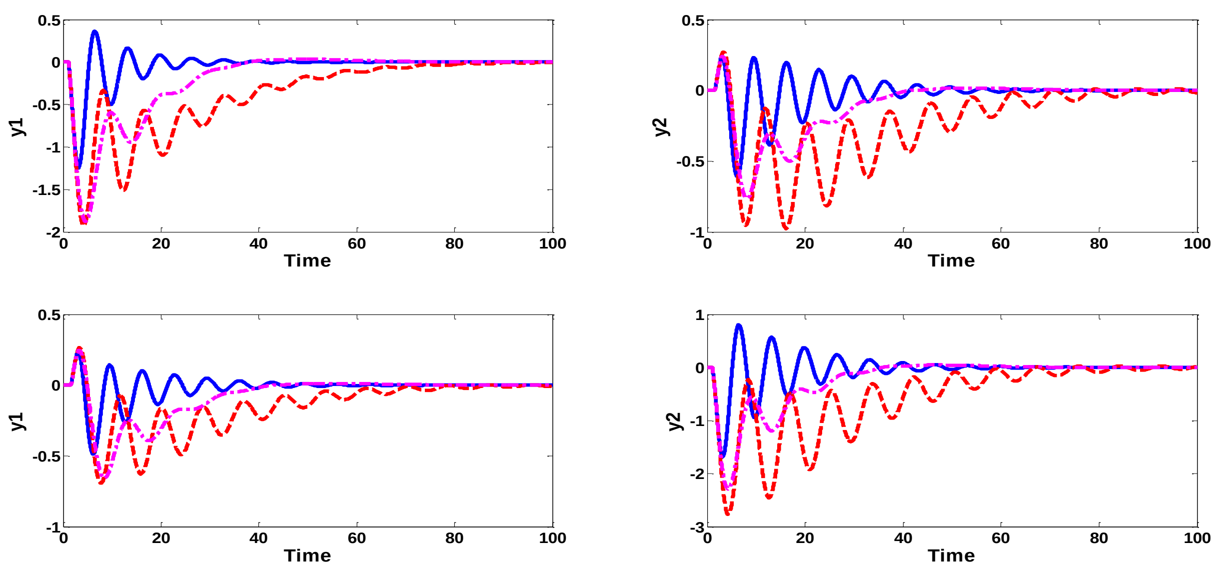

Figure 7.34 Output and control action behaviour for regulatory under 10% uncertainties to time delay for y_1 and y_2 for example 8, solid – Present work, dash – (Hazarika and Chidambaram, 2014), dash dot– (Dasari et al., 2016)

Table 7.4 Analysis of IAE values for servo responses for example-8.

perturbations	Proposed method				Hazarika et al.				Dasari et al.			
	loop1		loop2		loop1		loop2		loop1		loop2	
	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2
kp	2.66	0.04	0.02	2.85	13.96	0	0	14.10	12.42	0	0	11.90
1.1kp	2.66	0.03	0.02	2.82	15.98	0	0	16.77	13.54	0	0	13.07
0.9kp	2.66	0.04	0.02	3.28	11.85	0	0	11.92	11.11	0	0	10.57
τ	2.66	0.04	0.02	2.85	13.96	0	0	14.10	12.42	0	0	11.90
1.1τ	2.73	0.24	0.20	2.65	14	0.36	0.22	14.24	12.42	0.25	0.19	11.9
0.9τ	2.78	0.55	0.44	3.46	13.9	1.15	0.79	14.15	12.42	0.25	0.19	11.9
θ	2.66	0.04	0.02	2.85	13.96	0	0	14.10	12.42	0	0	11.90
1.1θ	3.36	0.71	0.60	5.68	13.96	0.57	0.39	14.3	12.42	0.04	0.03	11.9
0.9θ	2.62	0.14	0.11	2.38	13.96	0.09	0.06	14.13	9.12	0.01	0	7.74

Table 7.5 Analysis of IAE values for regulatory responses for example-8

	Proposed method				Hazarika et al.				Dasari et al.			
	loop1		loop2		loop1		loop2		loop1		loop2	
	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2
kp	4.23	2.49	2.08	6.64	36.54	22.81	14.64	53.12	21.54	10.64	8.96	24.01
1.1kp	4.10	2.45	2.10	5.78	36.35	22.64	14.58	52.73	20.86	10.34	8.73	23.2
0.9kp	4.64	2.82	2.07	7.99	36.8	23.22	14.7	54.33	22.78	11.19	9.39	25.48
τ	4.23	2.49	2.08	6.64	36.54	22.81	14.64	53.12	21.54	10.64	8.96	24.01
1.1τ	4.16	2.71	2.17	6.25	36.56	22.79	14.61	53.17	21.72	10.53	8.86	24.23
0.9τ	4.38	2.44	2.16	7.48	36.52	22.83	14.73	53.22	21.34	10.74	9.05	23.77
θ	4.23	2.49	2.08	6.64	36.54	22.81	14.64	53.12	21.54	10.64	8.96	24.01
1.1θ	6.38	5.74	3.79	13.79	36.53	23.04	14.76	53.46	21.47	10.67	9.02	23.94
0.9θ	3.88	2.33	1.98	4.95	36.55	22.71	14.53	53.14	21.62	10.61	8.91	24.11

Example 9:

Another TITO process (Hazarika and Chidambaram, 2014) is considered

$$G_P(S) = \begin{bmatrix} \frac{-1.6667e^{-s}}{-2.6s+1} & \frac{-1e^{-s}}{-1.6667s+1} \\ \frac{-0.8333e^{-s}}{-1.6667s+1} & \frac{-1.6667e^{-s}}{-1.6667s+1} \end{bmatrix} \tag{7.49}$$

Pairing is done based on RGA and NI

$$K = \begin{bmatrix} -1.6667 & -1 \\ -0.8333 & -1.6667 \end{bmatrix} \tag{7.50}$$

$$RGA = \begin{bmatrix} 1.4283 & -0.4283 \\ -0.4283 & 1.4283 \end{bmatrix} \tag{7.51}$$

NI for this system is 0.5833. From Hazarika and Chidambaram (2014) the tuning parameters are 2.0726, 2.0726 respectively. As the number of open loop unstable poles of $G_p(s)$ is different from $\overline{G}_p(s)$ = diag[gp,ii(s)]. A different paring criterion is used. As the calculated NI shows a positive value, interchanging of columns results in

$$G_P(S) = \begin{bmatrix} \frac{-1e^{-s}}{-1.6667s+1} & \frac{-1.6667e^{-s}}{-2.6s+1} \\ \frac{-1.6667e^{-s}}{-1.6667s+1} & \frac{-0.8333e^{-s}}{-1.6667s+1} \end{bmatrix} \tag{7.52}$$

Now the relative gain array is calculated as

$$RGA = \begin{bmatrix} -0.4283 & 1.4283 \\ 1.4283 & -0.4283 \end{bmatrix} \tag{7.53}$$

Equivalent transfer function is calculated as

$$\hat{G}_p = \begin{bmatrix} \frac{2.3348e^{-s}}{-1.6667s+1} & \frac{-1.16711e^{-s}}{-2.6s+1} \\ \frac{-1.16711e^{-s}}{-1.6667s+1} & \frac{1.9456e^{-s}}{-1.6667s+1} \end{bmatrix} \tag{7.54}$$

Decouplers are designed as

$$D(s)=\begin{bmatrix} 1 & -1.6667 \\ -2.001 & 1 \end{bmatrix} \tag{7.55}$$

Controller with lead lag filter designed by Dasari et al. (2016) is

$$G_c(s)=\begin{bmatrix} -0.0139(1+\frac{1}{0.6667s}+0.25) & 0 \\ 0 & -0.0199(1+\frac{1}{0.6667s}+0.25) \end{bmatrix} \tag{7.56}$$

$$F^{'}(s)=\begin{bmatrix} \frac{28.3073s+1}{0.3673s+1} & 0 \\ 0 & \frac{24.3746s+1}{0.3487s+1} \end{bmatrix} \tag{7.57}$$

This designed controller is applied with set point weighting of magnitudes that is for first loop β_1=0.18, γ_1=0.6 and for the second loop β_1=0.2, γ_1=0.63 present in the process. Figure 7.36 demonstrate the regulatory responses. The corresponding IAE values for different perturbations are given in Table 7.6. It is observed that the proposed method is robust. The IAE values for all three methods are given in Table 7.6 and 7.7 and as noticed in first example, the proposed method proved to perform better for this example as well.

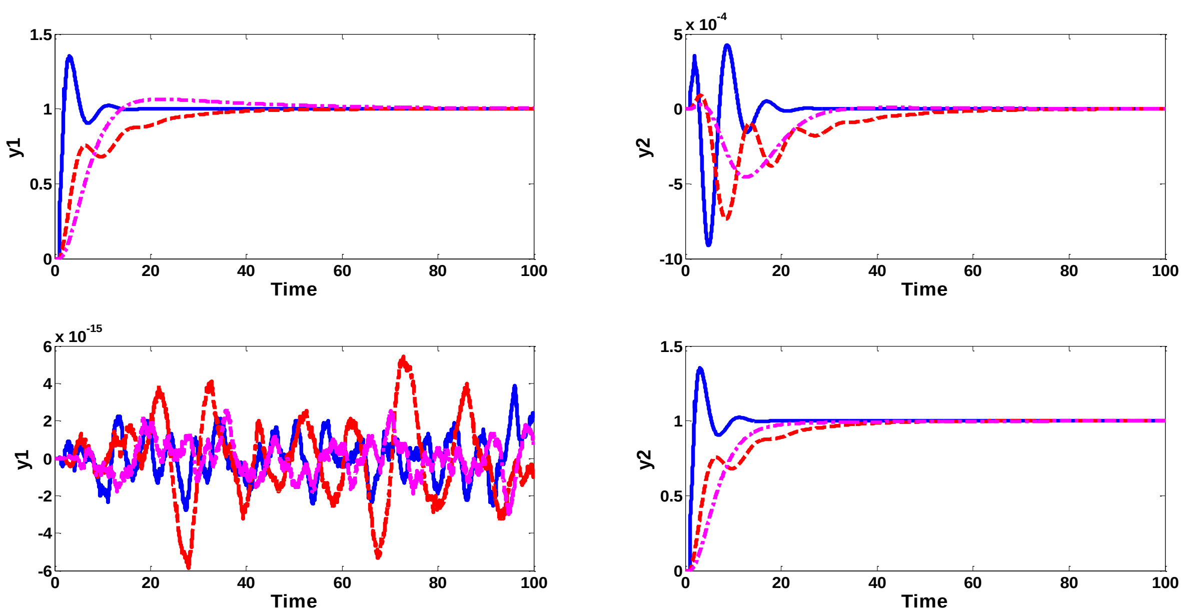

Figure 7.35 Output and control action behaviour for servo under exact model conditions for y_1 and y_2 for example 9, solid – Present work, dash – (Hazarika and Chidambaram, 2014), dash dot– (Dasari et al., 2016).

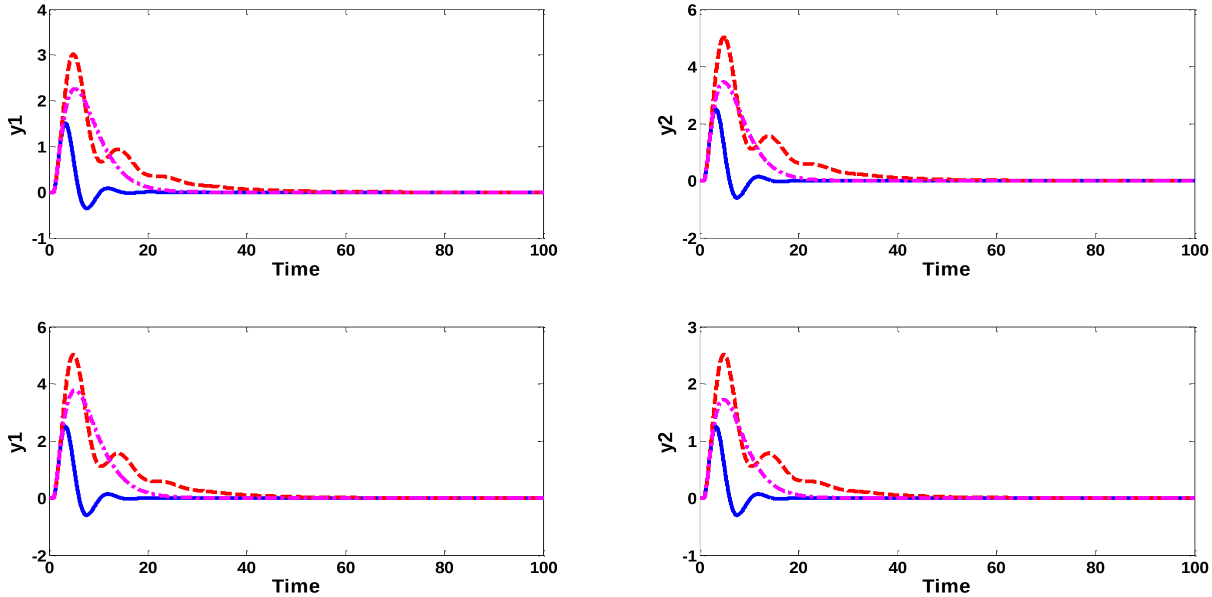

Figure 7.36 Output and control action behaviour for regulatory under exact model conditions for y_1 and y_2 for example 9, solid – Present work, dash – (Hazarika and Chidambaram, 2014), dash dot– (Dasari et al., 2016).

