Investigations on Synchrophasor assisted Power System State Estimation

Submitted in partial fulfillment of the requirements

for the award of the degree of

DOCTOR OF PHILOSOPHY

in

Electrical Engineering

by

Chejarla Madhu Kishore

Roll No: 701609

Supervisor:

Dr. Matam Sailaja Kumari

Professor

Department of Electrical Engineering

NATIONAL INSTITUTE OF TECHNOLOGY

WARANGAL-506 004, TELANGANA STATE, INDIA

December - 2021

Approval

This Thesis work entitled **”Investigations on Synchrophasor assisted Power System State Estimation”** by **Chejarla Madhu Kishore**, bearing **Roll No: 701609**, is approved for the degree of **Doctor of Philosophy** in **Electrical Engineering** from **National Institute of Technology, Warangal**.

Examiners

____

____

____

Supervisor

____

Dr. Matam Sailaja Kumari
Professor
Department of Electrical Engineering
NIT Warangal

Chairman

____

Dr. Matam Sailaja Kumari
Professor and Head
Department of Electrical Engineering
NIT Warangal

Date: __________

Place: NIT Warangal

DEPARTMENT OF ELECTRICAL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY

WARANGAL-506 004, TELANGANA STATE, INDIA

CERTIFICATE

This is to certify that the thesis entitled **"Investigations on Synchrophasor assisted Power System State Estimation"** which is being submitted by **Chejarla Madhu Kishore**, bearing **Roll No. 701609**, is a bonafide work submitted to **National Institute of Technology, Warangal** in partial fulfillment of the requirements for the award of the degree of **Doctor of Philosophy** in **Department of Electrical Engineering**. To the best of my knowledge, the work incorporated in this thesis has not been submitted elsewhere for the award of any degree.

Dr. Matam Sailaja Kumari

(Supervisor)

Professor

Department of Electrical Engineering

NIT Warangal

Date:

Place: NIT Warangal

DECLARATION

This is to certify that the work presented in the thesis entitled " Investigations on Synchrophasor assisted Power System State Estimation " is a bonafide work done by me under the supervision of **Dr. Matam Sailaja Kumari**, Department of Electrical Engineering, National Institute of Technology, Warangal, India and was not submitted elsewhere for the award of any degree.

I declare that this written submission represents my ideas in my own words and where others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above will be cause for disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

Chejarla Madhu Kishore
(Roll No.701609)

Date:____________
NIT Warangal

ACKNOWLEDGMENTS

I would like to express my gratitude to my supervisor, **Prof. Matam Sailaja Kumari**, for providing enough freedom and generating homely feeling throughout the journey, for her patience and valuable suggestions. It is impossible to accomplish this dissertation work with out her support, encouragement. Many thanks for her support and encouragement to achieve all my personal life ambitions too. Apart from all these its a honour for me to associate with a good human being like her.

I would like to thank **Dr. Matam Sailaja Kumari**, Head , Department of electrical engineering for providing all the required facilities and support.

I take the privilege to thank all my Doctoral Scrutiny Committee members, **Dr. M. Sydulu**, Professor(HAG), Department of Electrical Engineering, **Dr. Y. Chandra Sekhar**, Assistant Professor, Department of Electrical Engineering, **Dr. Altaf Q H Badar**, Assistant Professor, Department of Electrical Engineering and **Dr. R.B.V. Subramanyam**, Professor, Department of Computer Science Engineering for their detailed review, constructive suggestions and excellent advice during the progress of this research work.

I also appreciate the encouragement from teaching, non-teaching members and fraternity of Department of Electrical Engineering, NIT Warangal. They have always been motivating and supportive.

I wish to express my sincere thanks to **Prof. N. V. Ramana Rao**, Director, NIT Warangal for his official support and encouragement.

I would like to express my gratitude to my parents **Chejarla Lakshmi Narasimha Rao(Jangam Seethaiah Devara)** and **Malathi Devi**, my maternal uncle (**D. V. Uma Maheswara Rao**) and family members for their unconditional support, patience and encouragement throughout my journey, which helped me in completion of this dissertation work.

My sincere thanks to my M.Tech friends V. Rajasekhar Reddy, Rahul Prasad Dubey, M.

Vamsi Krishna, P. Sumanth and C. Purusotham Reddy for their unconditional love. You are one of the greatest gift I ever had in my life. Your influence on my personality transformation is huge. Thanks to my childhood friend Raavi Madhu Chowdary for being with me in odd times.

A special thanks to my co-scholar Anil Kumar Annam raju for his unconditional support throughout my research period and thanks to V. S. Sandeep Kumar Reddy, C. Bhanu Prasad and all co-scholars for their support and valuable suggestions.

Finally I would like to express my gratitude to National Institute of Technology Warangal for providing me with the opportunity to carry out research and I am indebted to the people of the country for indirectly supporting my work.

(Chejarla Madhu Kishore)

DEDICATION

The thesis is dedicated to my father Lingaikya Siva Sri Jangam Seethaiah Devara(great visionary and architect of my life).

My dear father without you I don't exist in any sense. The miracle in my life is taking birth as your son. The kind of mammoth love and affection which radiates from you I won't get from anybody else. The kind of paramananda and positivity I attain at your presence is enormous and there is no substitute for this too. I am very passionate about your way of doing things. Though you have very limited resources, your hand is very generous and charitable. Some villagers call you as "Dharma Raju , some call you as " Quick-tempered" and some others say: "Though our village has Eka Lingam, it is powerful." You transformed many lives by extending your helping hand. In my childhood I had a lot of fear talking with you. But as time passed I couldn't realize you had become my closest friend. In the journey with you, every time I experienced supreme qualities of you. It makes me very proud and amplified the feeling of respect and surrender towards you. Since my childhood I have been away from you for the sake of studies. By the time I reached a state to stay with you, you departed this world. I don't know how Lord Rama managed the grief of distancing away from Dhashardha. Losing you means losing myself from me. It is very hard to discover a day when I restrained myself from talking to you. You proved the statement "Great people never expect returns for their acts" by leaving me without taking anything from me. You used to say "Lucky man and Unlucky man both reside within us. But whom we are inviting decides what we are" and "If you want to create an empire don't look for help". Though you are not with me. I am still considering myself lucky man, because this world calls me your son. Atleast I will try to lead rest of my life without looking for help. If this document has any value. I am offering it as a flower at your feet.

ABSTRACT

Unbundling of vertically integrated electric utility, ever increasing load demand and integration of various intermittent resources made the electric grid more dynamic than ever. The current SCADA systems cannot provide the real time visibility of the system due to its low reporting rates. The measurement unit called Phasor Measurement Unit(PMU) invented by Prof. Arun Gandhi Phadke has the ability to monitor Electric Grid at higher sampling time than SCADA measurements. Therefore, their deployment and application in Power system state estimation becomes a need of the hour. To optimally integrate the PMUs and to use them in state estimation, existing methodologies and models were analyzed and proposed new strategies for deploying PMUs and new models for using deployed PMUs in PMU only assisted state estimation and in hybrid power system state estimation.

This dissertation describes two new methods for providing multiple solutions while deploying PMUs into the power system. The proposed strategies provide the entire feasible solution space which gives the observability of the power system by working on binary connectivity matrix of the system. From the feasible solution space, multiple optimal solutions are obtained. The drawback of topology based optimal PMU placement(OPP) method -I is overcame in topology based OPP method -II. In addition, normalized Bus observability index and normalized System observability redundancy index are presented to overcome the drawback of system observability redundancy index(SORI) and bus observability index(BOI).

Further, this dissertation introduces a new estimator called Quadratically decaying exponential criterion for a power system which is completely observable by PMUs. Quadratic-constant (QC) robust estimator has been improved to make it suitable for linear state estimation. The constant part of the estimator after the break-even point is changed to decaying exponential, termed as Quadratically decaying exponential criterion. The proposed Quadratically- decaying exponential criterion (QE) is tested on IEEE 14, 30, 57 and 118- bus systems for two cases. (a) measurement data having only Gaussian noise (b) with corrupt measurement data. It is demonstrated that the proposed estimator is able to suppress the bad data and exhibiting better estimation accuracy and

computational performance compared to least absolute value(LAV) and WLSE.

