IDENTIFICATION AND CONTROL OF A PROCESS USING RELAY FEEDBACK AND SUBSPACE APPROACH

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CHEMICAL ENGINEERING

By

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MAY 2019

Dedicated to my beloved Parents

NATIONAL INSTITUTE OF TECHNOLOGY WARANGAL

CERTIFICATE

This is to certify that the thesis entitled "**Identification and Control of a Process Using Relay Feedback and Subspace Approach"** being submitted by **Mr. D. KISHORE** in partial fulfillment for the award of the degree of Doctor of Philosophy (Ph.D) to the Department of Chemical Engineering, National Institute of Technology Warangal, India, is a record of the bonafide research work carried out by him under my supervision. The thesis has fulfilled the requirements according to the regulations of this Institute and in my opinion has reached the standards for submission. The results embodied in the thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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DECLARATION

This is to certify that the work presented in the thesis entitled "**Identification and Control of a Process Using Relay Feedback and Subspace Approach**" is a bonafide work done by me under the supervision of **Prof. ANAND KISHORE KOLA** and was not submitted elsewhere for award of any degree.

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(D.KISHORE)

ABSTRACT

Identification and control plays a crucial role in process control applications. For any process to control and optimize one needs to identify it first either from experimental data or by mathematical modeling through transfer function approach which requires in-depth knowledge of the process which is actually a difficult task. There are various methods available in literature for identification and control of a process. The main limitation of these methods are when the order of the system increases, it is difficult to identify the higher order systems with less information. The investigation carried out in the present research work focuses on developing a novel method to identify and control the processes.

In the present research, four objective works with benchmark case studies namely; Identification of linear SISO process with modified Relay methods, identification and control of non-linear process using Relay feedback approach, identification and control of 2x2 linear multi input and multi output process using Subspace method with Relay and identification and control of 3x3 MIMO process using Subspace method and Relay have been carried out.

A single asymmetric Relay feedback method has been used for identifying a SISO process with sinusoidal excitation to generate sustained oscillations. For effectiveness of the method adapted, it was compared with other existing methods in literature. The identified parameters for SISO system from the adapted test were used to design simple PID controller for different processes and the design method is compared with method proposed by Chidambaram et.al. It has been observed that the results of present work gave better identification accuracy in terms of integral square error (ISE) when compared with Chidambaram's method. A Relay feedback with subspace method has also been adapted to a SISO system. The relay test was carried out to generate input-output data which can replace experimental data for identification. It can be concluded that the proposed method works well for the identification of a process without any prior knowledge and may be extended for identification and control of higher order systems including unstable systems.

A simple approach is proposed to identify and control the non-linear systems using Relay feedback approach. A relay with hysteresis and combination of the low-pass filter is used to reduce the effect of noise in the process. From the identified model parameters, additional parameters were obtained by solving the set of non-linear equations using MATLAB.

The effectiveness of the proposed method is best illustrated by considering the two non-linear systems such as Hammerstein and Wiener processes.

The identification method adapted for SISO process is also extended for 2X2 MIMO to determine the system transfer function matrix by state space model using N4SID algorithm from system identification toolbox. The proposed identification method neither requires prior knowledge to carry out identification unlike subspace identification method nor involves excessive calculations unlike auto-tuning using relay-feedback method. Examples based on stable transfer functions were considered to observe the efficacy of the proposed method. Identification is followed by control. Subsequently a normalized de-coupler was designed for estimated transfer function matrix of the MIMO system based on RGA-NI-RNGA criterion. The controller parameters were obtained using Skogestad's IMC (SIMC) with PI/PID tuning rules. It has been observed that the accuracy is much better compared to relay-feedback method even for higher order system. The accuracy for system without disturbance approximates very close to parameters of systems but for systems with disturbance it varies depending on the order of the system & other factors. The closed-loop control scheme also showed that the performance of present control yields faster and stable response compared to other schemes.

The same method was also extended to 3x3 MIMO of Orgunnaike and Ray distillation column with N4SID algorithm. For the identified MIMO process, RGA-RNGA casual, stable and proper decoupler is derived that reduces the interaction and converts 3×3 MIMO system to '3' individual SISO systems. Skogestad's tuning rule was employed to design PI/PID controller parameters. This method is found to be more efficient, less time-consuming and easy for calculations for identification of multivariable systems, since N4SID algorithm is used to carry out matrix operations. Unlike the subspace method that requires pre-designed controller to carry out subspace identification, it doesn't require predesigned controller, which is replaced by the relay. The fundamental Z–N tuning method was used to determine another set of parameters and the effectiveness of control parameters obtained by both methods were compared. Here, the Skogestad's tuning rule showed better performance over Z–N PID in terms of minimum IAE.

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LIST OF ABBREVIATIONS

FOPTD	First order plus time delay
SOPTD	Second order plus time Delay
SISO	Single Input and single out put
MIMO	Multi input and Multi out put
PID	Proportional Integral Derivative
Z-N	Zigler-Nicholos
SID	Subspace identification method
CVA	Canonical Variate analysis
MOESP	Multivariable output error state space
N4SID	Numerical algorithms for subspace sate space system Identification
MON4SID	Modified algorithms for subspace state space system Identification
ISE	Integral square error
IAE	Integral Absolute error
ITAE	Integral Time absolute error
CSTR	Continuous stirred tank reactor
DA	Describing function analysis

Chapter 1

INTRODUCTION

1.0 General

Identification is the process of developing relationship between an input and an output measurement data response. Identification and control plays an important role in the process control community. For any process to optimize and control, one needs to identify it. Identification can be carried out by two ways; one by using the mathematical modeling or experimental data. Mathematical modeling involves obtaining the process transfer function using the first principles or empirical laws which involves the rigorous calculations and lot of assumptions.

1.1 Motivation for the Study

The extensive and critical literature review on the present research area motivated to identify the gaps and the following gaps have been identified.

- Most of the methods developed so far are based on extensive mathematical modeling and involves lot of computational efforts.
- Existing methods require more in-depth knowledge of the process.
- Existing methods require more trial and error iterations to identify and estimate the process parameters while solving analytical relations.
- Complexity is more when the order of the system increases and requires more approximations.
- Based on the gaps identified from the literature, a simple and new method namely subspace method has been proposed to overcome the shortcomings of the existing techniques.

The following objectives have been framed and completed in this research work with few case studies.

1.2 Objectives of the Study

- To identify linear SISO and MIMO processes using modified Relay methods.
- To identify non-linear systems with modified Relay feedback methods.
- To propose a simple method for identification of process parameters using subspace methods.
- To use a combination of subspace methods with Relay methods for identifying and control of 2x2 linear MIMO process.

- To apply Simulation tool for identification and control of 3x3 MIMO process using Subspace and Relay method.

Based on the objectives framed, the following works have been carried out in the present research.

- Identification of SISO process with Modified Relay methods
- Identification and control of non-linear process using Relay Feedback approach
- Identification and control of 2x2 linear multi input and multi output) process using Subspace method with Relay
- Identification and control of 3x3 MIMO process using Subspace method and Relay

1.3 Organization of the Thesis

The thesis has been organized into five different chapters as follows.

Chapter 1 gives general introduction on different conventional and advanced methods of process identification and control.

Chapter 2 presents the review of literature. The review of literatures pertains to the Relay identification, auto-tuning subspace methods.

Chapter 3 summarizes different identification methods; their importance and comparison of the existing identification methods with the proposed methods used in this work.

Chapter 4 explains the results and discussion of the works done in the thesis with detailed explanation.

Chapter 5 brings out conclusions of all case studies considered in the thesis and recommendations for future work are also presented.

1.4 Introduction to Identification methods

The identification methods can be classified broadly into open loop and closed loop identifications. The closed-loop identifications are preferred to open-loop methods as the

Feedback-loop need not be cut and production need not be disturbed. The identification requires both input and output data along with noise present in the process. Identification can be carried out in the open loop as well as closed loop but large systems with delay cannot be identified in open loop owing to large estimation error and takes long time to converge whereas closed loop identification requires less time to converge. In open-loop identification an excitation is introduced in the input variable of the process to obtain the output responses. Using these output responses, the transfer function matrix can be identified. The open-loop identification is simple but the test is sensitive to the disturbances and it is not applicable for the unstable systems. Generally, a step test is performed in terms of an open-loop structure and initiated when the process to be identified is at zero initial state or moved into the desired operating region especially around the operating point, namely, the set point as shown in figure 1.1. To prevent the process output from drifting too far away from the set point, as required from the operation of many industrial processes such as the continuous stirred tank reactors (CSTRs) and heating boilers, closed-loop identification test is usually adopted in engineering practice to keep the output deviation in an admissible working range. There are two types of closed-loop identification tests that are widely used in engineering applications, closed-loop step test (fig1.2) and relay feedback test (fig 1.3). For the use of a closed-loop step test, the closed-loop controller needs to be specified beforehand for maintaining the closed-loop stability, which may bring difficulty to the closed-loop configuration since the process is to be identified. In order to overcome the difficulty, recently relay methods got prominent role for identification and control of a process. Relay methods are based on the fact that from the two key parameters; ultimate gain and ultimate period, the other parameters can easily be obtained which does not require any other tests for identification and control design perspective.

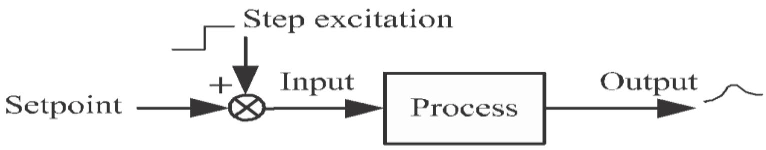

Fig 1.1 Schematic diagram of Open-loop test

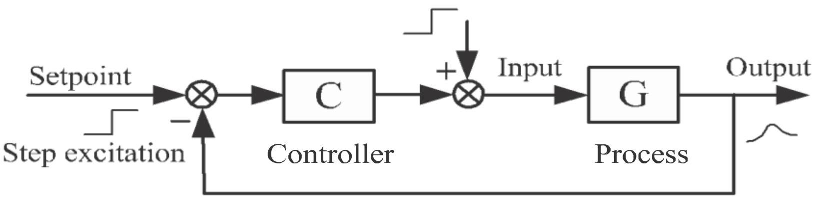

Fig 1.2 Schematic diagram of Closed-loop test

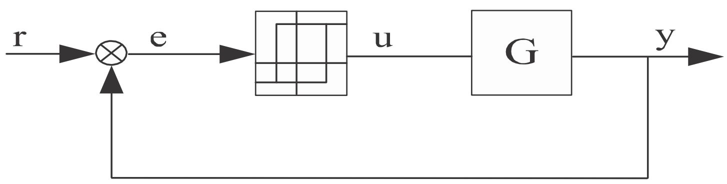

Fig1.3 Schematic diagram of Relay feedback test

With precise knowledge one can identify two key parameters those are used for controller design purpose. Relay feedback method plays vital role in the process industries to identify two important ultimate properties namely ultimate period and ultimate frequency using the closed loop method. Åstrom and Hagglund introduced Relay feedback method by conducting single relay feedback test to obtain ultimate gain and ultimate period. Later on, Luyben used the technique for tuning online PID parameters of simple processes. Relay feedback method offers a simple closed loop technique to identify the ultimate gain and ultimate period without disturbing the process/production, when compared to Ziegler-Nichols's method which requires trial and error method to obtain the ultimate period and ultimate gain which drives the process towards oscillatory behavior and instability. The model based control requires specific PID tuning where the exact mathematical model structure is used obtained from an open-loop or closed-loop test. Relay feedback method plays a simple approach to identify the two parameters without knowing the rigorous mathematical analysis of the process. Due to simplicity of the method, relay feedback is extensively used in the process industries in order to estimate the two parameters cited above. The input excitation is added to the process in very small amplitude so as to perturb the process to get just realizable distinct oscillations around its operating steady state. A relay with hysteresis mimics a real situation and is used to reduce the noise in estimated parameters. These parameters are used to formulate a model structure that helps in tuning model based PID parameters. According to a survey by Åstrom&Hagglund (1984), almost 95% of loops in process industries work on PID mode due to its ease in maintenance and implementation. Auto-tuning of PID parameters plays an important role in identification and control of process systems. Periodic and oscillatory signals of input & output, in relay feedback, are subjected to describing function analysis that result in harmonics at different stages and finally yield ultimate Parameters. Compared to other tuning methods like frequency response and time integral approach methods this helps for obtaining accurate parameters. The frequency response method such as FFT and curve fitting methods involve extensive simulation and mathematical analysis that needs much of the process knowledge. In process control community all the chemical systems can be classified into single input and single output (SISO), FOPTD systems, SOPTD systems, integrating systems, linear systems, non-linear systems, stable/unstable systems, multi input and multi output systems (MIMO) based on the dynamics of the process. Systems to which principle of superposition (if two different inputs are applied simultaneously to the system, the resultant output is the sum of the two inputs) is applicable are called linear systems if not they are called as non-linear systems. Analysis of a non-linear system is difficult due to nonlinear behavior but shows better performance than linear systems. Sometimes, non-linearity is introduced intentionally for improved performance. One of the simplest cases is relay-controlled system or on-off system.

If the non-linear system behavior can be expressed through a time dependent differential equation, it is called dynamic non-linearity otherwise static non-linearity. Relay is like on-off controller or two-state controllers. When connected to stable systems, it generates stable limit cycles or sustained oscillations as system response. An enormous amount of work has been carried out in the area of identification and control of process systems ever since its inception. Recently, one more method was proposed namely Subspace method for identification of all the processes including stable and unstable process.

Nowadays, the Subspace methods have attracted good popularity and more attention towards process control community as the subspace identification method has well defined algorithms when compared to other methods. The Subspace method, the name of which reflects the fact that the linear models can be obtained from row and column spaces of certain matrices calculated from input-output data. Typically, the column space of such data matrices contains information about the model, while the row space allows obtaining a state sequence directly from the input-output data. There is no need for an explicit parameterization. Another advantage is that it uses orthogonal projection to eliminate noise effect. For identification to be a subspace, one needs to represent the system in terms of state space at first.

$$X(k+1) = Ax(k) + Bu(k) + w(k) \qquad (1.1)$$

$$Y(k) \quad = \quad Cx(k) + Du(k) + v(k) \qquad (1.2)$$

$$E\left[\begin{pmatrix} wp \\ vp \end{pmatrix} \begin{pmatrix} wq^T & vq^T \end{pmatrix}\right] = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \delta pq \qquad (1.3)$$

The vectors u (k) of R^{mx1} and y (k) of R^{Lx1} are k^{th} measurements at time instant k with m inputs and L outputs of the process. The vector x (k) is the state vector of the process at discrete time instant k, V (k) Є R^{Lx1} and Wk Є R^{nx1} are unobserved vector signals, v (k) is called the measurement noise and wk is called the process noise. It is assumed that they are zero mean, stationary white noise vector sequences and uncorrelated with the inputs u (k). A Є $R^{nxn}$is the system matrix, B Є $R^{nxm}$is the input coefficient matrix, C Є R^{Lxn} is the output coefficient matrix while D Є R^{Lxn} is the direct feed-through matrix. The matrices Q Є R^{nxn}, S Є R^{nxL} and R Є R^{LxL} are the covariance matrices of the noise sequences W (k) and V (k). In subspace identification it is typically assumed that the number of available data points goes to infinity, and that the data is ergodic. The main problem of identification is arranged as follows: Given a large number of measurements of the input u (k) and the output y (k) generated by the unknown system described by equation (1). The task is to determine the order n of the unknown system, the system matrices A, B, C, D up to within a similarity transformation and an estimate the matrices Q, S and R. Schematic representation of Subspace identification algorithm is provided in Figure1.4The procedure consists of two steps as presented below

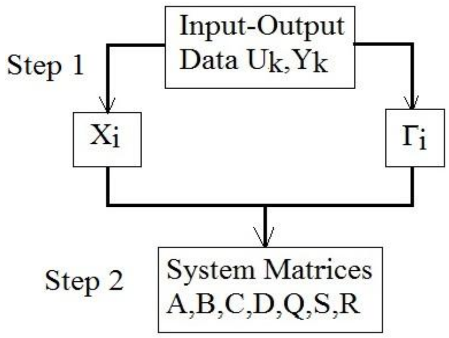

Fig. 1.4 Flow chart of Subspace algorithm

Projection, LQ factorization and singular value decomposition. “Subspace" method reflects the fact that the linear methods can be obtained from row and column spaces of certain matrices calculated from the input-output data. There is no need for an explicit model parameterization. The subspace algorithm has the elegancy and computational efficiency. The method is a non-iterative one [Keesman, (2011)]. The Subspace identification methods aim at directly estimating the system matrices A, B, C, and D in a state-space model structure from the noisy input-output data. All the algorithms can be interpreted as a singular value decomposition of a weighted matrix (Qin, 2006).

Block Henkel matrices play an important role in the subspace identification algorithms. These matrices can be easily constructed from the given input-output data. The subspace Identification algorithms make extensive use of their structures. The method uses Projection algorithms by which the system order and the extended observability matrix can be extracted from a singular value decomposition of an appropriate matrix [Van Overschee and De Moor, 1994)]. Subspace identification algorithms are often based on the geometric concepts. It should be noted that these geometric operations can be easily implemented using the QR decomposition [(Van Overschee and De Moor, 1996)].The subspace based methods such as Multivariable Output Error State (MOESP) [(Verhaegen, 1994)], Numerical algorithms for Subspace State Space System Identification (N4SID) [(Van Overschee and De Moor, 1996)] and Canonical Variate Analysis (CVA) [(Larimore, 1990)] are competing with the conventional prediction error methods in their performance and have shown several good properties, which make them favorable in industrial applications. It has several advantages such as numerical robustness, few user specified parameters, model order reduction and easier extension to MIMO system identification. There is no need for the complex parameterization even for multi input multi output systems because these methods are identifying a state space model. Subspace identification algorithms are reliable in the sense that a model is obtained solely by using numerically reliable matrix and vector manipulations such as projections and singular value decompositions, which in contrast with the classical predictor error methods involving minimization of a non-convex cost-function, with no guarantee that the obtained local minimum yields a good model.

Relay methods alone can be used to identify the SISO processes such as FOPTD, SOPTD, non-linear processes and yields the satisfactory results, when the relay method extended to MIMO (Multi input and Multi out processes) and order of the system increases the numerical complexity.Increases and tedious calculations is to be carried out. This complexity is overcome by addition of subspace methods for effective identification process. In this work relay methods combined with subspace methods have been used.

Chapter 2

LITERATURE REVIEW

This chapter discusses different identification methods such as open-loop and closed-loop identification methods. Reported works based on Relay and Subspace identification have been provided in this chapter. .it covers the sources and background of different kinds of relay and subspace methods. A brief overview of the closed-loop subspace methods and significance of the control design aspects along with the motivation for the method adapted for the present research have been presented here.

2.1 Open-loop identification methods

Open-loop identification methods were the first introduced one to identify the process. In open-loop methods an excitation (Ljung, 1999) is introduced in the input variable of the process. Using the output responses, the transfer function matrix is to be identified. The open-loop identification method is simple but test is sensitive to disturbances.

The examples of the open-loop identification methods are; step identification and process reaction curve methods. The drawback associated with open-loop methods are overcome by the introduction of popular methods like Z-N, continuous cycling and relay methods. Out of three methods; relay methods are simple owing to the fact that other two methods are trial and error methods and consume lot of time.

2.2 Review on Relay Identification methods

Relay feedback is one of the promising tools for identification of model structure of the unknown process. It is based on the observation that when the output lags behind the input by π radians, closed loop system may oscillate with a period of P_u. A relay of magnitude "h" is inserted in the feedback loop as shown in figure 2.1 and hence, the name relay feedback test. Initially, the input u (t) is increased by "h". Once, the output y (t) starts increasing after a time delay (D), the relay switches to opposite direction. Since, there is a phase lag of $-\pi$, a limit cycle of amplitude "a" is generated as shown in figure 2.2. The period of limit cycle is ultimate period, $\square_u$. The approximate ultimate gain, K_u and ultimate frequency, ω_u are given by equations (2.1) and (2.2) respectively.

$$k_u = \frac{4h}{\pi a} \tag{2.1}$$

$$w_u = \frac{2\pi}{p_u} \tag{2.2}$$

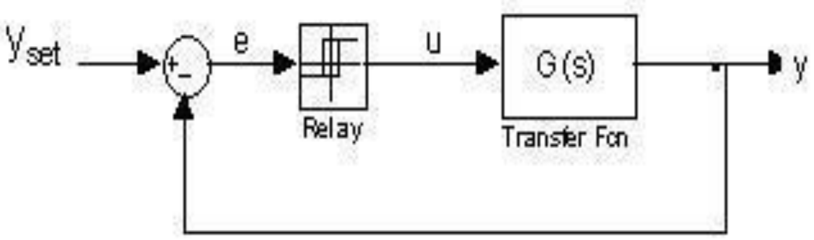

Fig 2.1 Block diagram for biased relay feedback test

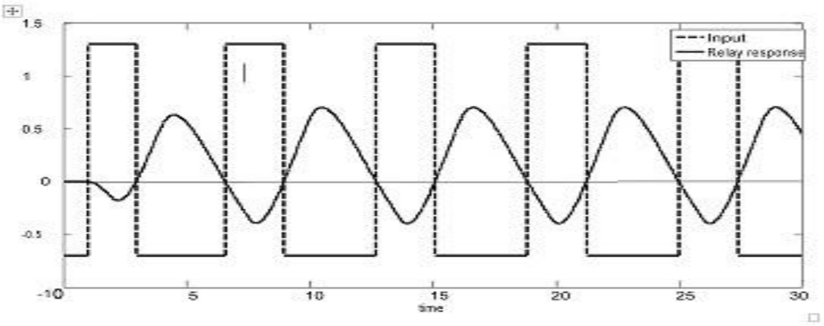

Fig 2.2 Biased relay feedback response

2.3 Advantages of using relay feedback test

There is no need for prior knowledge of the system time constants. This method automatically results in sustained oscillations at the critical frequency of the process. The only parameter to be specified is the height of the relay. Unlike step test, relay test is a closed loop test and hence, the process will not drift away from the set point. This keeps the process in the linear region, where we are trying to find the transfer functions. Hence, relay feedback precisely works on highly nonlinear processes. Accurate information is obtained around the important frequency. In contrast, the pulse testing tries to extract information for the wide range of frequencies; hence, pulse testing is inherently less accurate.

2.4 Literature on Relay Feedback method

K.J.Astrom and T.Hagglund et al [1984] have proposed simple methods for tuning PID regulators using relay tests which is robust and easy to use. Many commercial auto-tuners are designed based on ultimate gain and ultimate period obtained from relay test. William L. Luyben et al [2001] is among the first to employ the relay feedback test to get additional informationabout the process dynamics using the so called shape factor. The progress in relay feedback auto tuning up to 2006 is comprehensively documented in the book by C. C. Yu [2006]. T. Thyagarajan and C.C. Yu et al [2003] presented the identification procedure for different processes such as FOPDT, SOPDT and HO systems. R. C. Panda and C. C. Yu et al [2003] have derived time domain model equations for first, second, third and higher order processes. T. Thyagarajan, C. Esakkiappan and V. Sujatha [2010] presented the method for modeling of inverse response system with time delay by using Relay Feedback test. C. Esakkiappan, T. Thyagarajan and P. BlessyHepsiba [2012] presented the mathematical modelling, system identification and controller design of an over damped SOPDT process. Shih-Haur Shen and C. C. Yu [1994] have proposed a sequential identification-design procedure for the autotuning of multivariable systems. V. Sujatha, R. C. Panda [2012] have presented the modeling of off-diagonal closed loop transfer function of 2 X 2 Multi Input Multi Output (MIMO) system in time domain using relay feedback test. V. Sujatha, R. C. Panda [2012] have derived time domain model equations for closed loop transfer function of interacting loops of 2 X 2 Multi Input Multi Output (MIMO) processes. William L. Luyben [1987] has implemented a simple and easy procedure for controller tuning in multivariable systems. Dwyer A.O [1999] proposed PID and PI controller tuning rules for time delay processes. R. C. Panda et al [2006] had estimated the parameters of under-damped second order plus dead time process by using limit cycle data of relay responses obtained from single relay feedback test.Tao Liu et.al [2013] presented the review works on different identification methods and emphasized the need of relay feedback methods and explored the new directions in the identification and control using Relay feedback methods.

2.5 Review on Subspace methods

The subspace method, the name of which reflects the fact that the linear models can be obtained from row and column spaces of certain matrices calculated from input-output data. Typically, the column space such data matrices contain information about the model, while the row space allows obtaining a state sequence, directly from the input-output data.

There is no need for an explicit parametrization. A second advantage is the elegance and computational efficiency of the algorithm of the subspace algorithms. Subspace identification algorithms are based on concepts from system theory, linear algebra and statistics such as projections (orthogonal and oblique Projections) using QR decomposition.

Subspace identification methods (SIMs) have gained popularity in the field of system identification since they can identify state space model directly from the input-output data. Subspace methods are based on robust numerical tools such as QR factorization and singular value decomposition (SVD) which makes them attractive from the numerical point of view.

The most common classical subspace identification methods are Canonical Variate Analysis (CVA), Multivariable Output Error State Space (MOESP) and Numerical Subspace State Space System Identification (N4SID). These algorithms are different from each other due to its implementation, concept and interpretation.

The review works on the subspace identification algorithms are reported in literature (Overschee and Moor [1996], Katayama [2005& 2014], Ljung (1998),

All subspace identification methods consist of two stages. The first step performs a weighted projection of the row space of the constructed data Hankel matrices. From this projection, the observability matrix (X_i) and/or an estimate X_f of the state sequence can be retrieved. In the second Step, it computes the system matrices A, B, C and Subspace method is implemented with N4SID algorithm.

2.6 Numerical Subspace State Space System Identification (N4SID)

N4SID method is introduced by Overschee and Moor (1994). This method uses an oblique projection of future outputs along future inputs onto the past data. Order of the unknown system and the state sequences are obtained from SVD which is performed on the oblique projection. Then the system matrices are computed by LS regression. The significant works carried out on relay and subspace methods are tabulated in the table 2.1.

Table 2.1 Significant works carried out in literature

S. No.	Research Paper/Book Chapter	Investigators	Research findings
1.	Automatic Tuning of Simple regulators with specifications on Phase and angle margins (1984).	Astrom and Haggulnd	It is a simple method to identify and control the simple FOPTD process with ultimate properties of Ultimate gain k_u and Ultimate Period P_u and also proposed the auto –tuning of the simple regulators with specifications on phase and gain margins. Introduced relay feedback test as an alternative to the continuous cycling techniques.
2.	Relay Auto tuning for Identification and Control, First Edition Cambridge Press (2014).	M. Chidambaram &VivekSathe	The method introduced is based on the Fourier series and Laplace transform analysis involving the higher order harmonics and the lower order harmonics only. Proposed methods are to identify the various stable, FOPTD, SOPTD and MIMO using relay feedback approach.

3.	A tutorial review on process identification from step or relay feedback tests, 2013.	Tau &Lio,	Presented the review article on various methods and research findings in the area of Identification and control of the processes The main focus of the paper is consolidation of the various identification methods and explored the possibilities of the future research findings.
4.	Advanced Control Theory –A Relay feedback approach Second Edition - (2014).	SomanathMajhi& Research Group.	Proposed the identification of different processes such as stable, unstable, FOPTD, and SOPTD using state space approach. This sate space method involves more computational complexity and extensive calculations.
5.	Subspace identification of transfer function models for an unstable bioreactor, Chemical Engineering Communication, 202, 1296-1303 (2015).	M. Chidambaram & C. Sankar Rao	Presented the modified MON4SID method to identify different stable and unstable processes and also different unstable systems using Subspace methods.

But, the main drawback associated with the relay feedback and subspace method is that relay feedback is not much accurate on account of using DA analysis and neglecting higher order harmonics, and, in the subspace method much more extensive simulations are encountered. In order to overcome those difficulties a new method is proposed here which is a combination of both Relay and Subspace identification methods. Further, the drawbacks of the above methods have been simplified here by (i) designing controller using Relay auto tuning procedure (roughly) and then (ii) identifying process parameters by the subspace method and finally (iii) retuning the controllers using IMC–PID tuning. As it is advantageous to find systems ultimate properties and identify the plant in closed-loop, the method can be applicable to MIMO systems.

In spite of different kinds of methods proposed for identification and control, still there is so much scope for identification of higher order process. In this study, an attempt is made by introducing the new concept of the relay + subspace for effective simulation using the proposed method. By performing the relay feedback test different processes such as stable, FOPTD and SOPTD are identified and controllers are designed.

There are other methods of identification available in literature based on relay feedback methods such as describing function method, time integral approach and curve fitting approach. Out of all methods, the describing function method is widely used on account of accuracy while estimating the process information around the set point value.In subspace method there are different algorithms are available in the literature such as N4SID, MOESP and Modified MON4SID but out of all those methods the simplest is the N4SID and in this work it along with Relay feedback method was used in order to generate sustained oscillations and input-output data.

Chapter 3

IDENTIFICATION METHODS

This chapter summarizes different identification methods that exist in process control applications. In the previous chapter, different identification methods were outlined. In this chapter, a detailed write-up on conventional methods of identification and the methods (relay subspace) employed in this research work along with the merits and demerits of each one are emphasized. Also, the necessary background behind Subspace methods has been presented for better understanding.

3.1 Conventional identification methods

3.1.1 Step Test

This direct method is simple, easy and open-loop experiment. The parameters are extracted by fitting the step response data obtained by giving step input to the process. Nevertheless, it is only an approximate method which gives a rough model of the process. First Order plus Dead Time (FOPDT) process is excited by step input as shown in the Figure 3.1 and the corresponding step response is shown in Figure 3.2. This method cannot be précised for higher order systems due to its high sensitivity to non-linearity.

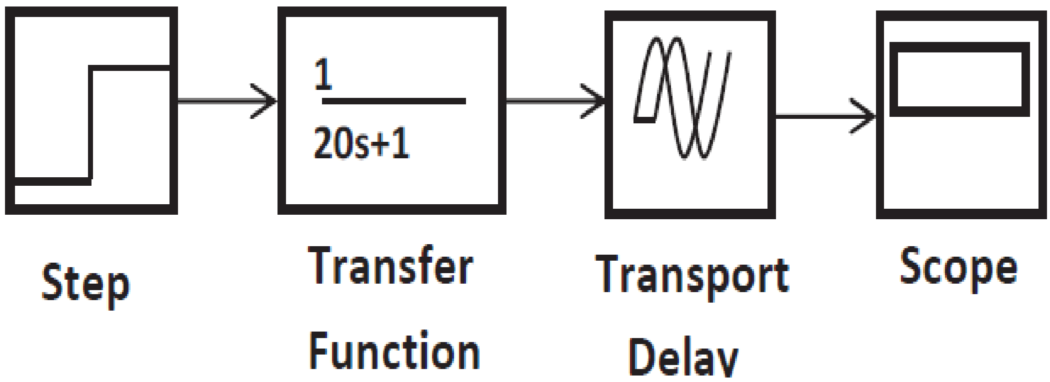

Fig. 3.1 Simulink model for step test of FOPDT process

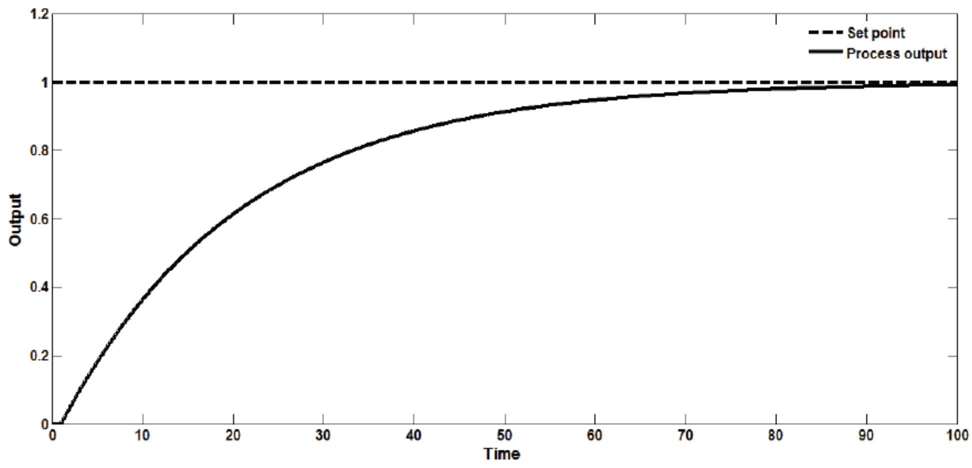

Fig. 3.2 Step response of FOPDT process

3.1.2 Sine Test

The excitation to the system is given in the form of a sinusoidal signal. A steady oscillatory output is established after all the transients are vanished. The amplitude ratios and phase angles at different frequenciesareestimated. Thus, the entire frequency cycle is traced out experimentally for different frequency ranges. The frequency response of FOPDT process is shown in Figure 3.3. The precise dynamic data is obtained by direct sine wave testing,which is used for noisy systems. However, it takes long time when applied to chemical process, as it is needed to wait for the transients to be vanished.