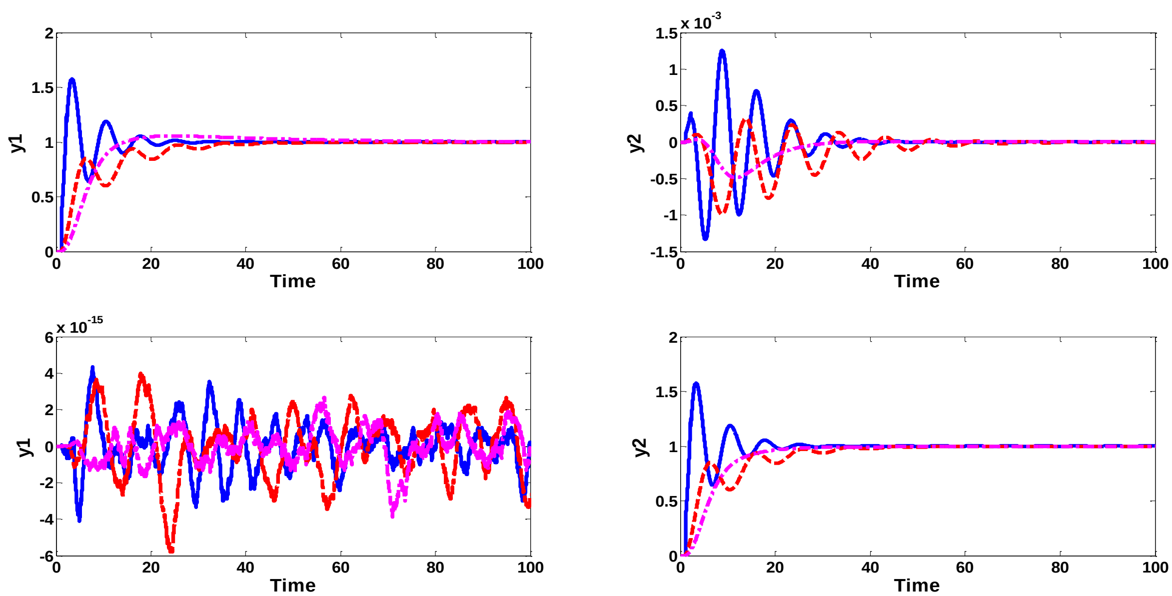

Figure 7.37 Output and control action behaviour for servo under 10% uncertainties to time delay for y_1 and y_2 for example 9, solid – Present work, dash – (Hazarika and Chidambaram, 2014), dash dot– (Dasari et al., 2016).

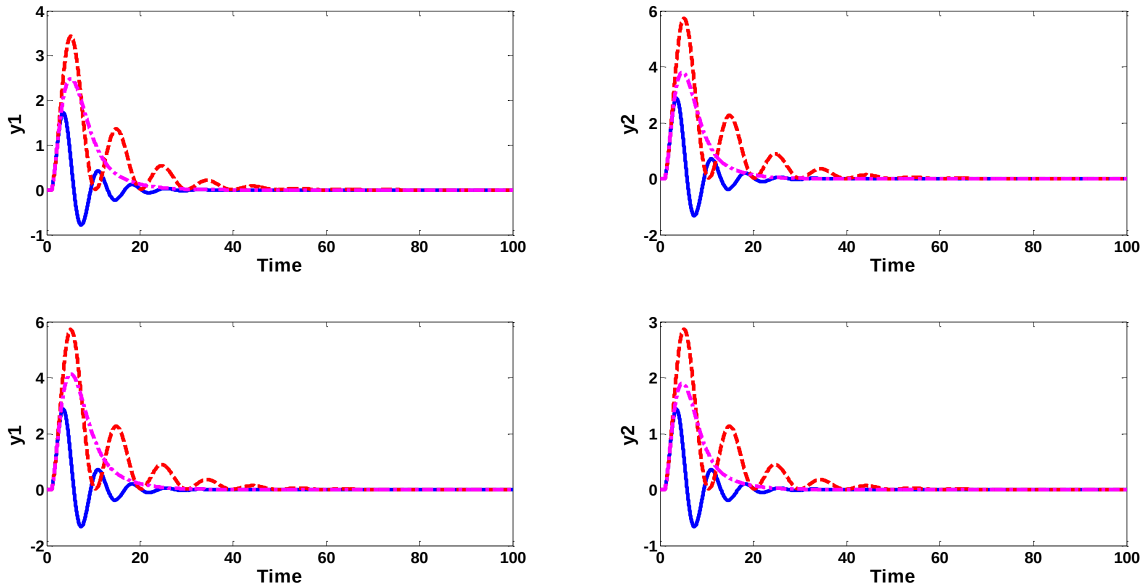

Figure 7.38 Output and control action behaviour for regulatory under 10% uncertainties to time delay for y_1 and y_2 for example 9, solid – Present work, dash – (Hazarika and Chidambaram, 2014), dash dot– (Dasari et al., 2016).

Table 7.6 Analysis of IAE values for servo responses for example 9

Perturbations	**Proposed method**				**Hazarika et al.**				**Dasari et al.**			
	loop1		**loop2**		**loop1**		**loop2**		**loop1**		**loop2**	
	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2
kp	2.41	0.00	0.00	2.41	7.91	0	0	7.89	8.8	0.01	0	7.734
1.1kp	2.33	0.00	0.00	2.33	10.43	0.01	0	10.42	9.22	0.01	0	9.30
0.9kp	3.80	0.00	0.00	3.81	4.82	0.01	0	4.8	9.87	0.011	0	7.39
τ	2.41	0.00	0.00	2.41	7.91	0	0	7.89	8.8	0.01	0	7.734
1.1τ	2.44	0.00	0.00	2.45	7.91	0.01	0	7.89	9.31	0.01	0	7.74
0.9τ	2.99	0.00	0.00	2.99	7.91	0.01	0	7.89	8.35	0.01	0	7.74
θ	2.41	0.00	0.00	2.41	7.91	0	0	7.89	8.8	0.01	0	7.734
1.1θ	4.39	0.01	0.00	4.39	7.91	0.01	0	7.89	8.47	0.01	0	7.74
0.9θ	1.85	0.00	0.00	1.85	7.91	0.01	0	7.89	9.12	0.01	0	7.74

Table 7.7 Analysis of IAE values for regulatory responses for example 9

Perturbations	Proposed method				Hazarika et al.				Dasari et al.			
	loop1		loop2		loop1		loop2		loop1		loop2	
	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2
kp	5.6	9.35	9.34	4.66	27.81	46.37	46.35	23.17	20.58	28.75	34.29	14.37
1.1kp	4.5	7.51	7.5	3.75	27.79	46.34	46.32	23.16	20.58	28.75	34.29	14.37
0.9kp	8.39	14	13.9	6.9	27.81	46.37	46.35	23.17	27.44	37.05	45.73	18.47
τ	5.6	9.35	9.34	4.66	27.81	46.37	46.35	23.17	20.58	28.75	34.29	14.37
1.1τ	5.85	9.78	9.76	4.88	27.81	46.37	46.35	23.17	20.86	29.15	34.77	14.56
0.9τ	5.91	9.87	9.86	4.93	27.81	46.37	46.34	23.17	20.58	28.75	34.29	14.37
θ	5.6	9.35	9.34	4.66	27.81	46.37	46.35	23.17	20.58	28.75	34.29	14.37
1.1θ	8.69	14.5	14.49	7.24	27.81	46.37	46.34	23.17	20.58	28.75	34.29	14.37
0.9θ	5.06	8.45	8.43	4.21	27.81	46.37	46.35	23.17	20.72	28.96	34.54	14.47

7.8 Summary

The determination of set point weighting parameters β and γ of a PID controller for unstable first and second order systems with time delay and TITO unstable process is found to be simple using the technique presented. Comprehending simulation results shown on different transfer function models, it is recognized that the overshoot and undershoot are reduced significantly in the present work. Assessment of the results with Nasution et al. (2011) and Hazarika and Chidambaram (2014) emphasized that, the present technique is easy to implement because of the simple calculation procedure to obtain the set weighting parameters. The same is observed for TITO systems when a comparison is made between Hazarika and Chidambaram (2014) method and Dasari et al. (2016) method. Moreover, an improvement is observed based on the ability to give a stable result even when the process parameters are subjected to varied perturbations. Further, the IAE and TV values are used to provide a quantitative assessment of the results obtained.

Chapter 8

Summary and Conclusions

Chapter 8

Summary and Conclusions

8.1 Summary

In this research, an enhanced design of optimal H_2 PID controller for unstable process with time delay is addressed. A simple PID design method with and without set point filter and set point weighting are proposed. An experimental application of these PID design strategies is performed on an inverted pendulum. The proposed control strategy is applied to cascade control structure. Finally, a method is proposed for the set-point weighted PID controllers for SISO and MIMO unstable time-delay systems. The results obtained in each section are summarized below.

8.1.1 Analytical PID tuning rules for unstable processes

H_2 minimization theory in combination with internal model control (IMC) is used to analytically derive novel PID controller settings which can be used as ready reference like look-up tables. These analytical settings are developed for a defined range of time delay to time constant ratio. Maximum sensitivity (M_s) is used for evaluating the robustness of the closed loop systems. Case studies are considered for unstable systems to evaluate the closed loop performances for set point variations and separately for load variations. Robustness is evaluated for uncertainities in the process model. Recently published methods in the literature are considered for performance comparison with the proposed method. Based on several simulation results, it is observed that the current methodology provides significantly enhanced performances when compared with those techniques available in the recent literature.

8.1.2 Experimental implementation on an inverted pendulum

A H_2 minimization based IMC-PID controller has been proposed for controlling the angle of an inverted pendulum. The identification of the process has been carried out based on available model parameters and the controller is designed based on the model. The designed controller provides a good set point tracking and a good disturbance rejection. The proposed controller also shows good performances under experimental implementation.

8.1.3 Design of cascade control systems for unstable processes with time delay

H_2 minimization based IMC-PID controller design for enhanced control of unstable series cascade scheme is proposed. The scheme consists of secondary loop and primary loop. The secondary loop controller is designed based on the IMC principles. The primary controller is designed to obtain the H_2 optimal performance. From the simulation studies, it is observed that proposed method provides enhanced closed loop performance when compared to the reported method in the literature. To analyze the robustness, the closed loop performances of the process model were analyzed in the presence of uncertainties.

8.1.4 PID controller design for multivariable unstable processes

IMC based PID controller with lead-lag filter is designed for multivariable unstable processes. The design is based on H_2 optimal closed loop transfer function for set point changes and step input disturbances. The individual controllers are designed based on the corresponding equivalent transfer function (ETF) model. Simplified decouplers are designed based on the ETF model. Two examples are considered to show the closed loop responses. The proposed method provides significantly improved closed loop performances for regulatory problem when compared to the methods in the literature.

8.1.5 Enhanced set-point weighting design for unstable systems

A method is proposed to design the set-point weighting parameters which is relatively simple and also reduces the overshoot significantly. Weighting is considered for both proportional (β) and derivative (γ) terms in the PID control law. In the closed loop relation for set-point tracking, the coefficients of 's' and separately 's^3' both in the numerator and denominator are made equal in order to find β and γ. The obtained expressions for β and γ are simple and depends on the controller parameters. The design is carried out first for single input single output (SISO) unstable first order and second order processes with time delays and then for multi input multi output (MIMO) unstable processes. In MIMO process control, decouplers are considered to ensure that the loops have minimum interactions. With the designed values, the closed loop performance is evaluated for different SISO and MIMO unstable processes with time delay. The present method is also compared with the recent methods proposed in the literature and it is observed that enhanced closed loop performances are achieved with the proposed method.

8.2 Conclusions

8.2.1. Robust analytical relations for PID controller are developed for time delayed unstable systems. These rules can be used like look-up tables by the operators for tuning of PID controllers. For unstable systems, it is very crucial to select the tuning parameters to acquire stable responses. Robustness always requires lower values of Ms which is usually not easy to achieve for such systems. The tuning parameter is selected to achieve minimum possible value of M_s and analytical formula is given to calculate λ. Further, the developed simple tuning formulae provide fair and enhanced performances. The present methods can be utilized as look up tables for selection of the PID controller tuning parameters. Analytical formula is provided to determine λ based on μ/n. The current methodology is relatively simple and can be applied for any system with the right half plane pole. Comparative analysis has also been carried out using IAE and TV. The evaluated responses of the current design reduces 30% IAE values when compared with existing techniques, especially when μ/n is significant. One more important asset of the current methodology is that the controller output responses are not sluggish and provides minimal TV values.