Next, a linear sequential hybrid state estimator is developed for integrating both SCADA and PMU measurements. In stage 1, the state of the power system is obtained using only SCADA measurements, with the help of the proposed linear model. In stage 2, state of power system is estimated using both PMU measurements and the intermediate state vector obtained in the first stage. During the instances when only PMU measurements are available, SCADA pseudo measurements are computed with the help of the previous instant state vector. The proposed method is tested on IEEE 14-bus, IEEE 30-bus, IEEE 57-bus, and IEEE 118-bus systems. The proposed method estimation accuracy is superior compared with traditional state estimator, pseudo measurement, and ANN based methods and competing with two stage hybrid state estimators. The computational efficiency of the proposed method is far superior than traditional state estimator, pseudo measurement-based method and two stage hybrid state estimators.

Finally, a linear single stage hybrid state estimator is developed for utilizing both SCADA and PMU measurements in the power system state estimation. With the help of previous instant state vector, SCADA measurements are transformed. The transformed measurements show a linear relationship with the state vector like PMU measurements. The transformed measurements and PMU measurements are used for estimating the states with the help of a linear model. The proposed state estimator is tested on IEEE 14- Bus, 30-Bus, 57- Bus and 118-Bus test systems. From the results, It is evident that the proposed estimator exhibits better accuracy and less computational time compared with traditional SCADA based WLS estimator and two stage hybrid state estimator. The proposed method is also able to track the system state in all PMU reporting rates.

Contents

List of Figures

List of Tables

List of Abbreviations

BOI	Bus Observability Index
BSP	Binary Semi-definite Programming
BPSO	Binary Particle Swarm Optimization
GPS	Global Positioning System
ILP	Integer Linear Programming
LAV	Least Absolute Value
LTS	Least Trimmed Squares
LMR	Least Measurements Rejected
LSHSE	Linear Sequential Hybrid State Estimator
LHSE	Linear Hybrid State Estimator
MST	Minimum Spanning Tree
MES	Maximum Exponential Square
MEAV	Maximum Exponential Absolute Value
MNMR	Maximum Normal Measurement Rate
MIP	Mixed Integer Linear Programming
MINP	Mixed Integer Non-linear Programming
MCS	Maximum Constraints Satisfaction

MSE	Mean Square Error
NSORI	Normalized SORI
NBOI	Normalized BOI
NZV	Non Zero Vector
OPP	Optimal PMU Placement
PMU	Phasor Measurement Unit
PSO	Particle Swarm Optimization
QL	Quadratic - Linear
QC	Quadratic - Constant
QE	Quadratically- decaying Exponential criterion
SCADA	Supervisory Control and Data Acquisition
SMT	Synchronized Measurement Technology
SORI	System Observability Redundancy Index
SHSE	Sequential Hybrid State Estimator
SCADASE	SCADA based State Estimation
WAMPAC	Wide-Area Monitoring, Protection and Control
WAMS	Wide Area Measurement System
WLS	Weighted Least Squares
WLSE	Weighted Least Squares Estimator
WLAV	Weighted Least Absolute Value
2HSE	Two stage hybrid state estimator

List of Symbols

C_{PMU_i}	Installation cost of PMU at i^{th} bus
N	Number of buses
H_1	Design matrix
BC	Binary connectivity matrix
b	unit vector having Nx1 size
n	node vector
d	Depth of penetration vector
s	Solution vector
ϵ	error vector
X	State vector
H_1	Design matrix
r_i	Measurement residual
τ	Break even point
Z^r	Real part of the PMU measurement set
Z^i	Imaginary part of the PMU measurement set
Z^s	SCADA measurements
P_p	Real power injection at p^{th} bus

Q_p	Reactive power injection at p^{th} bus
P_{pq}	Branch real power flow
Q_{pq}	Branch reactive power flow
V_p	Voltage magnitude at p^{th} bus
H	Jacobian matrix
X_{int}	Intermediate state vector
Z_s^{new}	Transformed SCADA measurements
Z_{new}	Combined transformed SCADA and PMU measurement set
W_1	Inverse co-variance matrix of SCADA measurements
W_2	Inverse co-variance matrix of PMU measurements

Chapter 1

Introduction

Unbundling of vertically integrated electrical utility, participation of various independent players in the electricity markets, integration of renewables for meeting ever increasing load demand and the necessity to decrease carbon footprints are making power grid more dynamic than ever before. Therefore, real time wide-area monitoring, protection and control (WAMPAC) of power system have become very essential. The preparedness of WAMPAC is not adequate with reference to traditional Supervisory Control and Data Acquisition (SCADA) system and its associated asynchronized measurements, because of its low reporting rate.

Synchronized Measurement Technology (SMT) with the help of Global Positioning System (GPS) and advancement in phasor estimation techniques has led to innovation in Phasor Measurement Unit (PMU). PMU will further enhance the propensity of WAMPAC.

A time synchronized phasor is called Synchrophasor. The measurement device which measures the synchrophasor is called Phasor Measurement Unit(PMU). PMU is essentially a digital recorder with synchronizing capability. A PMU installed at any node measures voltage phasor at that node, some or all branch current phasors incident to the node based on the availability of channel capacity, frequency and rate of change of frequency. Placement of multiple PMUs can enable coordinated system wide measurements. PMUs can also time stamp, record and store the phasor measurements of power system events. The basic features associated with PMU compared with the SCADA measurements are given in table1.1. Therefore, the deployment of PMUs into the system and their assistance in power system state estimation process will enhance the visibility of power system.

Table 1.1: Comparison of SCADA and PMU data

SCADA Data	PMU Data
Scan rate:2s	Scan rate:25-30 samples/s
Gives only magnitude measurements	Gives Phasor measurements
Not fast enough to respond to dynamic behavior	Fast enough to depict the system behavior
Time stamping for specific values and instances	Completely time tagged data with GPS synchronization

This chapter presents the literature review on PMU deployment methods into the power system and synchrophasor assisted power system state estimation.

The following investigations have been carried out:

1. Literature review on optimal PMU placement(OPP) strategies
2. Literature review on PMU only assisted power state estimation techniques
3. Literature review on hybrid power system state estimation techniques

1.1 Literature review on optimal PMU placement strategies

Depending on the availability of channels, PMUs measure time synchronized phasor of voltage at the installed bus and all or some of the current phasors of incident lines . Due to high installation costs, non- availability of communication infrastructure, it is not necessary to install PMUs at each bus to make the system observable. Also data management associated with the PMUs due to their high reporting rate, is also limiting the use of PMUs. Therefore, for optimal PMU placement, a suitable methodology is needed.

In recent past, many researchers have attempted the OPP problem either by using traditional techniques or meta- heuristic techniques for providing solution for optimal PMU placement problem.

In [1], a two stage method was proposed. In the first stage, a bisecting search is used to choose the number of PMUs and in the second stage, simulated annealing technique is used to select the location of PMUs which makes the system observable. Non dominated sorting genetic algorithm is used in [2] for obtaining pareto optimal solutions to the OPP problem considering minimization of number of PMUs and maximization of redundancy as objectives. It is first of its kind with reference to both multi-objective formulation and providing pareto optimal solutions.

In paper [3] , with the help of the topology of the network, binary connectivity matrix BC is formulated and with this, constraints of the objective function i.e minimization of number of PMUs are formed. It also discusses the procedure for formulation of additional constraints such as, conventional measurements and zero injection measurements. Integer linear programming is used for solving the formulated objective with constraints.

In [4] constraint formulation for conventional power flow, injection measurements and zero injection buses is proposed for OPP problem. In [5] constraints formulation is presented by relaxing the assumption of unlimited channel capacity of PMUs and the results are presented for different channel limits. It is the first article which considered channel limits for solving OPP problem.

[6] proposed an OPP method by considering minimization of the number of PMUs and maximization of redundancy. These two objectives are transformed into a single quadratic function. This quadratic objective function with constraints is solved using integer quadratic programming. It also describes how to modify constraints with reference to N-1 line outage and PMU outage. A weighted sum multi-objective approach is reported in [7] by considering two objectives which are mentioned in [6]. This multi-objective formulation is solved using binary particle swarm optimization.