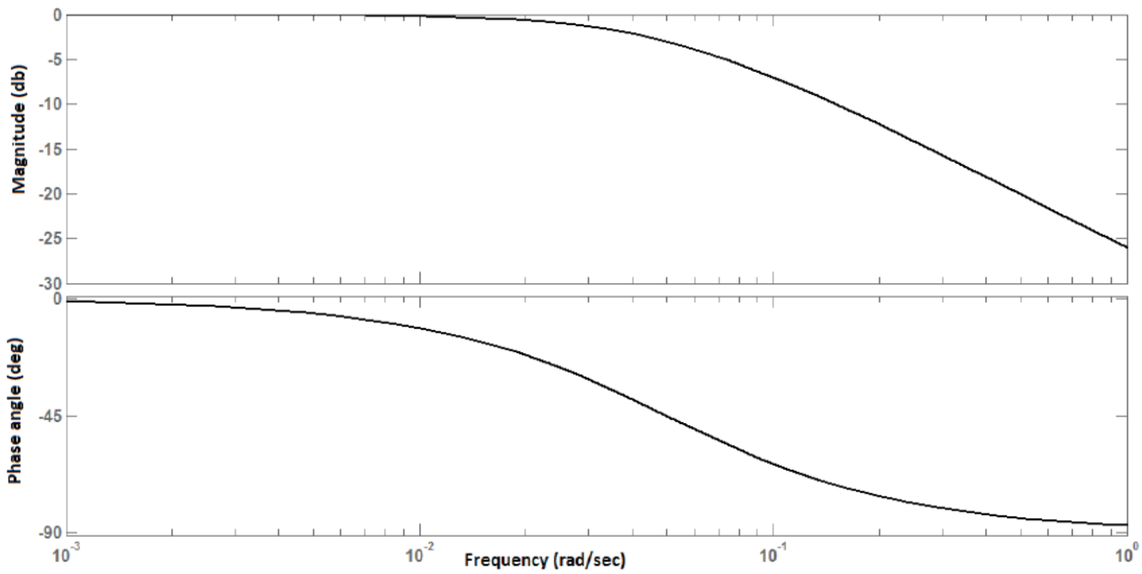

Fig. 3.3 Frequency response of FOPDT process

3.1.3 Pulse Test

Pulse test is one of the useful methods to obtain dynamic data from the chemical process. Input pulse of fairly arbitrary in shape is put into process. The pulse starts and ends at the same value and is often just a square pulse eventually returns to its original value. This square pulse is used as input to the plant and the output is recorded. The response of the third order system excited by an impulse input is shown in Figure 3.4. The pulse testare used to extract reasonably accurate frequency response curves within the fraction of the time that is taken by direct sine wave test. Still, it is not suitable for highly nonlinear systems when load disturbances occur during pulse test.The output may not returnto its initial value due to the effect of load disturbances.

Relay feedback test is an alternative to minimize the difficulties encountered in the conventional system identification methods.

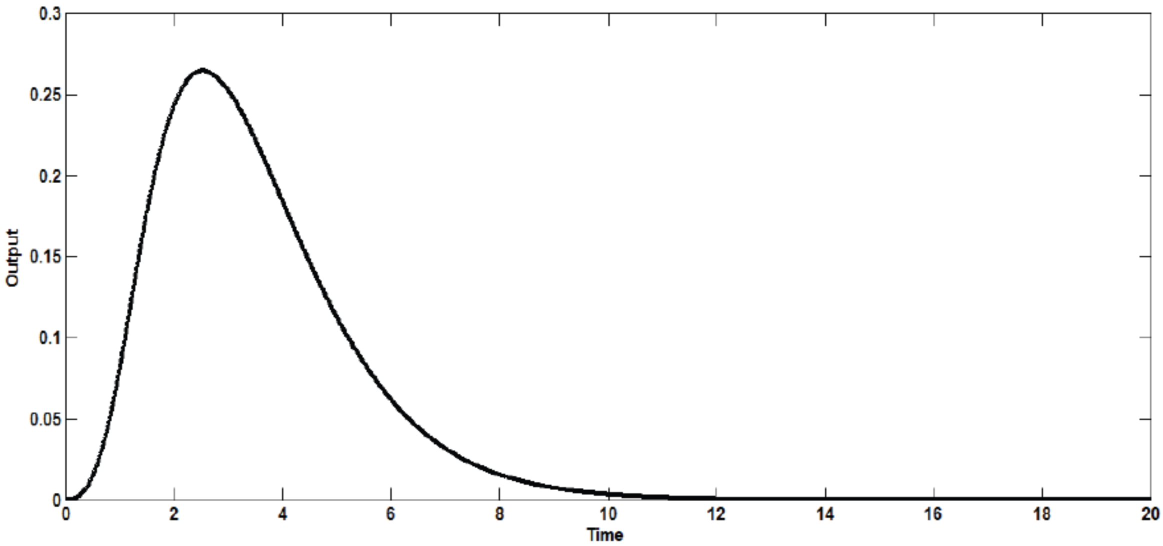

Fig. 3.4 Impulse response of third order system

3.1.4 Relay feedback test

Relay feedback test is acapable tool to identify the model structure and estimatethe model parameters of unknown process. It works on the principle that when the output lags behind the input by π radians, closed loop system may oscillate with a period of Pu. A relay of amplitude ‘h’ is placed in the feedback loop shown in Figure 3.5, which is known as relay feedback test.

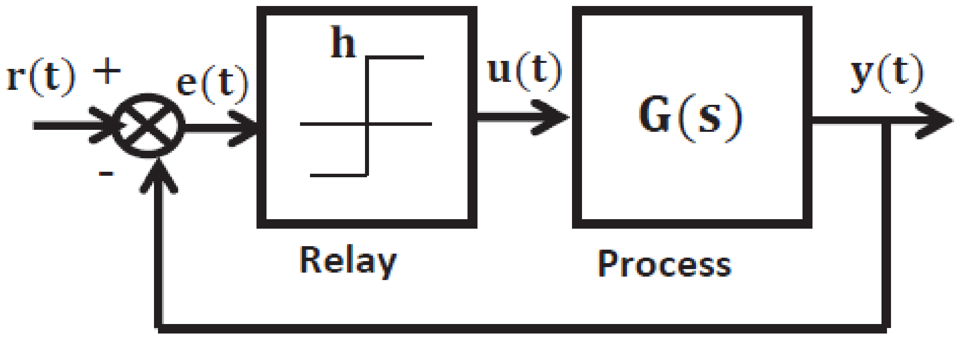

Fig. 3.5 Block diagram of Relay feedback test

The input u(t) is decreased by ‘h’ as shown in Figure 3.6 (a). As soon as the output y(t) starts decreasing after a time delay ‘D’, the relay shifts to the opposite directionas shown in Figure 3.6 (b). Since, there is a phase lag of ‘π’, a limit cycle of amplitude ‘a’ is generated in the output. The relations used in the relay feedback and its advantages over other methods are explained in Chapter 2.

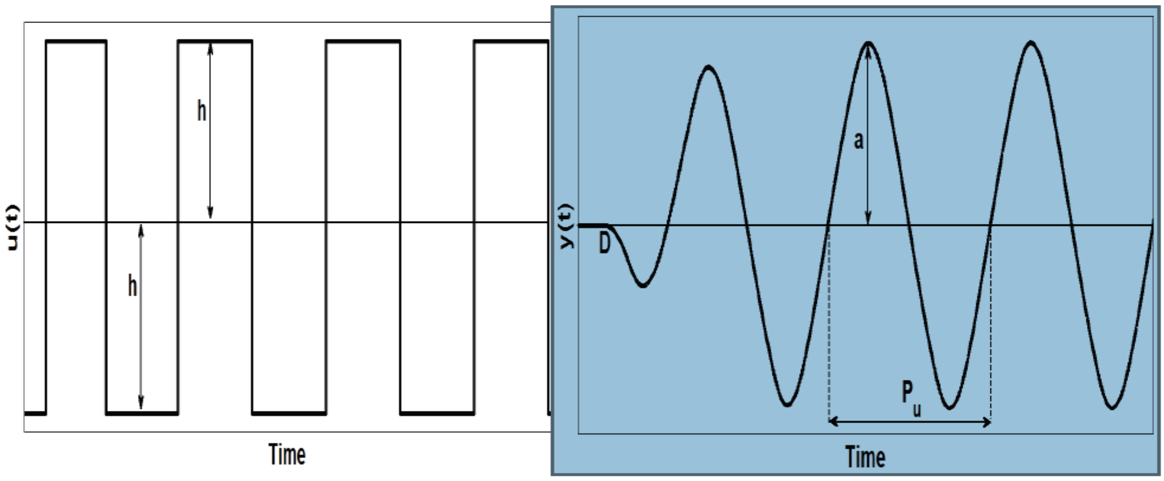

Fig. 3.6(a) Relay output Fig. 3.6(b) Process output

In the present work, the relay feedback test is used for the identification and estimation of model parameters of different benchmark processes. The relay feedback methods quite good for single input and single output (SISO) when it is applicable for higher order multi input and multi output (MIMO). The complexity increases while identifying the process to overcome this problem as an alternative closed loop method Subspace method is combines with relay to identify the process from closed loop data.

3.1.5 Subspace identification method

A linear time invariant dynamic system is represented using state space model as:

$$X(K) = Ax(k) + Bu(k) + w(k) \quad (3.1)$$

$$Y(k) = Cx(k) + Du(k) + v(k) \quad (3.2)$$

$$E\left[\begin{pmatrix} wp \\ vp \end{pmatrix}\begin{pmatrix} wq^T & vq^T \end{pmatrix}\right] = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \delta pq \quad (3.3)$$

A Є R^{nxn} is the system matrix, B Є R^{nxm} is the input coefficient matrix, C Є R^{Lxn} is the output coefficient matrix while D Є R^{Lon} is the direct feed-through matrix. The matrices Q Є R^{nxn}, S Є R^{nxL} and R Є R^{LxL} are the covariance matrices of the noise sequences W (k) and V (k). The main problem of identification is arranged as follows: given a large number of measurements of the input u (k) and the output y (k) generated by the unknown system described by equation. The task is to determine the order n of the unknown system, the system matrices A, B, C, D up to within a similarity transformation and an estimate the matrices Q, S and R. In subspace

Identification problems, order of unknown system and system related matrices are estimated from given input and output data.

The first in each subspace identification algorithm is the conversation of state space model into a singular linear matrix equation. This can be done by the recursive substitution of state equation in the output equation. The linear matrix equation is written as (Overschee and Moor, 1996)

$$Y_{f=}\Gamma_i X_f + H_i^d U_f + H_i^d E_f \quad (3.4)$$

Where, f is future data horizon indices. Superscripts d and s denote deterministic and stochastic. matrix,H_i^d and H_i^s are the lower triangular block-Toeplitz matrices for deterministic and stochastic part respectively. All the subspace identification algorithms start with the constructing the block Hankel matrices from the given input and output data. The Block Hankel matrices are given by;

$$U_f = U_{iI2i-1} = \begin{pmatrix} u_i & u_{i+1} \\ u_{i+1} & u_{i+2} \end{pmatrix} \quad (3.5)$$

Similar definitions hold for$Y_p, E_f, U_f, Y_f and\ E_p$

The input-output data are collected from closed loop system. The Block Hankel matrices are constructed from the input and output measurement data. The second and third term of the right hand side of the linear matrix equation are known as the input noise term respectively. In order to get the extended observability as the input and state sequence, the input and noise terms should be eliminated. These terms are eliminated by using the projections.N4SID uses the oblique projection to get the state sequences. The projections are estimated by efficient QR decomposition method. The QR decomposition can be viewed as a data Compression step. The data matrix having a large number of columns is compressed to a usually much smaller lower triangular R which contains all the relevant information of the system for which the data is generated.

3.1.6 Hankel Matrix

Given a sequence of data $u(t) \in R^{rxc} \forall$ 0, 1, 2 ….. $t_{0,}t_{0+1}$….. Where r is the number of rows in u (t) and C is the number of columns in u (t).

$$U_{0Ii-1} = \begin{pmatrix} u_0 & u_1 \cdots & u_{j-1} \\ u_1 & u_2 \cdots. & u_j \\ u_0 & u_1 \cdots.. & u_{j-1} \end{pmatrix} \qquad (3.6)$$

This is defined as a Hankel matrix because of the Special structure. The integer number 0, I, J defined as follows.

- 0 start index or initial state time in the sequence which is upper left block in the Hankel matrix.
- I is the number r-block rows in $U_{0I(i-1)}$
- j is the number of C-block columns in $U_{0I(i-1)}$

A Hankel matrix is symmetric and the elements are constant across the anti-diagonals.

3.1.7 Fundamental Subspaces

Definition: Let M be a subset of V. if M itself is closed with respect to the vector addition and scalar multiplication defined for V then M is a subspace of V (Sankar, 2015). Vectors spaces may be formed from subsets of other vectors spaces. These are called subspaces. Here four fundamental subspaces such as column space, row space, null space and left null space are explained with an example. Let A∈ R^{mxn} be a matrix of mXn dimension.it defines a linear transformation from vector spaceR^m.

Definition: (Sankar, 2015). Let V be a vector space and S= $(v_1, v_{2....}v_n)$ be a subset of V. we say that S spans V if every vector V can be represented as linear Combination of Vectors in S. V=$c_1v_1 + c_2v_2 + \ldots. c_nv_n$

- The vector space spanned by the columns of the matrix A is a subspace of R^m it is called the column space of the matrix A and it is denoted by range (A).The column space is made from the column of the matrix and it is the linear span of the columns the columns are vectors in the m dimensional space so the column space is naturally a subspace ofR^m. The columns of the mxn matrix a corresponding to the basic variables form a basis of the column of a (Sankar, 2014).
- Row of matrix of a span a vector space. This is subspace is called row space of the matrix A and it is denoted by range(A^T). The basis of the row space consists of (i) the nonzero rows of row reduced echelon matrix are independent and (ii) R(A)=R(U) if U is the row echelon form A.(sankar ,2014)
- The null space or kernel consists of all vectors x∈ R^n that satisfy Ax=0 and it is denoted by Ker (A).
- The left null space, denoted by ker (A^T) , consists of all vectors Y∈ R^m that satisfy A^Ty=0.

3.1.8 Orthogonal projection

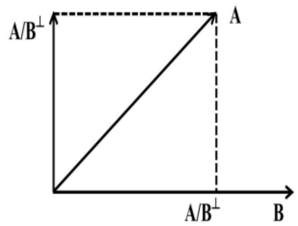

Fig. 3.7: Interpretation of the orthogonal projection in J-dimensional space

A/B is formed by projecting the row space A on the row space of B. on the other hand is formed by projecting the row space of A on the orthogonal compliment of the row space of note the boldface notation for the row space onto which one projects.

The geometric interpretation of the oblique projection is shown in the figure 3.7. The orthogonal projection is used to asymptotically to eliminate the influence of the deterministic signal. The projection of row space of the matrix A on the row space of the matrix is given by (Overschee and Moor, 1996).

$$A/B = A\ B^T(BB^T)B \qquad (3.7)$$

Matrix A can be decomposed as following

$$A = A/B + A/B^{\perp} \qquad (3.8)$$

Where $A/B^{\perp}$ s the projection of row space of matrix A onto the orthogonal complement of the row space of the matrix B.

3.1.9 Oblique Projection

Instead of decomposing 'A'intoa linear combination of two orthogonal matrices (B and B^1), it can also be decomposed into linear combination of two non-orthongal matrices (B and C) or the orthogonal complement of B and C. The geometric interpretation of the oblique projection is shown in figure 3.7. The rows of a matrix 'A' are decomposed into the linear combination of rows of B and C. It can be written as (Overschee and Moor, 1996):

$$A=A/B^{C}+A/c^{B}+A/\binom{B}{C} \qquad (3.9)$$

At the same time, the influence of disturbance and deterministic input signal is removed.

$$A/B^{C}=A(\ C^{T}B^{T})\ \begin{pmatrix} C\,C^{T} & CB^{T} \\ BC^{T} & B\,B^{T} \end{pmatrix} C \quad (3.10)$$

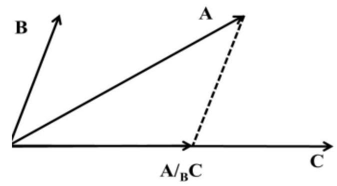

Fig. 3.8 Interpretation of oblique projection in J-dimensional space (j=2, in this case)

The oblique projection is formed by projecting the row space of A along with row space of C.

3.1.10 QR decomposition

The excellent reviews on QR decomposition are given by katayama (2005) and overschee and Moor (1996) .Any real square matrix may be decomposed as A=QR where Q is an orthogonal matrix (i.eQ^TQ=I), and R is an upper triangular matrix also called right triangular matrix. If A is nonsingular then factorization is Unique if the diagonal elements of R is of positive .However, this So-called QR decomposition or QR factorization also exists for rectangular matrices. The orthogonal projection is determined as follows

$$\binom{B}{A}=\begin{pmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} Q1^{T} \\ Q2^{T} \end{pmatrix} \qquad (3.12)$$

From above equation matrix A can be written as

$$A=L_{21}Q_1+L_{22}Q_2 \qquad (3.13)$$

From equation 3.6 and equation 3.10 the following equations can be written as.

$$A/B=L_{21}Q_1^T \qquad (3.14)$$

$$A/B=\ Q_2^T L_{22} \qquad (3.15)$$

Oblique projection can be calculated by LQ factorization as;

$$\begin{pmatrix} B \\ C \\ A \end{pmatrix} = \begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix} \qquad (3.16)$$

$$\text{A}/B^C = L_{32} L_{22} \text{C} = = (L_{21} L_{22}) \begin{pmatrix} Q_1^T \\ Q_2^T \end{pmatrix} \qquad (3.17)$$

3.1.11 Singular Value Decomposition (SVD)

It is known that the Eigen value decomposition is limited to square matrices only. The singular value decomposition (SVD) (Sankar 2015) is an important factorization of a rectangular matrix. It has several applications in signal processing and statics. The SVD technique decomposes A into $A = US\,V^T$where U and V are the orthogonal matrices of dimensions NXN and PXP respectively, such that $U^T\,V = I_N$ and $V^T\,V = I_P$. S is the N XP matrix which contains singular values. The columns of U are called the left singular matrix and those of V are called right singular vectors. The singular values are unique but the U and V are not unique.

3.1.12 Over View of Subspace identification method

In subspace identification method there are two types are exists one is deterministic subspace identification next one is stochastic subspace identification in stochastic subspace identification one needs to consider the white noise where as in Deterministic identification only input and output date consider.

A linear time invariant state space model for the deterministic system is given by

$$X_{K+1} = Ax_K + Bu_K \qquad (3.18)$$

$$y_K = C\,x_K + Du_K \qquad (3.19)$$

The input –output subspace matrix equation formed from the recursive substitution of the output equation in the state equation is written as follows (Overschee and Moor, 1996).

$$y_{p=} \Gamma_i X_p + H_i U_P \qquad (3.20)$$

Performing orthogonal projection of the row space of the past input block Hankel matrix $U_p$on to the row space of the past output block Hankel matrix Y_P Equation becomes

$$y_p u_P = \Gamma_i x_f U_p + H_i U_P \qquad (3.21)$$

The second term in the right hand side of the above equation 3.18 is reduced due to orthogonal projection. Therefore, the above equation is written as:

$$y_p u_p = \Gamma_i X \qquad (3.22)$$

In the above equation 3.22, $X = X_P U_P$ calculates the Kalman filter state. Equation 3.19 represents the column space of Γ_i which can be found out by singular value decomposition (SVD) of Γ_i . It is obtained from a simple LQ factorization of a matrix developed from block-Hankel matrix U_p and y_p as:

$$\begin{pmatrix} U_P \\ Y_P \end{pmatrix} = \begin{pmatrix} L_{11} & 0 \\ & L_{21} L_{22} \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \qquad (3.23)$$

$$Y_P = L_{21} Q_1 + L_{22} \qquad (3.24)$$

And the orthogonal projection in the left side of equation 3.21 can be calculated by matrix L_{22} . The SVD of L_{22} is given as (Katayama, 2005):

$$L_{22} = (U_1 U_2) = \begin{pmatrix} s_1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} v_1^T \\ v_2^T \end{pmatrix} = U_1 s_1 V_1^T \qquad (3.25)$$

The number of dominant singular values gives the order of the system. The left singular vectors corresponding to non-zero singular values give the information about the extended observability matrix and the right singular vectors will give the information about the state sequences.

The extended observability matrix and state sequences are calculated as;

$$\Gamma_i = U_1 s_1^{1/2} \qquad (3.26)$$

$$X_f = s_1^{1/2} v_1^T \qquad (3.27)$$

By defining the states x_{i+1} and x_i and input and output matrices x_{i+1} and x_i and input and output matrices U_i and y_i the state space model i.e. A, B, C and D are computed through the least square method.

$$x_{i+1=[} x_{i+1} \;\ldots\ldots\ldots\; x_{i+j-1]} \qquad (3.28)$$

$$X_i = [x_i \ldots \ldots .. x_{i+j-2}] \qquad (3.29)$$

$$U_{i/i} = [U_i \ldots\ldots.. U_{i+j-2]} \qquad (3.30)$$

$$y_{i/i} = [y_i \ldots \ldots \ldots .. y_{i+j-2}] \qquad (3.31)$$

$$s \begin{pmatrix} x_{i+1} \\ Y_{i/i} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} x_i \\ U_{i/i} \end{pmatrix} + \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} \qquad (3.32)$$

Chapter 4
RESULTS AND DISCUSSION

This chapter presents the research work carried out on identification of linear SISO process using modified Relay feedback method and followed by Relay feedback combined with subspace, identification and control of non-linear SISO process using Relay Feedback method, identification and control of 2x2 linear MIMO process using Relay combined with subspace method and finally identification and control of 3x3 MIMO process using Relay with subspace method.

Different case studies have been considered for carrying out the tasks cited above and are illustrated with the proposed methods followed by detailed discussion. The case studies considered are SISO, FOPTD, SOPTD, linear, non-linear and 2x2 & 3x3 MIMO distillation systems. Identification and control of all the above cited processes using proposed methods have been presented here. The general information on Relay feedback and subspace methods have been briefed in chapter 3.

4.0 Basic information on Identification and control of linear SISO processes:

Transfer function of any process can be identified from the experimental data for model based controller design. In most of the cases, obtaining the transfer function for tedious mathematical models is very much difficult. So, the system identification is the tool for identification of the lower order models based on input-output data. The relay feedback is the simplest method for extracting the oscillations of the process. The ultimate gain (ku) and ultimate frequency (wu) are obtained from the principal harmonics. As the Relay feedback test is closed loop, the sustained oscillations are generated from which the ultimate gain (k_u) and the ultimate frequency (ω_u) are obtained from the principal harmonics. One can approximate two key parameters as M. Chidambaram and Vivek Sathe (2014) have proposed the method to estimate and identify the parameters of different processes with different test signals as an excitation to the process. In this work, a modified asymmetric relay feedback method with sinusoidal excitation has been used in order to identify and calculate parameters of different processes such as FOPTD, SOPTD with time delay. Figure 4.1 shows the Simulink block diagram used in this work where the transfer function block is used to represent each of the different case studies followed by time delay. In this process an asymmetrical relay with heights +h and -2h were used to obtain sustained oscillations.

In this, the frequency of oscillation of the system output is used for auto tuning test. It is observed that Srinivasan and Chidambaram (2003) have used asymmetric relay test where along with asymmetric relay test and sinusoidal excitation. Different SISO processes have been considered from the literature to show the effusiveness of proposed method.

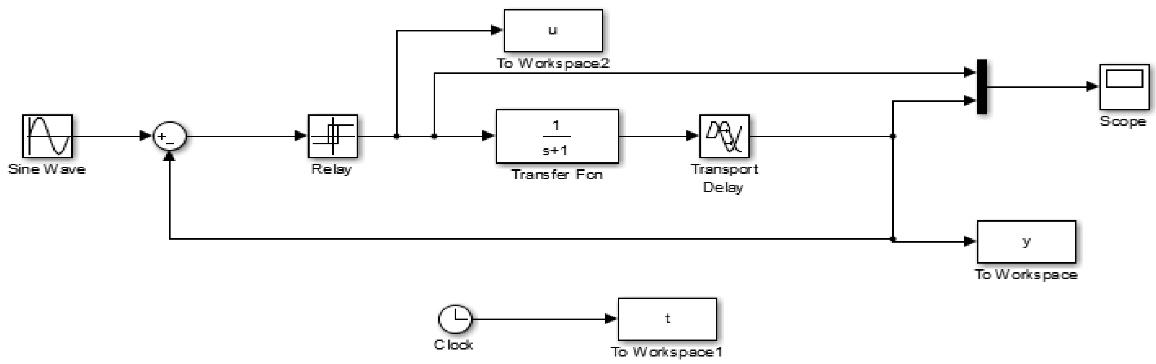

Fig 4.1 Simulink diagram of the Relay feedback system

4.1 **Determination of Kp, D and τ of linear SISO process using Modified Relay methods**

For any process to identify and control, one needs to determine the parameters such as process gain (Kp), dead time (D) and time constant (τ).The following numerical analysis is used in this work for the calculation of intermediate parameters which are explained here.

Laplace transform is given by;

$$Y(s) = \int_0^{\infty} y(t)e^{-st} d \qquad (4.1)$$

The interval y(s) can be calculated for s $=\frac{\delta}{t}$where't' is the time at which sequence of cycles of the oscillations exists in the output next to the initial transients die out. The usage of s=$\frac{\delta}{t}$ was due to as t > t_s being the small value of exp (-s), the effect of next term is ignored while obtaining the integral term. The integral of the resulting function is given by;

$$u(s) = \int_0^{\infty} u(t)e^{-st}\, dt (4.2)$$

Equation (4.2) can also be evaluated using s=$\frac{\delta}{t}$ with numerical values of y(s) and u(s). Let us consider the model, the following equation is formulated.

$$\frac{y(s)}{U(s)} = \frac{kp\, e^{-Ds1}}{\tau s+1} \qquad (4.3)$$

Padma Sree and Chidambaram are the pioneers in using the equation for obtaining PID controlled system response. Now, we are using it for relay auto tuning. As y (t) and u (t) are piecewise continuous and periodic, the Laplace transform of y (t) and u (t) can be written as (krez).

$$y(s) = \frac{1}{1-e^{-ps}} \int_0^p y(t)exp^{-st}\, dt \qquad (4.4)$$

$$u(s) = \frac{1}{1-exp^{-ps}} \int_0^p u(t)\, exp^{-st} dt \qquad (4.5)$$

From above equations (4.4) and (4.5)

$$G(s) = \frac{y(s)}{u(s)} = \frac{\int_0^p y(t)exp^{-st} dt}{\int_0^p u(t)exp^{-st} dt} \qquad (4.6)$$

$$p = \frac{2\pi}{\omega} \qquad (4.7)$$

Where

ω= is the angular frequency of the sustained oscillations.

P=Time period

By replacing s=Jω then applying the Euler's expansion for $\exp(-jw)$ in (4.6)

$$G(jw) = \frac{y(j\omega)}{U(J\omega)} \qquad (4.8)$$

For convenience of the analysis, equation (4.7) is rewritten as;

$$G(jw) = \frac{Y(j\omega)}{U(j\omega)} = \frac{c1-jd1}{c2-jd2} \qquad (4.9)$$

$$c_1 = \int_0^\infty y(t)\cos\omega t\, dt \qquad (4.10)$$

$$c2 = \int_0^\infty u(t)\cos\omega t dt \qquad (4.11)$$

$$d1 = \int_0^\infty y(t)\sin\omega t dt \qquad (4.12)$$

$$d2 = \int_0^\infty u(t) si\omega t dt \qquad (4.13)$$

Equations (4.10) and (4.11) are evaluated numerically using y (t) and u (t) using the information from auto-tuning test.

The equation will be rewritten as;

$$G(j\omega) = P + Jq \qquad (4.14)$$

Where

$$p = (c_1c_2+d_1d_2 / c_2+d_2) \qquad (4.15)$$

$$q = (d_2c_1 - d_1c_2)/(c_{2+}d_2) \qquad (4.16)$$

By replacing $s = J\omega$ in the equation we obtain as

We know the relation

$$e^{-\theta s} = \cos\theta s - j\sin\theta\, s \qquad (4.17)$$

Using the relation

$$G(jw) = \frac{(Kp\cos\,\theta s - J\,\sin\,\theta s)}{\tau j\omega\,+1} \qquad (4.18)$$

$$(P + Jq(\tau jw + 1) = Kp\cos\theta s - J\sin\theta s \qquad (4.19)$$

$p\tau j\omega + P - q\tau\omega + (p\tau\omega + jq)$(4.20)

$$P - q\tau\omega = Kp\cos\omega = 0 \qquad (4.21)$$

$$P\tau\omega + q + \sin(D\,\omega) = 0 \qquad (4.22)$$

By conducting the relay feedback test with sinusoidal excitation with different values of h= -2h, the two key parameters namely $K_p$and P_u are obtained and these two values are used to solve remaining parameters of Kp, D and τ.

4.1.1 **Identification and Control of a SISO process with Modified Relay method – Case studies**

In this subsection, the methods adapted for identification and control of various processes are discussed considering each one as a case study.

4.1.2 **Case study 1**

Consider the SOPTD (Second Order plus Dead Time) given by the following Transfer function.

$$G(s) = \frac{\exp(-2S)}{10s^2+11s+1} \qquad (4.1.1)$$

In this work, an asymmetric relay with h =-2h and sinusoidal excitation is used to get the sustained oscillations shown in fig 4.1.1 and from the auto tuning test, the ultimate gain (K_u) and ultimate Period P_u can be obtained using the numerical analysis. In section 4.1 the parameters are estimated subsequently and a simple PID controller is designed using Z-N settings. The relay response and controller response are shown in the figures 4.1.1 and 4.1.2 respectively.

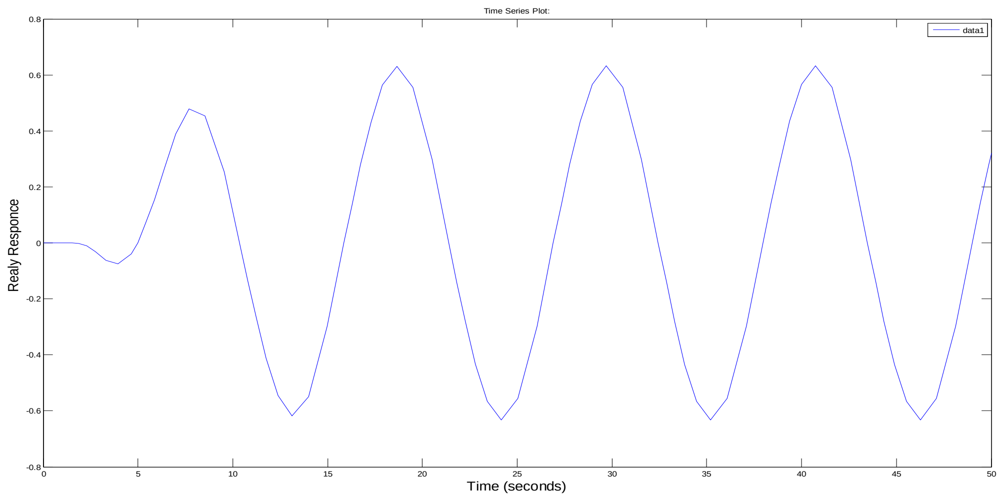

Fig. 4.1.1 Sustained oscillations generated from Relay feedback system for case study1

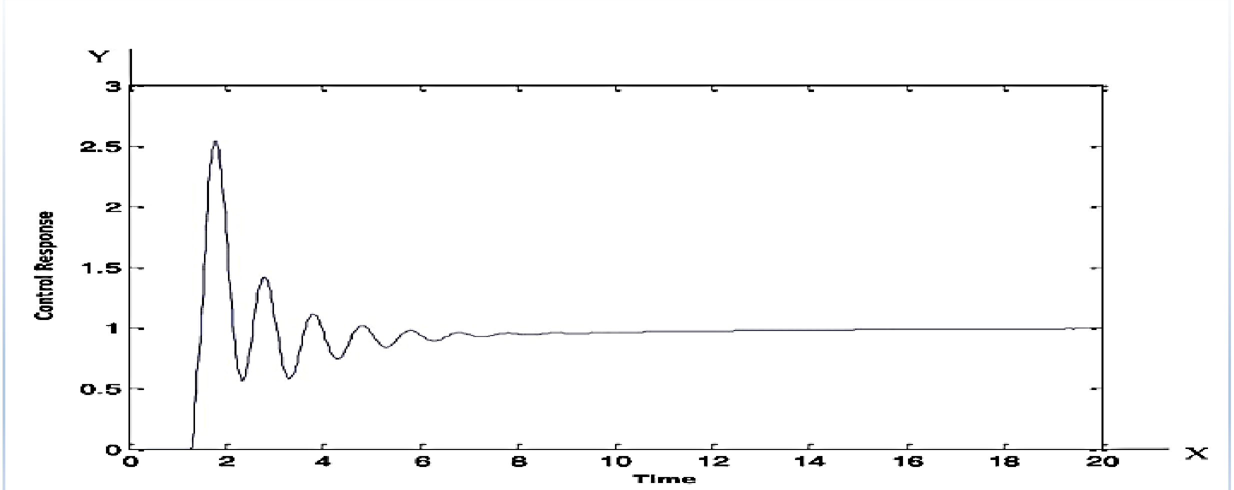

Fig. 4.1.2 Controller response for case study 1

4.1.3 **Results and Discussion on Case study 1**

In this case study, SOPTD transfer function is considered [VivekSathe et.al2014] and a modified asymmetric relay method is applied at input of the Simulink diagram shown in the figure 4.1 to perturb the processes with sinusoidal signal. This Simulink diagram is used for all case studies incorporated in this section to generate sustained oscillations for all the case studies considered. The sustained oscillations shown in figure 4.1.1 are very essential to estimate the parameters of ultimate gain K_u and ultimate periodP_U. The basic fundamentals of relay feedback test are presented in the chapter 3. The K_u andP_U are obtained by the describing function method. In addition to the above, the process parameters; gain $Kp,$ dead time D, and time constant τ are also estimated using the analysis given in section 4.1 and other intermediate parameters are also calculated which are useful for control design purpose and shown in table 4.1.