8.2.2 The Experimental implementation on an inverted pendulum to control pendulum rod angle control with PID controller shows good closed loop performances in tracking the set point with faster settling time (reduced by 64%) when compared to the method proposed by Begum et al. (2018) and Cho et al. (2014). The proposed method is also robust in the presence of disturbances affecting the cart position.

8.2.3 Enhanced design of controllers is proposed for unstable time delayed cascade processes. The proposed method consists of only two controllers whereas in the previous methods, at least two or three controllers were used. The present design is comparatively easy and can be implemented for any unstable cascade system. The ability to provide good stable closed loop response even when there are large amount of perturbations in the process parameters is a major advantage of the proposed method over previously existing methods. Quantitative comparison is carried out using IAE and TV values and the proposed method reduces 54% IAE values and 68% TV over the existing methods. Performance of the system for the proposed method is much better than that of the previously existing methods particularly for the disturbance rejection. One more main advatange of the proposed method is that the control action respones are smooth in all the examples and correspondingly provides low TV values which is recommended for any control system.

8.2.4 Multivariable PID controller is series with lead lag filter is applied based on the equivalent transfer function (ETF) model for unstable multivariable systems with time delay. The method uses simplified decouplers which decompose the unstable multi-loop systems into independent loops with ETFs as the resulting decoupled process model having unstable poles. To reduce the undesirable overshoot, set point weighting is used. Two simulation examples are studied and showed that the present method provides significantly (reduced by 56% IAE values) improved closed loop regulatory performances when compared to the methods in the literature. However, the set-point tracking performances are not better for the proposed method

8.2.5 The determination of set point weighting parameters β and γ of a PID controller for unstable first and second order systems with time delay and TITO unstable process is found to be simple using the technique presented. Comprehending the simulation results shown on different transfer function models, it is recognized that the overshoot, undershoot and 16% TV values are reduced compared with existing techniques. Assessment of the results with Nasution et al. (2011) and Hazarika and Chidambaram (2014) emphasized that, the present technique is easy to implement because of the simple calculation procedure to obtain the set weighting parameters. The same is observed for TITO systems when a comparison is made between Hazarika and Chidambaram, (2014) method and Dasari et al. (2016) method. Moreover, an improvement is observed based on the ability to give a stable result even when the process parameters are subjected to varied perturbations. Further, the IAE and TV values are used to provide a quantitative assessment of the results obtained.

8.3 Suggestions for Future Work

i. H_2 minimization based IMC-PID control technique can be extended to higher order unstable processes models.

ii. Parallel cascade control strategy with underdamped IMC filter structure can be developed for improved responses.

iii. Experimental implementation can also be carried out on the other unstable processes such as (a) ball and beam and (b) magnetic levitation systems.

iv. Control of Multivariable square as well as non-square unstable systems with more than two inputs and outputs can be studied.

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APPENDIX – A

Definition of constraints for section 3.2:

$a_1 = 18.167\,(T)^3 - 17.95\,(T)^2 + 5.6733\,(T) + 1.428$ for $0.1 \le (T) \le 0.4$

$a_1 = 0.1667\,(T)^3 - 0.3357\,(T)^2 - 0.053\,(T) + 2.0527$ for $0.4 < (T) \le 0.8$

$a_1 = -0.333\,(T)^3 + 0.8\,(T)^2 - 0.9167\,(T) + 2.272$ for $0.8 < (T) \le 1.2$

$b_1 = -2.88\,(T)^2 + 2.203\,(T) - 0.8444$ for $0.1 \le (T) \le 0.3$

$b_1 = -0.485\,(T)^2 + 0.8697\,(T) - 0.6597$ for $0.3 < (T) \le 0.6$

$b_1 = 0.066\,(T)^3 - 0.3245\,(T)^2 + 0.6276\,(T) - 0.5864$ for $0.6 < (T) \le 1.2$

$c_1 = -0.0455\,(T)^2 - 3.267\,(T) - 0.6616$ for $0.1 \le (T) \le 0.3$

$c_1 = -0.7633\,(T)^3 + 1.654\,(T)^2 - 1.544\,(T) - 0.426$ for $0.3 < (T) \le 0.7$

$c_1 = -0.2148\,(T)^3 - 0.272\,(T)^2 - 0.262\,(T) + 0.1358$ for $0.7 < (T) \le 1.2$

$a_2 = 0.2733$ for $0.1 \le (T) \le 0.3$

$a_2 = 0.8202$ for $0.3 < (T) \le 0.7$

$a_2 = -0.0116\,(T)^2 + 0.015\,(T) + 1.3621$ for $0.7 < (T) \le 1.2$

$b_2 = 2.682$ for $0.1 \le (T) \le 0.8$

$b_2 = 2.682$ for $0.8 < (T) \le 1.1$

$b_2 = 2.684$ for $1.1 < (T) \le 1.2$

$c_2 = 0.0905\,(T) + 0.7166$ for $0.1 \le (T) \le 0.3$

$c_2 = 0.276\,(T) + 2.1473$ for $0.3 < (T) \le 0.7$

$c_2 = 0.283\,(T)^2 + 0.0848\,(T) + 3.7066$ for $0.7 < (T) \le 1.2$

$a_3 = -0.8025\,(T)^2 + 1.2018\,(T) - 0.6698$ for $0.1 \le (T) \le 0.4$

$a_3 = -0.4936\,(T)^2 + 0.9362\,(T) - 0.6127$ for $0.4 < (T) \leq 0.8$

$a_3 = 0.275\,(T)^3 - 1.064\,(T)^2 + 1.3443\,(T) - 0.7144$ for $0.7 < (T) \leq 1.2$

$b_3 = -0.045\,(T)^2 + 0.3773\,(T) + 0.2469$ for $0.1 \leq (T) \leq 0.4$

$b_3 = 0.2667\,(T)^3 + 0.14\,(T)^2 + 0.3447\,(T) + 0.247$ for $0.4 < (T) \leq 0.8$

$b_3 = 0.625\,(T)^3 - 2.188\,(T)^2 + 2.3704\,(T) - 0.339$ for $0.8 < (T) \leq 1.2$

$c_3 = -0.525\,(T)^2 - 0.2445\,(T) + 0.3935$ for $0.1 \leq (T) \leq 0.4$

$c_3 = -0.505\,(T)^2 - 0.2009\,(T) + 0.382$ for $0.4 < (T) \leq 0.8$

$c_3 = -0.346\,(T)^2 + 0.0206\,(T) + 0.3038$ for $0.8 < (T) \leq 1.2$

APPENDIX – B

%......Closed loop response of UFOPTD system-Proposed Method ....%

%--------------**Example_1 for Chapter 3**---------------%

```
clc
clear all

steptime = 0;                % step time value
stepmag = 1;                 % step input change
Noise=0;

%Example 1
% km = -0.017; thetam =2.4; taum = 5.8;          % Process model parameters
% kp1 = -0.017; thetap1 = 2.4; taup1 = 5.8;
% % give lamda and mf values

% lamda = 3.87;mf1=1;mf2=1;mf3=1;
% epsilon = 1;%mean=0;variance=0.0025;
%
% flag=0;                    % Specify 0 for Perfect model and 1 for Perturbations
% if flag==0
%     tsim=120; disttime = 60; distmag =50;
% elseif flag==1
%     tsim=1000; disttime = 500; distmag = 20;
% end
% if flag==0
%     kp = -0.017; thetap = 2.4; taup = 5.8;
% elseif flag==1
%     kp = -0.017*0.9; thetap = 2.4*1.1; taup = 5.8*0.9;
% end


 %----------- Controller equations of the present method-----------%

 X=thetam/taum

if (X>=0.1) && (X<=0.4)
   a1   =18.167*X^3-17.95*X^2+5.6733*X+1.428;
   else if (X>0.4) && (X<=0.8)
   a1  = 0.1667*X^3-0.3357*X^2-0.053*X+2.0527;
   else if (X>0.8) && (X<=1.2)
   a1  =-0.3333*X^3+0.8*X^2-0.9167*X+2.272;
   else
```

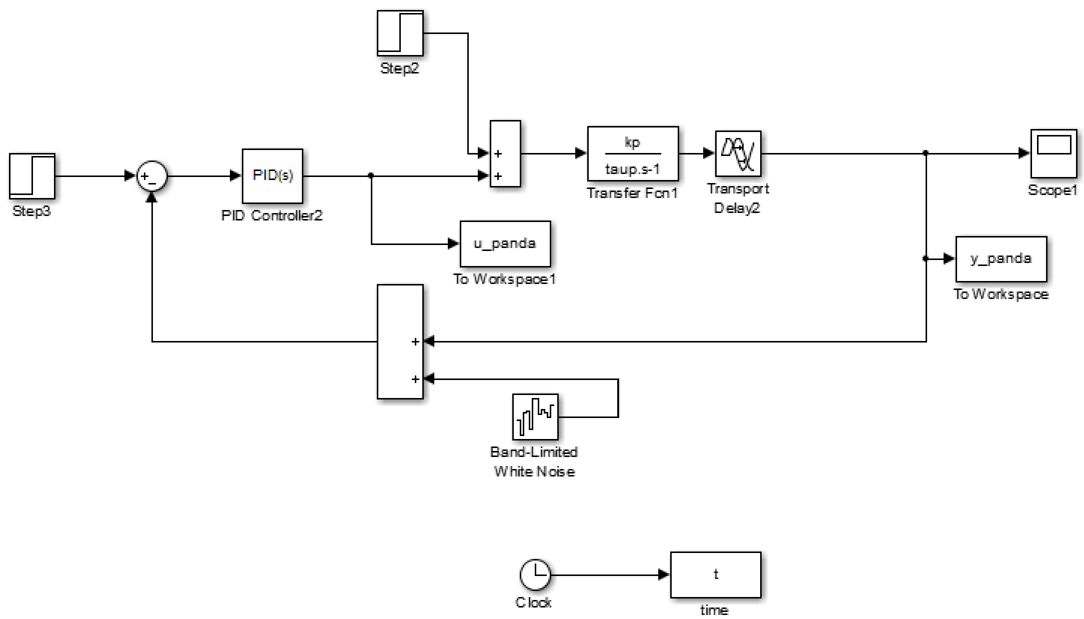
```
      disp('rules are applicable for only 0.1<=theta/taup<=1.2');
       end
       end
end


if (X>=0.1) && (X<=0.3)

      b1   =-2.88*X^2+2.203*X-0.8444;

else if (X>0.3) && (X<=0.6)

      b1   =-0.485*X^2+0.8697*X-0.6597;

else if (X>0.6) && (X<=1.2)
      b1   =0.0667*X^3-0.3245*X^2+0.6276*X-0.5864;

    end
    end
end

  if (X>=0.1) && (X<=0.3)
      c1   =0.0455*X^2-3.267*X+0.6616;
      else if (X>0.3) && (X<=0.7)
             c1=-0.7633*X^3+1.6539*X^2-1.5449*X+0.4259;
             else if (X>0.7) && (X<=1.2)
                    c1   =0.2148*X^3-0.2715*X^2-0.262*X+0.1358;
                end
          end
   end


if (X>=0.1) && (X<=0.3)
 a2   =0.2733;
else if (X>0.3) && (X<=0.7)
 a2   =0.8202;
else if (X>0.7) && (X<=1.2)

a2   =-0.0116*X^2+0.0151*X+1.3621;
end
    end
end
 if (X>=0.1) && (X<=0.8)
 b2   =2.682;
 else if (X>0.8) && (X<=1.1)
b2   =2.683;
```

```
else if (X>1.1) && (X<=1.2)
b2   =2.684;
end
    end
end

if (X>=0.1) && (X<=0.3)
 c2   =0.0905*X+0.7166;

else if (X>0.3) && (X<=0.7)
 c2   =0.276*X+2.1473;
 else if (X>0.7) && (X<=1.2)
 c2   =0.283*X^2+0.0848*X+3.7066;
 end
    end
end

if (X>=0.1) && (X<=0.4)

 a3   =-0.8025*X^2+1.2018*X-0.6698;
 else if (X>0.4) && (X<=0.8)
 a3   =-0.4936*X^2+0.9362*X-0.6127;
 else if (X>0.8) && (X<=1.2)
 a3   =0.275*X^3-1.0643*X^2+1.3443*X-0.7144;

        end
        end
end


if (X>=0.1) && (X<=0.4)

 b3   =-0.045*X^2+0.3773*X+0.2469;
else if (X>0.4) && (X<=0.8)
 b3   =-0.2667*X^3+0.14*X^2+0.3447*X+0.2473;
else if (X>0.8) && (X<=1.2)
 b3   =0.625*X^3-2.1886*X^2+2.3704*X-0.3396;

        end
        end
end

if (X>=0.1) && (X<=0.4)
c3   =0.525*X^2-0.2445*X+0.3935;
else if (X>0.4) && (X<=0.8)
c3   =0.505*X^2-0.2009*X+0.382;
```

```
else if (X>0.8) && (X<=1.2)
c3   =0.3468*X^2+0.0206*X+0.3038;

        end
        end
end
 kc= ((a1*((lamda/taum)^b1)+c1)/km)*mf1;
ti= ((a2*((lamda/taum)^b2)+c2)*taum)*mf2;
td=  ((a3*((lamda/taum)^b3)+c3)*taum)*mf3;

Gc_Present_Method = [kc ti td];

% %----------------------------Simulation(Simulink) --------------------------------%


sim('PresentMethod3',tsim);

% sim('Present Method',tsim);

%--------------------Calculation of IAE & TV----------------------------------%


IAE_Present=sum(abs(stepmag-y_PM)*0.01);
TV_Present=sum(abs(diff(u_PM)));
   ;         %...Total variation....%

subplot(2,1,1)
  plot(t,y_PM,'b','linewidth',2)
  hold on
  xlabel('Time'); ylabel('Outputs');
subplot(2,1,2)
  plot(t,u_PM,'b','linewidth',2)
    hold on
  xlabel('Time'); ylabel('Control actions');


%-------------------Calculationof Ms------------------%


% kp1 = 1; thetap1 = 0.5; taup1 = 1;
ww=logspace(-2,2,300);
for ii=1:300
    w=ww(ii);
    xx = j*w;
    Gp = kp1*exp(-thetap1*xx)/(taup1*xx-1); % Unstable FOPTD Process
    Gc = (kc*(1+(1/(ti*xx))+(td*xx)/(0.01*td*xx+1)));
```

```
   S(ii) = abs((1+Gc*Gp)^(-1)); % Sensitivity Function Description
 end
n = length(S);
for k=1:n
  if S(k+1)>=S(k)
     Mss=S(k+1);      % maxium value of sensitivity
  else
     break;
  end
end
Ms_Present = Mss

%-----------------------Display-------------------------------------%


ALL = [IAE_Present TV_Present Ms_Present];
disp('     PID Controller Settings Calculated          ');
disp('*********************************************************');
fprintf('\tKc\t        Ti\t      Td\n');
fprintf('%8.4f\t%9.4f\t %6.4f  \n',Gc_Present_Method);
disp('**********************************************************');
disp('     IAE, TV and Ms Values Calculated          ');
disp('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~');
fprintf('\tIAE_Present\t        TV_Present\t      Ms_Present\n');
fprintf('%12.4f\t%16.4f\t %13.4f  \n',ALL);
disp('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~');
```