The concept of criticality is introduced in [8] and it is proposed to identify the critical buses which need to be monitored. It also suggested that the selection of number of the critical buses should be low. PMUs are preassigned only at the identified critical buses. Objective function and constraints are formulated in such a way that, the identified critical buses set should be a sub set of an OPP set.

Modified binary PSO with integrated mutation strategy is proposed in [9] to optimize weighted sum multi-objective formulation for optimal PMU placement problem. Optimal solution for various cases such as channel limits, line outage and loss of PMU is also presented. Exponential binary PSO algorithm is proposed in [10] to provide multiple solutions to the optimal PMU placement problem. The main aim of this paper is to provide multiple solutions to the planning engineers in selecting the OPP set satisfying other applications. [11] proposed a recursive tabu search method for optimizing the single objective function consisting of weighted sum of the multi-objectives as discussed above. It also proposed a numerical observability method for checking observability. Bio-geography based optimization algorithm is used in [12] for providing pareto optimal solution to the multi-objective PMU placement problem. OPP problem is optimized using genetic algorithm in [13] by formulating six objectives and converting them into a single objective. The reliability of OPP solution is improved by accounting loss of measurement. In [14] OPP problem is formulated as a binary semi-definite programming problem. It is solved using binary integer programming. OPP solution to the same problem considering channel limits is reported in [15]. In [16] the concept of depth of unobservability is presented to find PMU placement sets for different depth of unobserabilities with the help of the graph theory based tree search technique. It also discusses the simulated annealing technique based PMU placement strategy considering communication constraint. The concept of multi-staging to the OPP problem is introduced for the first time. In [17] System observability redundancy index(SORI) and Bus observability index(BOI) are modeled. OPP solution is obtained using ILP. These indices are used for providing a multi-staging strategy which maximizes observability in the early installation stages.

Five indices such as Voltage stability monitoring, Tie-line oscillation monitoring, Angular stability monitoring, Improved state estimation and availability of communication are proposed in [18]. These five indices are evaluated for each PMU location and using fuzzy TOPSIS, ranking is given to each PMU location. Multi-staging installation of PMUs has been done based on the descending order of the ranking.

In [19] number of PMUs was selected and for the chosen PMUs, all possible combinations are generated. Each combination is checked for observability. Those combinations which satisfy observability are the optimal PMU placement solutions. If observability condition is not met, the number of PMUs is increased and the process is repeated. The main aim of this exhaustive search is to provide a benchmark model for OPP solutions.

Topological observability mayn't always lead to numerical observability. The proposed method in [20] first obtained OPP solutions which satisfy topological observability. Numerical observability of each solution was checked. The OPP solution which satisfied the numerical observability was taken as final solution.

In [21] PMU relocation scheme is proposed to maximize the percentage of observability at each stage. This scheme is proposed by considering the idea of residual amount of allocated budget in previous stage which is not sufficient for procurement of PMU. This is used for relocating PMU in order to maximize percentage of observability at that stage. In [22] optimal solution is obtained under normal case and under the presence of single or multiple flow measurements by applying improved binary artificial bee colony algorithm on placement problem. [23] Two objectives, observability and state estimation accuracy are taken for formulating multi-objective approach for providing solution to the OPP problem. [24] reformulated OPP problem by considering various contingencies and solved the problem using constriction factor- particle swarm optimization(CF-PSO). [25] solved the optimal PMU placement problem using hybridized PSO - Gravitational search algorithms. [26] optimal solution was obtained by considering state estimation accuracy variance as one of the objectives. [27] provided the optimal solution under preexisting condition of SCADA measurements by considering various contingencies.

All the earlier works reported have rendered a single solution considering system observability as the main criteria. Solutions dispensed by them are also restricted to only a single objective. In [10], the significance of multiple optimal solutions was well articulated and presented for OPP problem. But, this method fails to obtain all multiple optimal solutions available for the OPP problem. [28] proposed Binary Taguchi bat algorithm by uniting Taguchi and binary bat algorithms for providing multiple optimal solutions and optimal solution for unequal installation cost. In [29] Multiple optimal solutions are provided using binary particle swarm optimization. [30] proposed a two phase branch and bound algorithm for obtaining multiple optimal solutions. [31] proposed a new index called WAMS data traffic index. OPP problem considering data traffic index is solved for multiple optimal solutions using Genetic algorithm along with minimum spanning tree(MST) method. MST is used for modifying the unobservable solution into observable solutions. But, these methods fail to obtain all multiple optimal solutions available for OPP problem.

The following observations are made from the literature review on optimal PMU placement strategies. OPP solutions using classical and meta-heuristic techniques has been attempted by several researchers. OPP has also been attempted considering two or more objectives. The need for multiple optimal solutions was discussed and attempted, but has not been thoroughly explored.

1.2 Literature review on PMU assisted state estimation techniques

For obtaining reliable states of the power system in the energy management system, state estimation is an indispensable tool since its introduction by Fred Schweppe [32]. State estimation plays a major role in smart grid and its advancement is the need of the hour.

Weighted least squares state estimation(WLSE) technique is the predominantly employed technique because of its computationally inexpensive nature. WLSE provides very accurate estimation if the measurement set contains only Gaussian noise. But in the presence of bad data, it fails to provide reliable estimation and is unable to detect and eliminate bad data. For detecting and identifying bad data, various algorithms are proposed in [33–36]. The largest normalized residual test is the frequently employed technique [37].

Apart from these, many robust estimators have been proposed. The comparison of some alternative estimators such as Least absolute value(LAV) [38], Quadratic- Linear(QL), Quadratic-Constant(QC), Least trimmed squares (LTS), Least median of squares(LMS) [39] and Least measurements rejected (LMR) are presented in [40]. It is concluded that, LAV and QC estimators are computationally efficient. In the presence of Gaussian noise LMR and QL estimators are on par with WLSE. Under the presence of corrupt measurements QL and QC are performing better.

Initially, different solutions are obtained using different essential sets. The optimal solution out of these is the one which has maximum agreement with the remaining measurements [41]. In [42] measurement uncertainties are modeled as inequality constraints. While finding the solution, constraints satisfaction is maximized using Genetic algorithm. Though both methods are robust against leverage and bad measurements, they take heavy computational time and do not have noise filtering ability. Gaussian kernel is used as a objective function, maximization of which leads to maximization of correntropy between estimated solution and measurement set [43]. Par-zen window selection is deciding the optimality of the solution and convexity of the objective function. Normal measurement rate is optimized by formulating the objective function as a maximization problem and solved using interior point method [44]. Though it is a non-convex problem, it is designed as a two stage problem to overcome the local maxima problem. Local maxima is overcame to a large extent by using first stage solution as a initial solution in the next stage. Laplace kernel, which is based on the maximum correntropy criterion is used as a objective function. This enhanced the performance of Maximum exponential square(MES) but could not completely overcome the drawbacks of it [45]. Maximum exponential absolute value(MEAV) mathematical properties and theoretical basis is given and it is demonstrated that computational time is reduced by reducing the order of the correction equation and the number of fill-ins to make it suitable for large size systems. Maximum normal measurement rate(MNMR) algorithm is improved using a three step procedure: 1) measurements are maximized by approximately optimizing solution 2) To further maximize the normal measurement rate, consistency check has been done for abnormal measurements 3) To effectively filter noise, WLS is performed on normal measurements. Due to multiple steps, computational time of the proposed algorithm is very large [46]. In [47] the non-convexity problem of MNMR is removed by using Hyperbolic cosine function instead of sigmoid function, to achieve greater computational efficiency than MNMR. In [48] robust multi-objective state estimator is proposed by combining LAV and Mixed integer programming(MIP) to overcome the individual drawbacks associated with them. LAV is good at removing bad data and has the ability to filter measurements, MIP is good at removing leverage measurements. In [49] a practical rectangular pulse function is designed with the help of hyperbolic tangent function to maximize normal measurements. The computational efficiency of the rectangular pulse function method is higher compared with WLS, LAV, Mixed integer non-linear programming(MINP) and Maximum constraints satisfaction(MCS).

Though PMU is superior in performance compared to SCADA measurements, the cost of PMU and it's associated communication infrastructure cost has limited the deployment of PMU in power system. In recent times, power systems have been populating with PMUs. Under the assumption that, PMUs provide complete observability of the power system, the relation between the measurement set and state variables change from non-linear to linear. Because of this linearity, the iterative nature of WLSE changed to non-iterative, further decreasing the computational time. In a similar manner, computational time of the robust state estimators also decreased, interestingly competing with WLSE . This made the researchers to have a re-look on robust estimators.