Table 4.1 Intermediate parameters for the case studies considered

Case study	a	ωu	p_u	t_s	y (s1)	u (s1)	P	q
1	0.0192	0.5438	10.43	24.36	0.04123	0.4844	0.14412	-2.6e-4
2	0.1802	0.8432	7.2	39.60	0.0167	0.4708	-0.1146	0.0026
3	0.6162	2.011	3.1	24.75	0.2267	-0.1568	0.4276	0.0043
4	0.6012	2.01	2.9	23.67	0.21	-0.1438	0.3276	-0.039

With the help of two parameters; ultimate gain and ultimate period using Z-N settings, a simple PID controller is designed. The control response is shown in the figure 4.1.2. Chidambaram and Vivek Sathe proposed a method for identification and control of SOPTD process but it differs from Chidambaram method in terms of less ISE. A comparison is made between the present method and Chidamababaram method which is shown in table 4.2.

Table 4.2 Controller parameters obtained from estimated model other methods in the literature

Case study	Methods	$(k_c k_p)$	w_u	k_u	$\tau 1$ (Integral Time)	τd (Derivative time)	ISE (Integral Square Error)
$\frac{\exp(-2S)}{10s^2+11s+1}$	Proposed method	**6.9436**	**0.5691**	**3.945**	**4.963**	**1.0245**	**3.94**
	Chidambaram – Vivek method (C-V)	**7.0719**	**0.6010**	**4.3660**	**5.2217**	**1.13123**	**4.479**
$\frac{exp(-2s)}{(10s+1)}$	Proposed method	**7.9634**	**0.6935**	**4.982**	**3.012**	**0.493**	**3.00**
	C-V method	**8.0943**	**0.8453**	**5.1333**	**3.7162**	**0.9292**	**3.4949**
$\frac{\exp(-0.5s)}{(s-1)}$	Proposed Method	**2.4309**	**2.12**	**2.013**	**2.13**	**0.215**	**2.012**
	C-V method	**2.7119**	**2.3272**	**2.240**	**2.8507**	**0.2630**	**2.3971**
$\exp(-0.5s)$ / $(2s-1)(0.5s$ -	Proposed method	**0.5643**	**1.2**	**2.00**	**2.00**	**0.1345**	**2.2**
	C-V method	**0.8192**	**1.8964**	**2.014**	**2.1672**	**0.1645**	**2.866**

From the above table, it can be seen that proposed method yields good response for case study1 over the other methods. From the asymmetric relay feedback test using parameters of ultimate gain and ultimate period, PID Control response is obtained as shown in the figure 4.1.2. Initially the obtained response having an overshoot for set point change but as time progress the control response immediately reaches the desired set point with less time which is shown in figure clearly and it is evident that the designed PID control works well for set point change for given SOPTD process. Usually the second order systems possess more oscillations but the second order system obtained in this case gives the satisfactory response with less oscillations.

4.1.4 **Case study 2**

A First Order plus Time Delay Process (FOPTD) with the following transfer function has been considered.

$$G(s) = \exp(-2s) / (10s + 1) \qquad (4.1.2)$$

Identification has been carried out with sinusoidal input and asymmetrical relay feedback approach. The sustained oscillations obtained are shown in figure 4.1.3 and the simple PID controllers are shown in figure 4.1.4

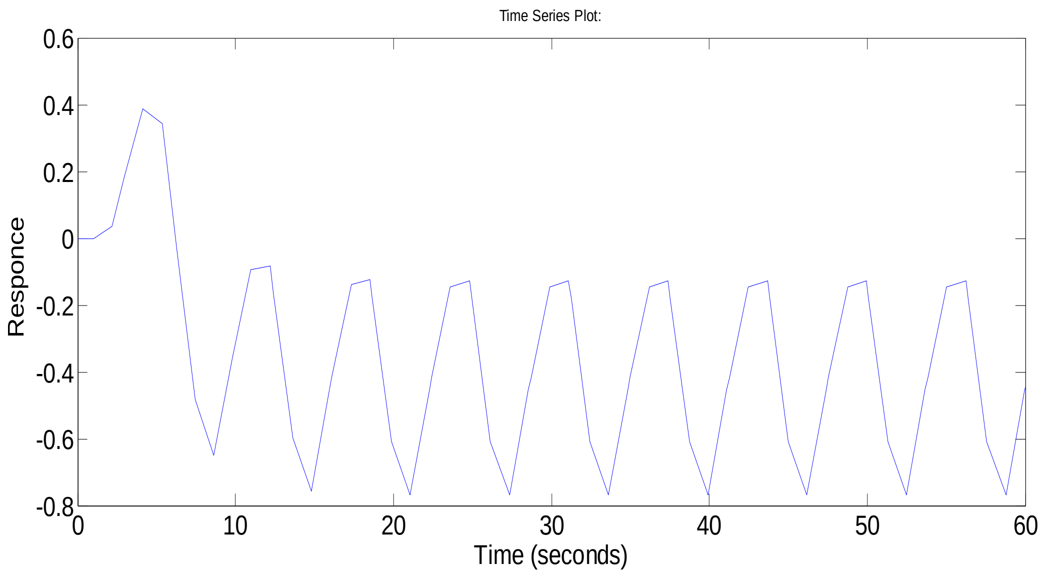

Fig. 4.1.3 Process output asymmetrical relay response

In this case study also, the similar numerical analysis which was carried out for the case study1 has been considered and the results are tabulated in table 4.1 and 4.2 respectively.

4.1.5 **Results and Discussion on Case study 2**

In this case study, FOPTD transfer function considered in many of the chemical processes such as CSTR, Bio-reactors, temperature control system, and inverted pendulum are approximated to First order plus time delay system for simplicity purpose. FOPTD process with transfer function is simulated in Simulink diagram shown in figure and sustained oscillation are generated with modified asymmetric relay method.

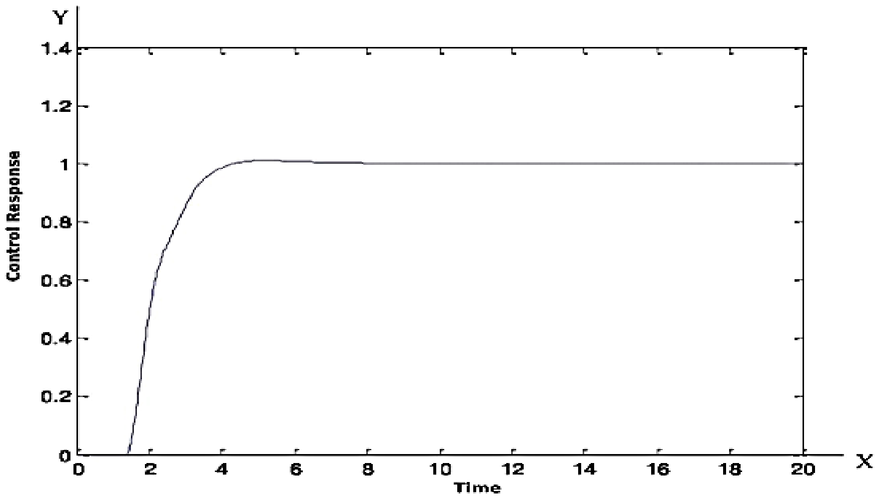

Fig. 4.1.4 Controller response for case study 2

From the sustained oscillations generated one can easily identify the ultimate gain and ultimate period. These two parameters are used in Z-N method to design simple PID controller which is shown in figure 4.1.4 and it shows very good response for the set point tracking for the designed PID controller using the Z-N method. For given step input signal, it quickly reaches to the desired state. The designed control response is compared with the method proposed by Chidambaram and Vivek Sathe. The present method shows the better response in terms of less Integral Square Error (ISE) which is shown in the table 4.1. The intermediate values necessary for estimating control parameters are also shown in table 4.2. The PID controller for the set point tracking shows good response for given FOPTD process. The smooth response which is shown in figure4.1.4 explains that the method adapted for this case study is satisfactorily working in the set point tracking.

4.1.6 **Case study 3**

An unstable FOPTD system with the following transfer function has been considered.

$$G(s) = \frac{\exp(-0.5s)}{(s-1)} \qquad (4.1.3)$$

The asymmetrical relay test is conducted the sustained oscillations are obtained are shown in the figure the same analysis extended to the this case study which is consider for the SOPTD process and FOPTD Process and MATLAB Simulink is used for this case study and corresponding response are plotted below figures. Subsequently the control response is also shown in the Figure 4.1.5

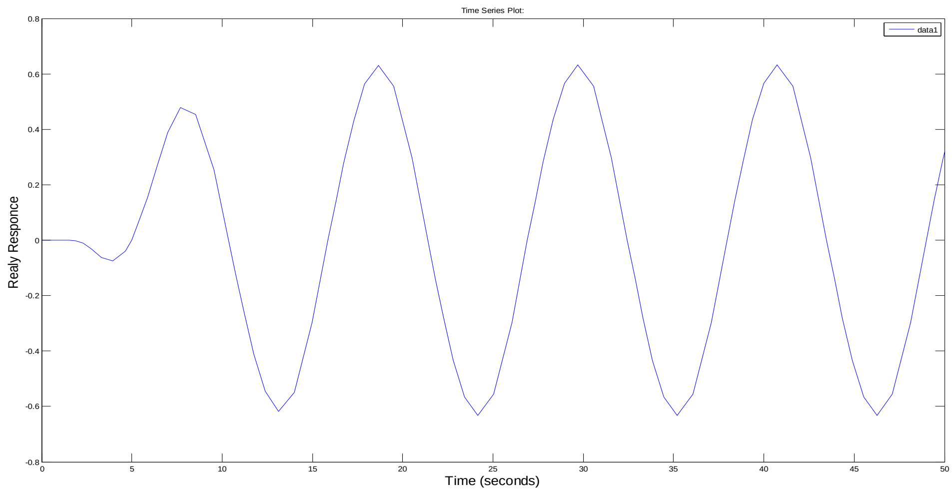

Fig. 4.1.5 process output asymmetrical relay response

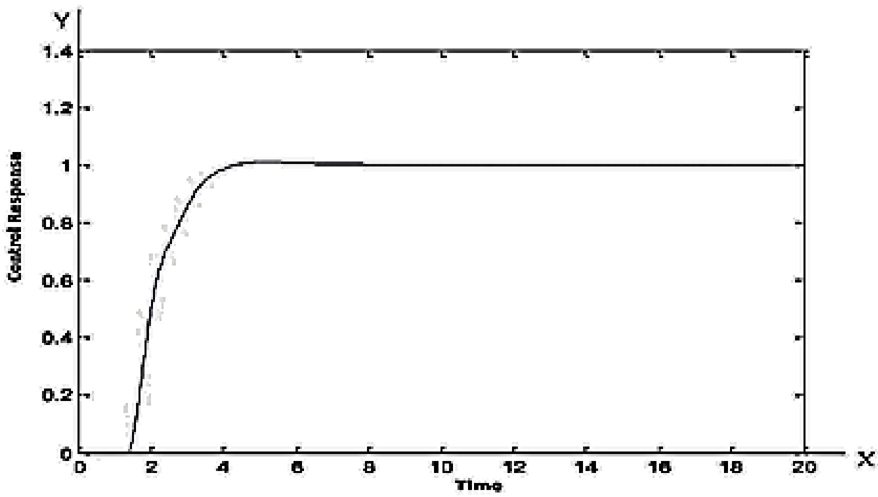

Fig. 4.1.6 Controller response for case study 3

4.1.7 Results and Discussion on Case study 3

In this case unlike the case studies considered above which are stable systems in this case study unstable system is considered unstable systems are defined as the system which are having at least one right pole lies right of the s-plane or for system which having unbounded output for bounded input. There are different practical examples of the unstable systems such as isothermal CSTR (Liou and Chien, 1991), bio reactor (Agarwal and Lim, 1989), dimerization reactor (Ali and Majhi, 2010) etc.

For unstable systems it is very difficult to get the sustained oscillations due to stabilization factor and more over there is a Constraint for unstable system if $\frac{\theta}{\tau} > 0.693$ the systems does not yield sustained oscillation. In this case an unstable system is considered given equation 4.1.3 and using the asymmetric relay sustained oscillations are generated which are shown in the figure 4.1.5 and subsequently a Simple PID Controller is designed as like in the case of stable systems.

Usually it is not easy to design Controller for Unstable systems in spite of the well-defined methods exists for unstable systems such as direct synthesis method, Equation Coefficient method, poleplacement methods, Optimization methods[ChintaSankar Rao et.al] in this case study a Simple PID Controller is designed with Z-N settings and response is shown in the figure 4.1.6

The control response shown in the figure 4.4.2 shows the satisfactory response for the given set point change even though it is unstable system in nature it yields the good response for servo changes and the designed PID Controller is compared with the Control design proposed by Chidambaram et.al and results are tabulated in the table 4.2 which shows that the designed control exhibits satisfactory response in terms less Integral square error (ISE).

4.1.8 Case study 4

In this case study, a second order process with time delay having one unstable has been considered with the following transfer function.

$$\frac{y(s)}{u(s)} = G(s) = \frac{\exp(-0.5s)}{(2s-1)(0.5s+1)} \qquad (4.1.3)$$

Same analysis that was considered for the previous case studies is extended to this case study also and the asymmetric relay method is used here to generate sustained oscillations and the responses corresponding to them is shown in the figure 4.1.7 and subsequently the controller response using Z-N method has been designed i.e. PID controller response shown in figure 4.1.8

4.1.9 Results and Discussion for Case Study 4

In this case study, second order system with one unstable is considered. Unstable second order systems frequently occur in chemical processes like fluidized bed reactor, crystallizer and gas phase polyolefin reactor [Chinta2015]. As the system considered in this case study is quite difficult as it is second order in nature followed by one unstable pole. For this, getting sustained oscillations does not easy owe to the fact that unstable pole and moreover there are limitations for generationof sustained oscillation for the unstable systems due to the constraint on the ratio of the dead to time constant.

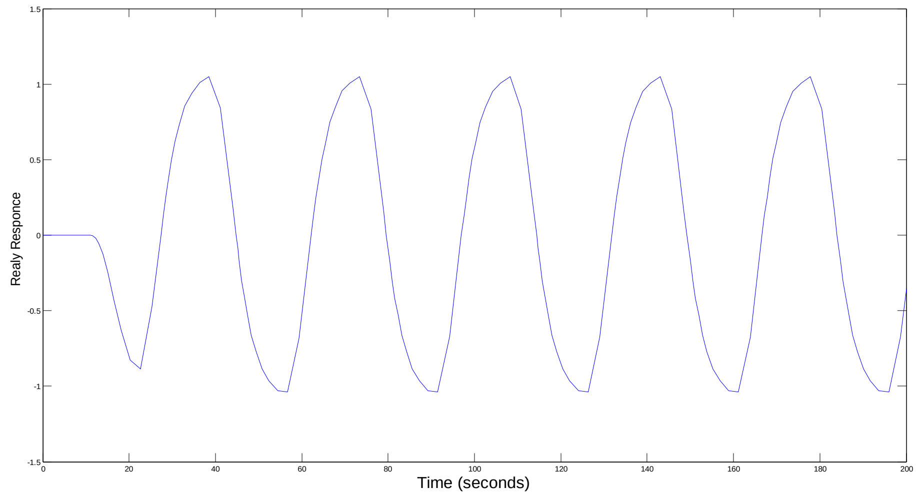

Fig. 4.1.7 Process output asymmetrical relay response of case study 4

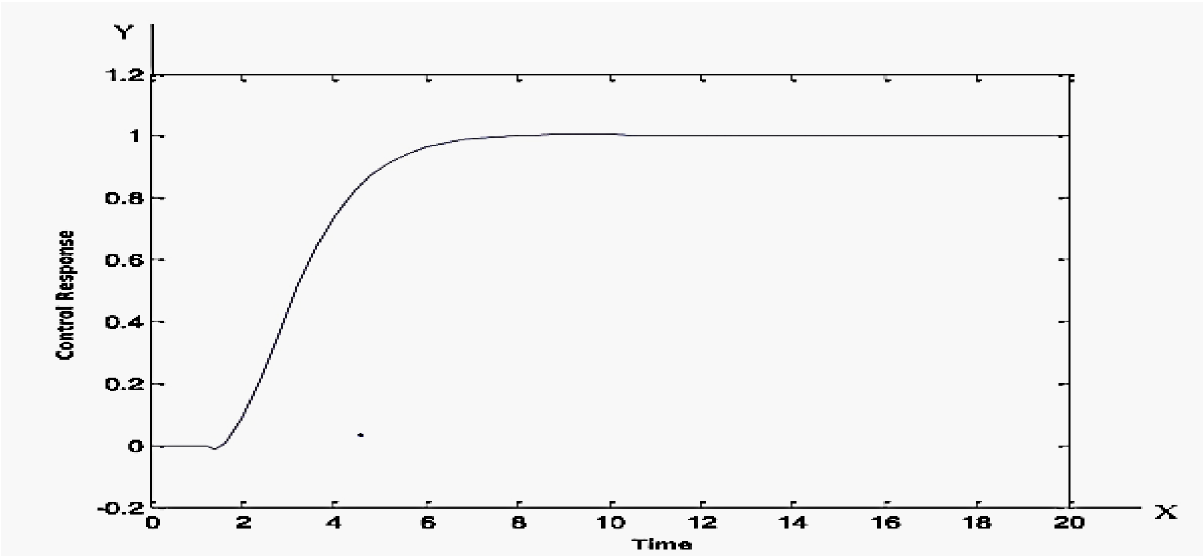

Fig. 4.1.8 Controller responses for case study 4

With the proposed method, sustained oscillations are obtained for the above transfer function which is shown in figure 4.1.7.The sustained oscillations are generated with the help of an asymmetric relay feedback method. When it comes to control design method for second order unstable systems even though a variety of methods are available in literature [Chinta Shankar 2015 et.al] for unstable systems, a simple PID controller is designed and the response is shown in figure 4.1.8. The response obtained is satisfactory for the designed controller even though a simple PID controller is used here for unstable process wherein it is actually very difficult to control the unstable systems due to their unstable behavior.

In this case study, a servo response is obtained for the set point tracking. The designed controller is compared with the method proposed by [Chidambaram at.al] and comparison table is given in table 4.2. It is seen from the table that the proposed designed method for control response yields less integral square error (ISE). For controlling second order unstable systems, there are different methods but due to simplicity of the relay feedback approach it is possible to get desired response for the second order unstable systems.

The figure 4.1.8 shows very smooth response for the given set point changes even though the second order systems possess damped oscillations. Here there is no such behavior for the obtained response. The control response shown in figure 4.1.8 reaches quickly to the steady sate without having any damping oscillations. The same analysis that was carried out in previous case studies has been incorporated here also while calculating other parameters which are shown in the table 4.1.

4.2 Identification & control of SISO process using Relay with Subspace:

The prerequisite concepts required for implementing the subspace and relay methods are discussed in the chapter 3.The benchmark example considered from literature here are linear SISO systems such as FOPTD and SOPTD. For these benchmark examples, a combination of relay and subspace methods have been adapted to generate Input-output data. A comparison between the identified process and actual was done and found that the results are in good agreement.

4.2.1 Subspace method

The subspace method has several advantages. The first advantage is that the linear models can be obtained from row and column spaces of certain matrices which are developed from input-output data. Typically, the column space consists of information about the model, while the row space allows obtaining a state sequence, directly from the input-output data. There is no need for an explicit parameterization. A second advantage is due to the elegance and computational efficiency of the algorithm of the subspace method. Subspace identification algorithms are based on concepts from system theory, linear algebra and statistics such as projections (orthogonal and oblique Projections) using QR decomposition. Subspace identification methods (SIMs) have gained popularity in the field of system identification since they can identify state space model directly from the input and output data. Subspace methods are based on robust numerical tools such as QR factorization and singular value decomposition (SVD) which makes them attractive from the numerical point of view. The most common classical subspace identification methods are CVA, MOESP and N4SID. In this work, relay and subspace algorithm are used to identify the process by taking different case studies from literature.

4.2.2 N4SID algorithm

In this research, N4SID has been introduced. Using the input-output data and the system identification tool box in MATLAB, the SISO process has been identified and compared with the actual process which is easier in terms of computational complexity and no prior information requirement. Relay feedback test is used to generate input-output data. Initially the data from the relay feedback test is generated in the form of sustained oscillations as shown in figure 4.2.1which shows the system identification tool box. The input and output data generated as shown in figure 4.2.1is fed to the N4SID algorithm to identify the process using the graphical user interface shown in figure 4.2.2

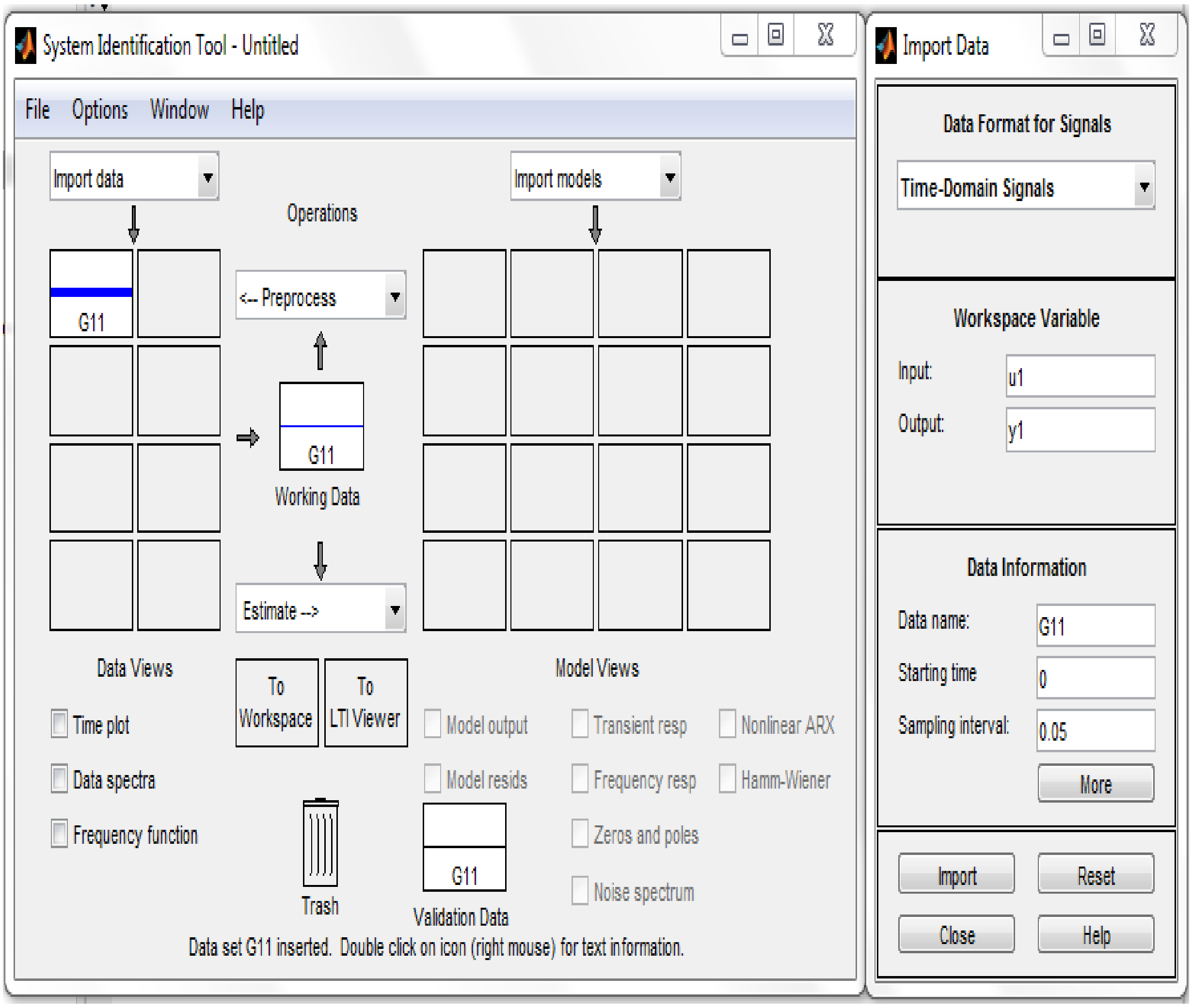

Fig.4.2.1 system identification Tool Box

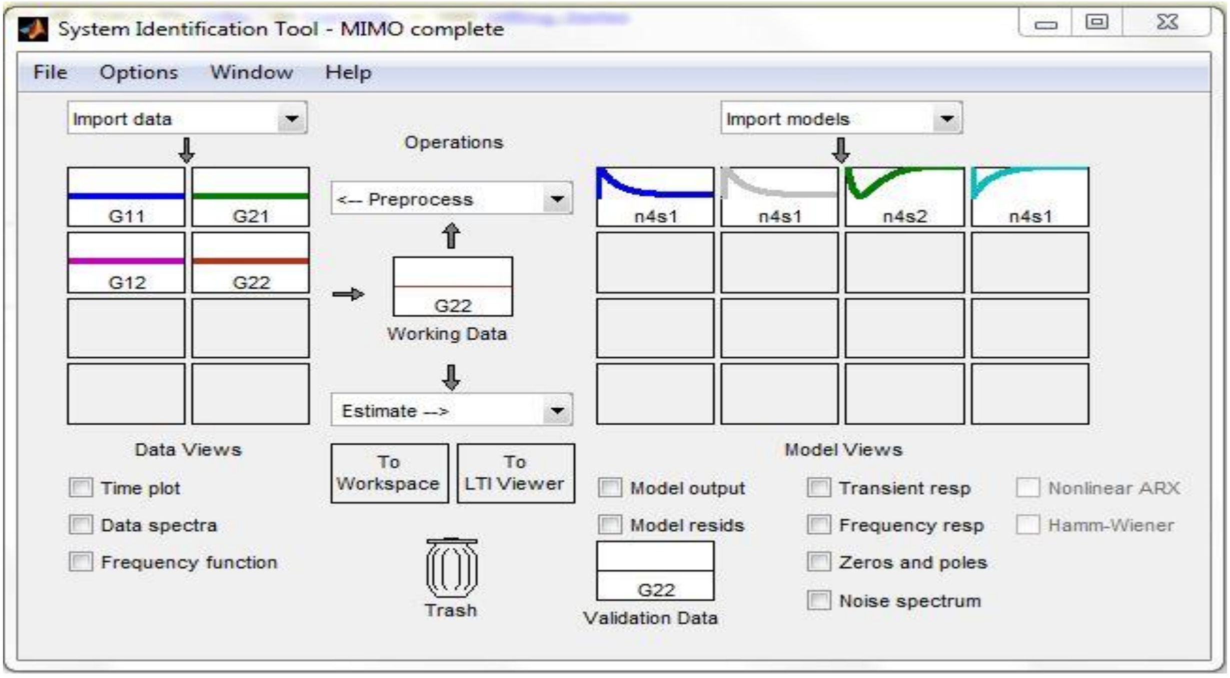

Fig.4.2.2 system Identification Tool box used for N4SID algorithm

4.2.3 Simulation Results

In this section different case studies from literature have been considered. The relay and subspace methods have been applied to identify the actual process**.**

4.2.3.1 Case study: EX - 1

A first order and dead time (FOPTD) dynamics shown below in equation 4.2.1 has been considered. Applying the Relay feedback test and subspace method, the following responses shown in figures 4.2.3 & 4.2.4have been obtained. Figure 4.2.3 shows sustained oscillations obtained from single relay feedback for the transfer function shown in the equation 4.2.1. The actual response for the system is shown in figure 4.2.4 and the identified process using the relay and subspace method obtained is shown in figure 4.2.5. From these figures, it is observed that there is good agreement between the actual process and identified responses.

$$G11 = \frac{0.126e^{-6s}}{(60s+1)} \qquad (4.2.1)$$

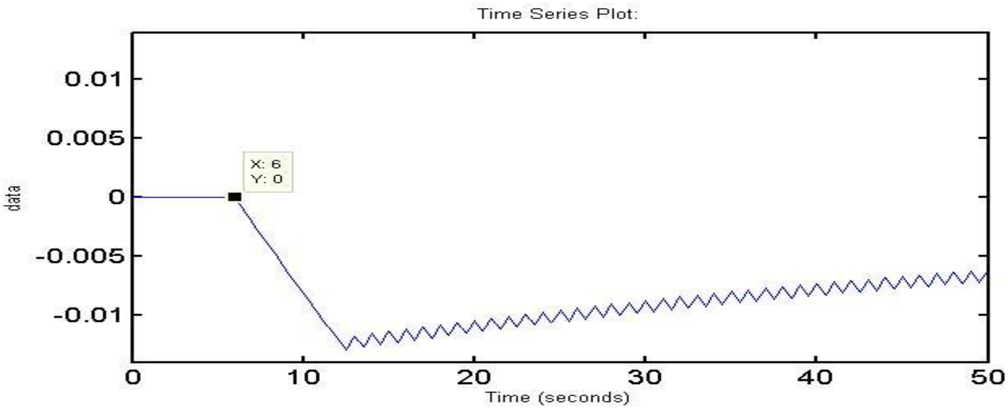

Figure 4.2.3 Relay Feedback response generated to be used for system identification

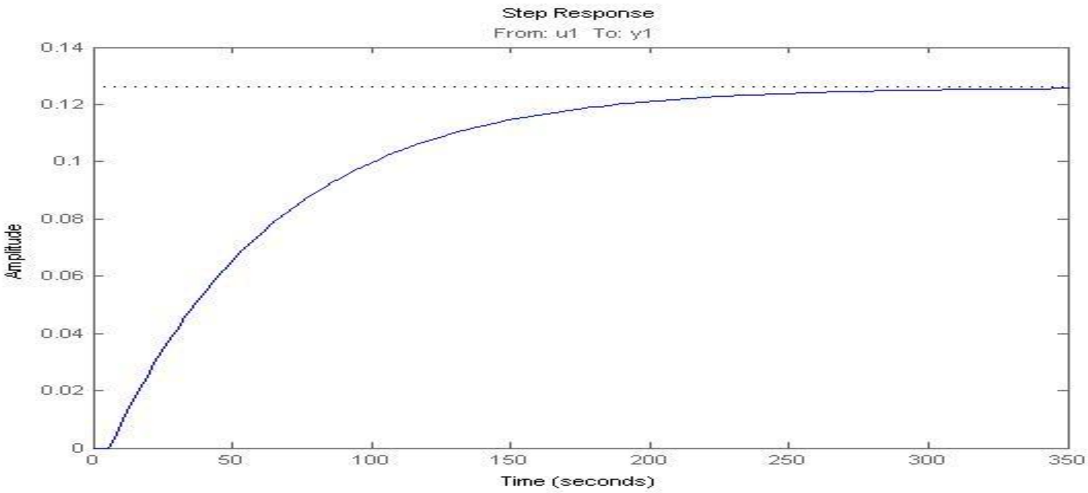

Figure 4.2.4 Actual response of the FOPDT system

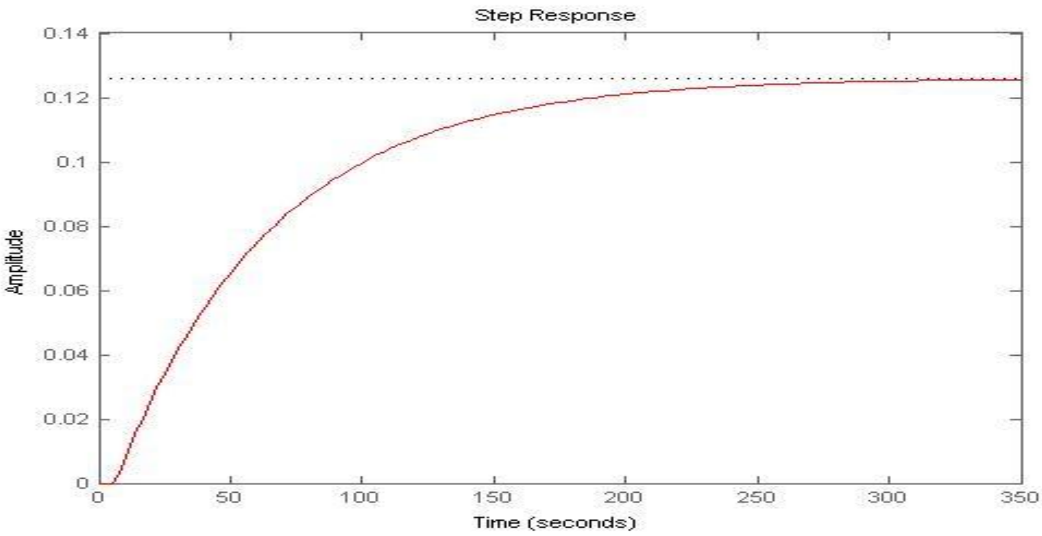

Figure 4.2.5 Step response from identified plant for Ex-1

For the identified process of case study 1, a simple PI controller has been designed and the response is shown in the figure 4.2.6. Figure 4.2.6 shows satisfactory result for the given set point change.