Simulink block diagram-UFOPTD_Ex1



APPENDIX – C

%......Optimal H_2 IMC-PID controller with set-point weighting for cascaded Time-Delayed Unstable process for SOUDP with two unstable pole.......%

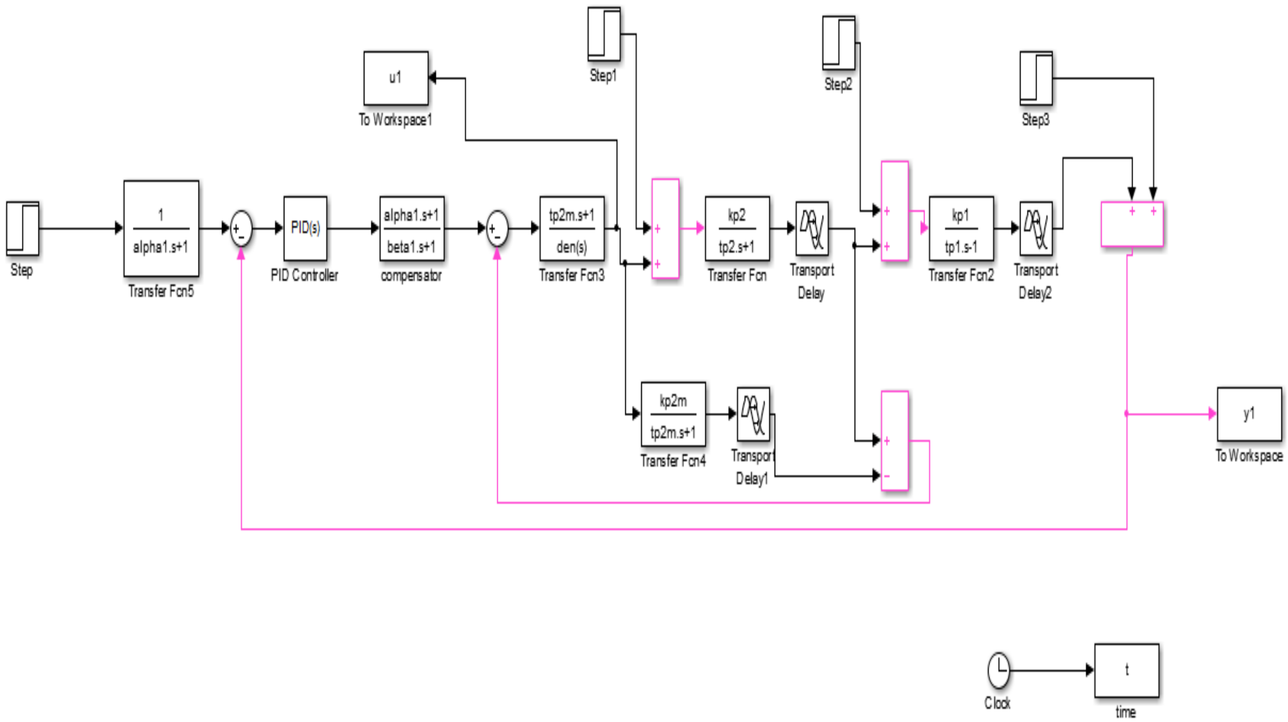
%--------------**Example_4 for chapter 5**---------------%

```
clear all
clc

% G2(s)=kp2*exp(-d2*s)/(tp2*s+1)%
% .... G(s)= kp*exp(-d*s)/(t1*s-1)(t2*s-1)......%

flag=1;    %flag = 0 (Perfect Model)
        %flag = 1 (Perturbations of )
steptime = 0; stepmag = 1;

if flag==0
   tsim=1400;
   disttime1 = 500;   % Secondary loop
   distmag1 = -0.5;
   disttime2 = 800;    % Primary loop before process
   distmag2 = -0.2;
   disttime3 = 0;      % Primary loop after the process
   distmag3 = 0;
elseif flag==1
   tsim=1400;
   disttime1 = 500;   % Secondary loop
   distmag1 = -0.5;
   disttime2 = 800;    % Primary loop before process
   distmag2 = -0.2;
   disttime3 = 0;      % Primary loop after te process
   distmag3 = 0;
end

if flag==0
   kp2 = 1; d2 = 0.5; tp2 = 3; % Secondary process
   kp1 = 3.433; d1 = 20; tp1 = 103.1; % Primary process
elseif flag==1
   kp2 = 1; d2 = 0.5*1.3; tp2 = 3; % Secondary process
   kp1 = 3.433; d1 = 20*1.3; tp1 = 103.1; % Primary process
end
```

```
%----------- Controller equations of the present method-----------%


kp2m = 1; d2m = 0.5; tp2m = 3;  % Secondary loop model

lamda2 = 0.5*d2;
lamda = 22;%6.6

kp = -3.433; d = 20.5; t1 = 103.1; t2 = -lamda2; % Primary loop model

%...........let us define a,b,x,y as follows for simplicity........%
a=(t2*(exp(d/t2)-1));
b=(t1*(exp(d/t1)-1));
x=(t1*t2*(a-b)*(1/(t1^2)))+(((t1*b)-(t2*a))*(1/t1))+(t1-t2);
y=(t1*t2*(a-b)*(1/(t2^2)))+(((t1*b)-(t2*a))*(1/t2))+(t1-t2);

%......filter coefficients alpha1 and alpha2 where
F(s)=(alpha2*(s^2)+alpha1*s+1)/((lamda*s+1)^4)....%

alpha1=(((((lamda/t1)+1)^4)*(t1-t2)*t1*(exp(d/t1))*(1/x)))-t1;
a1 = -(t1+t2);
a2 = (t1*t2);
D = t1-t2;
p = t1*t2*(t1-t2-(t1*(exp(d/t1)))+(t2*(exp(d/t2))));
q = ((t1^2)*(exp(d/t1))-(t2^2)*(exp(d/t2))+(t2^2)-(t1^2));
R = (4*lamda*D)+(d*D)-(alpha1*D)-(q);
X1=((6*D*(lamda^2))+(2*d*lamda*D)-(p)-(alpha1*q)+(0.5*d*q)+(0.5*d*D*alpha1))/R;
beta1 = (X1-a1-(0.5*d));
kc = (q/(kp*R));
tr = (q/D);
td = (p/q);
Gc = [kc tr td alpha1 beta1]

tff = 2/td;



% %----------------------------Simulation(Simulink) --------------------------------%

sim('Ex4_CSTR',tsim);

%%%%%% ERROR VALUES CALCULATION %%%%%%%%%%%%%%%%%%%%%%

st = 0.01; % sampling time
IAE = sum(abs(1-y1)*st)
TV = sum(abs(diff(u1)));         % Total Variation...%
r = [IAE;TV];
```

```
%______________________________________________________________________
%-----------------------Display-------------------------------------%

disp('    IAE and TV Values       ');
disp('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~');
fprintf('\tIAE\t        TV\t\n');
fprintf('%8.4f\t%8.4f\t   \n',r);
disp('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~');
%------------------------------------------------------------------%%
subplot(2,1,1)
   plot(t,y1,'r--','LineWidth',2)
   xlabel('Time','fontsize',12,'fontweight','b')
   ylabel('Closed loop output, y','fontsize',12,'fontweight','b')
hold on
   subplot(2,1,2)
   plot(t,u1,'r--','LineWidth',2)
   xlabel('Time','fontsize',12,'fontweight','b');
   ylabel('Control action, u','fontsize',12,'fontweight','b')
hold on
return
```