In [50], LAV based robust estimator performance with respect to completely PMU based power system state estimation is investigated. It is concluded that WLSE performs better computationally as well as from an estimation accuracy point of view than LAV under the presence of Gaussian noise. In the presence of corrupt measurements LAV is automatically suppresses bad data while showing superiority in computationally as well as in terms of estimation accuracy. With respect to leverage measurements, LAV too failed to give a good performance. Scaling method is introduced for removing PMU leverage measurements. Though QC has better bad data suppression ability, its potential with respect to PMU assisted power system state estimation has not been exploited.

LAV based estimator is proposed in [50] though performing better compared with WLSE.

The computational efficiency of LAV is very low compared with WLSE. Therefore, the search for a good robust estimator which has low computational time is the need of the hour.

The following observations are made from the literature review on PMU only assisted power system state estimation. The linear relation between PMU measurements and state estimation is drastically decreasing the computational time of the robust estimators. In the literature , only LAV based robust state estimator has been applied for PMU assisted power system state estimation. The efficacy of the other robust estimators was not explored with respect to PMU only assisted power system state estimation.

1.3 Literature review on hybrid power system state estimation techniques

Traditionally, state estimation is performed with the help of SCADA measurements. But, after the invention of phasor measurement unit, its deployment into the power system is happening all over the world. The PMUs deployed until now have not been able to provide complete observability of the system, as both SCADA and PMUs will coexist up to a certain period. Therefore, in order to exploit the best features associated with PMU measurements while doing state estimation with SCADA measurements, hybrid state estimation techniques are required. Many hybrid state estimation techniques are proposed in the literature to incorporate PMU measurements with traditional SCADA measurements.

In [51] two alternative approaches are proposed for including phasor measurements in state estimation. The first approach combines phasor measurements with SCADA measurements after converting measurements from polar to rectangular form. The second approach is formulated as a post processing linear estimation model considering estimated state vector from SCADA measurements and PMU measurements. It is concluded that, estimated state vector co-variances obtained from both methods are equal. For processing phasor measurements using second alternative approach requires some new software module in addition to the existing module. [52] proposed a two pass approach. The state vector obtained from the first pass using traditional measurements is mixed with phasor measurements in second pass to get the final states with the help of linear model. In [53] a novel non-linear hybrid state estimation model is proposed considering SCADA measurements, voltage phasors and branch current phasors. The procedure for transforming error co-variance of the received polar measurements into rectangular form is also presented.

In [54] a non-linear hybrid state estimator is proposed considering SCADA measurements and converted phasor voltage and current measurements into flow measurements. For estimating the state vector at only PMU reporting rates a pre estimating method is proposed by simulating a connection matrix between unobserved states and observed states. The pre estimated state is used as an initial state vector for improving convergence and reducing computational burden of the estimator. In [55] the aforementioned algorithm is compared with sequential estimator. In [56] three methods were proposed for inclusion of phasor current measurements in hybrid state estimator. 1. Inclusion of phasor current measurements in polar form 2. Inclusion of phasor current measurements in rectangular form 3. Inclusion of phasor current measurements in pseudo voltage measurements. It is concluded that inclusion of phasor current measurements in rectangular form(method 2) is most accurate of all. In [57] concept of pseudo flow measurements is proposed for integrating PMU measurements with traditional estimators.

[58] presented a constrained non-linear hybrid state estimator model considering buses observed by two PMUs as equality constraints. An unscented transformation approach is presented for transforming polar current phasors either to rectangular form or to pseudo voltage measurement form. This constrained non-linear hybrid state estimator is giving more accurate estimations than without constraints. In [59] two kinds of adaptive neural network based hybrid state estimation approaches were proposed for visualizing the power system in all PMU reporting rates. In [60] the ill-condition problem of Jacobian matrix during initial iteration of non-linear hybrid state estimator considering phasor measurements in polar form is eliminated by processing first iteration with rectangular phasor measurements.

In [61] a three stage state estimator is proposed. In the first stage, SCADA based state estimation has been carried out using the existing software module. The second stage PMU based linear state estimation has been done using distinct state estimation module. From both stages, estimated states are fusioned together to get the final estimate by applying Bar Shalom Campo fusion theory for fusioning multi rate sensors. In the second stage, full observability is obtained by using prior estimated states along with PMU measurements. Though the proposed method seems like a three stage procedure, computationally it takes very low time compared with simultaneous hybrid non-linear state estimator. In [62] a Tikhonov regularization approach is proposed for suppressing numerical ill-conditioning problem aroused in hybrid state estimators. In [63] in the first step, all available PMU measurements are used for generating pseudo measurements and updating the predicted pseudo measurements using Kalman filter. In the second step, non-linear estimator is used for obtaining states of the system considering conventional and pseudo measurements. [64] proposed a hybrid state estimator for effectively tracking the momentary changes in the power system. This is not possible only with SCADA measurements. When both PMU and SCADA measurements are available, it uses WLS estimator. When only PMU measurements are available, Weighted Least Absolute Value(WLAV) is used by considering some old SCADA measurements for making the system completely observable. In [65] an Unscented Kalman filter is used for predicting intermediate SCADA measurements in the not reporting intervals of SCADA measurements. These predicted measurements and PMU measurements are utilized in hybrid state estimation. An iterative kalman filter is used for predicting SCADA measurements in [66]. In [67] tracking hybrid state estimator is proposed. Critical pseudo measurements are identified for using them in SCADA not reporting intervals. These critical pseudo measurements are updated using Kalman filter based very short term load forecasting technique.

In [68] a hybrid tracking estimator is proposed with the help of interpolation matrix. Interpolation matrix is updated only if any abnormality is found in the system. This is done to reduce the computational complexity. In [69], a radial basis network is used to forecast the intermediate SCADA measurements. The states are estimated by using PMU measurements and forecasted SCADA measurements with the help of extended Kalman filter technique.In [70] three stage hybrid fast state estimator is proposed to reduce computational time and for improving estimation accuracy. Bad data processing has been done in the intermediate stage. Final estimates are obtained in the third stage using fusion theory. In [71] neural network based very short term load forecasting has been carried out and forecasted load is used in hybrid dynamic state estimation. In [72] pseduo voltage and current phasor measurements are used for performing linear state estimation in only PMU reporting rates. Non-linear state estimation has been done for estimating states when both SCADA and PMU data are available. [73] Least Absolute Value(LAV) based intermediate linear state estimation model is proposed for mixing both SCADA and PMU measurements. In [74] Sequential quadratic programming based hybrid state estimator is proposed for tracking the states of the system in the entire PMU measurement reporting intervals. In [75] the power flow and injection measurements are converted into branch current and node injection currents phasors. These converted measurements along with PMU measurements are used in linear robust model to estimate the state of the power system.

The following observations are made from the literature review on hybrid power system state estimation strategies. In the literature, the approximate linear variation in between two successive PMU reporting intervals has not been explored. The efficacy of robust estimators with respect to hybrid state estimator needs to be further explored.

1.4 Dissertation objectives and technical contributions

As discussed in the introduction section, for critical monitoring of the power system, there is a strong need of tools which are able to monitor the power system at a much higher rate than SCADA. PMU is the measurement device which plays a major role in revitalizing entire WAMPAC system. Therefore, in this thesis, the deployment of PMUs and its application in power system state estimation are investigated. Based on observations made from the literature review, the following objectives are considered:

1. To look for a PMU placement strategy which provides complete set of multiple optimal solutions.

2. To propose an effective PMU only assisted state estimation model which enhances key characteristics of state estimation.

3. To develop a linear hybrid state estimation model by integrating both PMU and SCADA measurements

By working on the above mentioned objectives, the following contributions are made:

- As a first contribution, two new techniques are proposed for providing multiple optimal solutions, which work only on binary connectivity matrix. The proposed techniques aim to provide complete set of multiple optimal solutions.
- As a second contribution, a state estimation model called Quadratically decaying exponential criterion is proposed for PMU only assisted power system state estimation.
- As a third contribution, a two stage linear sequential hybrid state estimation model is proposed for integrating both PMU and SCADA measurements.
- Finally, as a fourth contribution, a linear single stage hybrid state estimation model is proposed for integrating both PMU and SCADA measurements.