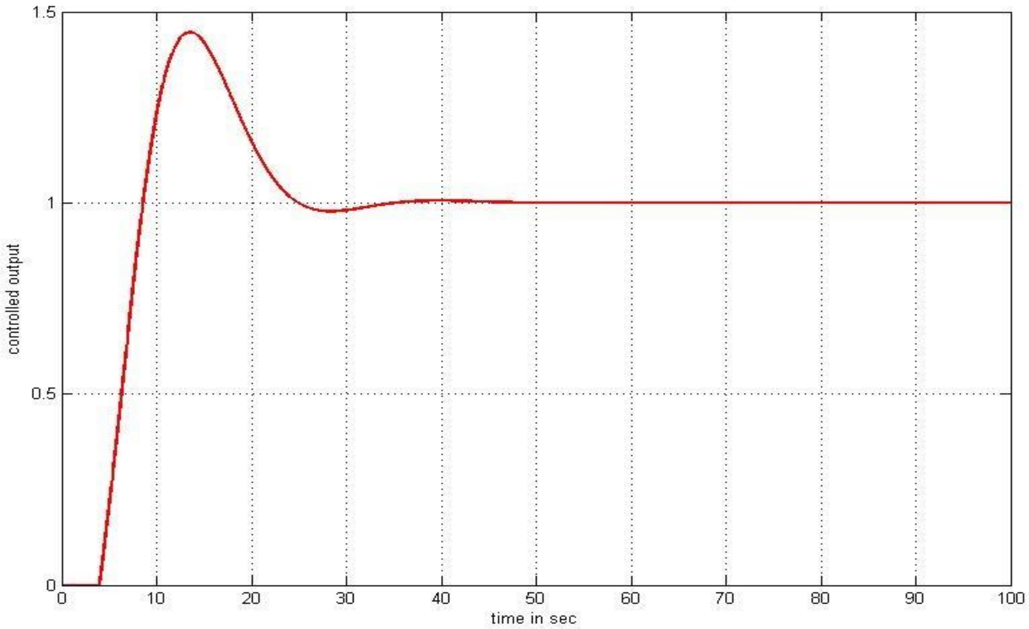

Fig. 4.2.6 Closed-loop controlled Response of the identified process Ex-1

4. 2.3.2. Case study: EX - 2

A FOPTD process with negative process gain has been considered here, where the actual process is given by the following transfer function shown in equation 4.2.2. The relay test is carried out to generate input-output data.

$$G22 = \frac{-0.1e^{-3s}}{(35s+1)} \qquad (4.2.2)$$

The sustained oscillations have been generated as shown in the figure 4.2.7. These sustained oscillations, as input and Output data are fed to the system identification tool box as shown in the figure 4.2.1 and 4.2.2. Using N4SID algorithm, the process is identified with less prior knowledge. The identified process is used to get step response which is shown in figure 4.2.9. There is much accuracy as we notice that there is very less error between responses of the actual and that of the Identified process. Step response of the identified process of Ex-2, is generated and is shown in Figure 4.2.9.

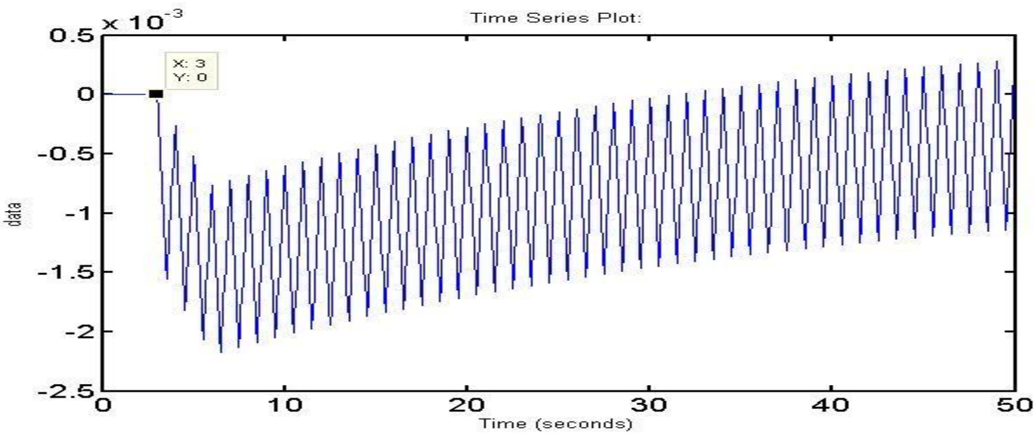

Fig.4.2.7 Relay feedback response of Ex-2

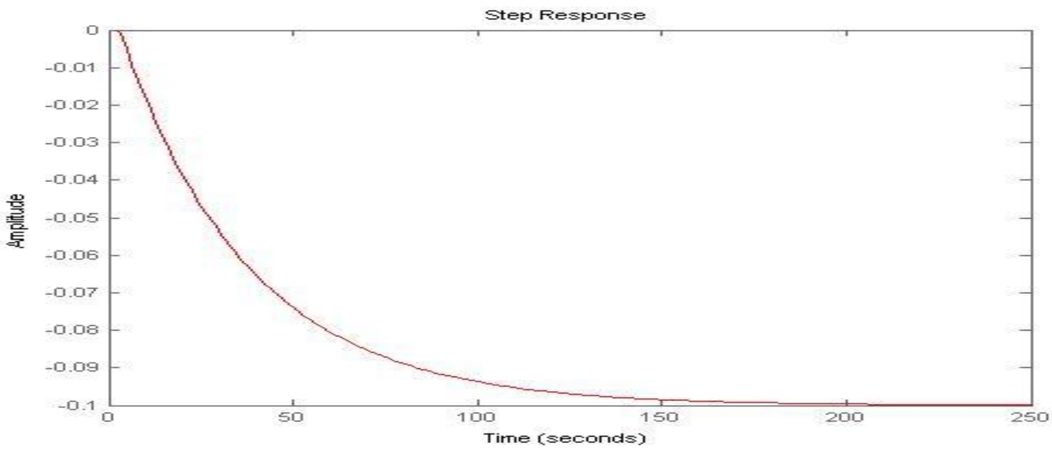

Fig.4.2.8 Actual step response of Ex-2

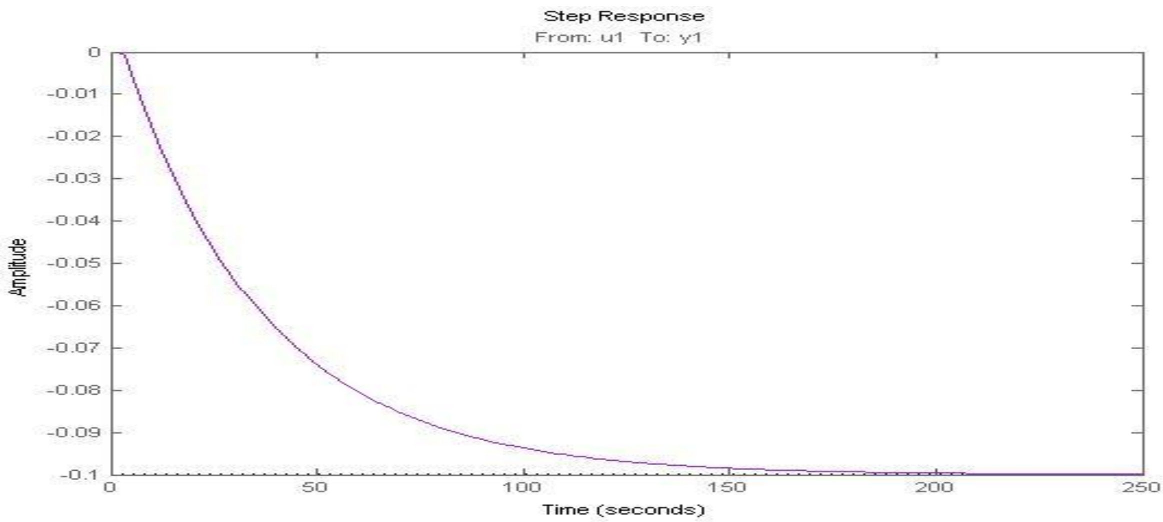

Fig.4.2.9 Step response of identified process of Ex-2

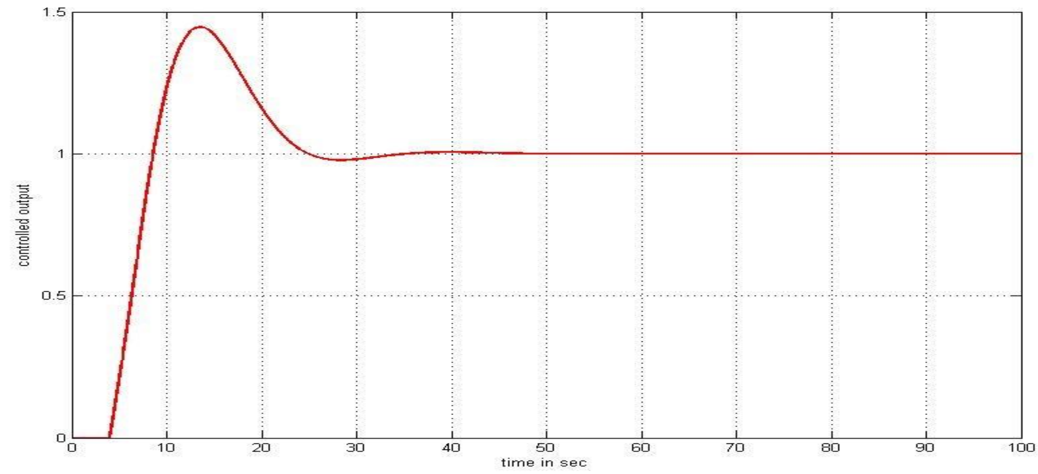

Fig.4.2.10 Closed-loop response of Ex-2 using PI controller

For the identified process of Ex-2, a simple PI controller is designed which is used to generate closed-loop response as shown in the figure 4.2.10. Figure 4.2.10 shows the satisfactory control response for the identified process.

4. 2.3.3 Case study: Ex-3

A process with second order plus dead time (SOPTD) dynamics ahs been consdered here. By carrying out similar exercise, relay response is generated for Ex-3. The input-output data is fed to subspace algorithm as described above. The sustained oscillations of Ex-3 are obtained and are shown in Figure 4.2.11. The sustained oscillations as input and output is fed to the system identification tool box as described in figures 4.2.1 &4.2.2.

The method has identified the process using the data obtained from the relay feedback test.

$$G12 = \frac{-0.1016 e^{-12s}}{(2160 s^2+93s+1)} \quad (4.2.3)$$

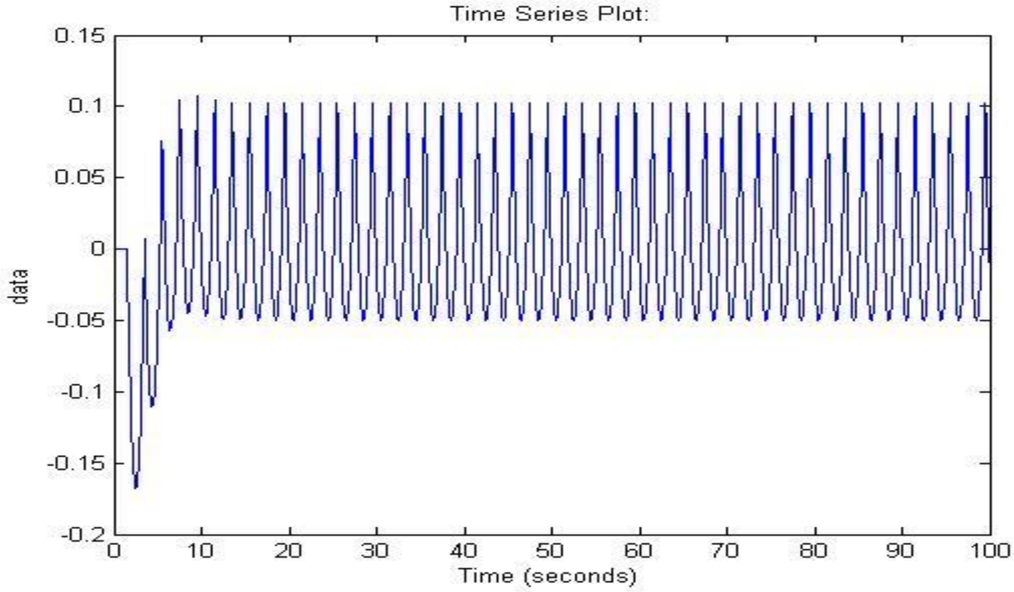

Fig.4.2.11 Relay Feedback response of Ex-3 showing sustained oscillation

The step responses of actual plant (Eq. 4.2.3) and identified transfer functions are shown in the figures 4.2.12 and 4.2.13 respectively. It can be found that there is a good agreement between the step responses of actual process and identified process.

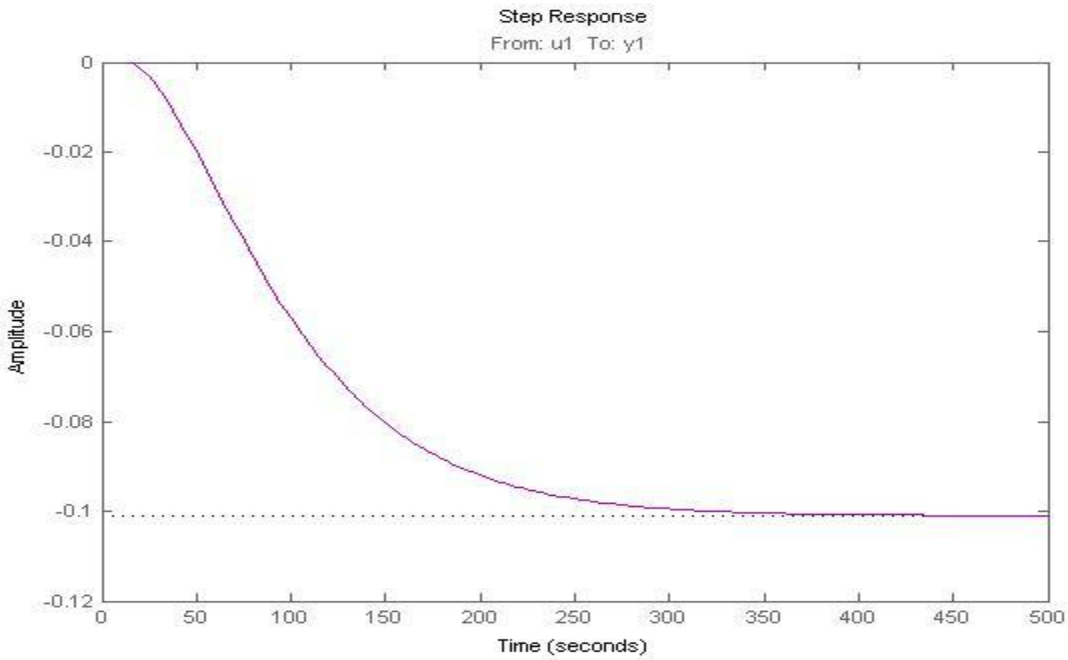

Fig.4.2.12 Step response of identified process of Ex-3

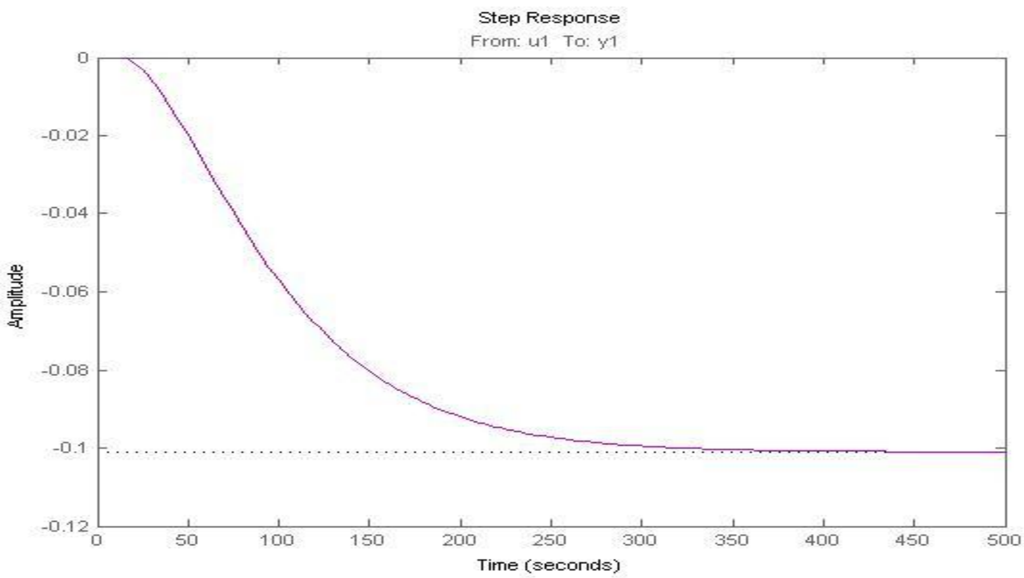

Fig.4.2.13 Step response of the actual response Ex-3

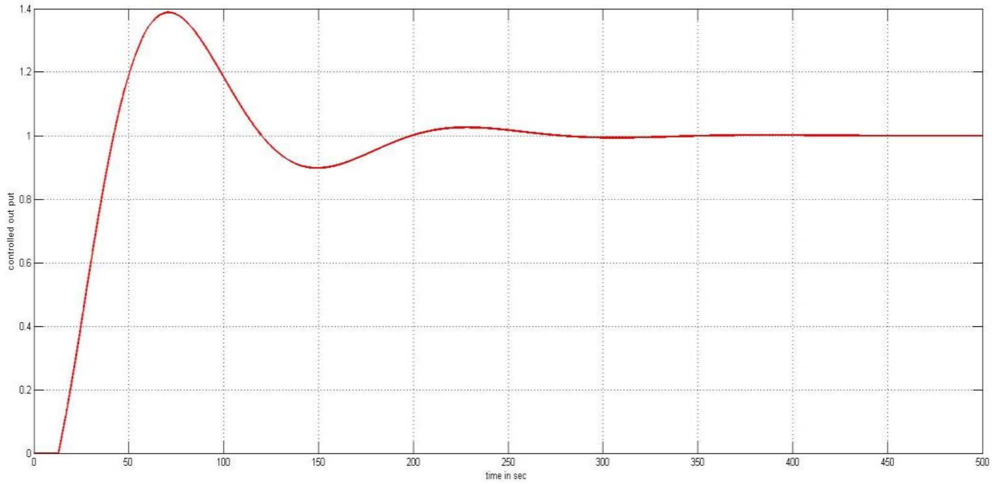

Fig. 4.2.14 Closed-loop response of Ex-3 using PID controller for a set-point change

For the identified process of Ex-3, a simple PID controller has been designed which is used to generate closed-loop response for a set point change, as shown in figure 4.2.14. It can be observed that the identified process is of second order dynamics with time delay which is having the damping oscillations. For this process, a simple PID controller is designed and the closed-loop system is simulated for set point tracking which shows the satisfactory response even with the simple PID controller. Table 4.2.1 shows the parameters for controller, and the performance index values. The performance index values for IAE are shown for three examples.

Table 4.2.1 Controller parameters and performance indices

Examples	Plant Transfer function	PID controller Parameters			IAE values
		Kc	τ_i	τ_d	
Ex-1	$G11 = \frac{0.126e^{-6s}}{(60s+1)}$	47.73	20	-	2.4
Ex-2	$G22 = \frac{-0.1e^{-3s}}{(35s+1)}$	-65.64	10	-	3.1
Ex-3	$G12 = \frac{-0.1016e^{-12s}}{(2160s^2+93s+1)}$	-44.95	55	13.75	2.67

4.3. Identification and control of non-linear SISO process using Relay Feedback approach:

Two case studies from the literature are considered here to illustrate the effectiveness of proposed method. Hammerstein and wiener process are considered with their transfer functions. A MATLAB Simulink approach is used to perform the relay feedback test followed by the control design for the respective processes.

4.3.1 General information on non-linear systems

Non-linear systems can be defined as the systems that do not obey the principle of superposition theorem. Non-linear systems are very common in Chemical process industries. All the chemical systems are non-linear in nature by virtue of their nature. The examples of non-linear system are pH of a process, distillation column, Continuous stirred tank reactor (CSTR), Bio-reactors, Heat Exchangers etc. All these non-linear processes are linearized at certain operating point and approximated to lower order processes for controller design. Usually the non-linear systems can be classified into two types viz. Wiener and Hammerstein type processes. A Wiener process is a linear system followed by non-linear element for Hammerstein process vice-versa.

4.3.2 Identification and control of Wiener-Process by relay Feedback approach:

A Wiener-type process given by the following transfer function with higher order linear subsystem followed by non-linear element [UtkalV.Mehta, 2011] has been considered.

$$G_1(s) = e^{-s}\frac{1}{(s+1)^2(2s+1)^3}$$

$$f_1(v) = 2(1 - e^{-0.693v}) \qquad (4.3.1)$$

An auto-tuning test is conducted with symmetric relay and the linear subsystem is identified which is given by the following transfer function.

$$G_p(s) = e^{-3.3013s}\frac{1}{(3.123s+1)^2} \qquad (4.3.2)$$

A Simulink diagram with added noise is used here to generate sustained oscillation. The process

is identified followed by the control design steps as given as in figures 4.3.1 & 4.3.2 that shows Sustained oscillations obtained from actual process. After identifying parameters, the controller is designed and is used to simulate actual process which is shown in figure 4.3.3. The closed loop oscillations are obtained as control response which is also obtained for actual process and identified process with noise and without noise. The oscillatory responses are shown in figures 4.3.1 and 4.3.2 respectively.

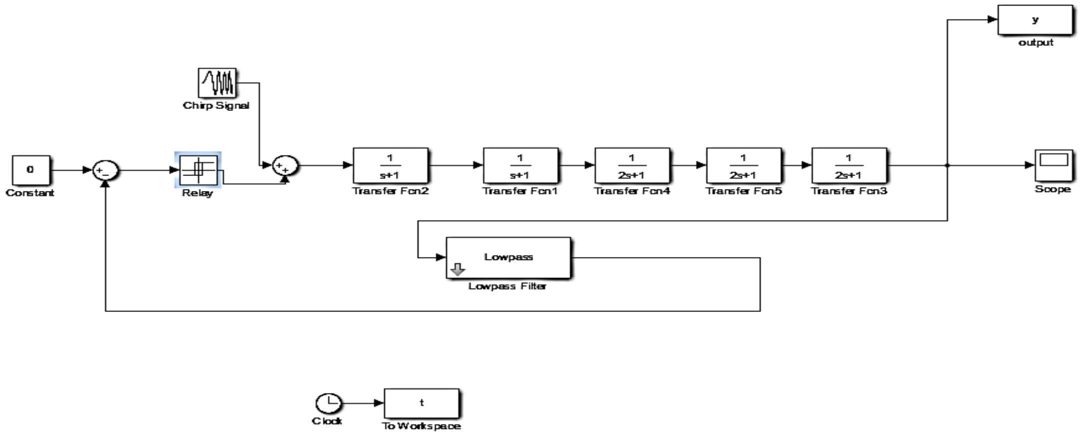

Fig. 4.3.1Simulink diagram for the identification of the wiener/Hammerstein process

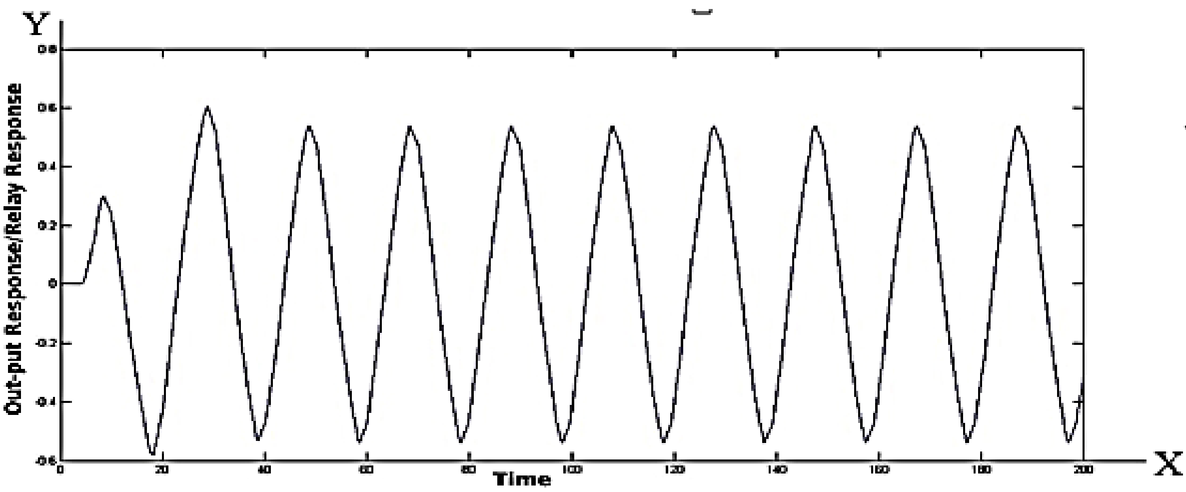

Fig. 4.3.2 Limit cycles of wiener process with chip signal (Noise signal)

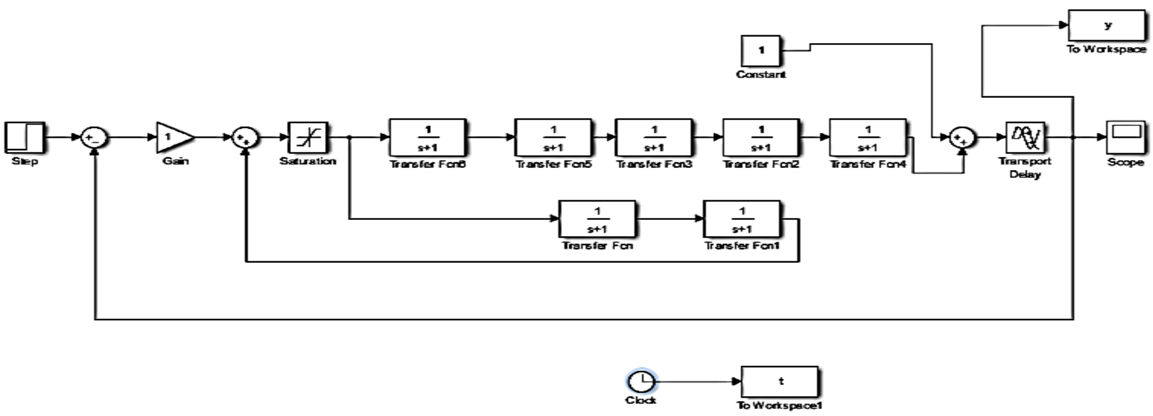

Fig. 4.3.3 Block diagram representation of closed-loop scheme for obtaining performance using identified process

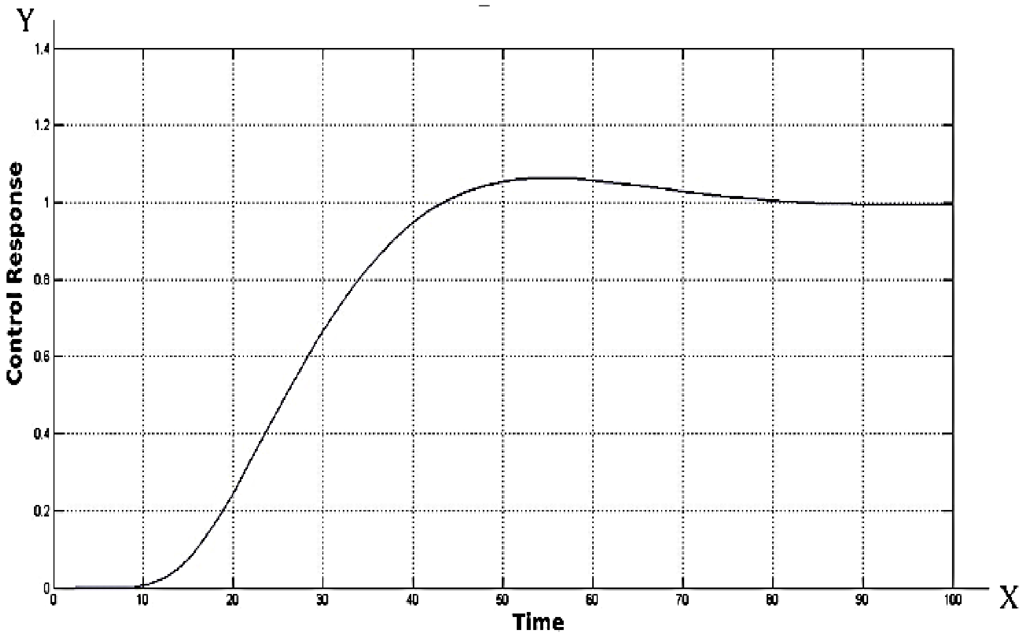

Fig. 4.3.4 Response of process variable using actual process with noise

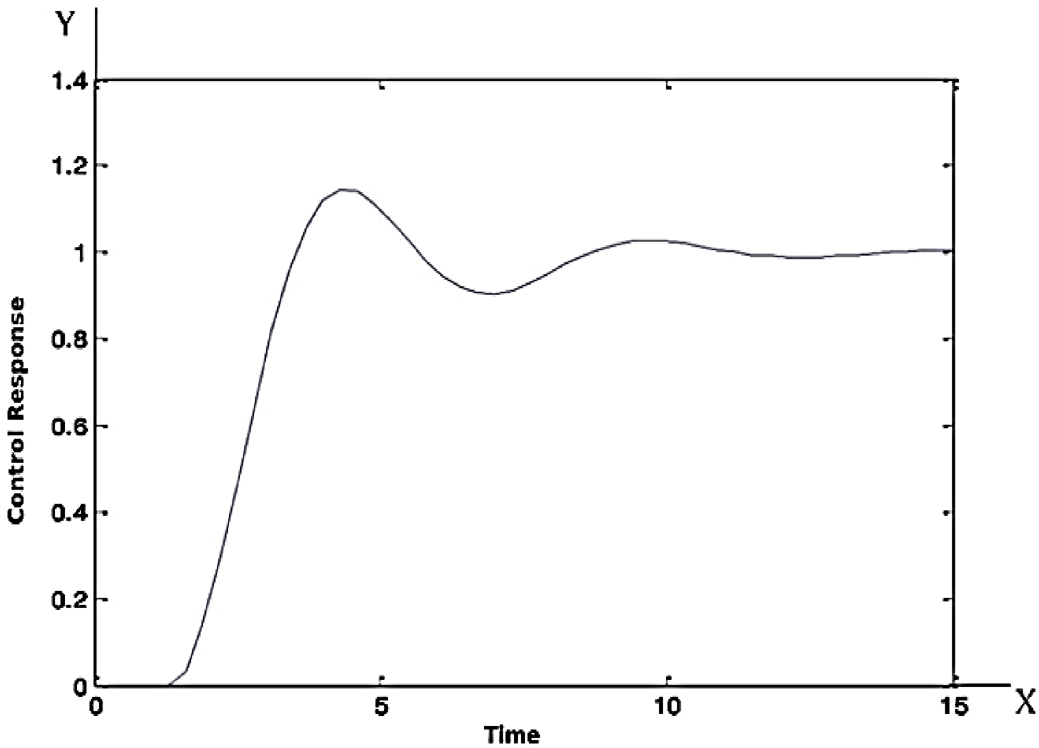

Fig. 4.3.5 Response of process variable using identified process with noise

4.3.3 Results and Discussion for Subsection

A non-linear Wiener process given by the transfer function cited above is considered here and an auto tuning test is carried out with help of Simulink block diagram shown in figure 4.3.1which is used again for carrying out auto-tuning test Hammerstein process also. It can be seen that a noise signal is inserted externally and low pass filter is fed to the feedback of Simulink approach to reduce the effect of noise.

Utkal V Metha (2011) used autotuning test to generate sustained oscillations and increase the height of the relay to reduce the effect of the noise. But, in this work instead of increasing the height of the relay which causes practical difficulties, a low pass filter is used to reduce the effect of noise.

The sustained oscillations are generated which are shown in the figure 4.3.2 by following the analysis adapted by Utkal V Metha (2011) and the simplified identified transfer function is obtained given by the transfer function shown in equation 4.3.2

A new proposed control structure shown in figure 4.3.3is used to obtain the responses of actual and identified processes which are given by the following figures 4.3.4 & figure 4.3.5. Figure 4.3.4 shows the response of the identified process using the proposed control response structure which shows the satisfactory response for the set point change even in the presence of noise. Similarly for identified process also, the control response is obtained in the presence of noise using the proposed control structure which shows the satisfactory response for servo change. In spite of having different controller design methods available in the literature for non-linear processes, here a simple method is adapted and control responses for the identified and actual process are plotted and shown in figures of 4.3.4 and 4.3.5 receptively.

4.3.4 Identification and control of Hammerstein process using Relay feedback approach

A non-linear transfer function given by the following equations [Utkal V Mehta, 2011] that has a non-linear static element followed by linear subsystem has been considered.

$$f_2(v) = \left(1.591 - e^{-(0.5u)}\right)|u| \qquad (4.3.3)$$

$$G(s) = e^{-s}\frac{1}{(s+1)(2s+1)^2} \qquad (4.3.4).$$

$$G(s) = e^{-3.898s}\frac{1}{(3.886s+1)^2} \qquad (4.3.5)$$

The identification is carried out in the presence of noise with noise signal and the new control structure is proposed for the identified and actual process which shows satisfactory response as shown in following figures in the presence of and absence of noise. The obtained results with the proposed control structure are shown in figure 4.3.6 which is obtained with MATLAB/Simulink approach. The transfer function of identified processes is given by equation 4.3.5.