Simulink block diagram-Cascade Control_Ex_4



APPENDIX – D

%......Optimal H_2 IMC-PID controller for unstable multivariable processes ....%

%--------------**Example_1_Servo Case for chapter 6**---------------%

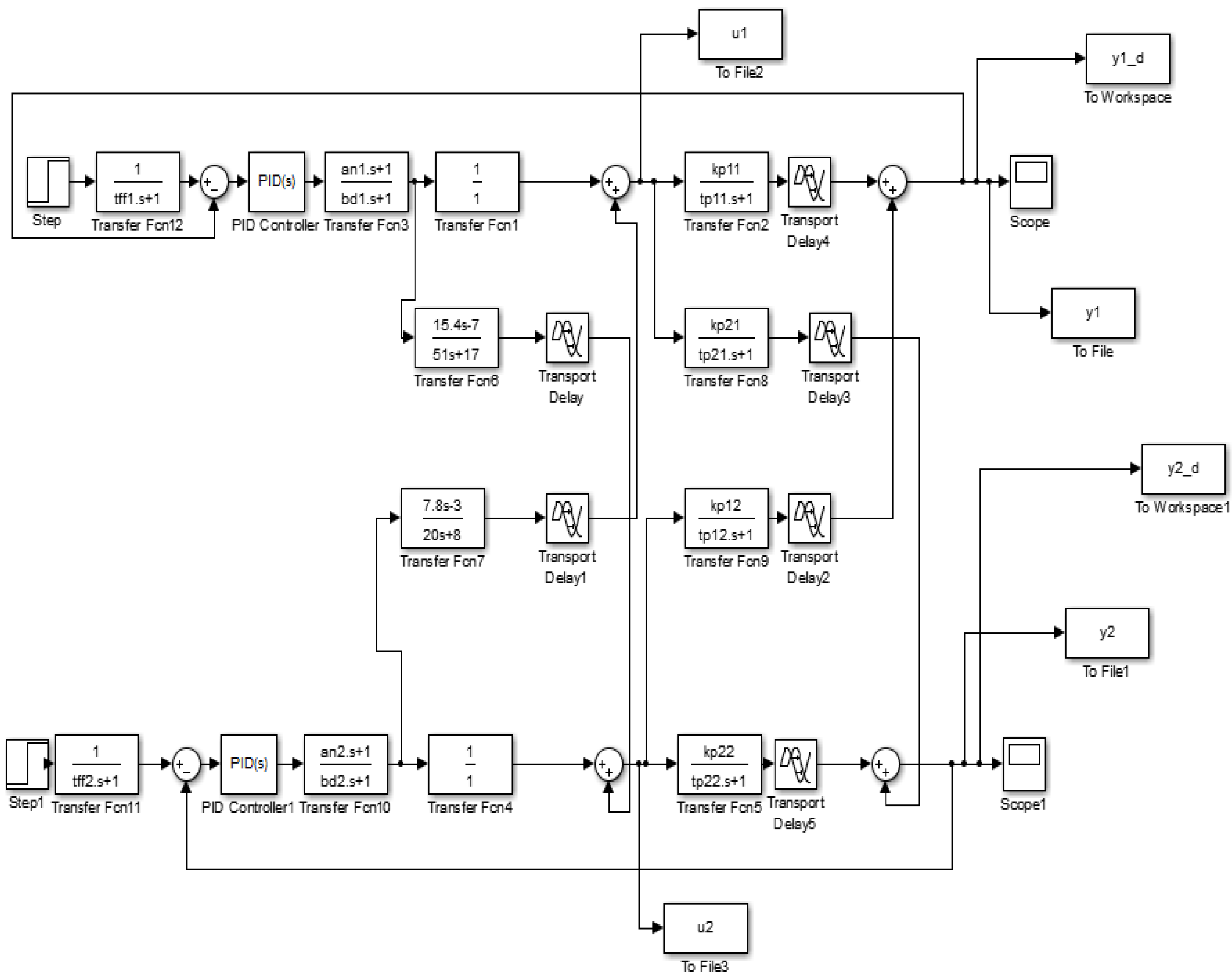
```
clear all;
clc;

kp11 = 1.6;
tp11 = -2.6;
thetap11 = 1*.1;

kp12 = 0.6;
tp12 = 2.5;
thetap12 = 1.5*1.1;

kp21 = 0.7;
tp21 = 3;
thetap21 = 1.5*1.1;

kp22 = 1.7;
tp22 = -2.2;
thetap22 = 1*1.1;

km1 = -1.3529; tm1 = 2.4396; thetam1 = 0.9383;
lamda1 = 2.7;   %% Recommonded value is 2.1

km2 = -1.437; tm2 = 2.0643; thetam2 = 0.9383;
lamda2 = 2.5;

tff1 = 25;
tff2 = 25;
tsim=100;
%%%%Proposed method%%%%%%%%

%----------- Controller equations of the present method-----------%


x1=(exp(thetam1/tm1)-1)*tm1;     %...for simplicity define x..%

ann1 =(lamda1)*((lamda1/tm1)^2+(3*(lamda1/tm1))+3) ;

an1 = 1*ann1;
```

```
kc1=-(4*thetam1)/(km1*((18*lamda1)+(6*thetam1)-(6*an1)-(6*x1)));

tau_i1=(2*thetam1)/3;

tau_d1=thetam1/4;

bdd1 =((18*(lamda1^2))+(12*lamda1*thetam1)+(thetam1^2)+(2*thetam1*an1)-
(6*an1*x1)+(2*thetam1*x1))/((18*lamda1)+(6*thetam1)-(6*an1)-(6*x1))+tm1-x1;
 bd1 = bdd1;

kcp1 = round(kc1*10000)/10000;
tau_ip1 = round(tau_i1*10000)/10000;
tau_dp1 = round(tau_d1*10000)/10000;
anpp1 = round(an1*10000)/10000;
bdp1 = round(bd1*10000)/10000;

 Gc_Proposed_rounded1 = [kcp1 tau_ip1 tau_dp1 anpp1 bdp1]
 Gc_proposed1 = [kc1 tau_i1 tau_d1 an1 bd1]




x2=(exp(thetam2/tm2)-1)*tm2;     %...for simplicity define x..%

ann2 =(lamda2)*((lamda2/tm2)^2+(3*(lamda2/tm2))+3) ;

an2 = 1*ann2;

kc2=-(4*thetam2)/(km2*((18*lamda2)+(6*thetam2)-(6*an2)-(6*x2)));

tau_i2=(2*thetam2)/3;

tau_d2=thetam2/4;

bdd2 =((18*(lamda2^2))+(12*lamda2*thetam2)+(thetam2^2)+(2*thetam2*an2)-
(6*an2*x2)+(2*thetam2*x2))/((18*lamda2)+(6*thetam2)-(6*an2)-(6*x2))+tm2-x2;
 bd2 = bdd2;

kcp2 = round(kc2*10000)/10000;
tau_ip2 = round(tau_i2*10000)/10000;
tau_dp2 = round(tau_d2*10000)/10000;
anpp2 = round(an2*10000)/10000;
bdp2 = round(bd2*10000)/10000;

 Gc_Proposed_rounded2 = [kcp2 tau_ip2 tau_dp2 anpp2 bdp2]
 Gc_proposed2 = [kc2 tau_i2 tau_d2 an2 bd2]
```

```
stepmag1 = 1;
stepmag2 = 0;
stepmag3 = 0;
stepmag4 = 0;

  sim('servoex1',tsim);
  load y1;
  subplot(2,2,1)
  plot(z(1,:),z(2,:),'linewidth',2);
  hold on

  sim('doublex1',tsim);
  load y1;
  subplot(2,2,1)
  plot(z(1,:),z(2,:),'r--','linewidth',2);
  xlabel('Time');
  ylabel('y1');

  sim('servoex1',tsim);
  load y2;
  subplot(2,2,2)
  plot(q(1,:),q(2,:),'linewidth',2);
  xlabel('Time');
  ylabel('y2');
  hold on

  sim('doublex1',tsim);
  load y2;
  subplot(2,2,2)
  plot(q(1,:),q(2,:),'r--','linewidth',2);


stepmag1 = 0;
stepmag2 = 1;
stepmag3 = 0;
stepmag4 = 0;

  sim('servoex1',tsim);
  load y1;
  subplot(2,2,3)
  plot(z(1,:),z(2,:),'linewidth',2);
  hold on

  sim('doublex1',tsim);
  load y1;
```

```
subplot(2,2,3)
plot(z(1,:),z(2,:),'r--','linewidth',2);
xlabel('Time');
ylabel('y1');

sim('servoex1',tsim);
load y2;
subplot(2,2,4)
plot(q(1,:),q(2,:),'linewidth',2);
xlabel('Time');
ylabel('y2');
hold on

sim('doublex1',tsim);
load y2;
subplot(2,2,4)
plot(q(1,:),q(2,:),'r--','linewidth',2);
```

Simulink block diagram-Unstable multivariable processes _Servo_Ex_1



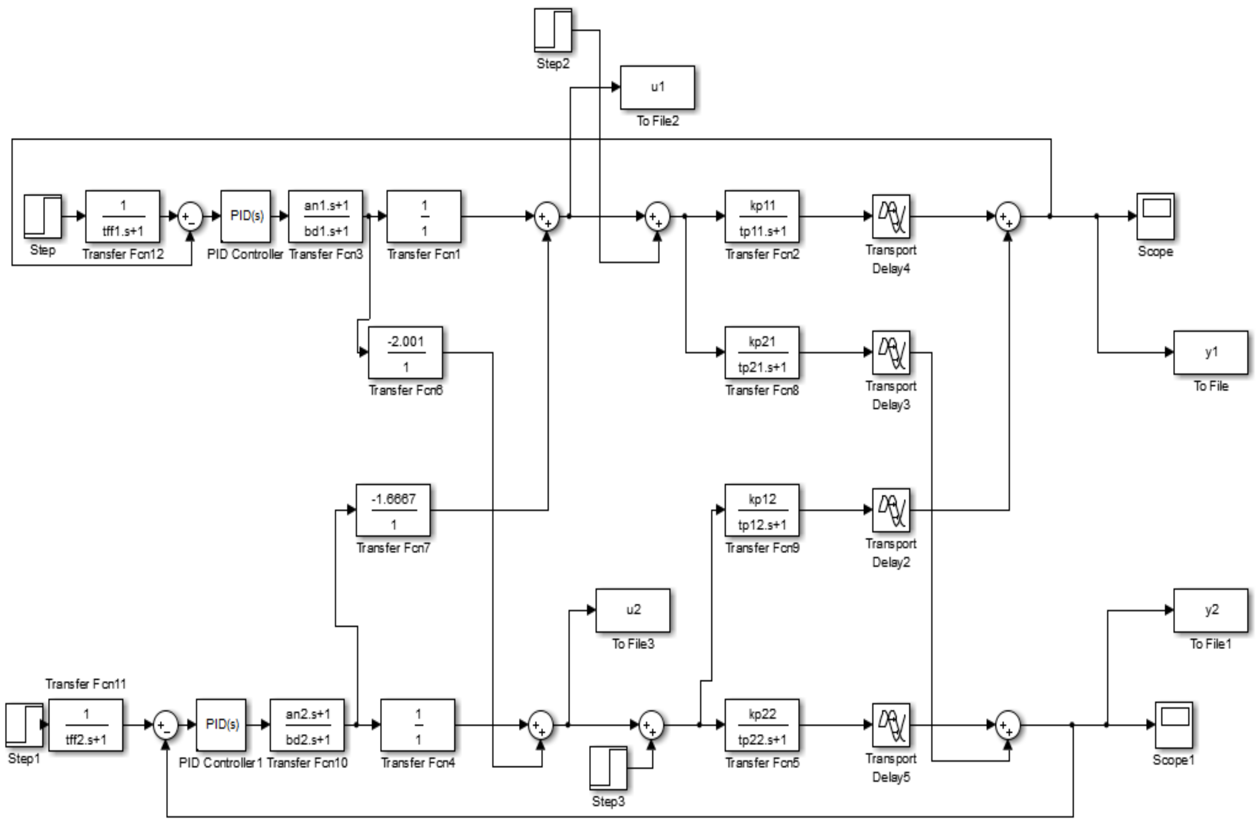
%--------------**Example_2_Regulatory Case for** U**nstable multivariable processes** ------------%

```
clear all
clc

kp11 = -1;
tp11 = -1.6667;
thetap11 = 1*1.1;

kp12 = -1.6667;
tp12 = -1.6667;
thetap12 = 1*1.1;

kp21 = -1.6667;
tp21 = -1.6667;
thetap21 = 1*1.1;

kp22 = -0.8333;
tp22 = -1.6667;
thetap22 = 1*1.1;

km1 = -2.3348; tm1 = 1.6667; thetam1 = 1;
lamda1 = 2.7;   %% Recommonded value is 2.1

km2 = -1.9456; tm2 = 1.6667; thetam2 = 1;
lamda2 = 2.5;

tff1 = 25;
tff2 = 25;
stepmag1 = 0;
stepmag2 = 0;
stepmag3=1;
stepmag4=0;
tsim=100;
%%%%Proposed method%%%%%%%%

x1=(exp(thetam1/tm1)-1)*tm1;     %...for simplicity define x..%

ann1 =(lamda1)*((lamda1/tm1)^2+(3*(lamda1/tm1))+3) ;

an1 = 1*ann1;

kc1=-(4*thetam1)/(km1*((18*lamda1)+(6*thetam1)-(6*an1)-(6*x1)));

tau_i1=(2*thetam1)/3;
```

```
tau_d1=thetam1/4;

bdd1 =((18*(lamda1^2))+(12*lamda1*thetam1)+(thetam1^2)+(2*thetam1*an1)-
(6*an1*x1)+(2*thetam1*x1))/((18*lamda1)+(6*thetam1)-(6*an1)-(6*x1))+tm1-x1;
 bd1 = bdd1;

kcp1 = round(kc1*10000)/10000;
tau_ip1 = round(tau_i1*10000)/10000;
tau_dp1 = round(tau_d1*10000)/10000;
anpp1 = round(an1*10000)/10000;
bdp1 = round(bd1*10000)/10000;

 Gc_Proposed_rounded1 = [kcp1 tau_ip1 tau_dp1 anpp1 bdp1]
 Gc_proposed1 = [kc1 tau_i1 tau_d1 an1 bd1]

x2=(exp(thetam2/tm2)-1)*tm2;     %...for simplicity define x..%

ann2 =(lamda2)*((lamda2/tm2)^2+(3*(lamda2/tm2))+3) ;

an2 = 1*ann2;

kc2=-(4*thetam2)/(km2*((18*lamda2)+(6*thetam2)-(6*an2)-(6*x2)));

tau_i2=(2*thetam2)/3;

tau_d2=thetam2/4;

bdd2 =((18*(lamda2^2))+(12*lamda2*thetam2)+(thetam2^2)+(2*thetam2*an2)-
(6*an2*x2)+(2*thetam2*x2))/((18*lamda2)+(6*thetam2)-(6*an2)-(6*x2))+tm2-x2;
 bd2 = bdd2;
 kcp2 = round(kc2*10000)/10000;
tau_ip2 = round(tau_i2*10000)/10000;
tau_dp2 = round(tau_d2*10000)/10000;
anpp2 = round(an2*10000)/10000;
bdp2 = round(bd2*10000)/10000;

 Gc_Proposed_rounded2 = [kcp2 tau_ip2 tau_dp2 anpp2 bdp2]
 Gc_proposed2 = [kc2 tau_i2 tau_d2 an2 bd2]
    sim('regulex2',tsim);
   load y1;
   subplot(2,2,1)
   plot(z(1,:),z(2,:),'linewidth',2);
   hold on
      load y2;
   subplot(2,2,2)
   plot(q(1,:),q(2,:),'linewidth',2);
```

```
  xlabel('Time');
  ylabel('y2');
  hold on
    sim('regul2ex2',tsim);
  load y1y;
  subplot(2,2,1)
  plot(y1y(1,:),y1y(2,:),'r--','linewidth',2);
  xlabel('Time');
  ylabel('y1');

  load y2y;
  subplot(2,2,2)
  plot(y2y(1,:),y2y(2,:),'r--','linewidth',2);

stepmag1 = 0;
stepmag2 = 0;
stepmag3=0;
stepmag4=1;
   sim('regulex2',tsim);
  load y1;
  subplot(2,2,3)
  plot(z(1,:),z(2,:),'linewidth',2);
  hold on

  load y2;
  subplot(2,2,4)
  plot(q(1,:),q(2,:),'linewidth',2);
  xlabel('Time');
  ylabel('y2');
  hold on

  sim('regul2ex2',tsim);
  load y1y;
  subplot(2,2,3)
  plot(y1y(1,:),y1y(2,:),'r--','linewidth',2);
  xlabel('Time');
  ylabel('y1');


  load y2y;
  subplot(2,2,4)
  plot(y2y(1,:),y2y(2,:),'r--','linewidth',2);
```