1.5 Organization of the dissertation

The dissertation is presented in seven chapters. The current chapter introduces synchrophasors and discusses the necessity of their deployment for application in power system state estimation. Literature review on optimal PMU placement strategies, PMU only assisted power system state estimation and hybrid power system state estimation techniques, their summary, research objectives of the thesis, its contributions and finally the organization of the thesis are presented.

Chapter 2 introduces a novel topology based method- I for optimal PMU placement problem for providing multiple optimal solutions.

Chapter 3 proposes a novel topology based method-II to overcome the drawback of topology based method-I and providing multiple optimal solutions for various cases such as line contingency, loss of PMU and considering channel limits and also presented modified BOI and SORI indices.

Chapter 4 introduces a new state estimation model called synchrophasor assisted power system state estimation using quadratically decaying exponential criterion and the comparison section to know the efficacy of the proposed method with the recently developed LAV based synchrophasor assisted power system state estimation.

Chapter 5 unveiled a new linear sequential hybrid model for integrating both PMU and SCADA measurements and its effectiveness in comparison with sequential hybrid state estimator and SCADA based traditional estimator.

Chapter 6 presents a new linear single stage hybrid model for integrating both PMU and SCADA measurements and its efficacy when compared with two stage hybrid state estimator and classical state estimator.

Finally, chapter 7 offers conclusions and future scope of the thesis.

Chapter 2

Multiple Solutions for Optimal PMU Placement Using a Topology-Based Method -I

2.1 Introduction

In [10], the significance of multiple optimal solutions for OPP problem was well articulated and resulting multiple optimal solutions were presented. This method however fails to obtain, all the multiple optimal solutions available for the OPP problem.

To overcome the above said drawback, this chapter proposes a novel topology based method-I for obtaining complete set of multiple optimal solutions available for the OPP problem. The outcome of the method is to present all the available multiple optimal solutions by working on the binary connectivity matrix without using any traditional or meta-heuristic technique.

2.2 Illustration of Topology Based OPP Method -I

This section presents the illustration of topology based method-I for optimal PMU placement problem. The methods available in the literature formulated OPP problem as a minimization problem. Typical objective function of the OPP problem reported in the literature is given in equation(2.1).

$$f(X) = \sum_{i=1}^{N} C_{PMU_i}.x_i \tag{2.1}$$

Subjected to

Rank(H_1)=N

(or)

BC.X=b

Where

C_{PMU_i}	Installation cost of PMU at i^{th} bus
x_i	$\begin{cases} 1 & PMU is installed at i^{th} bus \\ 0 & otherwise \end{cases}$
N	Number of buses
H	Design matrix
BC	Binary connectivity matrix
b	unit vector having Nx1 size
X	Nx1 size vector having x_i as an entity

The topology based OPP method-I is illustrated using a 7- bus system shown in figure. 2.1. The proposed method works on the binary connectivity matrix to generate the feasible solution space. Binary connectivity matrix of the 7-bus system is given in equation (2.3).

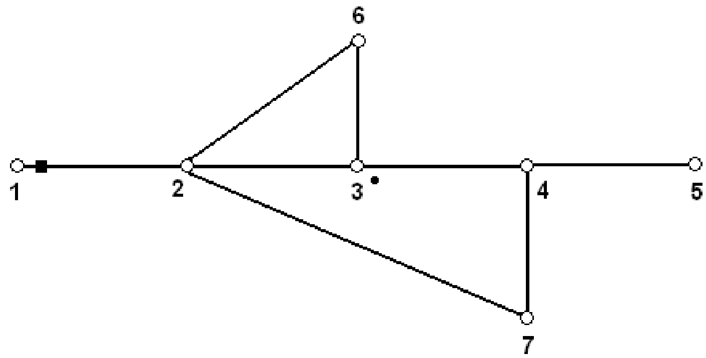

Figure 2.1: 7 BUS SYSTEM.

2.2.1 Formation of binary connectivity matrix

Binary connectivity matrix(BC) is a square matrix having size NxN. Where N is the number of buses of the system. The BC matrix for base case is given in equation(2.3). For a 2-3 line contingency case the BC(2,3) and BC(3,2) entities of the BC matrix are zero and is given in equation(2.4). At any bus the number of lines connected to the bus exceeds the channel limit of the PMU. In this case measuring of total line phasor measurements is not possible. Therefore ${}^{lc}c_{cl}$ ways are possible. Where lc is the number of lines connected and cl is the channel limit. For considering channel limit into formulation of BC matrix ${}^{lc}c_{cl}$ rows will add in place of corresponding row of BC matrix and is reported in [5]. For a 7 - bus system considering channel limit 3 at bus 2 will add 4 rows in place of row 2 and is given in equation(2.5).

$$BC_{ij} = \begin{cases} 1 \quad if \quad i = j \\ 1 \quad if \quad i \quad and \quad j \quad are \quad connected \\ 0 \quad otherwise \end{cases} \tag{2.2}$$

$$BC = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{2.3}$$

$$BC = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{2.4}$$

$$BC = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{2.5}$$

Starting from row 1 of the binary connectivity matrix given in equation(2.3). There are two PMU installation possibilities for making the bus 1 observable. i.e 1 and 2 . The number of possibilities of first row will decide how many trees need to be formed and act as roots of the solution trees. First form tree 1, placing PMU at bus 1 makes bus 1 observable and acts as a root node of the tree 1. Placing PMU at bus 1 makes bus 2 also observable. This can be sensed by checking the continuity of ones at that particular column of the placed PMU in the binary connectivity matrix. In this case, placing PMU at bus 1 has continuity of ones upto the second row. i.e placement of PMU at bus 1 makes observable both bus 1 and bus 2. In order to make 3rd bus also observable, look for number of non-zero entities(means number of possibilities) of the corresponding row of the binary connectivity matrix. In this case, number of possibilities is 4. These four possibilities will act as branch end nodes of the root node. Placing the PMU at any one of these possibilities makes bus 3 observable and again look for continuity of ones. The PMU thus placed will observe the corresponding row number buses upto where it has continuity of ones. The above said process repeats until it reaches last row of the binary connectivity matrix.

This process is repeated for all remaining identified root nodes nodes. Once the complete tree is formed, each path of the tree from start node to end node will act as a feasible solution to OPP problem. The formulated trees are shown in figure.2.2 and 2.3 for 7-bus system. The feasible solution space for the 7-bus system is given in table.2.1

Table 2.1: Feasible solution space for the 7-bus system

No of PMUs	No of solutions	Placement sites
2	2	2-4, 2-5
3	10	1-2-4, 1-2-5, 1-3-4, 1-4-6, 2-3-4, 2-3-5, 2-4-6, 2-5-6, 2-4-7, 2-5-7
4	21	1-2-3-4, 1-2-3-5, 1-2-4-6, 1-2-5-6, 1-2-4-7, 1-2-5-7, 1-3-4-7, 1-3-4-6, 1-3-4-5, 1-3-5-7, 1-4-6-7, 1-4-5-6, 1-5-6-7, 2-3-4-7, 2-3-4-6 2-3-4-5, 2-3-5-7, 2-3-5-6, 2-4-6-7, 2-4-5-6, 2-5-6-7
5	17	1-2-3-4-7, 1-2-3-4-6, 1-2-3-4-5, 1-2-3-5-7, 1-2-3-5-6, 1-2-4-6-7 1-2-4-5-6, 1-2-5-6-7, 1-3-4-6-7, 1-3-4-5-6, 1-3-5-6-7, 1-4-5-6-7 2-3-4-6-7, 2-3-4-5-6, 2-3-5-6-7, 2-3-4-5-7, 2-4-5-6-7
6	6	1-2-3-4-6-7, 1-2-3-4-6-7, 1-2-3-4-5-6, 1-2-3-5-6-7, 1-2-3-4-5-7, 1-2-4-5-6-7, 1-3-4-5-6-7

2.3 Algorithm of proposed topology based method- I

2.3.1 Description of the variables

Node vector(n): It is a vector which stores column number of non-zero elements corresponding to the selected row.