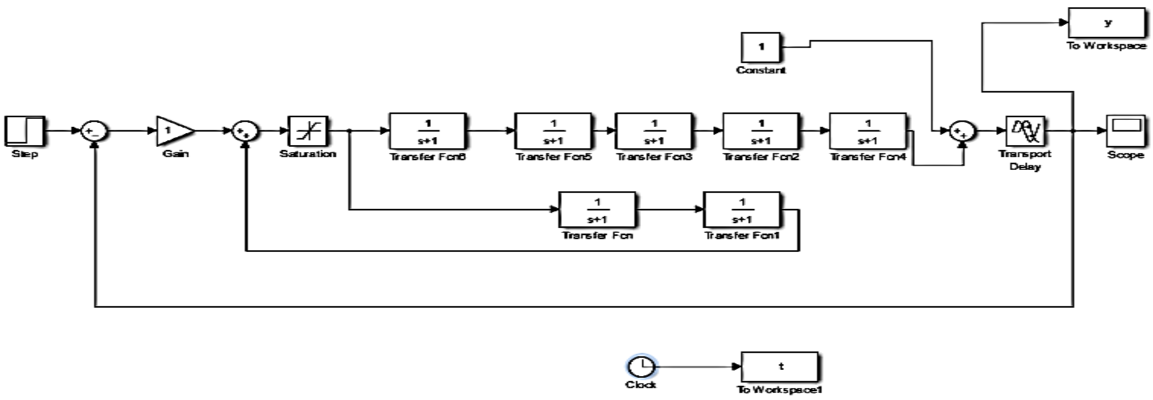

Fig. 4.3.6SchematicControl Diagram of the non-linear Hammerstein process

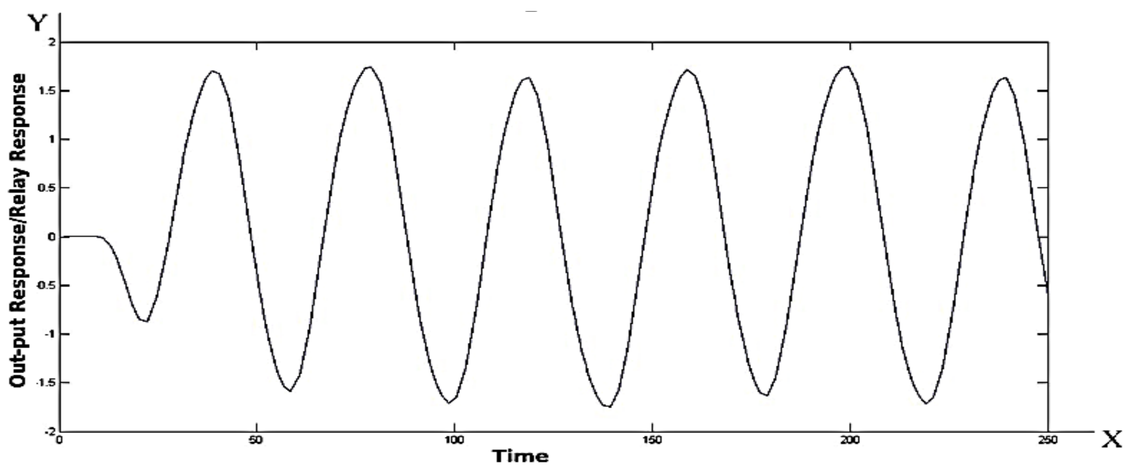

Fig. 4.3.7 Limit cycle oscillations of Hammerstein-type process with noise

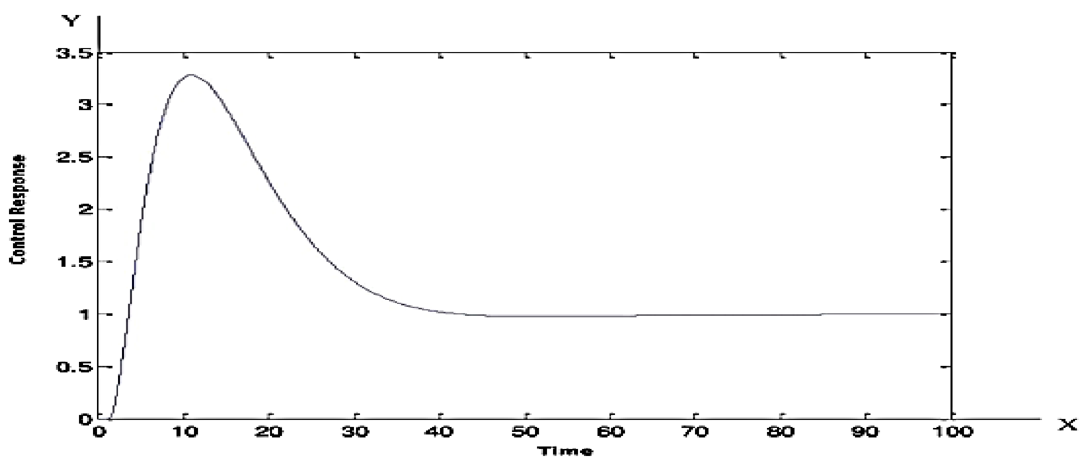

Fig. 4.3.8 Control response of the actual Hammerstein process with noise

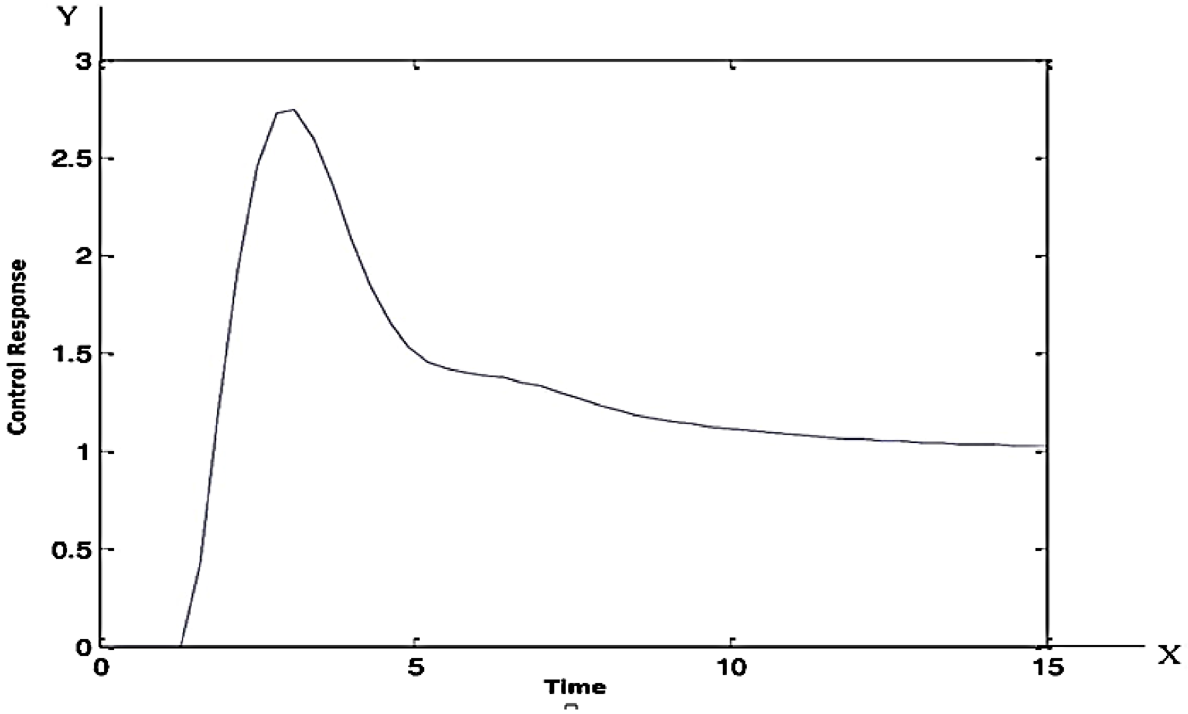

Fig. 4.3.9 Control response of the identified Hammerstein process with noise

In this section, a non-linear Hammerstein process is considered that is reverse of Wiener process and auto tuning test is carried out in the presence of noise using Simulink block diagram given by figure 4.3.6 The sustained oscillations are generated which are shown in figure 4.3.7 . In this a noise signal is externally added to see the effect of efficacy of the proposed method. For the identified and actual transfer function which are given by equations 4.3.3 and 4.3.4 respectively, the control responses are obtained which are shown in figures 4.3.8 and 4.3.9 respectively. UtkalV.Metha (2011) used auto tuning test to generate sustained oscillations and increase the height of the relay to reduce the noise effect of the noise. But in this work, instead of increasing the height of the relay which causes the practical difficulties the low pass filter is used to reduce the effect of noise.

The control responses which are obtained by the proposed control structure is given by figure 4.3.8 which shows the servo response for the actual process for given set point change. It gives the satisfactory response. Similarly, for the identified process also the proposed control structure is shown in figure 4.3.9. It can be observed that the identified response shows better value than the actual response even in the presence of noise which is externally added to the process.

Identified process is having less time to reach the steady sate when compared to the actual process. Even though different methods are available for the control of these non-linear processes, which are difficult, the present method does not require more analytical expressions for control processes. In this work a simple Simulink approach given in figure 4.3.6 gives good response for both the identified and actual processes.

4.4 identification and control of 2x2 linear MIMO process using Relay combined with subspace method:

In previous tasks, the discussions were on identification and control of linear and non-linear SISO processes. In this section, MIMO processes are considered for identification using Relay with subspace method. The necessary information on subspace method and its merits are presented in the third chapter. In this section, a case study on 2x2 distillation column is considered for illustrating the effectiveness of proposed method.

4.4.0 General

Most of the processes involved in industrial plants are multivariable in nature because these involve more than one input & output. Interaction between them makes much more difficult to control the process variables. Identification and control of MIMO systems by relay feedback methods (Sequential approach) got prominent role by using the analytical expression for estimating various parameters [Panda &Sujatha, 2012]. Moreover, when the order of the system increases, it is very difficult to obtain analytical expression using the methods proposed [Panda and Sujatha, 2013]. Model development is necessary for simulating system behaviour & to observe its control performance. In general, developing a model from first principles requires in-depth knowledge about the system as well as significant time and effort. In order to minimize this problem a new method is proposed and illustrated here with case study.

4.4.1 Identification Method Used:

In this work, simultaneous relay and subspace method has been introduced to identify a process and control it through N4SID algorithm step by step. For effective implementation of the proposed method, a 2x2 distillation column has been considered which is implemented in Simulink with the help of system identification tool box as shown in figure 4.4.1. The controllers are replaced by the Relay as per the specifications given below and sustained oscillations are generated individually for each transfer function. This data (input-output) is fed to the system identification toolbox with the help of N4SID algorithm and the parameters have been identified. This method needs less information but identifies parameters from the actual data for MIMO process.

4.4.2 Simultaneous Relay + Subspace test on 2×2 Wardle & Wood distillation column:

A Wardle & Wood (1969) transfer function matrix for separation of methanol and water given by equation 4.4.1 has been considered.

$$\begin{bmatrix} x_D \\ x_B \end{bmatrix} = \begin{bmatrix} \frac{0.126e^{-6s}}{60s+1} & \frac{-0.101e^{-12s}}{2160s^2+93s+1} \\ \frac{0.094e^{-7s}}{38s+1} & \frac{-0.1e^{-3s}}{35s+1} \end{bmatrix} \begin{bmatrix} L \\ V \end{bmatrix} \qquad (4.4.1)$$

Where

x_D = top composition x_B = bottom composition L = reflux V = reboiler steam flow rate this transfer function matrix is used for illustration.

A 2×2 Wardle & Wood system consists of two inputs (u1, u2) and two outputs (y1, y2) for methanol water separation. No pre-designed controller is required for this identification method. In the first step, specific settings for each block are set. Gaussian white &triangular wave is used as set point for the system. Signal r1 & r2 should be such that it will generate random input signal to excite the system dynamics to generate better & accurate data from system behavior. Specifications about these signals are given below followed by the simulation procedure (Fig 4.4.1). Initially the first loop is closed while the second loop is open. The response obtained y1 & y2 corresponds to G11 & G21. From the response, the time delays for G11 & G12 are obtained. Likewise, keeping the first loop open while second loop is closed, the response of G12 & G22 is obtained. From the response time delays for G12 & G22 are obtained. The stepwise procedure is detailed below.

Step-1: For first run, the first loop is closed while second loop is open.

Step-2: For second run, the second loop is closed while first loop is open

Y_{sp1}: Step input of magnitude 0.96

Y_{sp2}: Step input of magnitude 0.03

r1: white Gaussian noise, sample time 0.05, Noise power: [0.1].

r2: triangular repeating sequence, time values [0 2], output values [-1 1]

Relay height [1 -1], default values for others

Simout/to work space blocks: sample time [0.05 sec] to generate a large amount of data for better accuracy), format of saving data: [array], no disturbance is assumed throughout the system. Simulation time: 1000 second, the above identification procedure was carried out in two steps and the corresponding excitation signals were shown in the figures 4.4.2 to 4.4.5

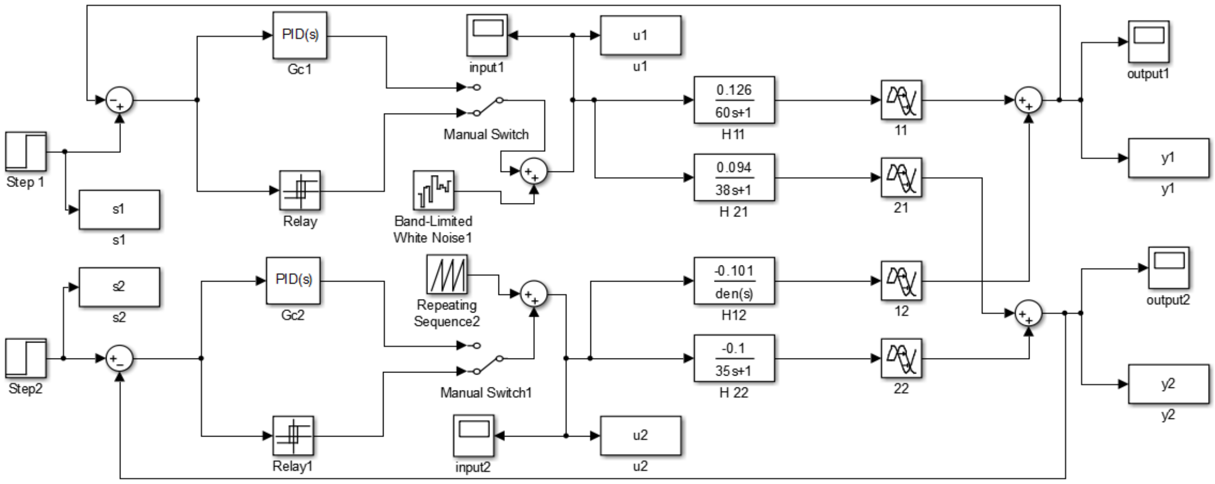

Fig. 4.4.1 Simulink diagram used for Identification

4.4.3 N4 SID algorithm

In this section, N4SID algorithm has been adapted to the input-output data obtained from Relay feedback test using system identification tool box in MATLAB and the MIMO process was identified and compared with the actual process which is easiest in terms of the computational complexity and no prior information is needed.

The following steps are involved inN4SID algorithm

Firstly make a Hankel matrix from I/O data (4.4.2)

- $H = \begin{bmatrix} u_P \\ y_P \end{bmatrix}$
- Compute LQ factorization of H. (4.4.3)
- $H = \begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}$ (4.4.4)

Where L is lower triangular metrics. Q is orthogonal matrix.

- $y_P = L_{21}Q_1 + L_{22}Q_2 = y_P/u_P + y_P/{u_p}^{\perp}$ (4.4.5)
- $L_{22} = y_P/{u_p}^{\perp}$ (4.4.6)
- Since $\mathbb{O}_h = y_P {u_p}^{\perp}$, the SVD of L_{22} = $U_1 {S_1}^{1/2} {S_1}^{1/2} V_1^T$ (4.4.7)
- Determine $\mathcal{O}_h = U_1 {S_1}^{1/2} and\ \widetilde{x}_f = {S_1}^{1/2} V_1^T$ (4.4.8)
- Now $\widetilde{x}_K$ & $\widetilde{x}_{K+1}$states are determined.
- $\begin{bmatrix} x_{K+1} & x_{K+2} & \cdots & x_{K+J-1} \\ y_K & y_{K+1} & \cdots & y_{K+j-2} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_K & x_{K+1} & \cdots & x_{K+j-2} \\ u_K & u_{K+1} & \cdots & u_{K+J-2} \end{bmatrix}$ (4.4.9)
- The model parameters A, B, C & D are calculated by least square method.

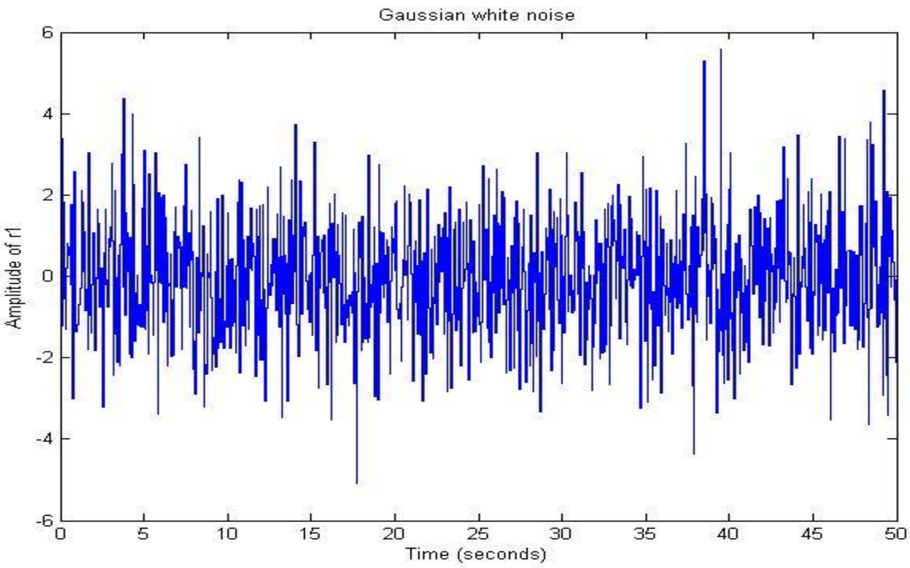

Fig. 4.4.2 Gaussian white noise signals used for excitation in step 1

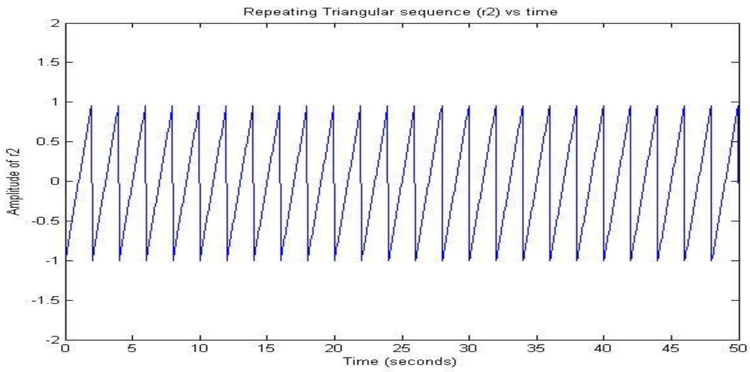

Fig. 4.4.3 Repeating triangular sequence (r2) used for excitation step 1

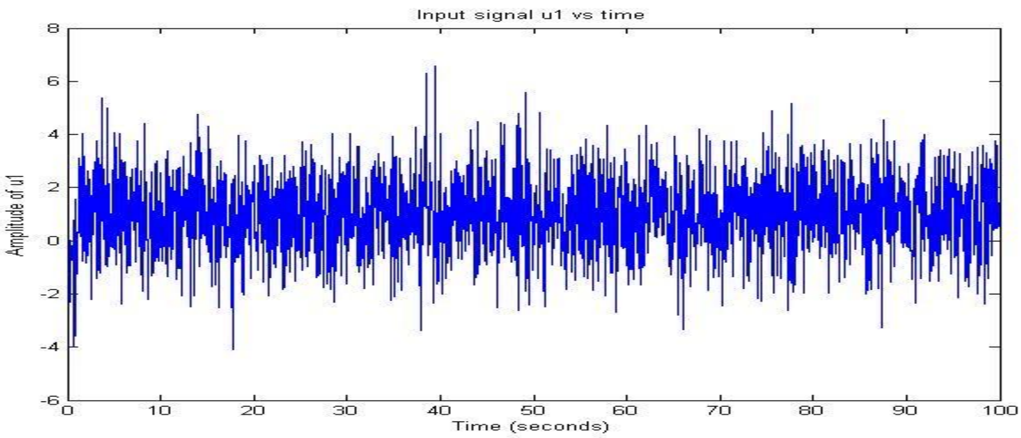

Fig. 4.4.4Gaussian white noise signal used for excitation in step2

Step-1

Upon completion of first simulation run, it generated u1, y1 & y2 data sets in workspace of MATLAB.

- The system identification toolbox is opened next as in figure 4.4.7 & 4.4.8
- Select time domain data in import data section.
- A new tab will open as shown in figure 4.4.8, Set input: [u1], output [y1], data name: [G11], sample time [0.05], starting time [0]. Then import the data.
- Now select estimate model & choose state space model. A new window will open (see fig 4.4.7).
- $x(t) = \mathrm{Ax(t) + Bu(t)}$
- $\mathrm{y(t) = Cx(t) + Du(t)}$
- Select as shown in 4.4.6,the system identification used in this work is shown in the figure 4.4.7
- Order 1:10 means system will decide the order of estimated model after simulating data.
- Method : n4sid
- Continuous signal
- Feed through : false (D matrix should be zero)
- Form: companion
- Input delay : same as output delay of G11(Delay is intentionally added to make a symmetric Hankel matrix from data points)
- K =0 since no disturbance is considered here.
- Then estimate the model. It will generate identified model and state space matrices A, B, C & D.
- The identified model parameters are determined by converting A, B, C & D matrices to transfer function in command window shown in the fig 4.4.7.
- Sys = SS (A, B, C, [ ]);   G11 = tf (Sys);   to get G11 identified model.
- Similarly to estimate G21, the data imported should be u1 & y2 instead of u1 & y1. The same procedure is to be carried out to estimate G21.the response of the inputs for identification procedure was shown in the figures4.4. 8 & 4.4.9 belowStep-1.
- After determination of G11 & G21, the first loop is opened, while 2nd loop is closed).Similar procedure is followed to determine G12 & G22 (Figure4.4.9). The response for each transfer function for two inputs figs are shown in the figures 4.4.10 & 4.4.11 below.

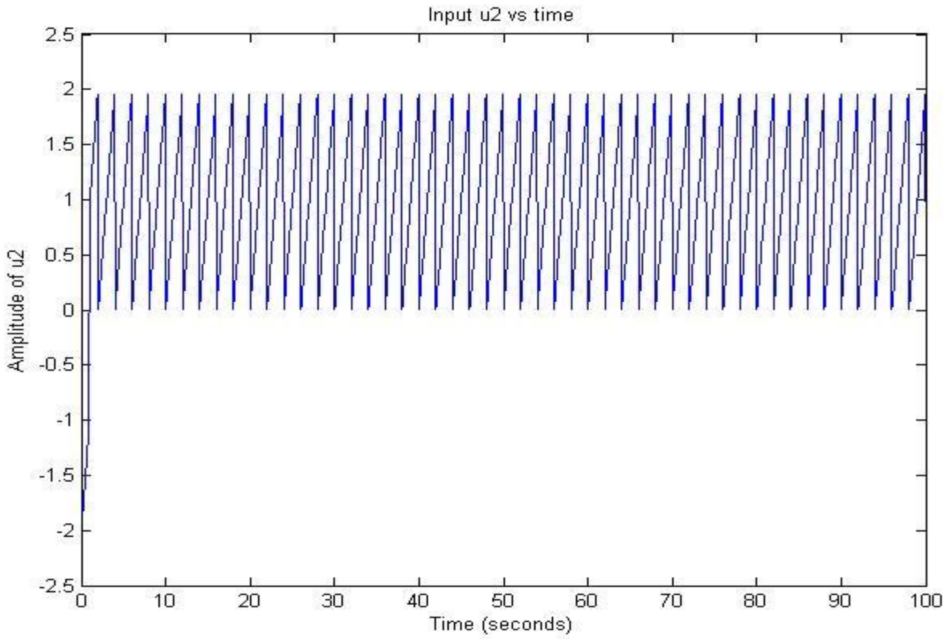

Fig. 4.4.5 Repeating triangular sequence (r2) used for excitation step 2

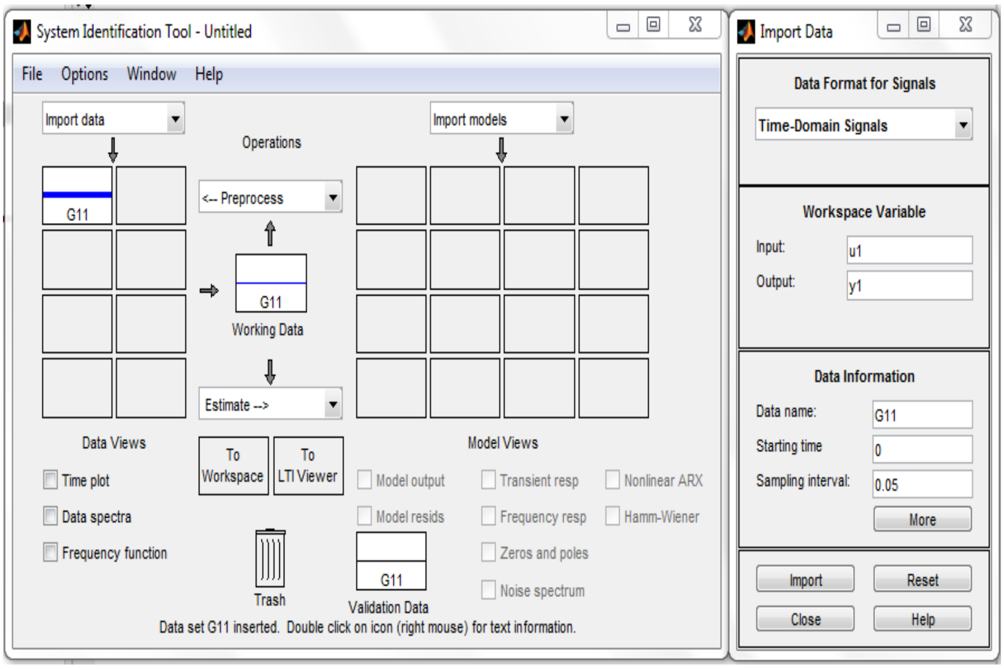

Fig. 4.4.6 System identification tool box

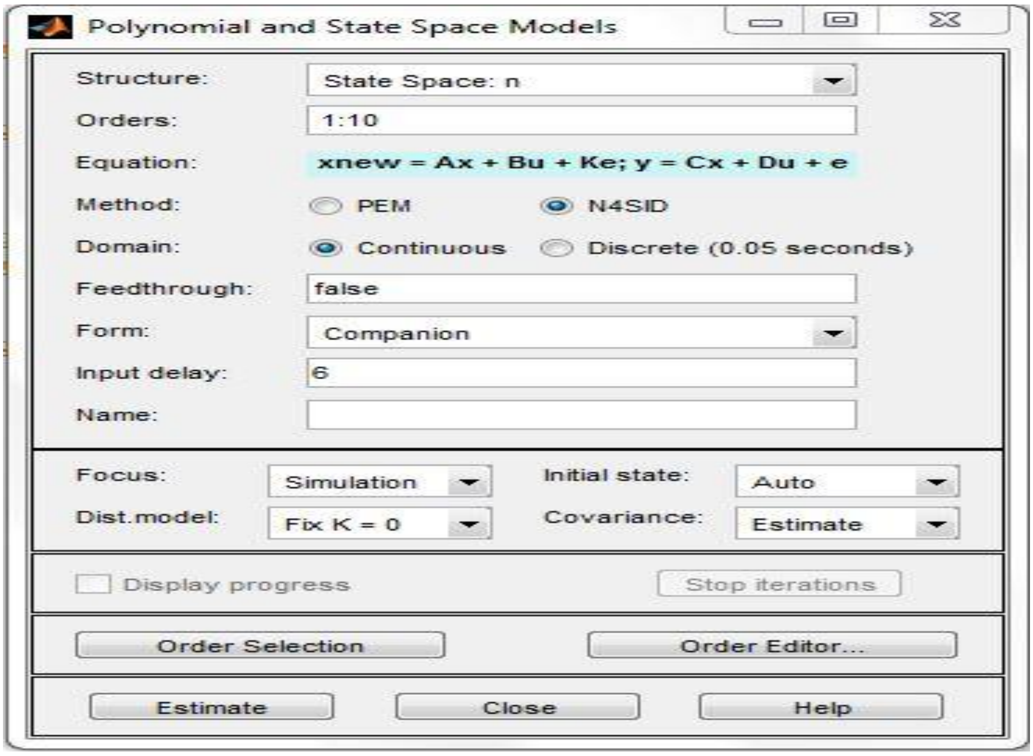

Fig. 4.4.7 State Space model windows

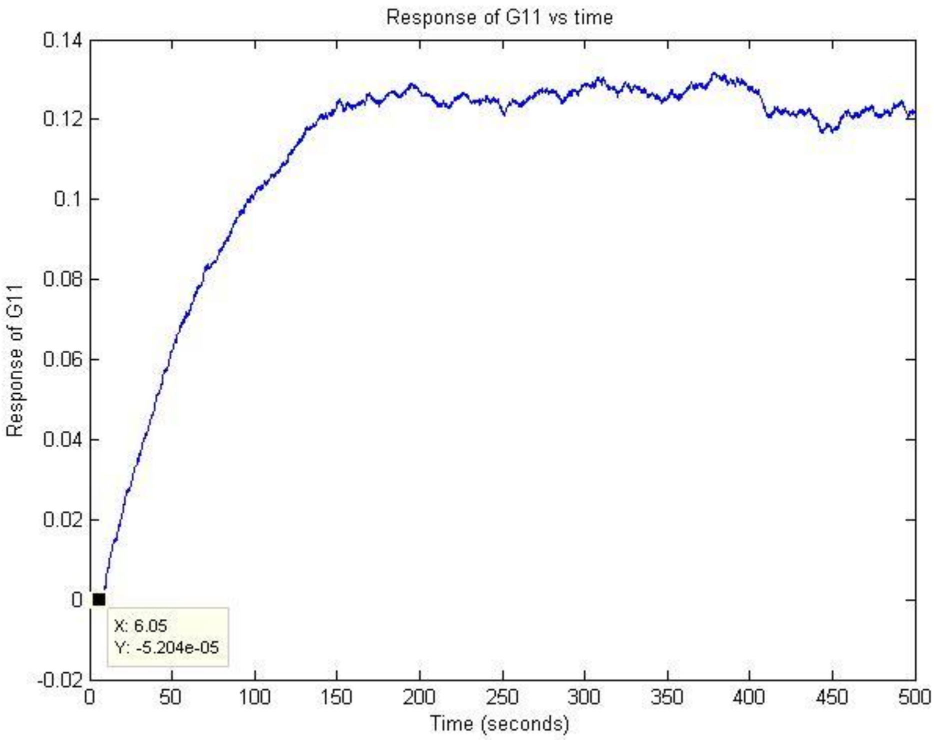

Fig. 4.4.8 Response of G11 for the input of the Gaussian white noise signal used in step 1 of identification

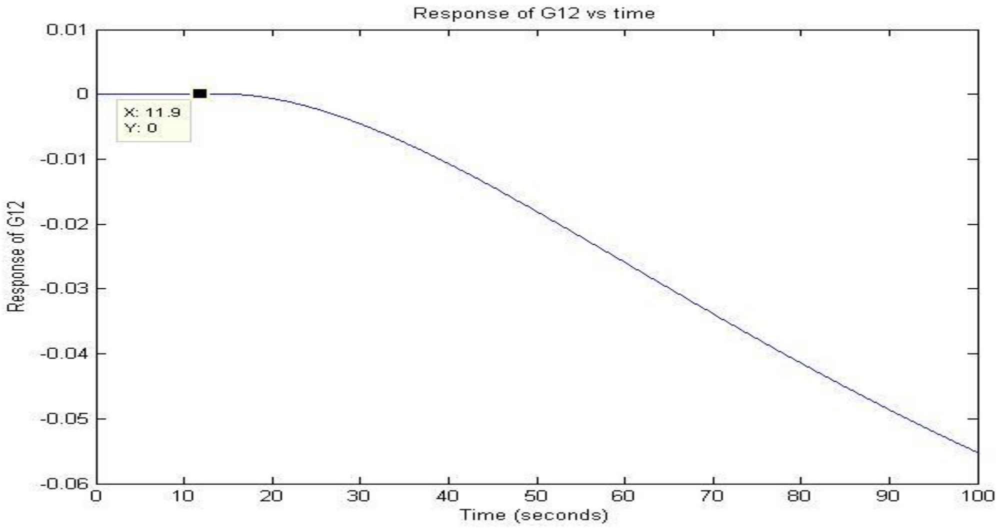

Fig. 4.4.9 Response of G12 for input of repeating triangular sequence signal used in step 1 of identification

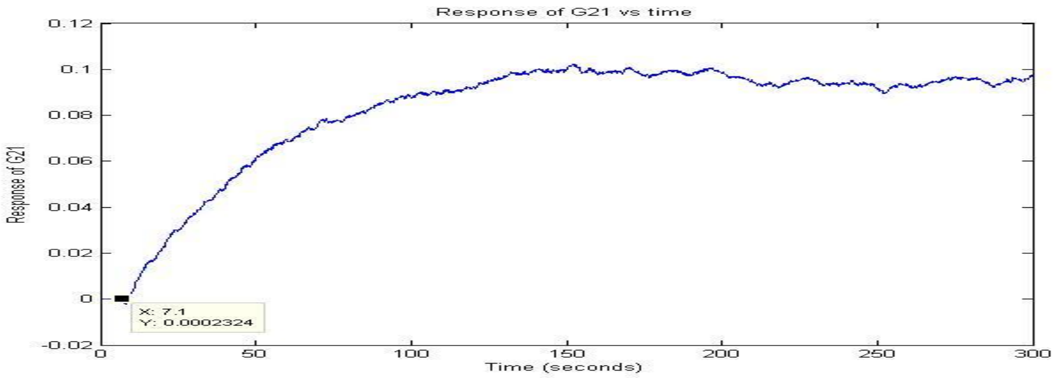

Fig. 4.4.10 Response of G21 for the input of the Gaussian white noise signal used in step 1 of identification

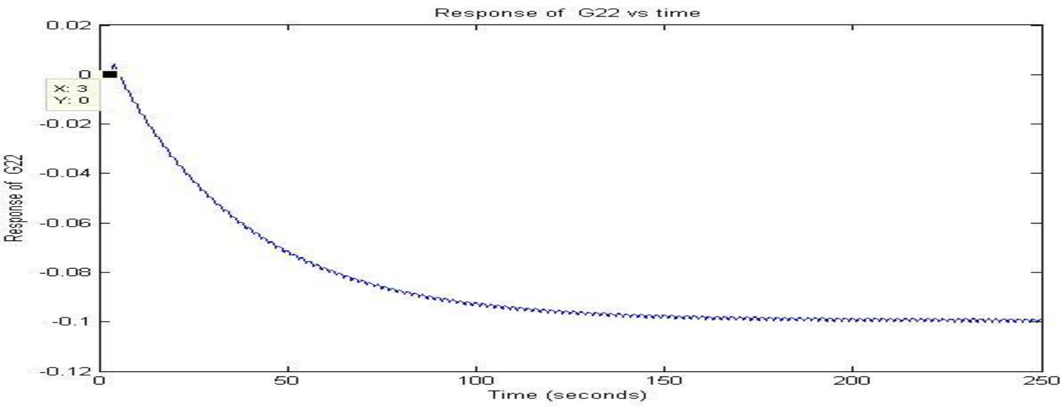

Fig. 4.4.11 Response of G22 response of G12 for input of repeating triangular sequence signal used in step 1 of identification.