Simulink block diagram-Unstable multivariable processes _Regulatory_Ex_2



APPENDIX – E

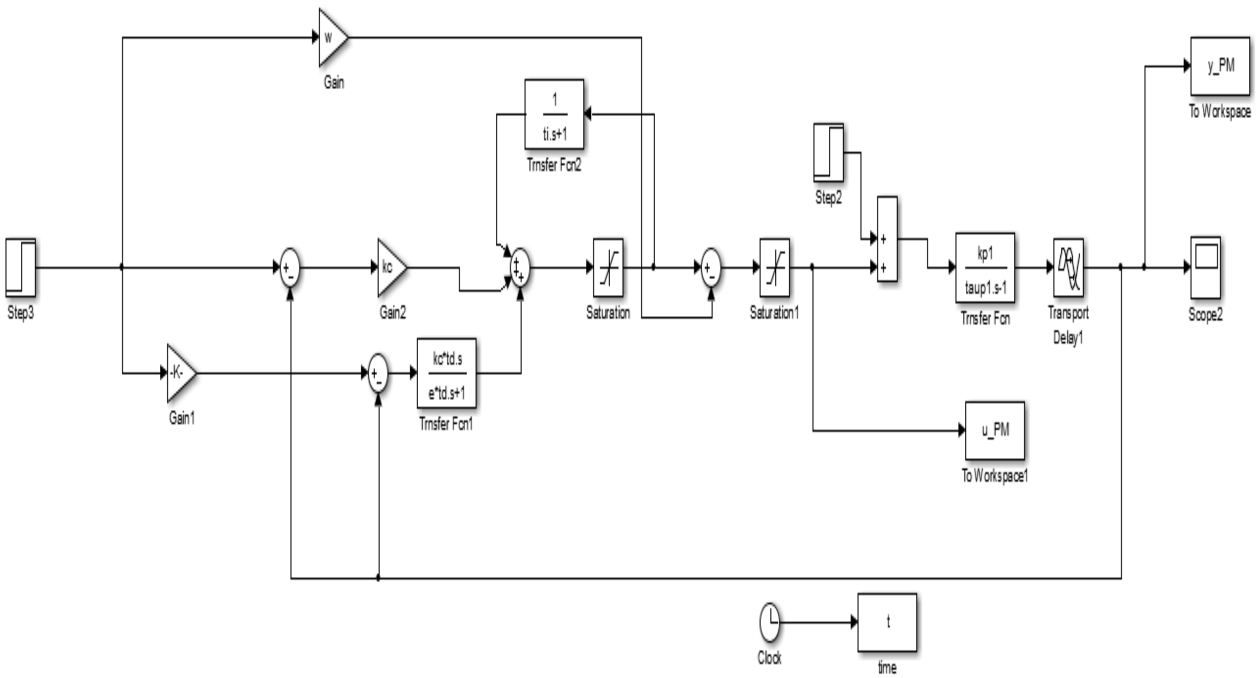
%......Dynamic Set-Point Weighting Design for SISO ....%

%--------------**Example_1 for chapter 7**---------------%

```
clc
clear all
steptime = 0; stepmag = 1;e=0.1;
 % Example:1
kp = 1; thetap = 1.2; taup = 1;
%tsim=50;
flag=1;    % Specify 0 for Perfect model and 1 for Perturbations
if flag==0
    tsim=80; disttime = 40; distmag =0.1;
elseif flag==1
    tsim=100; disttime = 50; distmag =0.1;
end
if flag==0
    kp1 = 1; thetap1 = 1.2; taup1 = 1;
elseif flag==1
    kp1 = 1; thetap1 = 1.2*1.05; taup1 = 1;
end
  lamda=3;
kc= 1.15;
ti= 63.5;
td=  0.56;
p1=0.7;
```

```
p2=0.3;

beta=(p1*(1-(1/(kc*kp))-(thetap/ti)))   %for FOPTD

% beta =(p1+(p1/(kc*kp))-((0.5*thetap)/ti)*(1+p1)+((p1-1)/ti))   %FOR USOPTD

% gama= p2*(1-(0.5*(1/ti)*(thetap+1))+(0.5*thetap)/(kc*kp))

% gama= p2*(1+(taup-(0.5*thetap))/(taup*kc*kp)-(((0.5*thetap)*(1+beta))/td)) %for% FOPTD
s^2


gama=   p2*(((taup)/((kc*kp)*td))-1) %for FOPTD s^3

% gama=0;

w =(1-beta)*kc;

% m=((kc*td)*(1-gama));

sim('PresentMethod_setpoint2',tsim);


IAE_Present=sum(abs(stepmag-y_PM)*0.01)

TV_Present = sum(abs(diff(u_PM)))          %...Total variation....%

ITAE_present= sum((abs(stepmag-y_PM))*tsim*0.01)

ISE_present= sum((abs(stepmag-y_PM).^2)*0.01)

a=[0:0.01:tsim];

S = stepinfo(y_PM,a)

 subplot(2,1,1)

 plot(t,y_PM,'b','linewidth',4)

  hold on

  xlabel('Time'); ylabel('Outputs');

subplot(2,1,2)

  plot(t,u_PM,'b','linewidth',4)

    hold on

  xlabel('Time'); ylabel('Control actions');
```

Simulink block diagram- Dynamic Set-Point Weighting Design for SISO _Ex_1



%......Dynamic Set-Point Weighting Design for MIMO ....%

%--------------**Example_1 for chapter 7**---------------%

```
clear all
clc

% example 1
pb1=1;pb2=1;pb3=1;
kp11 = 1.6*pb1;
tp11 = -2.6*pb2;
thetap11 = 1*pb3;

kp12 = 0.6*pb1;
tp12 = 2.5*pb2;
thetap12 = 1.5*pb3;

kp21 = 0.7*pb1;
tp21 = 3*pb2;
thetap21 = 1.5*pb3;

kp22 = 1.7*pb1;
tp22 = -2.2*pb2;
thetap22 = 1*pb3;

km1 = -1.3529; tm1 = 2.4396; thetam1 = 0.9383;
km2 = -1.437; tm2 = 2.0643; thetam2 = 0.9383;

e=0.1;
tff1=10;
tff2=15;

% lamda1 = 1.5;
% lamda2 = 1.5;
 mf1=1;mf2=1;mf3=1;


%-----------------------------------------------

X1=thetam1/tm1

if (X1>=0.1) && (X1<=0.4)
    a1    =18.167*X1^3-17.95*X1^2+5.6733*X1+1.428;
    else if (X1>0.4) && (X1<=0.8)
    a1   = 0.1667*X1^3-0.3357*X1^2-0.053*X1+2.0527;
    else if (X1>0.8) && (X1<=1.2)
```

```
    a1   =-0.3333*X1^3+0.8*X1^2-0.9167*X1+2.272;
    else
       disp('rules are applicable for only 0.1<=theta/taup<=1.2');
        end
        end
end


if (X1>=0.1) && (X1<=0.3)

       b1   =-2.88*X1^2+2.203*X1-0.8444;

else if (X1>0.3) && (X1<=0.6)

       b1   =-0.485*X1^2+0.8697*X1-0.6597;

else if (X1>0.6) && (X1<=1.2)
       b1   =0.0667*X1^3-0.3245*X1^2+0.6276*X1-0.5864;

    end
    end
end

  if (X1>=0.1) && (X1<=0.3)
       c1   =0.0455*X1^2-3.267*X1+0.6616;
       else if (X1>0.3) && (X1<=0.7)
               c1=-0.7633*X1^3+1.6539*X1^2-1.5449*X1+0.4259;
               else if (X1>0.7) && (X1<=1.2)
                       c1   =0.2148*X1^3-0.2715*X1^2-0.262*X1+0.1358;
                   end
           end
   end




if (X1>=0.1) && (X1<=0.3)
 a2   =0.2733;
else if (X1>0.3) && (X1<=0.7)
 a2   =0.8202;
else if (X1>0.7) && (X1<=1.2)

a2   =-0.0116*X1^2+0.0151*X1+1.3621;
end
    end
end
```

```
 if (X1>=0.1) && (X1<=0.8)
 b2   =2.682;
 else if (X1>0.8) && (X1<=1.1)
b2   =2.683;
else if (X1>1.1) && (X1<=1.2)
b2   =2.684;
end
    end
end

if (X1>=0.1) && (X1<=0.3)
 c2   =0.0905*X1+0.7166;

else if (X1>0.3) && (X1<=0.7)
 c2   =0.276*X1+2.1473;
 else if (X1>0.7) && (X1<=1.2)
 c2   =0.283*X1^2+0.0848*X1+3.7066;
 end
    end
end


if (X1>=0.1) && (X1<=0.4)

 a3   =-0.8025*X1^2+1.2018*X1-0.6698;
 else if (X1>0.4) && (X1<=0.8)
 a3   =-0.4936*X1^2+0.9362*X1-0.6127;
 else if (X1>0.8) && (X1<=1.2)
 a3   =0.275*X1^3-1.0643*X1^2+1.3443*X1-0.7144;

        end
        end
end


if (X1>=0.1) && (X1<=0.4)

 b3   =-0.045*X1^2+0.3773*X1+0.2469;
else if (X1>0.4) && (X1<=0.8)
 b3   =-0.2667*X1^3+0.14*X1^2+0.3447*X1+0.2473;
else if (X1>0.8) && (X1<=1.2)
 b3   =0.625*X1^3-2.1886*X1^2+2.3704*X1-0.3396;

        end
        end
end
```

```
 if (X1>=0.1) && (X1<=0.4)
c3   =0.525*X1^2-0.2445*X1+0.3935;
else if (X1>0.4) && (X1<=0.8)
c3   =0.505*X1^2-0.2009*X1+0.382;
else if (X1>0.8) && (X1<=1.2)
c3   =0.3468*X1^2+0.0206*X1+0.3038;

        end
        end
end


lamda1 = (2.0957*(X1)^2 + 0.9634*(X1) - 0.0889)*(tm1)
kc1= ((a1*((lamda1/tm1)^b1)+c1)/km1)*mf1;
tau_i1= ((a2*((lamda1/tm1)^b2)+c2)*tm1)*mf2;
tau_d1=  ((a3*((lamda1/tm1)^b3)+c3)*tm1)*mf3;

Gc_Present_Method1 = [kc1 tau_i1 tau_d1];


% kc1=((kca*((lamda1/tm1)^kcb)+kcc)/km1)*mf1
% tau_i1=((tia*((lamda1/tm1)^tib)+tic)*tm1)*mf2
% tau_d1=((tda*((lamda1/tm1)^tdb)+tdc)*tm1)*mf3

%-------------------------------------------------------------------------------
X2=thetam2/tm2

if (X2>=0.1) && (X2<=0.4)
    a11    =18.167*X2^3-17.95*X2^2+5.6733*X2+1.428;
    else if (X2>0.4) && (X2<=0.8)
    a11   = 0.1667*X2^3-0.3357*X2^2-0.053*X2+2.0527;
    else if (X2>0.8) && (X2<=1.2)
    a11   =-0.3333*X2^3+0.8*X2^2-0.9167*X2+2.272;
    else
       disp('rules are applicable for only 0.1<=theta/taup<=1.2');
        end
        end
end


if (X2>=0.1) && (X2<=0.3)

       b11   =-2.88*X2^2+2.203*X2-0.8444;

else if (X2>0.3) && (X2<=0.6)
```

```
      b11   =-0.485*X2^2+0.8697*X2-0.6597;

else if (X2>0.6) && (X2<=1.2)
      b11   =0.0667*X2^3-0.3245*X2^2+0.6276*X2-0.5864;

   end
   end
end

  if (X2>=0.1) && (X2<=0.3)
      c11   =0.0455*X2^2-3.267*X2+0.6616;
      else if (X2>0.3) && (X2<=0.7)
             c11=-0.7633*X2^3+1.6539*X2^2-1.5449*X2+0.4259;
             else if (X2>0.7) && (X2<=1.2)
                   c11   =0.2148*X2^3-0.2715*X2^2-0.262*X2+0.1358;
                end
         end
  end



if (X2>=0.1) && (X2<=0.3)
 a22   =0.2733;
else if (X2>0.3) && (X2<=0.7)
 a22   =0.8202;
else if (X2>0.7) && (X2<=1.2)

a22   =-0.0116*X2^2+0.0151*X2+1.3621;
end
   end
end


if (X2>=0.1) && (X2<=0.8)
 b22   =2.682;
 else if (X2>0.8) && (X2<=1.1)
b22   =2.683;
else if (X2>1.1) && (X2<=1.2)
b22   =2.684;
end
   end
end

if (X2>=0.1) && (X2<=0.3)
 c22   =0.0905*X2+0.7166;
```