Ex: n=[1 2 0 0 0 0 0]

Depth of penetration vector(d): It is a vector which stores row number upto which stored column number in ’n’ is having continuity of ones. Apart from that it also stores the information about how many times this vector needs to be visited in d(nc, N+1) entity and how many times it has already visited in d(nc, N+2) entity

Ex: d=[2 3 0 0 0 0 0 2 0]

Temporary solution updater(s): Initially s is initialized as a 1xN zero vector, Which stores solution temporarily.

Ex: s=[1 0 0 0 0 0 0]

Final solution updater(fs): Initially fs is initialized as a 1xN zero vector, Which stores final solution.

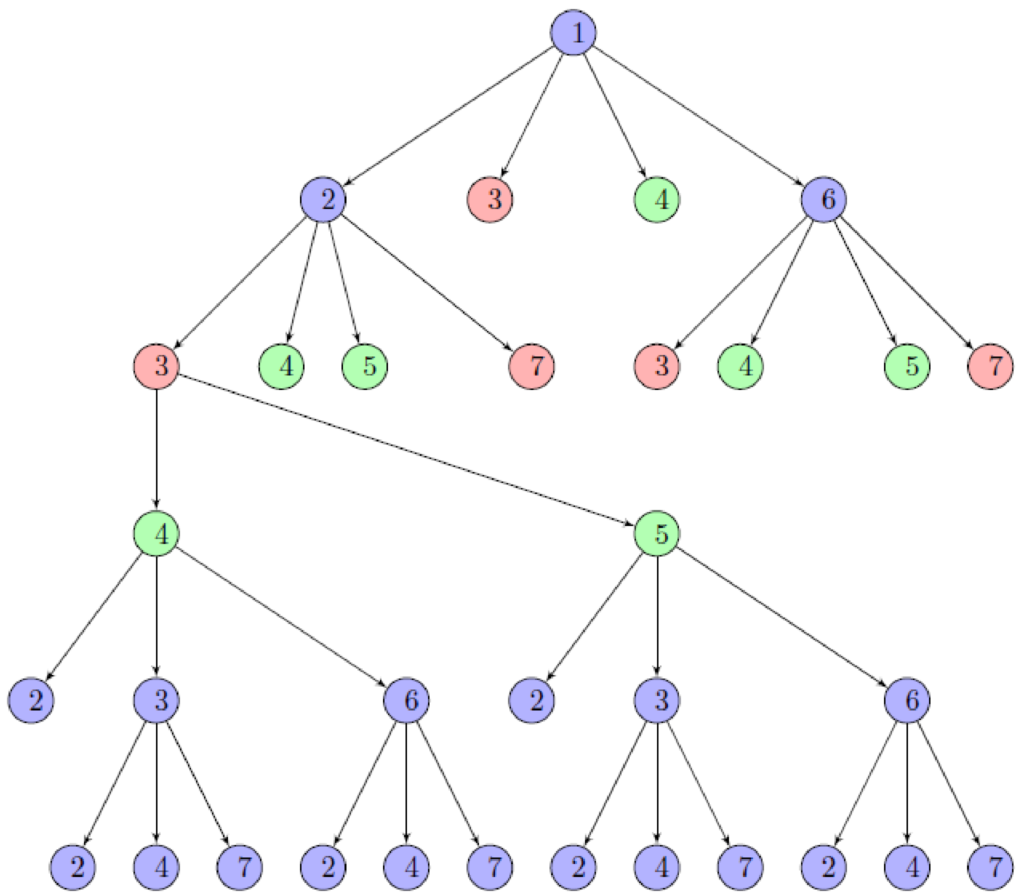

Figure 2.2: Tree 1

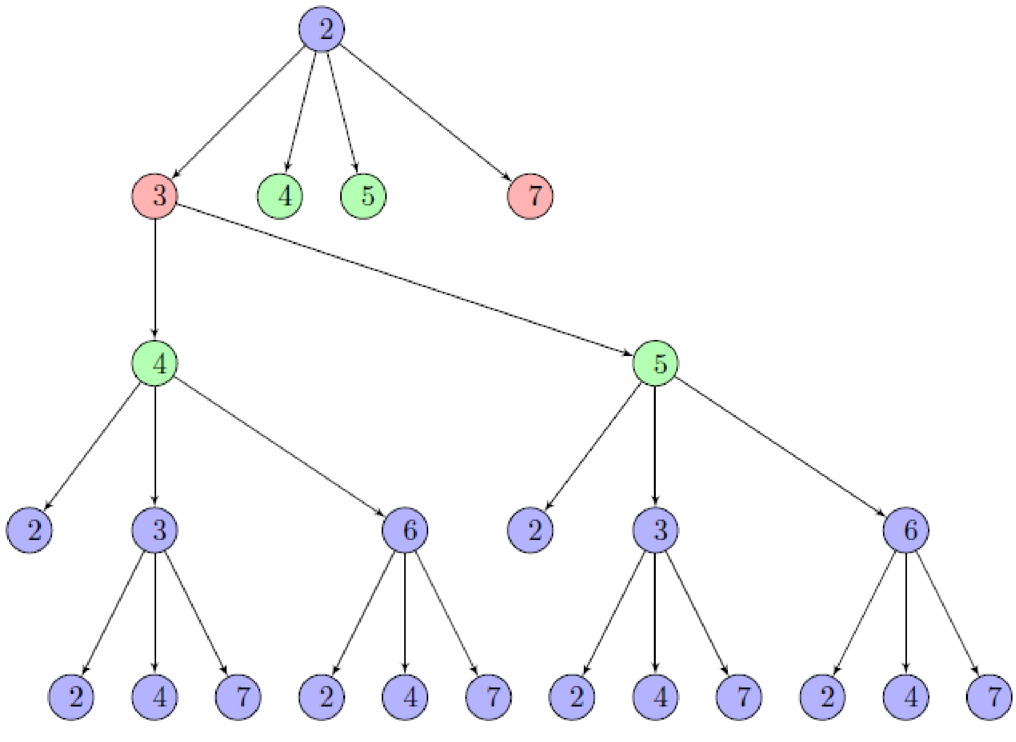

Figure 2.3: Tree 2

The flow chart of the proposed topology based OPP method-I is given in figure.2.4.

2.4 Case study and Results

The efficacy of the proposed method is tested on IEEE 14-Bus and 30-Bus systems for base case. For coding the proposed algorithm MATLAB software is used.

From table.2.2 , the proposed method is providing all 5 optimal solutions for IEEE-14 Bus system where as EBPSO method provided only four optimal solutions. From the results, it is observed that the proposed method has the ability to provide complete set of multiple optimal solutions.