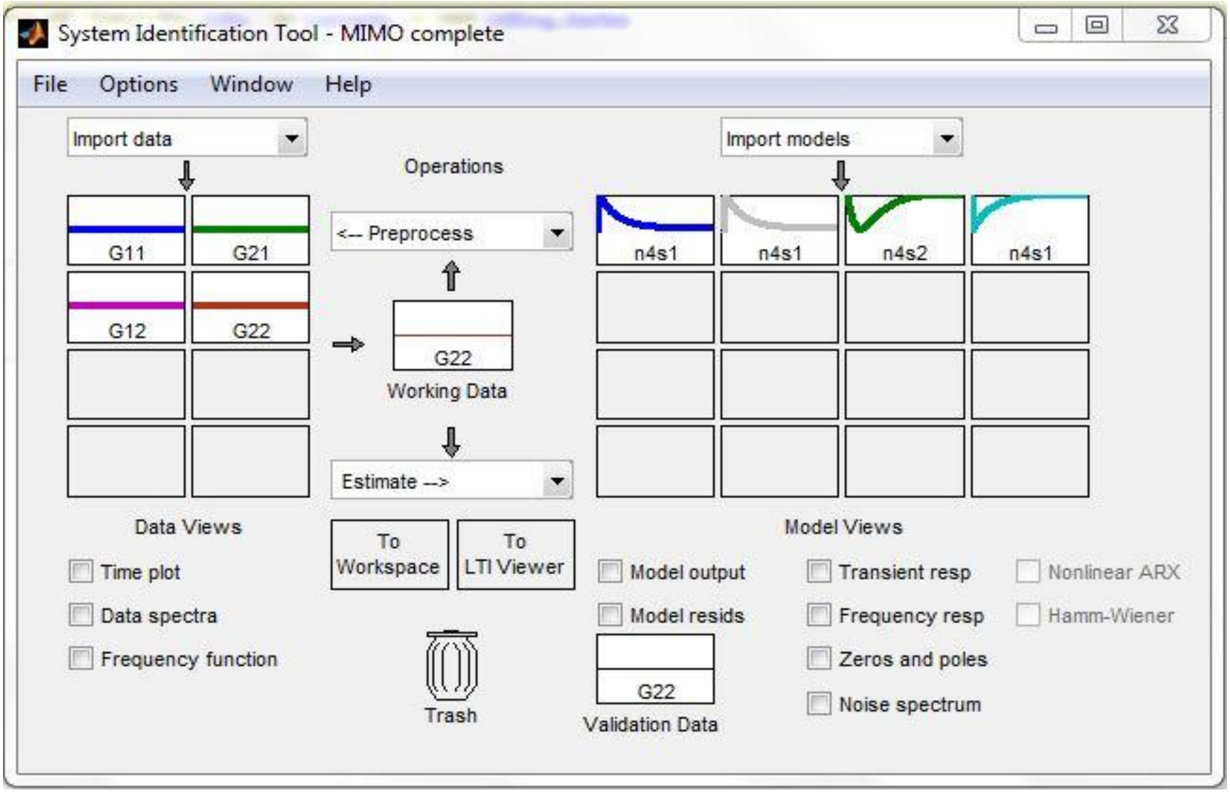

Fig. 4.4.12 Screen print of final identified 2×2 system

The time delay for each transfer function is determined from the above four responses & is listed in Table 4.4.1.

Table 4.4.1 Time delays of respective transfer function

Transfer function	Time delay
G11	6.05
G12	11.9
G21	7.1
G22	3

The estimated model parameters from the input and output data of MIMO system with N4SID algorithm is shown in the Table 4.4.2

Table 4.4.2 Estimated state space model matrices (MIMO system)

	G11	G12	G21	G22
A	$[-0.01667]$	$\begin{bmatrix} 0 & -0.000463 \\ 1 & -0.04306 \end{bmatrix}$	$[-0.02632]$	$[-0.02857]$
B	$[1]$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$[1]$	$[1]$
C	$[0.0021]$	$[3.365e-15 \quad 4.676e-$	$[0.002474]$	$[-0.002857]$
D	$[0]$	$[0]$	$[0]$	$[0]$
K	$[0]$	$[0]$	$[0]$	$[0]$
FPE	$2.023e-28$	$1.786e-24$	$6.198e-29$	$4.288e-27$
MSE	$2.295e-28$	$2.382-24$	$7.611e-29$	$4.89e-27$
Fit to the estimated data	100%	100%	100%	100%

Where

FPE: Akaike's Final Prediction Error, It describes the quality of model,

MSE: Mean squared error.

Table 4.4.3 Comparison of Estimated model & True process

S. No	Actual process	Estimated process by sequential relay-feedback	Estimated process by proposed method
I	$G11 = \frac{0.126e^{-6s}}{60s+1}$	$G11 = \frac{0.1267e^{-6s}}{60.35s+1}$	$G11 = \frac{0.12597e^{-6.05s}}{60.35s+1}$
II	$G12 = \frac{-0.101e^{-12s}}{2160s^2+93s+1}$	$G12 = \frac{-0.1018e^{-12s}}{2203.025s^2+93.92s+}$	$G12 = \frac{-0.10099e^{-11.9s}}{2159.827s^2+93.002s+1}$
III	$G21 = \frac{0.094e^{-7s}}{38s+1}$	$G21 = \frac{0.0948e^{-7s}}{38.2005s+1}$	$G21 = \frac{0.09388e^{-7.1s}}{37.99s+1}$
IV	$G22 = \frac{-0.1e^{-3s}}{35s+1}$	$G22 = \frac{-0.1202e^{-3s}}{35.02s+1}$	$G22 = \frac{-0.1e^{-3s}}{35.0017s+1}$

4.4.4 Controller Design

Normalized decoupler design method (Yuling Shen, 2010) is used to design decouplers. Thus the 2×2 MIMO system is separated into two 1×1 SISO system by eliminating the cross-interaction. Then, SIMC method is used to design decentralized controller for each of the SISO systems separately. The configuration of the simplest decentralized control structure is adopted.

$$G_{p(s)} = \begin{bmatrix} \frac{0.126e^{-6s}}{60s+1} & \frac{-0.101e^{-12s}}{2160s^2+93s+1} \\ \frac{0.094e^{-7s}}{38s+1} & \frac{-0.1e^{-3s}}{35s+1} \end{bmatrix} = g_{ij}(s) = \frac{k_{ij}}{\tau_{ij}s+1} e^{-\theta_{ij}S}, \quad i,j = 1,2,3 \dots n,$$

$$K = \begin{bmatrix} 0.126 & -0.101 \\ 0.094 & -0.1 \end{bmatrix} = steadystategainmatrix$$

$$K_N = \begin{bmatrix} k_{N,11} & k_{N,12} & \cdots & k_{N,1n} \\ k_{N,21} & k_{N,22} & \cdots & k_{N,2n} \\ \vdots & \vdots & \cdots & \vdots \\ k_{N,n1} & k_{N,n2} & \cdots & k_{N,nn} \end{bmatrix}$$

$$k_{N,ij} = \frac{k_{ij}}{\tau_{ij} + \theta_{ij}}, \quad i,j = 1,2 \dots\dots n$$

Where

K_N is the normalized gain matrix

$k_{N,ij}$ is the normalized gain

$\tau_{ij} + \theta_{ij}$ Is the average residence time of loops i-j

$$K_N = 10^{-3} \begin{bmatrix} 1.909 & -0.962 \\ 2.089 & -2.632 \end{bmatrix}$$

$$K_N^{-T} = \begin{bmatrix} 873.0061 & -692.89 \\ -319.0851 & -633.195 \end{bmatrix}$$

$$\Lambda(s) = \begin{bmatrix} 4.064 & -3.064 \\ -3.064 & 4.064 \end{bmatrix} = relativegainarraymatrix$$

$$\Phi = K_N \otimes K_N^{-T}$$

$$\Phi = \begin{bmatrix} 1.66 & -0.66 \\ -0.66 & 1.66 \end{bmatrix}$$

Where

Operator $\bigotimes$ = Hadamard product

Φ = Relative Normalized Gain Array (RNGA)

$$\Gamma = \Phi \odot \Lambda = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \cdots & \Phi_{1n} \\ \Phi_{21} & \Phi_{22} & \cdots & \Phi_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \Phi_{n1} & \Phi_{n2} & \cdots & \Phi_{nn} \end{bmatrix} \odot \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_{n1} & \lambda_{n2} & \cdots & \lambda_{nn} \end{bmatrix}$$

$$= \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1n} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \cdots & \gamma_{nn} \end{bmatrix}$$

Where

λ_{ij} = Relative Gain Array element

Λ = relative gain array matrix

Operator $\odot$ is the Hardamard division

$$\gamma_{ij} = \frac{\Phi_{ij}}{\lambda_{ij}} == \text{ relative average residence time}$$

$\Gamma =$ dynamic relative gain array

$$\Gamma = \Phi \odot \Lambda = \begin{bmatrix} 0.408 & 0.2154 \\ 0.215 & 0.4085 \end{bmatrix}$$

$$\hat{g}_{ij}(s) = \frac{k_{ij}/\lambda_{ij}}{\gamma_{ij}\tau_{ij}s + 1} e^{-\gamma_{ij}\theta_{ij}S}$$

$$\hat{g}_{11}(s) = \frac{0.031}{0.408 \times 60s + 1} e^{-0.408 \times 6S} = \frac{0.031}{24.48s + 1} e^{-2.448S}$$

$$\hat{g}_{12}(s) = \frac{0.0329}{0.215 \times 70.5s + 1} e^{-0.215 \times 34.5S} = \frac{0.0329}{15.16s + 1} e^{-7.42S}$$

$$\hat{g}_{21}(s) = \frac{-0.037}{0.215 \times 38s + 1} e^{-0.215 \times 7S} = \frac{0.037}{8.2s + 1} e^{-1.505S}$$

$$\hat{g}_{11}(s) = \frac{0.0326}{0.408 \times 35s + 1} e^{-0.408 \times 3S} = \frac{0.031}{14.3s + 1} e^{-1.224S}$$

$$\hat{G}^T = \begin{bmatrix} 1/\hat{g}_{11}(s) & 1/\hat{g}_{12}(s) \\ 1/\hat{g}_{21}(s) & 1/\hat{g}_{22}(s) \end{bmatrix}$$

$$\hat{G}^T = \begin{bmatrix} \frac{(24.48s + 1)e^{2.45s}}{0.031} & \frac{(8.2s + 1)e^{1.505s}}{-0.037} \\ \frac{(15.16s + 1)e^{7.42s}}{0.0329} & \frac{(14.3s + 1)e^{1.224s}}{-0.0326} \end{bmatrix}$$

$$G_R(s) = \begin{bmatrix} \frac{e^{-7.42s}}{(24.48s + 1)} & 0 \\ 0 & \frac{e^{-1.505s}}{(14.3s + 1)} \end{bmatrix}$$

$$G_R(s) = \begin{bmatrix} \frac{e^{-7.42s}}{(24.48s + 1)} & 0 \\ 0 & \frac{e^{-1.505s}}{(14.3s + 1)} \end{bmatrix}$$

In table 4.4.4 GI(s) = stable decouplers matrix and $G_C(s)$ = decentralized controller matrix Gp(s) is the process transfer function. Applying SIMC method of tuning, consequently the PI controllers obtained are shown below and comparison between the two identification control strategies is shown in the table 6.5 which shows that Relay + Subspace method provides better response in terms of minimum values of IAE for the control of top and bottom products of distillation column.

$$k_C = \frac{1}{1} \times \frac{24.48}{2 \times 7.48} = 1.63$$

$$\tau_I = 24.48$$

$$G_{C11}(s) = 1.63\left(1 + \frac{1}{24.48}\right) = 1.63 + \frac{0.066}{s}$$

$$k_C = \frac{1}{1} \times \frac{14.3}{2 \times 1.505} = 4.75$$

$$\tau_I = 14.04$$

$$G_{C22}(s) = 4.75\left(1 + \frac{1}{12.04s}\right) = 4.75 + \frac{0.394}{s}$$

Table 4.4.4Decouplers& Controller matrix

	Transfer function matrix		Transfer function matrix
$\hat{G}^T(s)$	$\begin{bmatrix} \frac{(24.48s+1)e^{2.45s}}{0.031} & \frac{(8.2s+1)e^{1.505}}{-0.037} \\ \frac{(15.16s+1)e^{7.42s}}{0.0329} & \frac{(14.3s+1)e^{1.22}}{-0.0326} \end{bmatrix}$	$G_I(s)$	$\begin{bmatrix} 32.26e^{-4.97S} & -\frac{221.65S+27.0}{14.3S+1} \\ \frac{460.864S+30.4}{24.48S+1} & -30.7e^{-0.281S} \end{bmatrix}$
$G_R(s)$	$\begin{bmatrix} \frac{e^{-7.42s}}{(24.48s+1)} & 0 \\ 0 & \frac{e^{-1.505s}}{(14.3s+1)} \end{bmatrix}$	$G_C(s)$	$\begin{bmatrix} 1.63\left(1+\frac{1}{24.48}\right) & 0 \\ 0 & 4.75\left(1+\frac{1}{12.04s}\right) \end{bmatrix}$

Table 4.4.5 Controller parameters for 2x2 systems

	G_{C11}			G_{C22}		
	K_c	τ_I	τ_D	K_c	τ_I	τ_D
SIMC	1.63	14.925		4.75	12.04	
Z-N tuning	3.4334	13.374	3.3435	9.157	2.892	0.723

Table 4.4.6 Performance values of closed-loop

Identification/Control strategy	IAE for y1 (top)	IAE for y2 (bottom)
Sequential relay	8.263	0.2312
Relay + subspace	6.702	0.1032

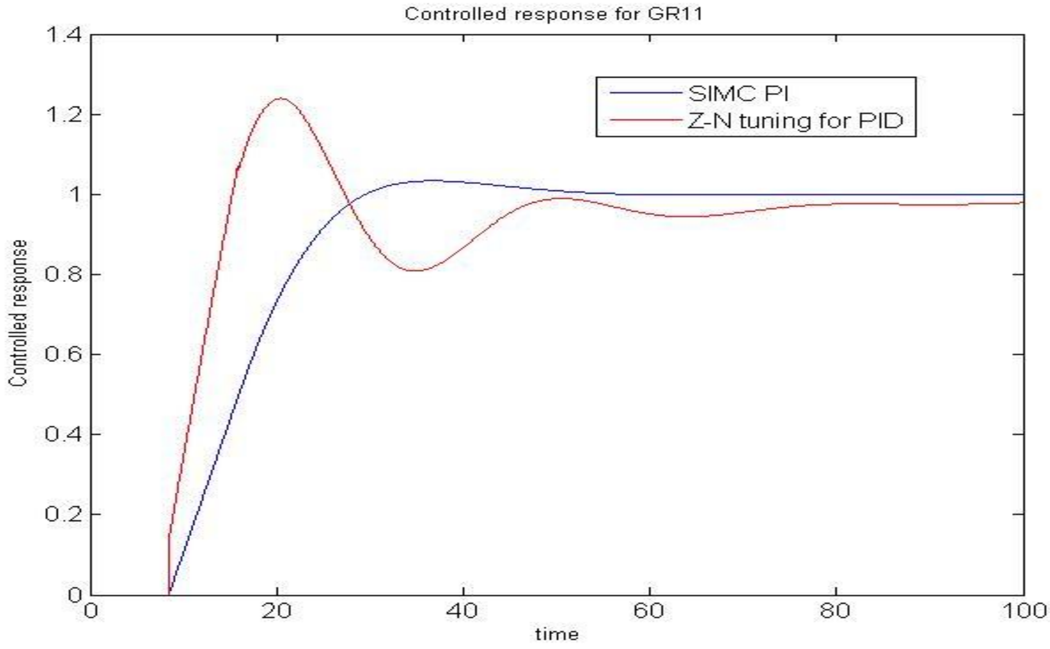

Fig. 4.4.13 Controlled Output for top product composition

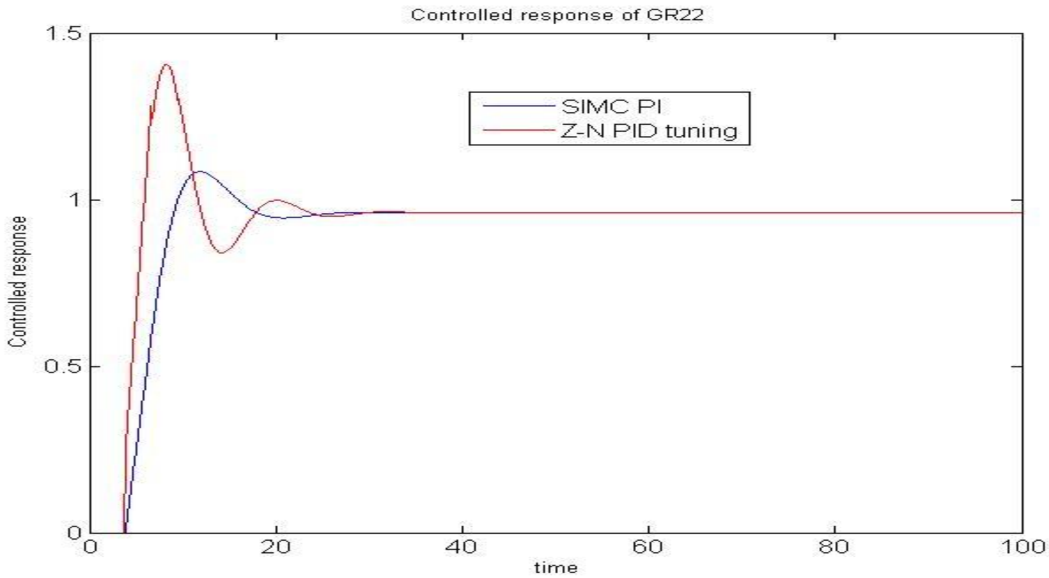

Fig 4.4.14 Controlled output for bottom product composition

In this subsection, identification and control of 2x2 MIMO processes is considered and with the help of Relay and subspace identification method input and this input and output data is fed to theN4SID is used to convert the state space matrix of the input and output data. The state space matrices are converted into transfer function matrix. For the converted transfer functions, simple PI controllers are designed. Figure 4.4.1 shows the Simulink representation of bench mark example i.e. Warde &Wood distillation column (1969). The Simulink diagram is as per the block diagram shown in figure 4.4.1 to generate two inputs and two outputs for water-methanol processes.

The random signal and triangular waves are shown in figures 4.4.2 & 4.4.3. These are the excitation signals that are used to feed to the process as inputs shown in figures 4.4.2 &4.4.3 the reason for choosing these signals are easy excitation and the same is shown in figures 4.4.2 to 4.4.5 for identification procedure.

The figures 4.4.2 and 4.4.3 show the response of the Gaussian white noise signal and repeating triangular signal used in the Simulink diagram to excite the process to generate input-output data in the 2nd step of identification. It is to be noted that these are only meant for excitation of the process using Simulink diagram shown in figure 4.4.1

The identification step is carried-out in two stages when loop 1 is running with the excitation signals shown in figure 4.4.1 while loop 2 is open. In similar way, the figures 4.4.5 and 4.4.6 are used in the second step for excitation process when the second loop is to excite the first loop is opened and the second loop is exited to generate the input and output data in the second step of identification.

Figures 4.4.7 & 4.4.8 explain the system identification tool box used in identification procedure. The figures 4.4.8 and 4.4.9 show the response of G11 for the input Gaussian white noise signal which is used in the step 1 of identification. G11 response as in the form of input and out data fed to the system identification tool box with the help of N4SID algorithm to convert it into state space model in step 1. In similar way, response G12 for input of the repeating triangular sequence signal used in step of identification in step 1 of identification procedure. As it is MIMO process, there are four responses G21 and G22 due to interactions for the second step of excitation given in figures 4.4.10 & 4.4.11

The response of each transfer function for two inputs and two outputs are shown figures 4.4.7 &4.4.8. The response attributed between the two figures is due to the presence of strong interaction among the manipulated and controlled variables.

The response from each transfer function of the process in this work is shown in following figures. So, by giving the excitation in two inputs, the outputs [4.4.8 to 4.4.11] are generated. N4SIDalgorithm was employed for estimating the state matrix where the corresponding pictorial representation [Fig 4.4.12] shows the scheme of identification procedure.

During the process of estimation of transfer function, MATLAB solver is used through N4SID algorithm to find A, B, C & D matrix. Feed-forward matrix D is taken as zero by default to make the output y (t) independent of input u (t). Then A, B, C matrices are determined accordingly depending on the type of realization selected for state space model.

As no disturbance is used in the system, the "K" matrix (Equation 4.4.9) is zero. FPE & MSE is also determined by solver automatically which ascertain the quality of the model developed from the identification procedure. In addition to the estimation of parameters by the proposed method, different methods from literature have been compared and the results are shown in Table 4.4.4. It can be found that the 3rd column (present method) shows better or nearer values to actual parameters compared to the 2nd column (sequential relay).

By carrying out the analysis, the exact transfer function model has been identified using combination of both relay feedback and subspace which yields accurate result and is shown in the above table 4.4.4. The PI controllers are obtained for the same process applying SIMC method of tuning. In this work RGA analysis is used for paring purpose.

The responses of PI controllers for top and bottom product compositions are shown in figures 4.4.13 & 4.4.14 respectively. For a step input of magnitude 0.96 in reflux, the controlled response for top product composition is obtained by Z-N PID tuning & Skogestad's PI tuning as shown in fig.4.4.13. For a step input of magnitude 0.96 in vapour flow rate, the controlled response for bottom product composition is obtained by Z-N PID tuning & Skogestad's PI tuning as shown in fig. 4.4.14

The controller methods; Z-N PID and Skogestad's PI tuning used in this work are compared and the results are shown in figures 4.4.13 &4.4.13 respectively for set point changes. From these figures, it can be seen that Skogestad's PI tuning method yields best response for controlling of the top and bottom product compositions of distillation column.

It can be seen from Table 4.4.5 that the performances of closed-loop control also is better in the case of present method as it yields IAE of 6.702 compared to 8.263 for bottom product, while, IAE of 0.1032 compared to 0.2312 for bottom product composition.

4.5 Identification and control of 3x3 MIMO process using Relay with subspace method:

This is an extension to the previous section wherein the methods employed for identification and control of 2x2 systems are also extended to 3x3 system of distillation column with an example. Orgunnaike and Ray (OR) 3x3 system distillation column (Sujata & Panda, 2013) with three inputs (u1, u2 & u3) and three outputs (y1, y2 & y3) for ethanol-water separation system has been considered here.

The distillation column has got 19 trays with variable feed, side stream draw-off with binary ethanol-water system. Y1 is overhead ethanol mole fraction, side-stream product ethanol-mole fraction is y2 and bottom product temperature becomes y3. Similarly, u1 is reflux flow-rate, u2 is flow-rate of side-draw and u3 is steam pressure at reboiler. Disturbance transfer function is not considered in this study and no pre-designed controller is required for this identification method. Now, first step is to set specific settings for each block. The excitation signals which are used in previous section are used in this work also.

Gaussian white noise is used as the reference signal r1, r2 & r3 for the system identification. Reference signals should be such that it generate a random input signal to excite the system dynamics to generate better & accurate data from system behavior. The ultimate properties of diagonal system are evaluated as: $K_{Uii} = [7.128 \quad -1.386 \quad 2.452]$ and $P_{Uii} = [9.144 \quad 10.032 \quad 3.696]$.

The system identification tool box is used along with the relay method for generation of sustained oscillations. Collected input-output data is used to identify the system using subspace method. For the identified process using RGA analysis, paring is carried out and the process decouplers are designed using the decentralized controller structure. Using Z-N and SIMC method, PI controllers are designed for three set points. The transfer function of 3×3 Orgunnaike and Ray distillation column can be written as;

$$\begin{bmatrix} \text{Over head mole fraction of ethanol} \\ \text{Sidestream mole fraction of ethanol} \\ \text{Tempreature of 19th tray} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{0.66e^{-2.6s}}{6.7s+1} & -\dfrac{0.61e^{-3.6s}}{8.64s+1} & -\dfrac{0.0049e^{-s}}{9.06s+1} \\ \dfrac{1.11e^{-6.5s}}{3.25s+1} & -\dfrac{2.36e^{-3s}}{5s+1} & -\dfrac{0.01e^{-1.2s}}{7.09s+1} \\ -\dfrac{34.68e^{-9.2s}}{8.15s+1} & \dfrac{46.2e^{-9.42s}}{10.9+1} & \dfrac{0.87(11.61s+1)e^{-s}}{(3.89s+1)(18.8s+1)} \end{bmatrix} \begin{bmatrix} \text{Reflux flow rate} \\ \text{Side stream drawoff rate} \\ \text{Reboiler stream pressure} \end{bmatrix}$$

Where;

y1=Overhead molefraction of ethanol, u1 = Reflux flow rate

y2 =Side stream molefraction of ethanol, u2 = Side stream draw off rate

y3 =$Temperatureof\ 19thtray$, u3 = $Reboilerstreampressure$

4.5.1 Estimation of Time Delay

In the first part of identification, the time delays for each transfer function are obtained using system identification tool box and N4SID algorithm. The excitation signals used for this process are shown in previous section from above transfer functions and nine responses are listed sequentially in Table 4.5.1 the plots for the each transfer function are shown in figures 4.5.1 to 4.5.9.