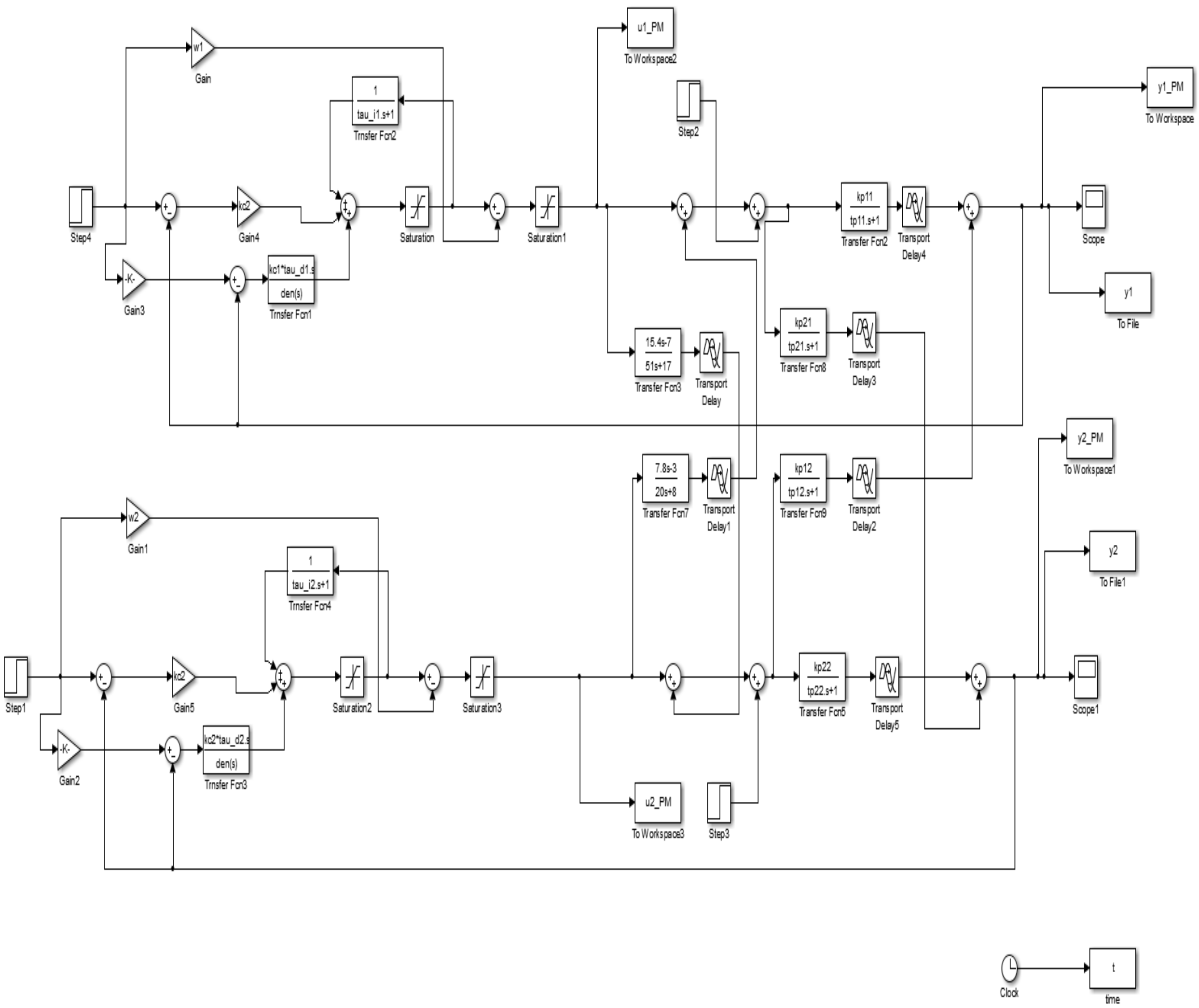
```
else if (X2>0.3) && (X2<=0.7)
 c22   =0.276*X2+2.1473;
 else if (X2>0.7) && (X2<=1.2)
 c22   =0.283*X2^2+0.0848*X2+3.7066;
 end
    end
end


if (X2>=0.1) && (X2<=0.4)

 a33   =-0.8025*X2^2+1.2018*X2-0.6698;
 else if (X2>0.4) && (X2<=0.8)
 a33   =-0.4936*X2^2+0.9362*X2-0.6127;
 else if (X2>0.8) && (X2<=1.2)
 a33   =0.275*X2^3-1.0643*X2^2+1.3443*X2-0.7144;

        end
        end
end


if (X2>=0.1) && (X2<=0.4)

 b33   =-0.045*X2^2+0.3773*X2+0.2469;
else if (X2>0.4) && (X2<=0.8)
 b33   =-0.2667*X2^3+0.14*X2^2+0.3447*X2+0.2473;
else if (X2>0.8) && (X2<=1.2)
 b33   =0.625*X2^3-2.1886*X2^2+2.3704*X2-0.3396;

        end
        end
end

if (X2>=0.1) && (X2<=0.4)
c33   =0.525*X2^2-0.2445*X2+0.3935;
else if (X2>0.4) && (X2<=0.8)
c33   =0.505*X2^2-0.2009*X2+0.382;
else if (X2>0.8) && (X2<=1.2)
c33   =0.3468*X2^2+0.0206*X2+0.3038;

        end
        end
end
```

```
 lamda2 = (2.0957*(X2)^2 + 0.9634*(X2) - 0.0889)*(tm2)
kc2= ((a11*((lamda2/tm2)^b11)+c11)/km2)*mf1;
tau_i2= ((a22*((lamda2/tm2)^b22)+c22)*tm2)*mf2;
tau_d2=  ((a33*((lamda2/tm2)^b33)+c33)*tm2)*mf3;

Gc_Present_Method2 = [kc2 tau_i2 tau_d2];

% kc2=((kca*((lamda2/tm2)^kcb)+kcc)/km2)*mf1
% tau_i2=((tia*((lamda2/tm2)^tib)+tic)*tm2)*mf2
% tau_d2=((tda*((lamda2/tm2)^tdb)+tdc)*tm2)*mf3
%-------------------------------------------------------------


p1=0.7;
p2=0.3;

beta1=(p1*(1-(1/(kc1*km1))-(thetam1/tau_i1)))
gama1=   p2*(((tm1)/((kc1*km1)*tau_d1))-1)
w1 =(1-beta1)*kc1;


beta2=(p1*(1-(1/(kc2*km2))-(thetam2/tau_i2)))
gama2=   p2*(((tm2)/((kc2*km2)*tau_d2))-1)
w2 =(1-beta2)*kc2;


stepmag1=1;
stepmag2=0;
stepmag3=0;
stepmag4=0;
tsim=100;

sim('setpoint_ex33',tsim);
  load y1;
  subplot(2,2,1)
  plot(z(1,:),z(2,:),'b','linewidth',4);
  hold on

  sim('doublex_11',tsim);
  load y1;
  subplot(2,2,1)
  plot(z(1,:),z(2,:),'r--','linewidth',4);
  xlabel('Time');
  ylabel('y1');

  sim('setpoint_ex33',tsim);
```

```
   load y2;
   subplot(2,2,2)
   plot(q(1,:),q(2,:),'b','linewidth',4);
   xlabel('Time');
   ylabel('y2');
   hold on

   sim('doublex_11',tsim);
   load y2;
   subplot(2,2,2)
   plot(q(1,:),q(2,:),'r--','linewidth',4);
fprintf('------------------------unit step change in setpoint of  y1 only----------------');
   IAE_Present_y1=sum(abs(stepmag1-y1_PM)*0.01)
   IAE_hazarika_y1=sum(abs(stepmag1-y1_hz)*0.01)
   TV_Present_y1 = sum(abs(diff(u1_PM)))
   TV_hazarika_y1 = sum(abs(diff(u1_hz)))
   IAE_Present_y2=sum(abs(stepmag2-y2_PM)*0.01)
   IAE_hazarika_y2=sum(abs(stepmag2-y2_hz)*0.01)
   TV_Present_y2 = sum(abs(diff(u2_PM)))
   TV_hazarika_y2 = sum(abs(diff(u2_hz)))


stepmag1 = 0;
stepmag2 =1;
stepmag3=0;
stepmag4=0;

   sim('setpoint_ex33',tsim);
   load y1;
   subplot(2,2,3)
   plot(z(1,:),z(2,:),'b','linewidth',4);
   hold on

   sim('doublex_11',tsim);
   load y1;
   subplot(2,2,3)
   plot(z(1,:),z(2,:),'r--','linewidth',4);
   xlabel('Time');
   ylabel('y1');

   sim('setpoint_ex33',tsim);
   load y2;
   subplot(2,2,4)
   plot(q(1,:),q(2,:),'b','linewidth',4);
   xlabel('Time');
   ylabel('y2');
```

```
   hold on

   sim('doublex_11',tsim);
   load y2;
   subplot(2,2,4)
   plot(q(1,:),q(2,:),'r--','linewidth',4);

fprintf('------------------------unit step change in setpoint of  y2 only----------------');

   IAE_Present_y1_mag2=sum(abs(stepmag1-y1_PM)*0.01)
   IAE_hazarika_y1_mag2=sum(abs(stepmag1-y1_hz)*0.01)
   TV_Present_y1_mag2 = sum(abs(diff(u1_PM)))
   TV_hazarika_y1_mag2 = sum(abs(diff(u1_hz)))
   IAE_Present_y2_mag2=sum(abs(stepmag2-y2_PM)*0.01)
   IAE_hazarika_y2_mag2=sum(abs(stepmag2-y2_hz)*0.01)
   TV_Present_y2_mag2 = sum(abs(diff(u2_PM)))
   TV_hazarika_y2_mag2 = sum(abs(diff(u2_hz)))

   Gc_Present_Method2 = [kc2 tau_i2 tau_d2]
   Gc_Present_Method1 = [kc1 tau_i1 tau_d1]

   fprintf('%f %f %f
%f',IAE_Present_y1,IAE_Present_y2,IAE_Present_y1_mag2,IAE_Present_y2_mag2)

   fprintf('\n%f %f %f
%f',IAE_hazarika_y1,IAE_hazarika_y2,IAE_hazarika_y1_mag2,IAE_hazarika_y2_mag2)
```

Simulink block diagram- Dynamic Set-Point Weighting Design for MIMO _Ex_1



APPENDIX – F

Nasution et al. (2011) method:

Controller Design for an unstable first order processes

Figure 1 depicts the IMC structure, in which $G_p(s)$ is the unstable process considered as first order in nature. $G_m(s)$ is the corresponding model. Q_c is IMC controller. In this study, the dynamics of the UFOPTD element is considered as

$$G_p(s) = \frac{k_p e^{-\theta s}}{\tau s - 1} \qquad (F.1)$$

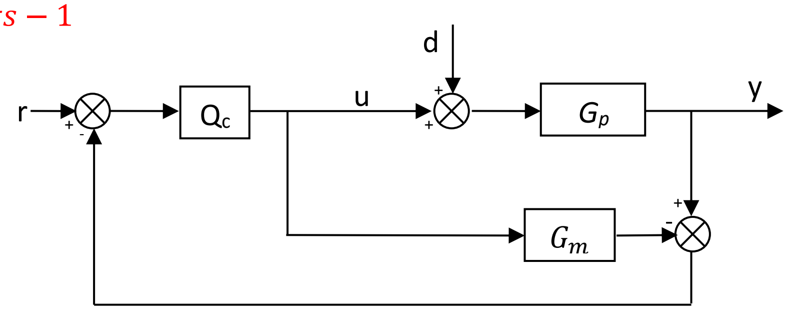


Figure F.1 Internal model control structure

As stated by the H_2 optimal controller design, the IMC controller Q_c is outlined as,

$$Q_C = \tilde{Q}_C F \qquad (F.2)$$

Where F is a filter which is used for altering the robustness of the controller. The filter structure should be selected such that the IMC controller Q_c is proper and realizable and also the control structure is internally stable. In addition to these requirements, it should be selected such that the resulting controller provides improved closed loop performance. $\tilde{Q}_c$ is designed for a specific type of step input type (v) to obtain H_2 optimal performance (Morari, M. and Zafiriou, 1989) and is based on the invertible portion of the process model. The process model and the input are divided as

$$G_m = G_{m-} G_{m+} \quad and \quad v = v_- v_+ \qquad (F.3)$$

where the sign ' – ' indicates invertible phase part and ' + ' indicates non invertible phase part where the Blaschke product of RHP poles of $G_m(s)$ and v(s) are denoted as

$$b_m = \prod_{i=1}^{k} \frac{-s+p_i}{s+\bar{p}_i} \quad and \quad b_v = \prod_{i=1}^{\tilde{k}} \frac{-s+p_i}{s+\bar{p}_i} \tag{F.4}$$

where p_i and $\bar{p}_i$ are the i[th] RHP pole and its conjugate, respectively. Based on this, the H_2 optimal controller is derived by using the following formula given by Morari and Zafiriou (1989).