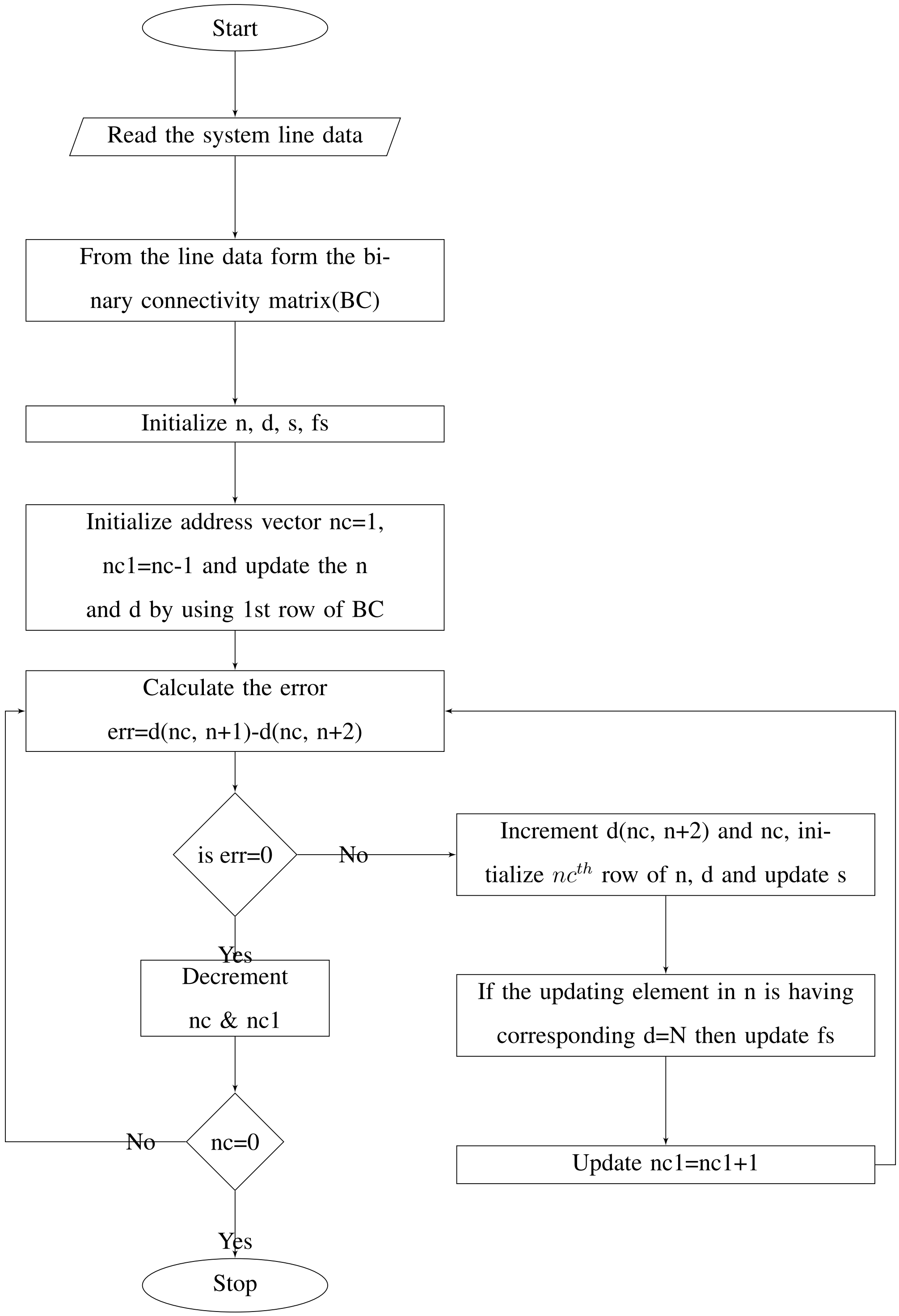

Figure 2.4: Flow chart of Topology based OPP method-I

Table 2.2: Comparison of proposed method with literature for IEEE-14 Bus system

Method	No of PMUs	No of solutions	Placement sites
Proposed method	4	5	2-6-7-9, 2-6-8-9, 2-7-11-13, 2-7-10-13, 2-8-10-13
EBPSO [9]	4	4	2-6-7-9, 2-6-8-9, 2-7-10-13, 2-8-10-13

From table.2.3, for IEEE-30 bus system, the proposed method is providing 734 solutions, Out of which only 5 solutions are presented in the table.2.3. Where as all the methods in the literature provided single optimal solution. All solutions provided in the literature are the subset of the multiple optimal solution space obtained by the proposed method.

Table 2.3: Comparison of proposed method with literature for IEEE-30 Bus system

Method	No of PMUs	No of solutions	Placement sites
Proposed method	10	734	3-5-8-9-10-12-19-23-25-30, 2-4-6-9-10-12-15-18-25-27, 1-2-6-9-10-12-15-19-25-27, 1-2-6-9-10-12-15-18-25-27, 2-4-6-9-10-12-15-19-25-27
ILP based [6], IMS MBPSO [10]	10	1	2-4-6-9-10-12-15-19-25-27
BPSO [7]	10	1	2-4-6-9-10-12-15-18-25-27
BSP [14]	10	1	1-2-6-9-10-12-15-18-25-27
Exhaustive search [19]	10	1	1-2-6-9-10-12-15-19-25-27

By running the proposed algorithm one will get complete feasible solution space to an OPP problem. This feasible solution space will be pruned to get complete set of multiple optimal solutions. Obtaining optimal solution using meta-heuristic techniques or classical techniques will give one solution in every run. Multiple runs may or may not give different optimal solutions.

The proposed method obtained multiple optimal location of PMUs without using any conventional or heuristic optimization techniques and by working only on the binary connectivity of the matrix. From the results, it is observed that the optimal PMU placement problem is having many global optimal points rather than single global optimal point. Therefore, OPP problem is a specialized optimization problem having many global optimal solutions. The proposed method guarantees complete set of multiple global optimal solutions. These multiple optimal solutions would give the operator freedom to choose an optimal solution which has a good fit to the considered other minor objectives.

2.5 Summary

This chapter proposed a novel topology based OPP method- I for providing multiple optimal solutions. The proposed method purely works on binary connectivity matrix without using any heuristic or conventional optimization technique. The proposed method has the ability to provide complete set of optimal solutions. Multiple optimal solutions would give the choice to the operators to choose an optimal solution which has good fit to the considered other minor objectives rather than providing a single optimal solution. From the results, it is observed that the optimal PMU placement problem is having many global optimal points rather than single global optimal point. Therefore, OPP problem is a specialized optimization problem having many global optimal solutions. The proposed method guarantees complete set of multiple global optimal solutions. As the algorithm progresses the solution vector length is increasing. Because of this, computers with low RAM capacity are not sufficient to run the algorithm for higher order systems. The drawback of the proposed method is that, it fails to get multiple solutions for different cases such as line contingency, loss of PMU and considering channel limit.

Chapter 3

Multiple Solutions for Optimal PMU Placement Using a Topology Based Method -II

This chapter proposes a novel topology based method -II for obtaining complete set of multiple optimal solutions available for the OPP problem and overcomes the drawback of Topology based method-I. This method come up with all available multiple optimal solutions by working on the binary connectivity matrix without using any traditional or meta-heuristic technique. Apart from that, this chapter also discusses the drawback of Bus Observability Index(BOI) and System Observability Redundancy Index(SORI) presented in [17] and to overcome, a normalized BOI(NBOI) and SORI(NSORI) are proposed.

3.1 Illustration of Topology based OPP Method -II

The topology based OPP method-II is illustrated using a 7- bus system as shown in Fig. 2.1. The proposed method works on the binary connectivity matrix to generate the feasible solution space. Binary connectivity matrix of the 7-bus system shown in figure.2.1 is given in equation (3.1).

$$BC = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{3.1}$$

Starting from the last row of the binary connectivity matrix. In order to make seventh bus of the system observable, number of possible PMU placements are 2,4 and 7. Therefore, the number of feasible solutions to make the seventh bus observable are three. S is the solution vector and is given in equation(3.2).

$$S = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.2}$$

In order to make 6^{th} bus observable, the number of possible PMU placements are 2,3 and 6, which is evident from the 6^{th} row of binary connectivity matrix. To make both 6^{th} and 7^{th} buses observable, the number of feasible solutions are 3x3 i.e 9. Therefore, the solution vector to make both buses observable is given in equation(3.3)

$$S = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \tag{3.3}$$

From the 5^{th} row of binary connectivity matrix and to make 5^{th}, 6^{th} and 7^{th} buses observable number of feasible solutions are 9x2 i.e 18. After removing the repeated solutions out of 18. the Solution vector S is given in equation(3.4)

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{3.4}$$

Similarly to make 4^{th}, 3^{rd}, 2^{nd} & 1^{st} buses observable along with 5^{th}, 6^{th} and 7^{th} number of feasible solutions are 63. The entire feasible set of 63 solutions for the 7 bus system is reported in Table.3.1. Once feasible PMU placement set is obtained for any given system, by considering utility requirements, one can choose optimal solution satisfying the desired objectives.