For the illustration purpose, the responses with figures are drawn below followed by the time delay of each transfer function as shown in table 4.5.1

Table.4.5.1 Time delays of respective transfer function

Transfer function	Time delay
G11	2.6
G12	3.6
G13	1
G21	6.5
G22	3
G23	1.2
G31	9.2
G32	9.42
G32	1

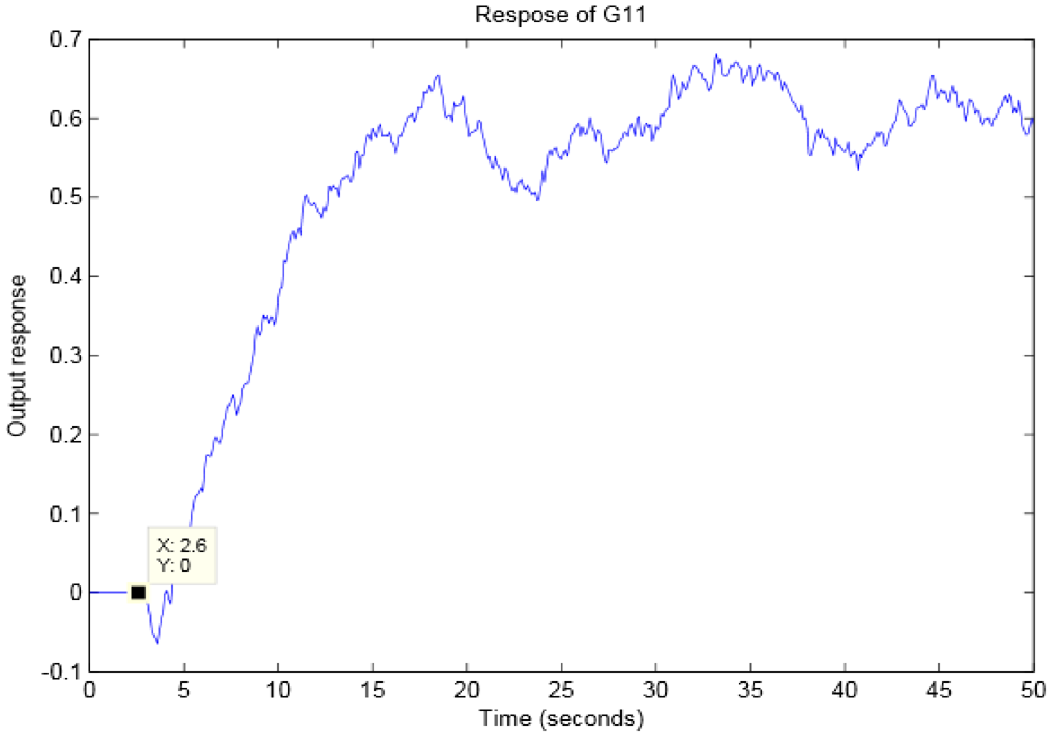

Fig 4.5.1estimated response of G11 transfer function

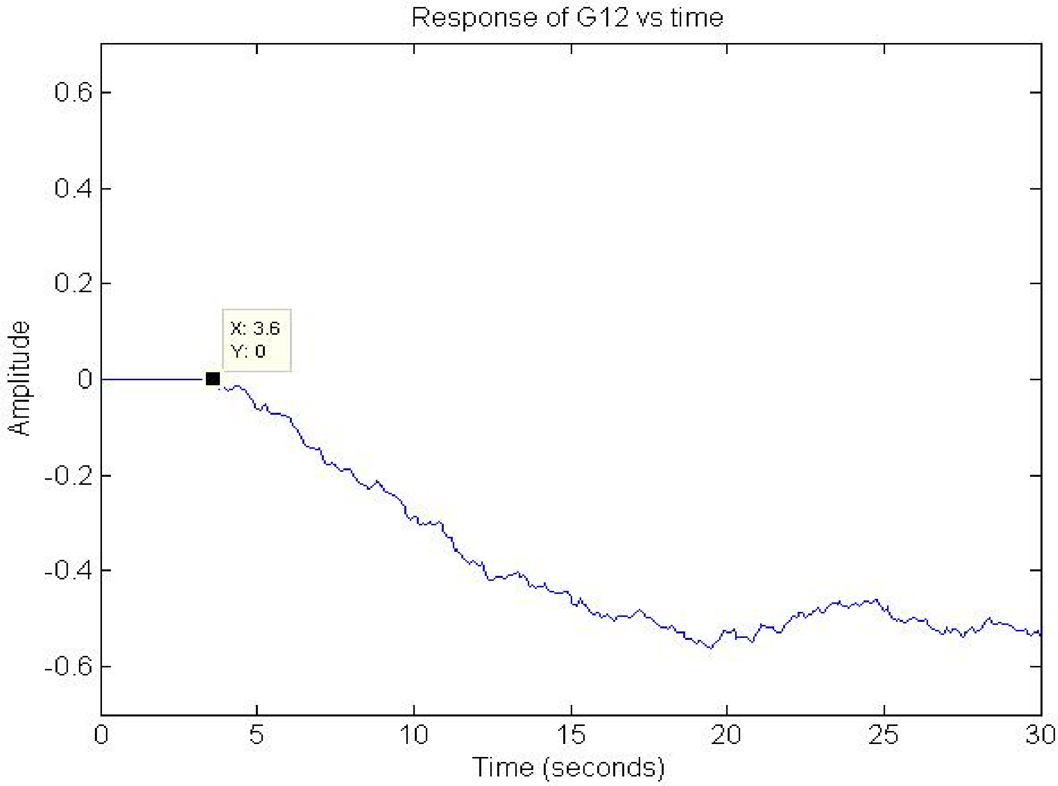

Fig 4.5.2 estimated response of G12 transfer function

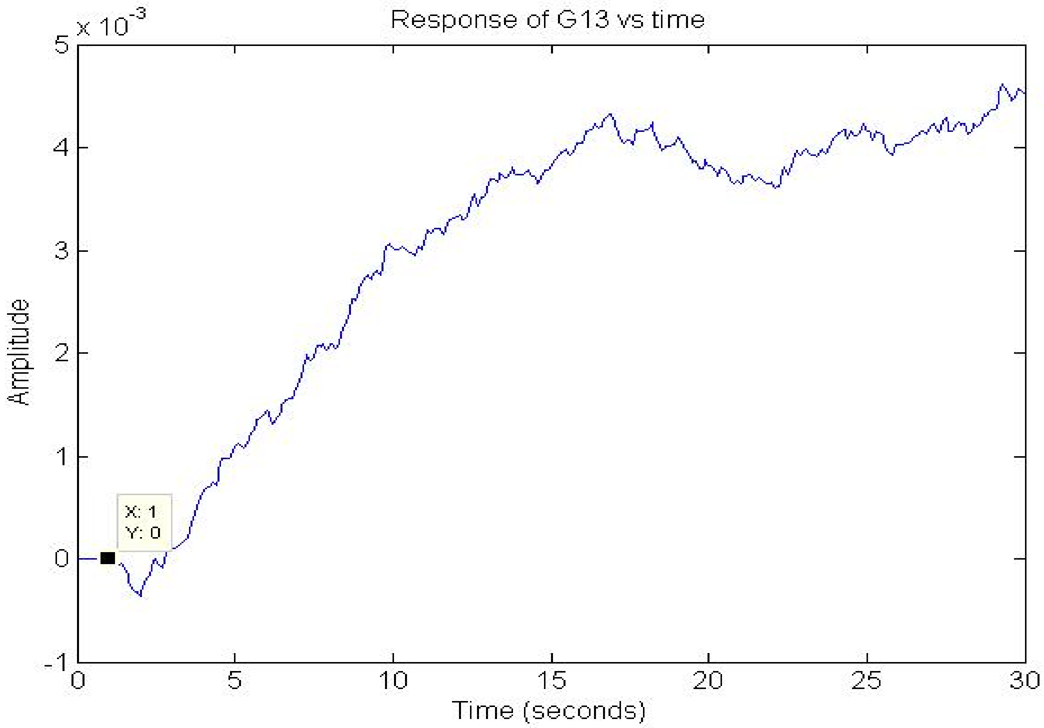

Fig 4.5.3 estimated response of G13 transfer function

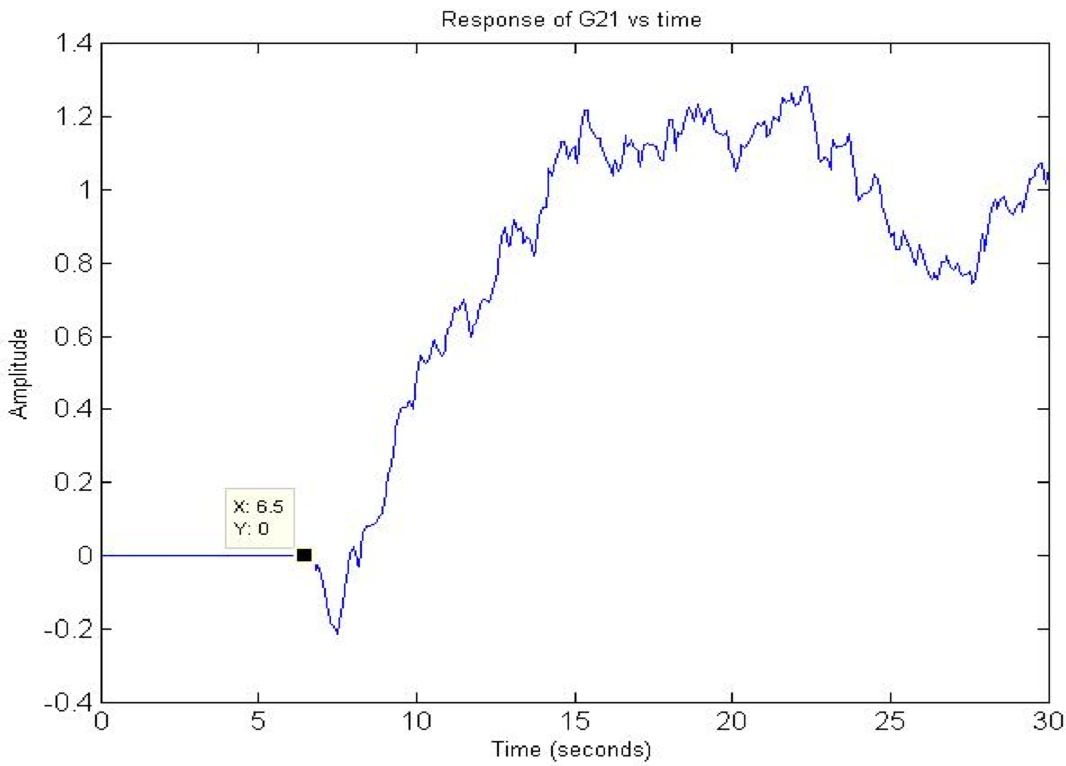

Fig 4.5.4 estimated response of G21 transfer function

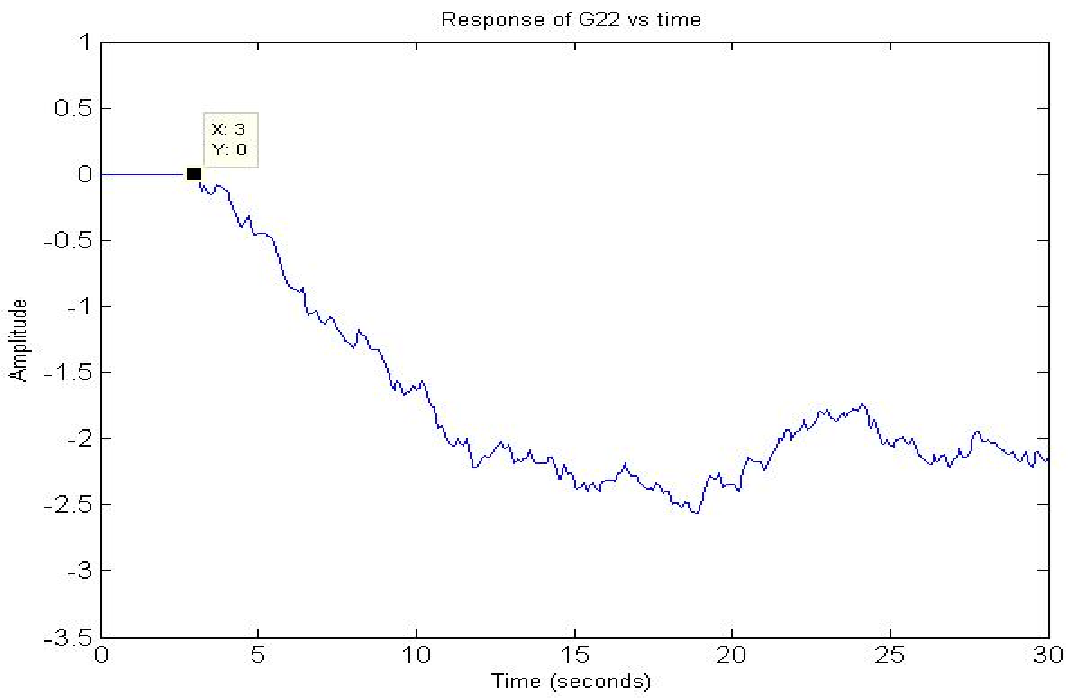

Fig 4.5.5 estimated response of G22 transfer function

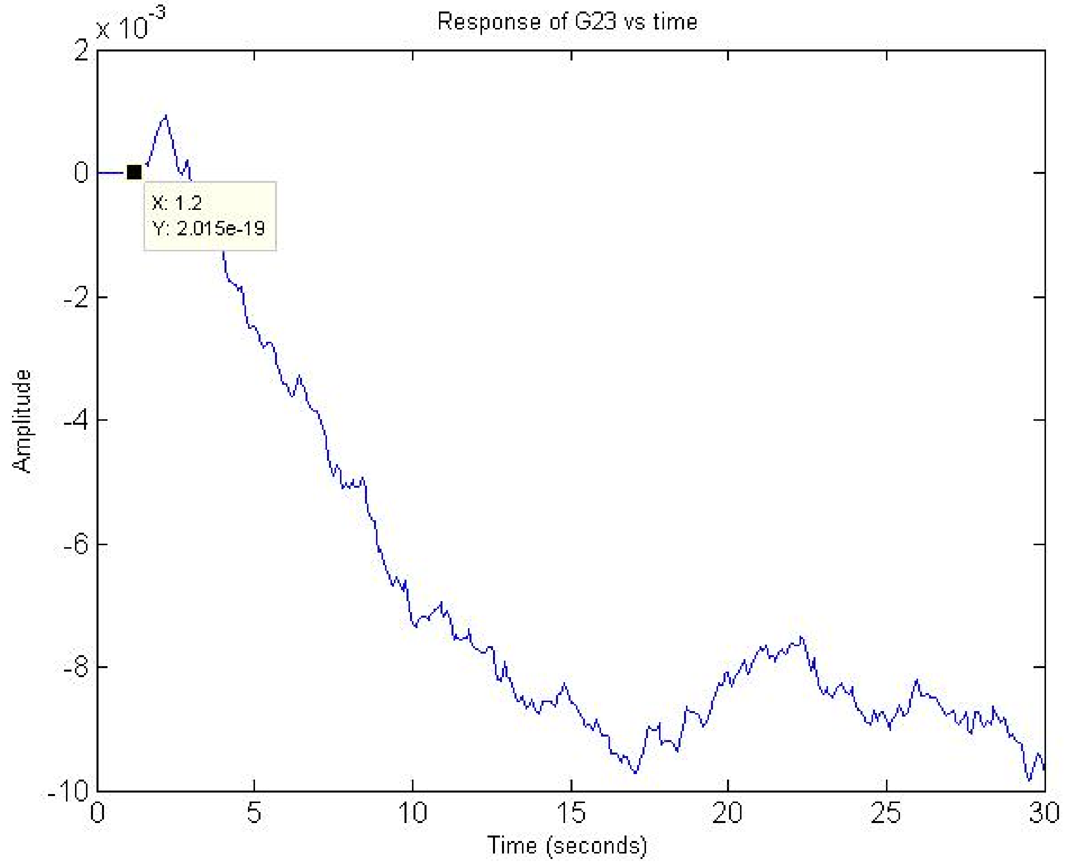

Fig 4.5.6 estimated response of G23 transfer function

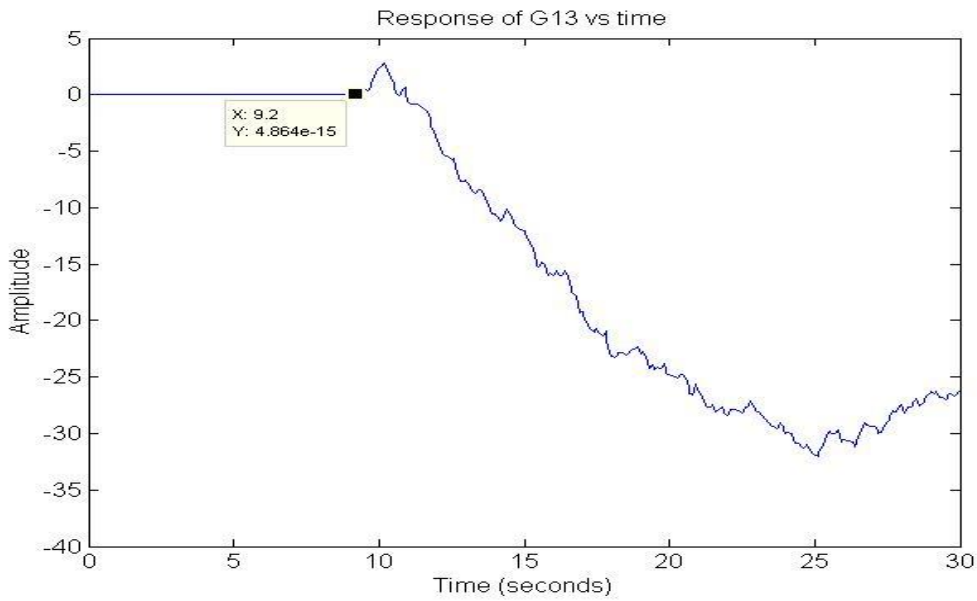

Fig 4.5.7 estimated response of G31 transfer function

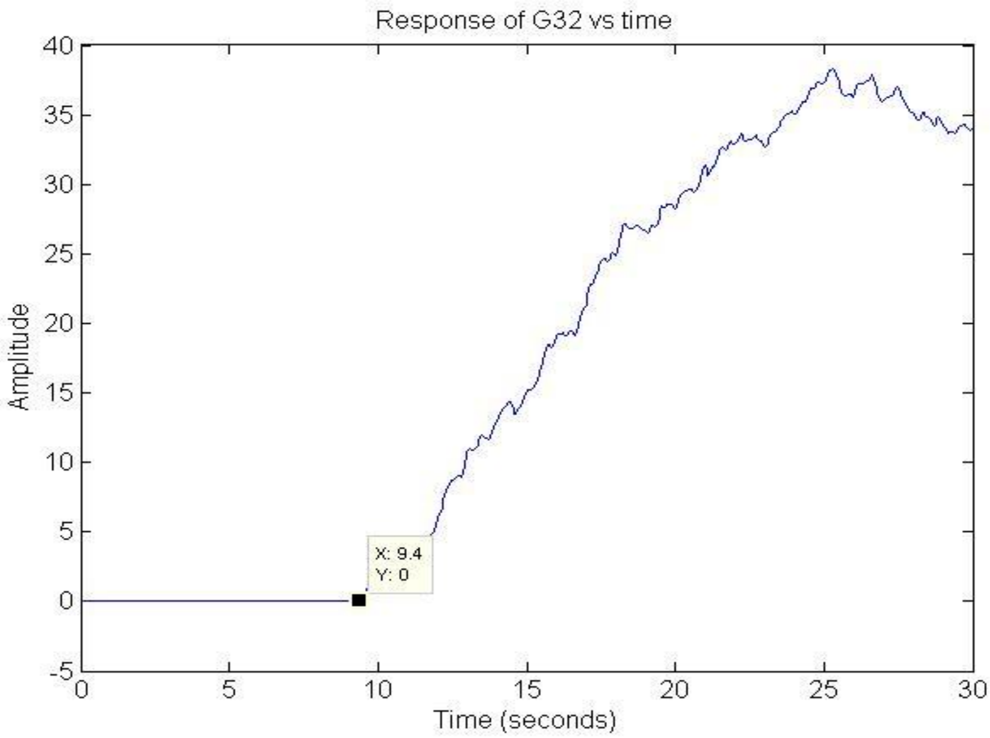

Fig 4.5.8 estimated response of G32 transfer function

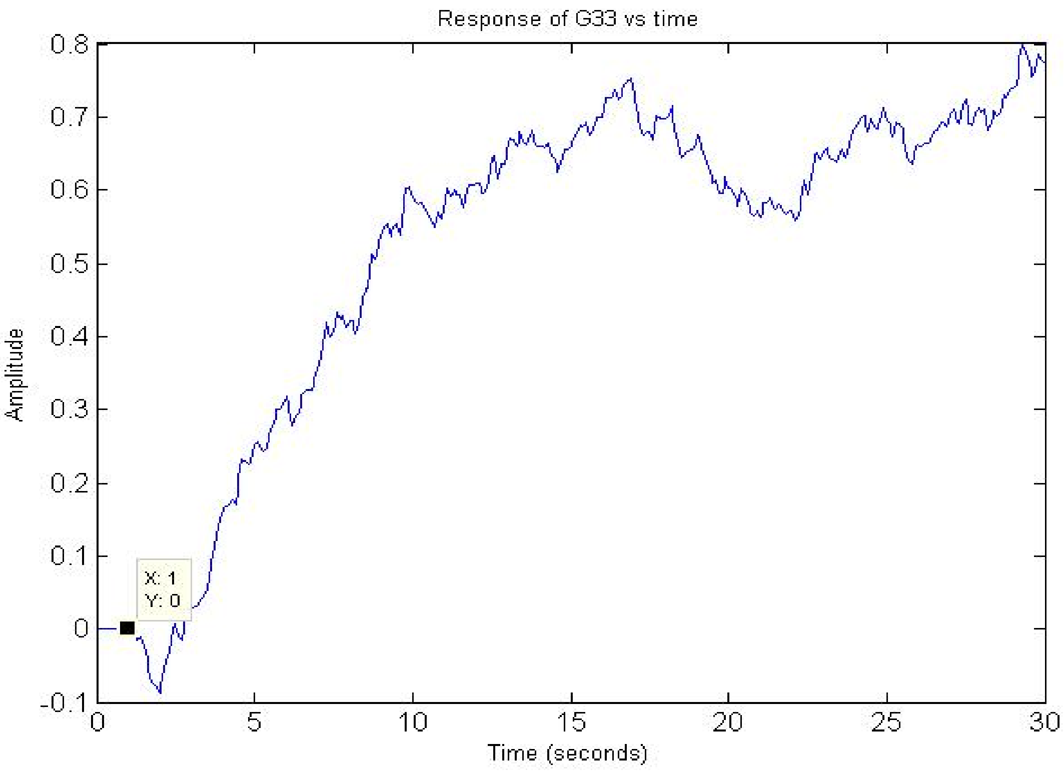

Fig. 4.5.9 estimated time response of G33 transfer function

Table 4.5.2 Estimated state space model matrices I

	G11	G21	G31	G12	G22
A	$[-0.1493]$	$[-0.3077]$	$[-0.1227]$	$[-0.1157]$	$[-0.2]$
B	$[1]$	$[1]$	$[1]$	$[1]$	$[1]$
C	$[0.09851]$	$[0.3415]$	$[-4.255]$	$[-0.0706]$	$[-0.472]$
D	$[0]$	$[0]$	$[0]$	$[0]$	$[0]$
K	$[0]$	$[0]$	$[0]$	$[0]$	$[0]$

FPE	$[6.838e-28]$	$[6.537e-26]$	$[2.332e-24]$	$[7.689e-28]$	$[1.226e-26]$
MSE	$[7.716e-28]$	$[6.498e-26]$	$[3.225e-24]$	$[9.556e-28]$	$[1.315e-26]$
Fit to the estimated data	100%	100%	100%	100%	100%

Table 4.5.3 Estimated state space model matrices II

	G32	G13	G23	G33
A	$[4.24]$	$[-0.1104]$	$[-0.141]$	$[-0.1625]$
B	$[1]$	$[1]$	$[1]$	$[1]$
C	$[0.0917]$	$[0.0005409]$	$[-0.00141]$	$[0.1406]$
D	$[0]$	$[0]$	$[0]$	$[0]$
K	$[0]$	$[0]$	$[0]$	$[0]$
FPE	$[4.312e-12]$	$[5.465e-11]$	$[1.918e-10]$	$[0.0004276]$
MSE	$[0.007028]$	$[3.46e-11]$	$[1.915e-10]$	$[0.0004326]$
Fit to the data	98.72%	98.9%	98.77%	78.86%

Table 4.5.4 Comparison between Identified model & True Process

Transfer function	Actual process model	Estimated model by proposed method
G11	$\frac{0.66e^{-2.6s}}{6.7s+1}$	$\frac{0.6598e^{-2.6s}}{6.69s+1}$
G12	$\frac{-0.61e^{-3.6s}}{8.64s+1}$	$\frac{-0.61e^{-3.6s}}{8.64s+1}$
G13	$\frac{-0.0049e^{-s}}{9.06s+1}$	$\frac{-0.00489e^{-s}}{9.057s+1}$
G21	$\frac{1.11e^{-6.5s}}{3.25s+1}$	$\frac{1.1098e^{-6.5s}}{3.249s+1}$
G22	$\frac{-2.36e^{-3s}}{5s+1}$	$\frac{-2.36e^{-3s}}{5s+1}$
G23	$\frac{-0.01e^{-1.2s}}{7.09s+1}$	$\frac{-0.01e^{-1.2s}}{7.09s+1}$
G31	$\frac{-34.68e^{-9.2s}}{8.15s+1}$	$\frac{-34.678e^{-9.2s}}{8.149s+1}$
G32	$\frac{46.2e^{-9.42s}}{10.9s+1}$	$\frac{46.237e^{-9.42s}}{10.805s+1}$
G33	$\frac{(0.1367s+0.012)e^{-s}}{s^2+.31s+0.01367}$	$\frac{(3.379s+82.52)e^{-s}}{s^2+551.5s+89.18}$

In estimation of transfer function, MATLAB solver uses N4SID algorithm to generate A, B, C & D matrix (Tables 4.5.2 & 4.5.3). Feed-forward matrix D is taken zero by default to make the output y(t) independent of input u(t) & then A, B, C matrix is determined accordingly depending on the type of realization selected for state space model. No disturbance is used in the system. Hence "K" matrix is zero. FPE & MSE is also determined by solver automatically. Estimated state space model matrices II is shown in table 4.5.3 and comparison is made between the identified and the true process as shown in table 4.5.4, the table shows that there is similarity between the actual and identified process identified by the proposed method.

4.5.2 Controller Design Methods:

Thus the 3x3 MIMO systems were separated into three 1×1 SISO systems by eliminating the cross-interaction. In this work, Skogestad's IMC-PID tuning and conventional Z-N PID tuning methods were used.A FOPTD process as given in equation 4.5.1 has been considered and decentralized control structure as shown in figure 4.5.10 are used along with RNGA-RGA method in the estimation of Steady state gains derived transfer function matrices as shown in table 4.5.5

$$G(s) = \frac{Ke^{-t_d s}}{(\tau s+1)} \qquad (4.5.1)$$

PID controller settings are given as follows

$$K_C = \frac{1}{K} \times \frac{\tau}{\tau_C + t_d} \qquad (4.5.2)$$

$$\tau_I = \min\{\tau, \ 4(\tau_C + t_d)\} \qquad (4.5.3)$$

For tight control i.e. good robustness

$$\tau_C = t_d \qquad (4.5.4)$$

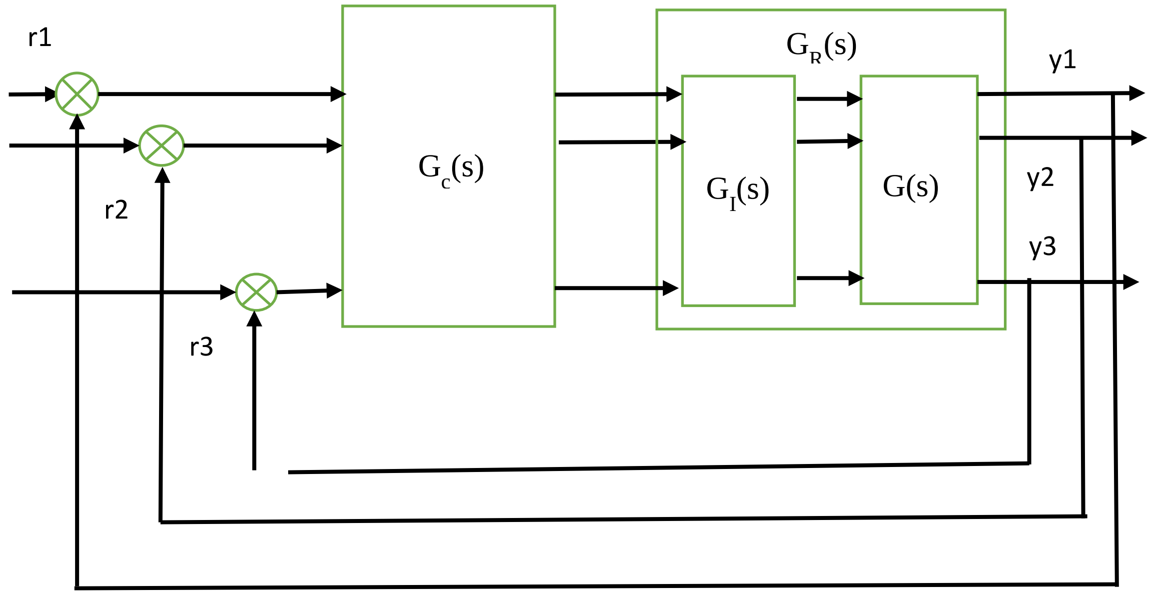

Fig. 4.5.10 Decentralized control structure

$$G(s) = \begin{bmatrix} \frac{0.66e^{-2.6s}}{6.7s+1} & \frac{-0.61e^{-3.6s}}{8.64s+1} & \frac{-0.0049e^{-s}}{9.06s+1} \\ \frac{1.11e^{-6.5s}}{3.25s+1} & \frac{-2.36e^{-3s}}{5s+1} & \frac{-0.01e^{-1.2s}}{7.09s+1} \\ \frac{-34.68e^{-9.2s}}{8.15s+1} & \frac{46.2e^{-9.42s}}{10.9s+1} & \frac{0.87(11.61s+1)e^{-s}}{(3.89s+1)(18.8s+1)} \end{bmatrix} \quad (4.5.5)$$

After approximating higher order transfer function to FOPTD

$$G(s) = \begin{bmatrix} \frac{0.66e^{-2.6s}}{6.7s+1} & \frac{-0.61e^{-3.6s}}{8.64s+1} & \frac{-0.0049e^{-s}}{9.06s+1} \\ \frac{1.11e^{-6.5s}}{3.25s+1} & \frac{-2.36e^{-3s}}{5s+1} & \frac{-0.01e^{-1.2s}}{7.09s+1} \\ \frac{-34.68e^{-9.2s}}{8.15s+1} & \frac{46.2e^{-9.42s}}{10.9s+1} & \frac{0.869e^{-s}}{6.989s+1} \end{bmatrix} \quad (4.5.6)$$

$$\text{Gainmatrix, K} = \begin{bmatrix} 0.66 & -0.61 & -0.0049 \\ 1.11 & -2.3 & -0.01 \\ -34.68 & 46.2 & 0.869 \end{bmatrix} K^{-T} =$$

$$\begin{bmatrix} 3.0436 & 1.1834 & 58.5466 \\ -0.5818 & -0.7732 & 17.8865 \\ 0.0105 & -0.0022 & 1.6867 \end{bmatrix} \quad (4.5.7)$$

$$Relative\text{ Gain Array} = RGA, \Lambda = \begin{bmatrix} 2.0088 & -0.7219 & -0.2629 \\ -0.6458 & 1.8247 & -0.1789 \\ -0.363 & -0.1028 & 1.4657 \end{bmatrix}$$

$$\text{ativeNormalizedGainArray} = \text{RNGA}, \Phi = \begin{bmatrix} 1.3776 & -0.3298 & -0.0478 \\ -0.2659 & 1.4229 & -0.1571 \\ -0.1117 & -0.0932 & 1.2049 \end{bmatrix} \quad (4.5.8)$$

$$\text{RelativeAvg. ResidenceTimeArray} = \text{RARTA} = \text{RNGA} \odot \text{RGA} =$$

$$\begin{bmatrix} 0.6858 & 0.4568 & 0.1667 \\ 0.4117 & 0.7798 & 0.8781 \\ 0.3077 & 0.9069 & 0.822 \end{bmatrix} \quad (4.5.9)$$

$$\hat{g}_{11}(s) = \frac{0.66/2.088}{0.686\times6.7s+1} e^{-2.6\times0.686S} = \frac{0.316}{4.596s+1} e^{-1.784S} \quad (4.5.10)$$

$$\hat{g}_{12}(s) = \frac{-0.61/-0.722}{0.457\times6.7s+1} e^{-3\times0.457S} = \frac{0.845}{3.95s+1} e^{-1.371S} \quad (4.5.11)$$

$$\hat{g}_{13}(s) = \frac{-0.005/0.263}{0.167\times9.06s+1} e^{-1\times0.167S} = \frac{-0.019}{1.513s+1} e^{-0.167S} \quad (4.5.12)$$

$$\hat{g}_{21}(s) = \frac{1.11/-0.646}{0.417\times3.25s+1} e^{-6.5\times0.417S} = \frac{-1.72}{1.355s+1} e^{-2.71S} \quad (4.5.13)$$

$$\hat{g}_{22}(s) = \frac{-2.36/1.825}{0.779\times5s+1} e^{-3\times0.779S} = \frac{-1.3}{3.9s+1} e^{-2.34S} \quad (4.5.14)$$

$$\hat{g}_{23}(s) = \frac{-0.01/-0.179}{0.878\times7.09s+1} e^{-1.2\times0.878S} = \frac{0.056}{6.225s+1} e^{-1.054S} \quad (4.4.15)$$

$$\hat{g}_{31}(s) = \frac{-34.68/-0.363}{0.307\times8.15s+1} e^{-9.2\times0.307S} = \frac{95.54}{2.5s+1} e^{-2.824S} \quad (4.4.16)$$

$$\hat{g}_{32}(s) = \frac{46.2/-0.13}{0.907\times10.9s+1} e^{-9.42\times0.907S} = \frac{-355.4}{9.88s+1} e^{-8.54S} \quad (4.4.17)$$

$$\hat{g}_{33}(s) = \frac{0.869/1.466}{0.822\times6.989s+1} e^{-0.992\times0.0.822S} = \frac{0.6}{5.745s+1} e^{-0.815S} \quad (4.4.18)$$

$$\widehat{G}(s) = \begin{bmatrix} \frac{(4.596s+1)e^{1.784s}}{0.316} & \frac{(3.95s+1)e^{1.371s}}{0.845} & \frac{(1.513s+1)e^{0.167s}}{-0.019} \\ \frac{(1.355s+1)e^{2.71s}}{-1.72} & \frac{(3.9s+1)e^{2.34s}}{-1.3} & \frac{(6.22s+1)e^{1.054s}}{0.056} \\ \frac{(2.5s+1)e^{2.824s}}{95.54} & \frac{(9.88s+1)e^{8.54s}}{355.4} & \frac{(5.745s+1)e^{0.815}}{0.6} \end{bmatrix} \quad (4.4.19)$$

Table 4.5.5 Derived transfer function matrices

	Transfer function matrix
$\hat{G}^T(s)$	$\begin{bmatrix} \frac{(4.596s+1)e^{1.784s}}{0.316} & \frac{(1.355s+1)e^{2.71s}}{-1.72} & \frac{(2.5s+1)e^{2.824s}}{95.54} \\ \frac{(3.95s+1)e^{1.371s}}{0.845} & \frac{(3.9s+1)e^{2.34s}}{-1.3} & \frac{(9.88s+1)e^{8.54s}}{355.4} \\ \frac{(1.513s+1)e^{0.167s}}{-0.019} & \frac{(6.22s+1)e^{1.054s}}{0.056} & \frac{(5.745s+1)e^{0.815}}{0.6} \end{bmatrix}$
$G_R(s)$	$\begin{bmatrix} \frac{e^{-1.784s}}{(4.596s+1)} & 0 & 0 \\ 0 & \frac{e^{-2.71s}}{(6.22s+1)} & 0 \\ 0 & 0 & \frac{e^{-8.54s}}{(9.88s+1)} \end{bmatrix}$
$G_I(s)$	$\begin{bmatrix} 3.164 & \frac{-0.581(1.355s+1)e^{-7.169s}}{(6.22s+1)} & \frac{0.0105(2.5s+1)e^{-5.7}}{(9.88s+1)} \\ \frac{1.183(3.95s+1)e^{-0.413s}}{(4.596s+1)} & \frac{-0.77(3.9s+1)e^{-0.37s}}{(6.22s+1)} & 2.814\times 10^{-3} \\ \frac{-52.63(1.513s+1)e^{-1.617s}}{(4.596s+1)} & 17.86e^{-1.656s} & \frac{1.67(5.745s+1)e^{-7.7}}{(9.88s+1)} \end{bmatrix}$
$G_C(s)$	$\begin{bmatrix} 1.288+\frac{0.234}{s} & 0 & 0 \\ 0 & 1.148+\frac{0.16}{s} & 0 \\ 0 & 0 & 0.578+\frac{0.101}{s} \end{bmatrix}$