$$\tilde{Q}_C = b_m (G_{m-} b_v v_-)^{-1} \{(b_m G_{m+})^{-1} b_v v_-\}|_* \tag{F.5}$$

where $\{....\}|_*$ is defined as the operator that operates by omitting all terms involving the poles of $(G_{m+})^{-1}$ after taking the partial fraction expansion. This idea is applied successfully by Nasution et al. (2011) for deriving the IMC based PID controller. The same derivation for obtaining the IMC controller Q_c for UFOPTD processes is given here for clear understanding. Considering perfect model case i.e., $G_p = G_m$, split the process model and input into minimum and non-minimum phase parts as

$$G_{m-} = \frac{-k_p}{\tau(-s+(1/\tau))} \; and \quad G_{m+} = e^{-\theta s} \tag{F.6}$$

$$v_- = \frac{-k_p}{\tau(-s+(1/\tau))s} \; and \quad v_+ = 1 \tag{F.7}$$

Based on Eq. F.6 and Eq. F.7, the Blaschke product is obtained as

$$b_m = \frac{(-s+(1/\tau))}{(s+(1/\tau))} \quad \text{and} \; b_v = \frac{(-s+(1/\tau))}{(s+(1/\tau))} \tag{F.8}$$

By using Eqs. F.6-F.8 in Eq. F.5, the controller is obtained

$$\tilde{Q}_c = \frac{(\tau s - 1)}{k_p} \left\{ \left(e^{\frac{\theta}{\tau}} - 1 \right) \tau s + 1 \right\} \tag{F.9}$$

Now, the filter F(s) has to be chosen properly as the closed loop performance and robustness is dependent on the form of the filter used. The new optimal filter is considered as

$$F(s) = (\alpha s + 1)/((\lambda^2 s^2 + 2\varsigma\lambda s + 1)(\lambda s + 1)) \tag{F.10}$$

The IMC controller as per Eq. F.2 is obtained as

$$Q_c = ((\tau s - 1)\{(e^{\frac{\theta}{\tau}} - 1)\,\tau s + 1\}(\alpha s + 1))/k_p(\lambda^2 s^2 + 2\varsigma\lambda s + 1)(\lambda s + 1)) \qquad (F.11)$$

The corresponding controller (G_c) in conventional feedback form according to Figure F.2 is obtained as

$$G_c(s) = Q_c/(1 - Q_c G_m) \qquad (F.12a)$$

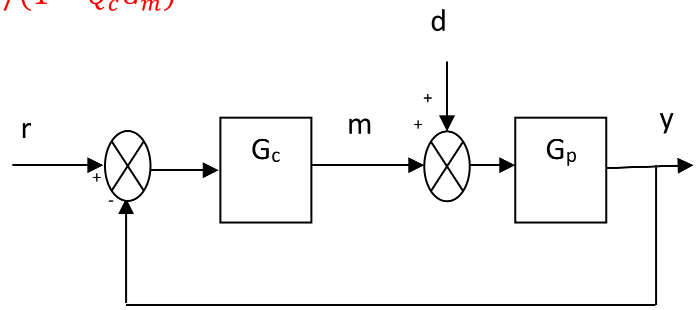


Figure F.2 Unity feedback system

After substituting all the terms, the controller G_c is derived as

$$G_c(s) = \frac{(\tau s - 1)\left\{\left(e^{\frac{\theta}{\tau}} - 1\right)\tau s + 1\right\}(\alpha s + 1)}{k_p\left[(\lambda s + 1)(\lambda^2 s^2 + 2\varsigma\lambda s + 1) - \left\{\left(e^{\frac{\theta}{\tau}} - 1\right)\tau s + 1\right\}(\alpha s + 1)e^{-\theta s}\right]} \qquad (F.12b)$$

However, Eq. F.12b is complex and need to be simplified for practical implementation. One can convert this into a simple controller of PID in nature or a PID in addition to a lead and lag filter by using suitable estimations.

Maclaurin series is used to convert the complex Eq. F.15b into a simple PID controller. To approximate using Maclaurin series, L(s) is defined as, $L(s) = sG_c(s)$. Expand L(s) using Maclaurin series

$$G_c(s) = \frac{1}{s}\left(L(0) + L'(0)s + \frac{L''(0)}{2!}s^2 + \cdots\right) \qquad (F.13)$$

The conventional controller is considered in the form of

$$G_c(s) = k_c(1 + \frac{1}{\tau_i s} + \tau_d s) \qquad \text{(F.14)}$$

On comparing Eqs. F.16 and F.17, the PID controller parameters are obtained as

$$k_c = L'(0), \quad \tau_i = \frac{L'(0)}{L(0)} and \quad \tau_d = \frac{L''(0)}{2L'(0)}$$

Wang et al. (2016) method:

Wang et al. (2016) have proposed a new IMC-PID tuning method based on pole zero conversion design and PID with a lead lag compensator is designed for first order plus integrating and second order unstable processes with time delay.

In this study, the dynamics of the UFOPTD element is considered as

$$G_p(s) = \frac{k_p e^{-\theta s}}{\tau s - 1} \qquad (F.15)$$

By using IMC-PID tuning method on pole zero conversion these tuning parameters find out

$$\alpha = \left(\lambda^2 + 2\lambda T + \tau T\right)/(T - \tau))$$

$$k_c = (\alpha + \lambda)/k_p(\alpha - 2\lambda - \tau)),$$

$$\tau_i = \alpha + \lambda$$

$$\tau_d = (0.5\alpha\lambda)/(\alpha + \lambda)),$$

Here imaginary filter is $(\lambda^2 + 2\lambda T + \tau T)/(Ts + 1)$

Hazarika and Chidambaram (2014) method:

Hazarika and Chidambaram (2014) designed multivariable proportional integral controllers for unstable multivariable systems and used equivalent transfer function model to design multivariable PI controllers for diagonal elements and simplified decouplers are used to decompose the unstable multi loop systems into independent loops and the double loop control structure is used to reduce the overshoot for unstable systems.

Design based ETF model development

The TITO block diagram with decouplers and controllers is shown in Figure F.3. If the second feedback controller is in the automatic mode, with y_{r2} = 0, then the overall closed-loop transfer function between y_1 and u_1 is

$$\frac{y_1}{u_1} = g_{p,11} - \frac{g_{p,12}g_{p,21}(g_{c,2}g_{p,22})}{(1+g_{c,2}g_{p,22})g_{p,22}} \qquad \text{(F.16)}$$

And similarly for the second loop, the relation can be written as

$$\frac{y_2}{u_2} = g_{p,22} - \frac{g_{p,21}g_{p,12}g_{c,1}g_{p,11}}{(1+g_{c,1}g_{p,11})g_{p,11}} \qquad \text{(F.17)}$$

Based on these relations, the ETF is derived as given in Hazarika and Chidambaram (2014).For obtaining ETFs, the controller need not be known apriori. Once the ETFs are obtained, the corresponding controller is designed.

Controller design

The open loop transfer function is

$$Y(s) = G_p(s)D(s)U(s) \qquad \text{(F.18)}$$

$$G_p(s)D(s) = \begin{bmatrix} g_{p,11} & g_{p,12} \\ g_{p,21} & g_{p,22} \end{bmatrix} \begin{bmatrix} 1 & d_{12} \\ d_{21} & 1 \end{bmatrix} = \begin{bmatrix} g^*_{p,11} & 0 \\ 0 & g^*_{p,22} \end{bmatrix} \qquad \text{(F.19)}$$

Where the simplified decoupler is designed as

$$d_{12}(s) = -\frac{g_{p,12}(s)}{g_{p,11}(s)}; d_{12}(s) = -\frac{g_{p,21}(s)}{g_{p,22}(s)} \tag{F.20}$$

For these systems, if there exist time delay, it may lead to unrealizable situations. Hence, an extra time delay (θ) is to be incorporated into the decoupler matrix which is further added to the corresponding ETF. In the presence of the decoupler, the TITO system behaves like two independent loops for which the controllers can be designed independently. In the present work, diagonal controllers are designed by optimal H_2 – IMC based method based on the corresponding unstable ETFs. ETFs are developed to take into account the loop interactions in the design of multi-loop control systems.

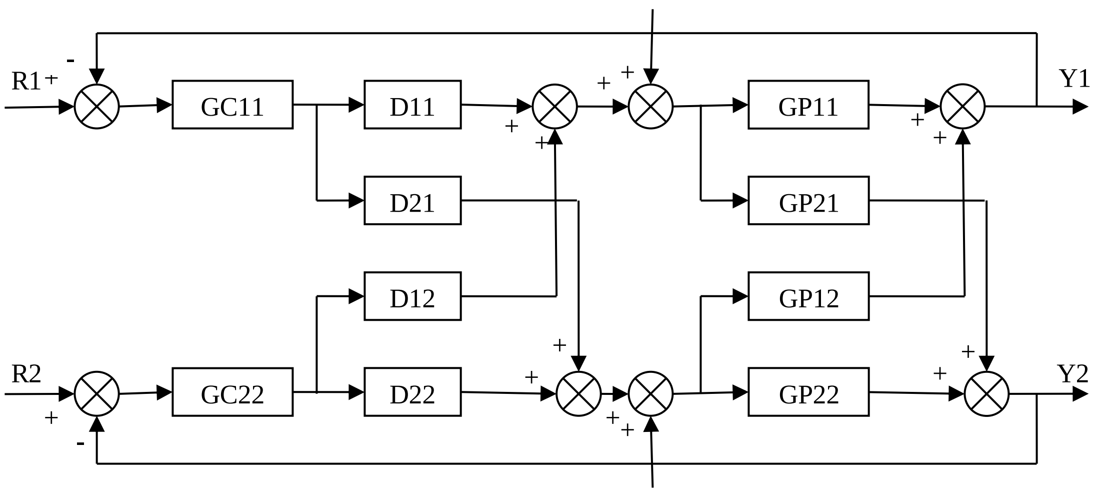


Figure F.3 Closed loop control for TITO system

Once the ETFs are derived, based on pairing using relative gain array and Neiderlinski Index, the corresponding controllers are designed. The design is based on unstable first order plus time delay system. Anusha and Rao (2012) developed a PID design method based on optimal-H_2 minimization concept for second order unstable processes.

List of Publications

International Journals

1. Purushottama Rao Dasari, Lavanya Alladi, A. Seshagiri Rao and ChangKyoo Yoo, Enhanced design of cascade control systems for unstable processes with time delay, Journal of Process Control, 2016, 45, 43-54.
2. Purushottama Rao Dasari, Raviteja K and A. Seshagiri Rao, Optimal H_2 - IMC based PID controller design for multivariable unstable processes, IFAC-PapersOnLine, 2016, 617–622.
3. Purushottama Rao Dasari, M. Chidambaram and A. Seshagiri Rao, Simple method of calculating dynamic set-point weighting parameters for time-delayed unstable processes, IFAC-PapersOnLine, 2018, 395–400.
4. Purushottama Rao Dasari and A. Seshagiri Rao, Enhanced dynamic set-point weighting design for two-input-two-output (TITO) unstable processes, Chemical Product and Process Modelling, Revision submitted, May 2019 (Under review).
5. Purushottama Rao Dasari and A. Seshagiri Rao, Enhanced dynamic set-point weighting design for SISO and MIMO unstable systems, Submitted to Chemical Engineering Research and Design, November 2018 (Under review).
6. Purushottama Rao Dasari and A. Seshagiri Rao, Development of Look-up Table like Analytical PID Rules for Control of Unstable Chemical and Biological Processes, Submitted to Chemical Engineering Communications, January 2019 (Under review).
7. Purushottama Rao Dasari, and A. Seshagiri Rao, A new method to design dynamic set-point weighting for unstable cascade processes, to submit to ISA Transactions, May 2019.

Conference Proceedings

1. Purushottama Rao Dasari, Raviteja K and A. Seshagiri Rao, Simple PID tuning method for unstable time delay processes, Indian Control Conference (ICC), IIT Guwahati, January 4 – 5, 2017. (IEEE Xplore, 2017, 403 – 408).
2. Purushottama Rao Dasari, Raviteja K, A. Seshagiri Rao, Improved control design for two input two output unstable processes, International Conference on Separation Technologies in Chemical, Biochemical, Petroleum, and Environmental Engineering (Technoscape), October 20-21,2016. (Resource Efficient Technologies, 2016, 2, 76–86).
3. Purushottama Rao Dasari, A. Seshagiri Rao, Effect of Enhanced dynamic set-point weighting design for two-input-two-output (TITO) unstable processes, International Conference on Advances and Challenges for Sustainable Eco Systems (ICACSE), December 6-8, 2018, NIT Tiruchirappalli, India.
4. Purushottama Rao Dasari, A. Seshagiri Rao, enhanced dynamic set-point weighting design for cascade control systems with unstable processes, International Conference on new Frontiers in Chemical, Energy, and Environmental Engineering (INCEEE), February15-16, 2019, NIT Warangal, India.

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M.Tech: Process control and instrumentation in Chemical Engineering from National Institute of Technology, Tiruchirappalli, Tamil Nadu, India, (2014).

B.Tech: Chemical Engineering from Andhra University, Visakhapatnam, Andhra Pradesh, India, (2012).

Awards and Honors:

- One among toppers in every standard consistently during schooling.
- Received Best Paper award in ACODS 2018 Appreciation for the project Titled "Simple method of calculating dynamic set-point weighting parameters for time delayed unstable processes" at Dr. APJ Abdul Kalam Missile Complex in the year 2018.