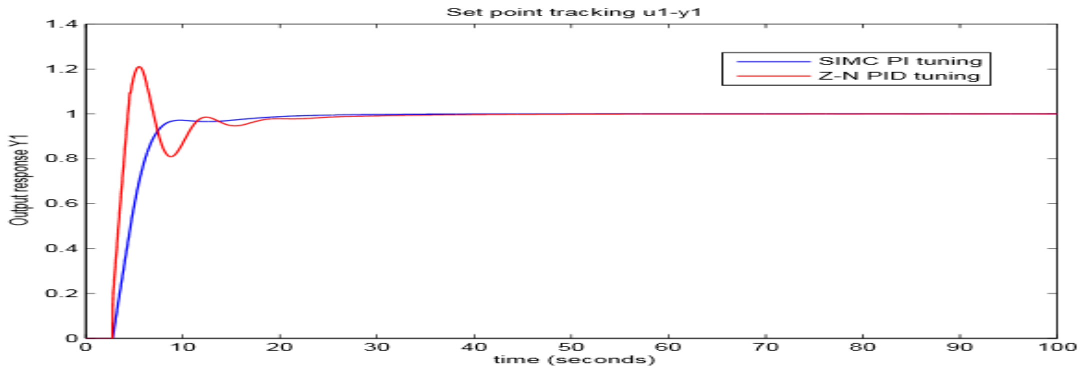

Fig. 4.5.11 Closed-loop process responses for desired output y1

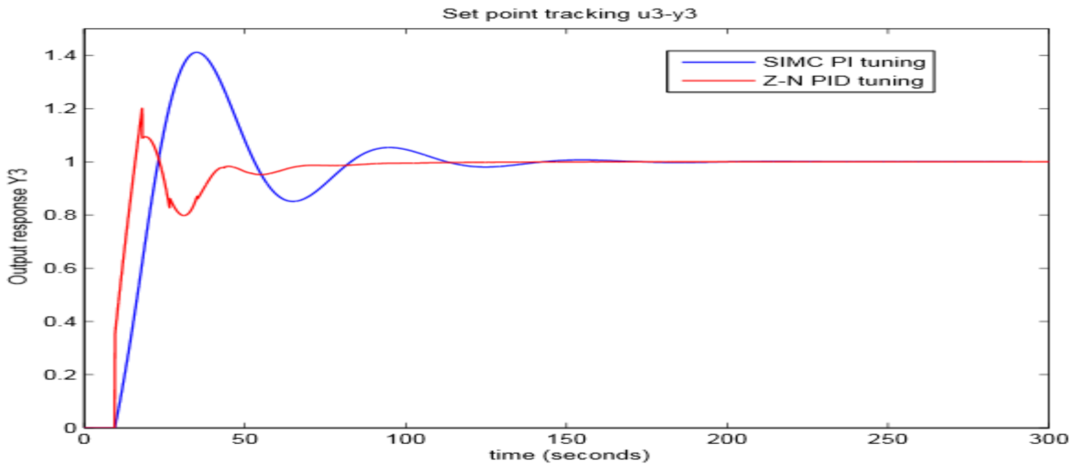

Fig. 4.5.12 Closed-loop process responses for desired output y2

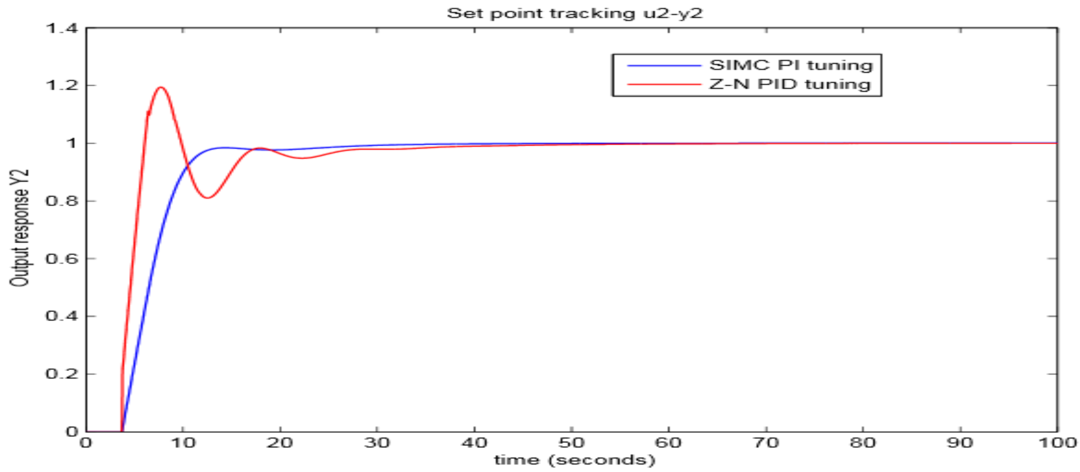

Fig 4.5.13 Closed-loop process responses for desired output y3

Table 4.5.6 Controller parameters for 3x3 systems

Method	G_{C11}			G_{C22}			G_{C33}			IAE
	K_c	τ_I(s)	τ_D(s)	K_c	τ_I(s)	τ_D(s)	K_c	τ_I(s)	τ_D(s)	2.86
SIMC	1.288	4.272		1.148	6.22		0.578	9.88		
Z-N tuning	2.768	3.14	0.785	2.51	4.71	1.177	1.471	13.54	3.384	10.34

For the 3 SISO processes controllers are designed for set point tracking and the results are shown in figures 4.5.11 – 4.5.13.A step change of magnitude 0.96 in the overhead reflux flow rate, the set point tracking for top product composition of ethanol is obtained by Z-N PID tuning & Skogestad's PI tuning are shown in the Figure 4.5.11 Both methods are compared in terms of IAE as shown in table 4.5.2 from which it can be seen that SIMC gives minimum error.

Similarly, for a step change of magnitude 0.96 in the side stream flow rate, the set point tracking for side stream composition of ethanol is obtained by Z-N PID tuning & Skogestad's PI tuning in as shown in fig 4.5.11 . For a step change of magnitude 0.96 in the Reboiler stream pressure, the set point tracking for temperature at 19^{th} tray is obtained by Z-N PID tuning & Skogestad's PI tuning in fig. 4.5.12 From figures 4.5.10 to 4.5.13, it can be observed that SIMC tuning yields much robust and stable closed-loop response and the value of IAE is minimum for SIMC method for set point change.

In Figure 4.5.12, the closed-loop response of Y3 is shown where Z-N tuning has shown better performance over the SIMC tuning. The performance comparisons of two methods are shown in table 4.5.2 which shows that SIMC method gives best in terms of minimum IAE when compared to Z-N method.

Chapter 5

CONCLUSIONS

This chapter presents the conclusions and the significant contributions brought out in this thesis work and recommendations for future works. For each objective discussed in thesis, the conclusions are drawn from the results and discussions carried out individually in previous chapters.

5.1 Identification and control of a SISO process with Modified Relay methods:

- In this work, a modified asymmetrical relay feedback method along with sinusoidal signal is used to identify various parameters of the stable, SOPTD and unstable systems.
- MATLAB Simulink approach is used to generate the sustained oscillations for each process considering each one as case study adapting numerical analysis.
- Limit cycle oscillations obtained in this work are further used to estimate other intermediate parameters besides using ultimate gain and ultimate period.
- The ultimate gain and period are used to design simple PID controller with Z-N settings for each case study.
- The improved method has been observed to give better identification accuracy in terms of ISE, when compared to other methods and simple PID controllers are designed using asymmetric relay feedback approach with Z-N settings and improved method is compared to the method proposed by other researchers.

5.2 Identification & control of SISO process using Relay with Subspace:

- The main conclusions that can be drawn from this study is the design and use of a simple method for identification of process model parameters of low order systems.
- The method does not require any prior knowledge of the process in terms of in-depth mathematical modeling.
- Relay test is carried out to generate input-output data which can replace need of the experimentation data for identification.
- The method can be applied in case of presence of measurement noise also. The simple relay feedback test is employed in this work to obtain the process information such as ultimate gain and ultimate period which are fed to the system identification tool box to synthesize controller parameters in the form of simple PI and PID Controllers.

- It can be conclude that the proposed method works well for the identification of the process with less knowledge and may be extended for the identification and control of the higher order systems, including unstable systems.

5.3 Identification and control of non-linear process using Relay Feedback approach:

- Two non-linear processes; Hammerstein and Wiener considered here were identified using the proposed method low-pass filter in the feedback path and sustained oscillations were generated.
- A MATLAB Simulink approach is adapted to obtain the sustained oscillations after incorporating low pass filter in the feedback path using ultimate properties.
- In this work, a new technique is introduced in order to reduce the effect of noise in the identification process using relay feedback.
- It is observed to be quite difficult with increasing height of relay for practical applications. Instead of increasing the height of relay, it is proposed to introduce low-pass filter in order to suppress the noisy components affecting the identification procedure.
- It is observed that the proposed control structure shows the satisfactory response both in the servo and regulatory responses with and without noise.
- Low pass filter with few specifications are used in this method to implement in the feedback path of the reported method which was not carried out in the literature.
- Using the proposed control structure, the control responses are obtained both in servo and regulatory responses which show the satisfactory response.

5.4 .Identification and control of 2x2 MIMO process using Relay and Subspace method:

- In this, a simple and novel method has been used for identification and control of 2x2 MIMO distillation column, combination of relay and subspace identification has been adapted for generating the input-output data.
- N4SID algorithm has been employed to obtain state space model parameters. Using normalized decoupling controller design rules casual, stable & proper decouplers is derived & Skogestad's tuning rule is employed to design PI/PID controller parameters and the result shows satisfactory responses for set point changes.
- Using the proposed method the estimated transfer functions are determined from input-output data. It has been observed that the accuracy is much better compared to relay-feedback method even for higher order system.

- The accuracy for system without disturbance approximates very close to parameters of systems but for systems with disturbance it varies depending on the order of the system & other factors.
- The advantage of this method is its simplicity and the closed-loop control scheme also showed that the performance of present control (proposed estimation followed by PI) yields faster and stable response compared to other scheme.

5.5 An Improved Identification and control of 3x3 MIMO system using Relay and Subspace method:

- In this work new method of identification which is the combination of Relay and subspace was introduced to identify and control of 3X3 distillation column.
- The relay was adapted to generate input-output data, from which subspace identification used with N4SID algorithm.
- For the MIMO identified process using RGA-RNGA, casual, stable & proper decoupler is derived which reduces the interaction and converts 3x3 MIMO system to ‘3’ individual SISO systems.
- Skogestad’s tuning rule is employed to design PI/PID controller parameters.
- The advantage of this method is its simplicity, also by increasing the relay height the effect of noise can be minimized which is a major problem in the closed loop identification processes.
- The proposed method is more efficient, less time-consuming & requires fewer calculations for identification of multivariable systems, since N4SID algorithm is used to carry out matrix operations.
- Unlike subspace method which requires pre-designed controller to carry out subspace identification, it doesn’t require pre-designed controller, which is replaced by the relay.
- The proposed method is more efficient, less time-consuming & calculations for identification of multivariable systems.
- The fundamental Z-N tuning method is used to determine another set of parameters and the effectiveness of control parameters is obtained by both the methods are compared.
- Here the Skogestad’s tuning rule showed better performance over Z-N PID in terms of minimum IAE.

5.6 Significant Contributions made to the research work:

The significant contributions that are made in this thesis are as follows.

- A simple method is developed for identification and control of 2x2 and 3x3 MIMO Processes.
- Relay method in combination with subspace methods are developed for identification of higher order process without using complex calculations.
- N4SID algorithm has been adapted for conversion of states space model into transfer function model using system identification tool box which is a simple identification process.
- The process can be identified with precise knowledge without much mathematical modeling and input-output data can be easily generated.
- In addition to the above, significant work on identification and control of SISO process with modified relay methods have been carried out.
- Relay Feedback methods have extended to identification and control of non-linear process which are very significant in chemical processes.
- A new control structure is extended to the control of non-linear process which shows the satisfactory response in both servo and regulatory cases.
- Noise component is also considered here which is suppressed with low pass filter and shows less impact on identification process.

5.7 Recommendations for future work:

- Subspace methods along with Relay can be used to identify unstable MIMO processes.
- Relay with subspace methods can be used to identify and control the non-linear unstable systems.
- Relay with subspace methods with addition of noise can be used to identify a process since noise was not used in the closed loop data of present work.
- Experimental validation for the simulated results can be carried out for benchmark examples in the literature.

REFERENCES

1. Astrom, KJ &Haagglund, T 1984, 'Automatic tuning of simple regulators with specifications on phase and amplitude margins', Automatica, vol. 20, pp.645-651.
2. Astrom, KJ &Hagglund, T 2000, 'Benchmark system for PID control', Preprint of IFAC PID2000 Workshop, Terrassa, Spain.
3. Atherton, DP &Majhi, S 1998, 'Plant parameter identification under relay control', in Proc. IEEE Conference on Decision & Control, Tampa, Florida, USA.
4. Atherton, DP 2006, 'Relay autotuning: An overview and alternative approach', Ind. Eng. Chem. Res., vol. 45, pp. 4075-4080.
5. Bi, Q, Wang, QG & Hang CC 1997, 'Relay-based estimation of multiple points on process frequency response', Automatica, vol. 33, no. 9, pp. 1753-1757.
6. Chang, JW & Yu, CC 1990, 'The Relative gain for non-square multivariable systems', Chemical Engineering Science, vol.45(5), pp. 1309-1323.
7. Chang, R, Shen, S & Yu, CC 1992, 'Derivation of transfer function from relay feedback systems', Ind. Eng. Chem. Res, vol.31, pp. 855- 860.
8. Chen Juan, He Ze-Fang, Xin Qi 2011, 'A new control method for MIMO first order time delay non-square systems', Journal of Process Control, vol.21, pp.538-546.
9. Cheng, YC & Yu, CC 2000, 'Relay feedback identification for actuators with hysteresis', Ind. Eng. Chem. Res., vol.39, pp.4239-4249.
10. Chiang, RC & Yu, CC 1993, 'Monitoring Procedure for Intelligent Control: On-line Identification of Maximum Closed-Loop Log Modulus' Ind. Eng. Chem. Res, vol.32, pp.90-99.
11. Chidambaram, M &VivekSathe 2014, 'Relay Autotuning for Identification and Control', Cambridge University Press.
12. Esakkiappan, C,Thyagarajan, T &Sharmila, GV 2012, 'Identification of model structure from the relay response using artificial intelligence based pattern recognition techniques', European Journal of scientific research, vol. 74, no. 1, pp. 61-68.
13. Friman, M & Waller, KV 1997, 'A two-channel relay for autotuning', Ind. Eng. Chem. Res., vol. 36, pp. 2662-2671.
14. Gu, D, Ou, L, Wang, P & Zhang, W 2006, 'Relay feedback autotuning method for integrating processes with inverse response and time delay', Ind. Eng. Chem. Res., vol. 45, pp. 3119-3132.
15. Hang, CC, Astrom, KJ & Wang, and QG 2002, 'Relay feedback autotuning of process controllers - A tutorial review', Journal of Process Control, vol. 12, pp. 143-162.

16. Hord, M &Skogestad, S 1994, ‘Improved independent design of robust decentralized controllers’, Modelling, Identification and Control, vol.15, pp.119-125.
17. Huang, H, Chen, C, Lai, C & Wang, G 1996, ‘Autotuning for model based PID controllers’, AIChE Journal, vol. 42, pp. 2687-2691.
18. Kaya, I & Atherton, DP 1998, ‘An improved parameter estimation method using limit cycle data’, in Proc. UKACC International Conference on Control, Sussex, UK.
19. Kaya, I & Atherton, DP 2001, ‘Parameter estimation from relay auto tuning with asymmetric limit cycle data’, Journal of Process control, vol. 11, pp. 429-439.
20. Lin, YD, Huang, HP & Yu, CC 2006, ‘Relay feedback tests for highly nonlinear processes: Reactive distillation’, Ind. Eng. Chem. Res., vol. 45, pp. 4081-4092.
21. Liu, T &GAO, F 2008, ‘Identification of integrating and unstable processes from relay feedback’, Computers and Chemical Engineering, vol. 32, pp. 3038-3056.
22. Liu, T, Gao, F & Wang, Y 2008, ‘A systematic approach for on-line identification of second-order process model from relay feedback test’, AIChE Journal, vol. 54, no. 6, pp.1560-1578.
23. Luyben, WL 1987, ‘Derivation of transfer functions for highly nonlinear distillation columns’, Ind. Eng. Chem. Res., vol.26, pp. 2490-2495.
24. Luyben, WL 2001, ‘Getting more information from relay feedback tests’, Ind. Eng. Chem. Res., vol. 40, pp.4391-4402.
25. Majhi, S & Atherton, DP 1999, ‘Autotuning and controller design for processes with small time delays’, IEE Proc. -Control Theory Appl., vol. 146, no. 5, pp. 415-425.
26. Majhi, S & Atherton, DP 2000, ‘Obtaining controller parameters for a new Smith predictor using autotuning’ Automatica, vol. 36, pp. 1651-1658.
27. Majhi, S & Atherton, DP 2000, ‘Online tuning of controllers for an unstable FOPDT process’, IEE Proc.-Control Theory and Appl., vol. 147, no. 4, pp. 421-427.
28. Majhi, S &Litz, L 2003, ‘Relay based estimation of process model parameters’, in Proc. American Control Conference, Denver, Colorado.
29. Majhi, S 2007, ‘Relay based identification of processes with time delay’, Journal of Process Control, vol. 17, pp. 93-101.
30. Ogunnaike, BA & Ray, WH, 1994, ‘Process dynamics, modeling and control’, Oxford Univ. Press, Oxford/ New York.
31. Panda, RC & Yu, CC 2002, ‘Exact Analytical Expressions for Relay Feedback Responses—Second Order Plus Dead Time Processes’, 4th Asian Control Conf., Singapore.

32. Panda, RC & Yu, CC 2003, 'Analytical expressions for relay feedback responses', Journal of Process Control, vol.13, pp.489–501.
33. Panda, RC & Yu, CC 2005, 'Shape factor of relay response curves and its use in autotuning', Journal of Process Control, vol. 15, pp. 893-906.
34. Panda, RC 2006, 'Estimation of parameters of under-damped second order plus dead time processes using relay feedback', Computers and Chemical Engg, vol. 30, pp. 832-837.
35. Ramakrishnan, V & Chidambaram, M 2003, 'Estimation of a SOPTD transfer function model using a single asymmetrical relay feedback test', Computers and Chemical Engineering, vol. 27, pp. 1779-1784.
36. Selvakumar, C & Panda, RC 2010, 'Modeling relay responses for multivariable processes', Indian chemical engineer, vol. 52, no.4, pp 315-324.
37. Shen, SH & Yu, CC 1994, 'Use of Relay – Feedback Test for Automatic Tuning of Multivariable System', AIChe Journal, vol. 40, no. 4, pp.627-646.
38. Shen, SH, Yu, HD and Yu, CC 1996, 'Use of saturation-relay feedback for auto tune identification', Chemical Engineering Science, vol. 51, no. 8, pp. 1187-1198.
39. Sigurd Skogestad's 2003, 'Simple analytic rules for model reduction and PID controller tuning', Journal of Process Control, vol.13, pp.291–309.
40. Siva Kumar, E, Vivek, S & Chidambaram, M 2005, 'Improved saturation relay test for systems with large dead time', Ind. Eng. Chem. Res., vol. 44, pp. 2183-2190.
41. Soltesz, K, Hagglund, T &Astrom,KJ 2010, 'Transfer function parameter identification by modified relay feedback', in Proc. American Control Conference, Baltimore, USA.
42. Srinivasan, K, & Chidambaram, M 2003, 'Modified relay feedback method for improved system identification', Computers and Chemical Engineering, vol. 27, pp. 727-732.
43. Sujatha, V & Panda, RC 2012, 'Relay Feedback-Based Time Domain Modelling of Linear 3-by-3 MIMO System', American Journal of Systems science, vol.1, no. 1, pp.1-6.
44. Sujatha, V & Panda, RC 2013, 'Estimation of parameters for disturbance model of 3-by-3 MIMO Process Using Relay Feedback', Columbia International Publishing, Journal of Modeling, Simulation, Identification and Control, vol. 1, pp.27-40.
45. Sung, SW & Lee, IB 1997, 'Enhanced relay feedback method', Ind. Eng. Chem. Res., vol. 36, pp. 5526-5530.
46. Thyagarajan, T & Yu, CC 2003, 'Improved Autotuning Using the Shape factor from Relay Feedback', Industrial Engineering Chemistry Research vol.42, no. 20, pp.4425-4440.

47. Thyagarajan, T, Yu, CC & Huang, HP 2003, ‘Assessment of controller performance: A relay feedback approach’, Chemical Engineering Science, vol. 58, pp. 497-512.
48. Vivek, S & Chidambaram, M 2005, ‘Identification using single symmetrical relay feedback test’, Computers and Chemical Engineering, vol. 29, pp. 1625-1630.
49. Vivek, S. and Chidambaram, M. ‘An improved relay auto tuning of PID controllers for unstable FOPTD systems’, Computers and Chemical Engineering, Vol. 29, pp. 2060-2068, 2005.
50. Wang, QG, Hang, CC. and Zou, B 1997, ‘Low-order modeling from relay feedback’, Ind. Eng. Chem. Res., vol. 36, pp. 375-381.
51. Wood, RK & Berry, MW 1973, ‘Terminal Composition Control of a Binary Distillation Column’, Chem. Eng. Sci. vol.28, pp.1707–1717.
52. Yu, CC 2006, ‘Auto-Tuning of PID Controllers’, Springer –Verlag, London.
53. T. Liu, Q. G. Wang, H. P. Huang. , 2013, ‘A tutorial review on Process identification from step or relay feedback test,’ Journal of Process Control, 2013, 23(10): 1597 – 1623.
54. K. J. Åstr Om, P. Eykhoff. 1997, ‘System identification - A survey.Automatica’ 1971, 7(2): 123 – 162.
55. Q. G. Wang, C. C. Hang, B. Zou., 1997, Low order modeling from Relay feedback systems. Industrial and Engineering Chemistry Research, 1997, 36(2): 375 – 381.
56. Atherton, MajhiS. 1998, ‘Plant parameter identification under relay control ‘In: Proceedings of the37thIEEEconferenceondecision&control, Tampa, FL, USA, .pp1272–12777.
57. C. C. Yu. 1991, Introduction. ‘Autotuning of PID controllers – A relay feedback Approach’. London: Springer-Verlag, 2nd Edition, 1 – 8.
58. C. C. Hang, K. J.Åstr Om, Q. G. 2002, ‘Wang. Relay feedback autotuning process controllers a - tutorial review’. Journal of Process Control, 12(1): 143 – 162.
59. W. Li, E. Eskinat, W. L. Luyben, 1991, ‘An improved auto tune identification Method’. Industrial and Engineering Chemistry Research, 30(7): 1530 – 1541.
60. S. Majhi, D. P. 1999, Atherton. Auto tuning and controller design for processes with small time delays. IEE Proceedings on Control Theory and Applications, 146(5): 415 – 424.
61. S. Majhi, 2005, ’On-line PI control of stable processes ‘Journal of Process Control, 15(8): 859 -867.
62. S. Majhi. Relay, 2007, ‘based identification of processes with time delay. Journal of Process Control, 2007’ 17(2): 93-99.

63. T. Thyagarajan, C. C. Yu. ‘Improved auto tuning using shape factor from Relay feedback’ Industrial and Engineering Chemistry Research, 2003, 42(20): 4425 – 4440.
64. R. C. Panda, V. Vijayan, V. Sujatha, 2011, et al.’ Parameter estimation of integrating and time delay processes using single relay feedback test’ ISA Transactions, 50(4): 529 – 537.
65. Chidambaram, and VivekSathe. 2014 ‘Relay Auto tuning for Identification and control’.
66. V Ramakrishnan, and M. Chidambaram. 2003 ‘Estimation of a SOPTD transfer function model using the single asymmetric relay feedback test ‘, Computers &Chem.Engg.27 (12**)**: 1799-764.
67. Srinivasan K, and Chidambaram. 2003, ‘Modified relay feedback method for improved system identification.’ Computers& Chemical Engineering, computers &Chem.Engg27 (5): 727-732.
68. Overschee, P.V and Moor, B.D. (1994),N4SID: ‘Subspace Algorithms for the Identification of Combined Deterministic-Stochastic Systems’, Automatica, and Statistical signal Processing and Control, Tarrytown, NY, USA, 30(1), pp. 75–9
69. Overschee, P.V. and Moor, B.D, (1996), ‘Subspace Identification for Linear Systems’ KLUWER Academic, Boston.
70. Katayama, T. (2005) ‘Subspace Method for System Identification’ Springer, London.
71. Bakke, M. (2009),’Subspace Identification Using Closed Loop Data’ MS Thesis” Norwegian University of Science and Technology, NTNU, Norway (2009).
72. Ljung, L. (1998) ‘System Identification Toolbox’ MATLAB, Natick, MA, PP. 508–647, the Math Works.
73. Shen, Y.,Cai, W.-J and Li, S., (2010) ‘Normalized Decoupling Control for High Dimensional MIMO Process for Application in Room Temperature Control HVAC Systems’ 2nd edition n, Springer, London, pp. 652–654.
74. Panda, R. C. and Sujatha, V, 2012, ‘Identification and Control of Multivariable Systems-Role of Relay Feedback’, Introduction to PID Controller-Theory, Tuning and Applications to Frontier Areas.
75. Chidambaram, M. and Pad ma Sree, R. (2017), ‘Control of Unstable and Single and Multi-Variable System’, Narosa Publishers, New Delhi.
76. Chidambaram, M. and Rao, C. S. A, 2015, Thesis Submitted on ‘Subspace Identification of Unstable Systems.’ IIT Madras, Chennai.
77. Kalapana D., Thygarajan, T. and Thenral, R., 2015 ‘Improved Identification and Control of 2-by-2 MIMO System Using Relay Feedback’, IEEE, 17, pp. 23–32.

78. Bijeesh, K., Kalpana, D. and Thyagarajan, T., 2014, ‘Identification of 3-by 3 MIMO System Using Relay Feedback’ IEEE International Conference on Advanced Communications, Control and Computing Technologies, Ramanathapuram, pp. 157–162.
79. Akaike, H., 1974 ‘Stochastic Theory of Minimal Realization’, IEEE, Automatic Control, 19(6), pp. 667–664.
80. He, M.-J., Cai, W.-J., Ni, W. and Xie, L.-H., 2009 ‘RNGA Based Control System Configuration for Multivariable System’, J. Process Control, 19(6), pp. 1036–1042.
81. Lim, R.K., Phan, M.Q. and Longman, R.W.1998 ‘State Space System Identification with Identified Hankel Matrix, ‘Department of Mechanical and Aerospace Engineering Technical Report No. 3045, Princeton University, September.
82. Skogestad’s, S. 2011 ‘The SIMC Method for Smooth PID Controller Tuning’, J. Process. Control, 13, pp. 291–309.
83. Sujatha, V. and Panda, R.C., 2013 ‘Estimation Sujatha, V. and Panda, R.C., 2013 ‘Estimation of Parameter for Disturbance Model of 3-by-3 MIMO Process Using Relay Feedback Test’, J Modeling, Simulation, Identification and Control, 1, pp. 27–40.
84. Vivero, O. and Licega-Castro, J., 2008 ‘MIMO Toolbox for MATLAB. 37th IEEE Annual conference ‘CDC, Auckland, New-Zealand, pp. 1–5.
85. Wang, Q.-G., Hang, C.-C. And Zou, B., 2000 ‘Multivariable Process Identification and Control from Decentralized Relay Feedback’, Int. J. Mod. & Simulation, 20(4) pp.341–348
86. Stephanopoulos, G. 1984 ‘Chemical Process Control’ Pearson Education, Englewood Cliffs, NJ.
87. Bakke, M. 2009. ‘Subspace Identification using closed loop data’ Norway. Cock, K. D., Moor,
88. H, A. (1974). ‘Stochastic Theory of Minimal Realization’ IEEE, Automatic Control, 667-664.
89. He, M.-J., Cai, W.-J., Ni, W., &Xie, L.-H. 2009. ‘RNGA Based Control System Configuration for Multivariable System’. Journal of Process Control, 1036-1042.
90. Katayama, T. 2005,’ Subspace Method for System Identification’. Verlag, London: Springer.
91. Wang, Q.-G., Chang-Chieh, & Zou, B. 2000. ‘Multivariable Process Identification and Control from Decentralized Relay Feedback’. IEEE, Vol. 20 No.4.
92. C. Sankar Rao, M. Chidambaram, 2014 ‘Subspace identification of transfer function models for an unstable bioreactor,’ Chemical Engineering Communication, PP, 1296-1303.

93. C. Sankar Rao, M. Chidambaram, 2015.' Subspace identification of unstable transfer function models', Indian Chemical Engineer', Vol 59, Issue-1, 2017.
94. Tao Liu, Qing-Guowang, Hsia-ping Huang, 2013 'A tutorial review on process identification from step or relay feedback test' Journal Process Control.
95. Rames.C.Panda, V.Sujatha, 2012 'Identification and control Multivariable systems –"Role of relay Feedback' Introduction to PID controller-Theory", Tuning and Applications to frontier Areas. DOI 10.5772/31509, In Tech Publications.
96. Shankar Prasad, Yaddanapudi Jaya, 2009 'Process Identification: Identification and control of Relay-Stabilizable Processes' VDM VERILAG Publishers,USA, Aug 21, 09.
97. Utkalk.V.Metha, 2011, 'process modeling using the Relay Feedback Approach' Ph.D. Thesis, IIT Guwahati.
98. Luyben, W.L. 2003, 'Identification and tuning of integrating process with dead time and Inverse response', Ind.Eng.Chem.Res. Vol.42, pp.3030-3035.
99. Thyagarajan, T, Esakkiappan, C and Sujatha,V ,2010, 'Modeling and Control of Inverse Response process with time delay using Relay Feedback Test', Proceedings of the International Conference on Modelling, identification and Control, Okayama, Japan.
100. Esakkiappan, C. Thyagarajan ,T and Blessy Hasina, P ,2012, 'Identification of Over Damped Second Order Plus Dead Time Process Using Relay Feed Back Test', IEEE-International Conference on advances in Engineering, Science and Management (ICAESM -2012) ,March 30.
101. Sujatha, V and Panda, R. C. 2011, 'Analytical Expressions for Relay Responses of Systems', IEEE Conference on Communication, Computing and Control CCCA-2.
102. Thyagarajan, T, Esakkiappan, C and Sujatha,V ,2010, 'Modeling and Control of Inverse Response process with time delay using Relay Feedback Test', Proceedings of the International Conference on Modelling, identification and Control, Okayama, Japan.
103. William L. Luyben, 1986, 'Simple Method for Tuning SISO Controllers in Multivariable Systems', Ind. Eng. Chem. Process Des. Dev. Vol.25, pp. 654-660.129.
104. Panda. R.C, 2006, 'Estimation of parameters of under damped SOPDT processes using relay feedback', Computers and Chemical Engineering, Vol.30, pp. 832-837.
105. O'Dwyer, A 1999, 'PI and PID controller tuning rules for time delay processes: A summary', in Proc. Irish Signals and Systems Conference, National University of Ireland, Dublin.
106. D.Kalpana, 2016, "Modeling and Control of MIMO systems using relay Feedback" A Thesis Submitted to Anna University.

List of publications in Journals

- D. Kishore, K. Anand Kishore, RC Panda (2018), "Identification and control of process using the modified asymmetrical Relay Feedback method", Procedia Computer science, Volume 133, PP 1029-1034, 2018.
- D. Kishore, K. Anand Kishore & Rames C. Panda (2018), "An improved identification and control of 3 × 3 Multi-input Multi-output system using relay and subspace Method", Indian Chemical Engineer journal, Taylors & Francis group, Volume 61, Issue 1, PP 87-101.
- D. Kishore, K. Anand Kishore, R.C panda (2018), "Identification and control of MIMO systems using simultaneous relay and subspace Method", Journal of Modeling, Identification, simulation, Columbia International publishing (Accepted).

List of publications in Conferences

- D. Kishore, K. Anand Kishore, R.C panda, "Non-Linear system identification and control of Wiener process using Relay Feedback", International conference on Frontiers in Optimization Theory and Applications(FOTA), 24-25 November, 2016, Kolkata.
- D. Kishore, K. Anand Kishore, R.C panda, "Identification and control of Non-Linear systems using relay Feedback (a non-linear element) Approach", National Conference on Applied Nonlinear Dynamics, 21-22 December 2016, NIT Warangal.
- D. Kishore, K. Anand Kishore, R.C panda, "Stability analysis of SOPTD system with Numerator dynamics using Relay Feedback approach", CHECON 2016, IIT Madras, 27-30 December, 2016.
- D. Kishore, K. Anand Kishore, RC panda (2017), "An improved Identification of Low order process Using Relay Feedback approach", IEEE Explorer, DOI 10.1109/TIMA.2017.8064789.
- D. Kishore, K. Anand Kishore, RC panda, "Stability analysis and controller Design for unstable systems using Relay feedback approach", SOCPROS-2017(Springer) at IIT Bhubaneswar Dec 23-24, 2017.
- D. Kishore, K. Anand Kishore, RC Panda, "Identification of unstable process parameters using modified Relay Feedback Approach", Sixteenth International Conference on Science Engineering, Vellore Institute of Technology, Vellore, 02-03 April 2018.

- D. Kishore, K. Anand Kishore, RC panda, “Identification and control of cascade control system using simple Relay Feedback approach”, NCSCSTACE-2018, CSIR - NAL Bangalore, Jan 23-25, 2018.
- D. Kishore, K. Anand Kishore, RC Panda, “Identification and Control of SISO systems using relay and subspace method”, INCEEE-II, 15-16 Feb 2019, NIT Warangal.
- Gaikwad Rahul. B, D. Kishore, K. Anand Kishore, “Experimental Identification of Process Parameters Using Relay Feedback with State space Approach”, INCEEE-II, 15-16 Feb 2019, NIT Warangal.