Table 3.1: Feasible PMU placement space for 7 bus system

No of PMUs	No of solutions	Placement sites
2	2	2-4, 2-5
3	11	1-2-4, 1-2-5, 1-3-4, 1-4-6, 2-3-4, 2-3-5, 2-4-5, 2-4-6, 2-5-6, 2-4-7, 2-5-7
4	23	2-5-6-7, 2-4-6-7, 2-4-5-7, 2-4-5-6, 2-3-5-7, 2-3-5-6, 2-3-4-7, 2-3-4-6, 2-3-4-5, 1-5-6-7, 1-4-6-7, 1-4-5-6, 1-3-5-7, 1-3-4-7, 1-3-4-6, 1-3-4-5, 1-2-5-7, 1-2-5-6, 1-2-4-7, 1-2-4-6, 1-2-4-5, 1-2-3-5, 1-2-3-4
5	19	2-4-5-6-7, 2-3-5-6-7, 2-3-4-6-7, 2-3-4-5-7, 2-3-4-5-6, 1-4-5-6-7, 1-3-5-6-7, 1-3-4-6-7, 1-3-4-5-7, 1-3-4-5-6, 1-2-5-6-7, 1-2-4-6-7, 1-2-4-5-7, 1-2-4-5-6, 1-2-3-5-7, 1-2-3-5-6, 1-2-3-4-7, 1-2-3-4-6, 1-2-3-4-5
6	7	2-3-4-5-6-7, 1-3-4-5-6-7, 1-2-4-5-6-7, 1-2-3-5-6-7, 1-2-3-4-6-7, 1-2-3-4-5-7, 1-2-3-4-5-6
7	1	1-2-3-4-5-6-7

3.2 Algorithm of the proposed method

3.2.1 Description of Variables

Formation of binary connectivity matrix:

Binary connectivity matrix is a square matrix having size NXN, Where N is the number of buses in the system.

$$BC_{ij} = \begin{cases} 1 \quad if \quad i = j \\ 1 \quad if \quad i \quad and \quad j \quad are \quad connected \\ 0 \quad otherwise \end{cases} \tag{3.5}$$

Solution matrix (S):

Solution matrix is the matrix containing all the previous solutions and is updated according to a (Non Zero Vector) NZV. Initially it is initialized as a 1XN zeros matrix.

Non Zero Vector (NZV):

NZV is a vector which stores column position of the non zero elements of i^{th} row of a binary connectivity matrix.

3.2.2 Procedure for updating solution vector

Consider the 7 bus system as mentioned in above section and its binary connectivity matrix. The solution vector to make 7^{th} bus observable is given in equation(3.2) i.e

$$S = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.6}$$

NZV corresponding to 6^{th} row of binary connectivity matrix is

$$NZV = \begin{bmatrix} 2 & 3 & 6 \end{bmatrix} \tag{3.7}$$

NZV contains three elements. Therefore S needs to be updated three times

$$S_{old} = S \tag{3.8}$$

1^{st} **updation:**

$$S_{new} = S_{old} \tag{3.9}$$

Set the $NZV(1,1)$ column of S_{new} as unity column.

$$S_{new} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.10}$$

$$S = S_{new} \tag{3.11}$$

2^{nd} **updation:**

$$S_{new} = S_{old} \tag{3.12}$$

Set the $NZV(1,2)$ column of S_{new} as unity column.

$$S_{new} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.13}$$

Add S_{new} to S

$$S = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.14}$$

3^{rd} **updation:**

$$S_{new} = S_{old} \tag{3.15}$$

Set the $NZV(1,3)$ column of S_{new} as unity column.

$$S_{new} = S = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \tag{3.16}$$

Add S_{new} to S

$$S = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \tag{3.17}$$

Therefore S is completely updated for the NZV of 6^{th} row and is given in equation(3.17). Similarly do for the same for all remaining rows to get the complete feasible solution space.

The flowchart of the proposed algorithm for obtaining all feasible solutions is shown in figure.3.1. The proposed algorithm provides complete feasible solution space. The procedure for obtaining optimal multiple solutions from the feasible solutions for base case, single line contingency, loss of PMU and considering channel limits is given in subsections.3.2.3-3.2.6.

3.2.3 Procedure for obtaining multiple solutions having maximum redundancy

1. Prune the solutions which have minimum number of PMUs.
2. Calculate the NSORI of pruned solutions.
3. Prune solutions which have maximum NSORI
4. The Pruned solutions are the multiple solutions.

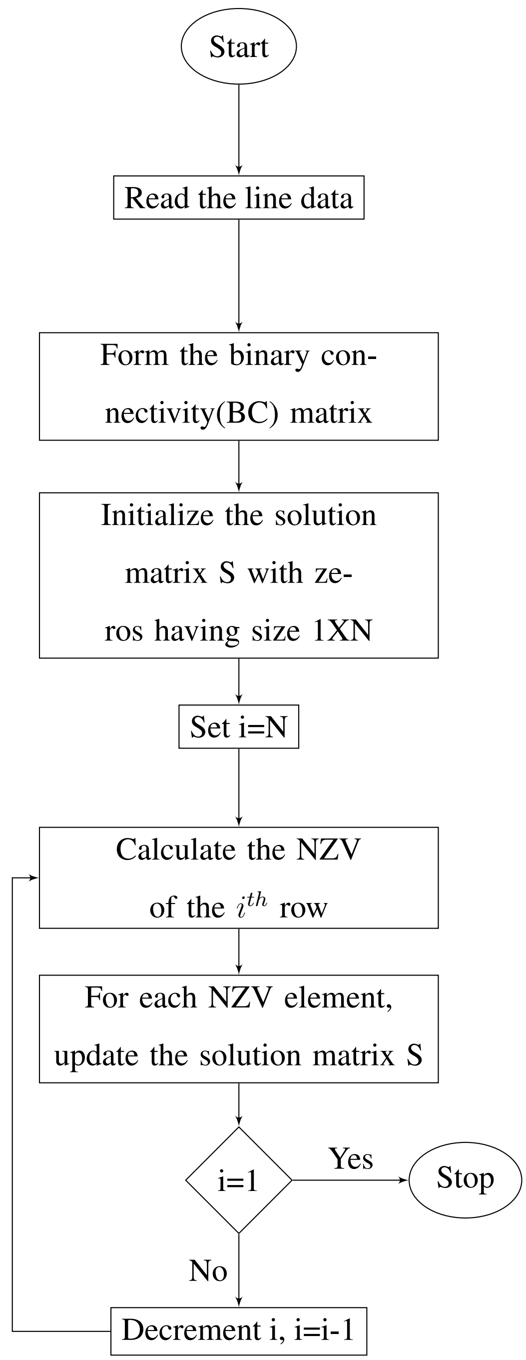

Figure 3.1: Flow chart of the proposed algorithm

3.2.4 Procedure for obtaining multiple solutions for N-1 line contingency

1. Generate N-1 line contingencies.
2. Formulate the BC matrix for each line contingency like as given subsection.2.2.1.
3. Prune the solutions which satisfy $BC.X = b$ from the feasible solutions.
4. Set feasible solution space as a pruned solution space and repeat the steps 2 & 3 for all remaining contingencies.
5. Follow the procedure given in 3.2.3 for obtaining multiple solutions having maximum redundancy for obtaining multiple solutions

3.2.5 Procedure for obtaining multiple solutions PMU loss case

1. Set all elements of b equal to 2
2. Prune the solutions which satisfy $BC.X = b$ from the feasible solutions.
3. Follow the procedure given in 3.2.3 for obtaining multiple solutions having maximum redundancy for obtaining multiple solutions

3.2.6 Procedure for obtaining multiple solutions considering channel limits

1. Form the BC matrix for the considered channel limit like as reported in [5] and reproduced in subsection.2.2.1.
2. Prune the solutions which satisfy $BC.X = b$ from the feasible solutions.
3. Follow the procedure given in 3.2.3 for obtaining multiple solutions having maximum redundancy for obtaining multiple solutions

3.2.7 NBOI and NSORI

The drawback of SORI is that, it is not aimed at maximizing individual redundancies. To overcome normalized SORI and BOI indices are proposed.

Normalized BOI(NBOI):

NBOI of i^{th} bus is the ratio of the number of PMUs observing the bus to the number of lines(l) connected to that bus plus one.

$$NBOI_i = p/(l_i + 1) \tag{3.18}$$

Where, p - Number of PMUs observing i^{th} bus

Normalized SORI(NSORI):

It is the cumulative sum of NBOI of all buses

$$NSORI = \sum_{i=1}^{N} NBOI_i \tag{3.19}$$

To intuitively understand this drawback, two feasible solutions 1-4-5-6 & 1-3-5-7 of 7 Bus system are given in table.2.1. From the figure.2.1, it is evident that buses 1, 2, 3, 4, 5, 6 & 7 can have maximum redundancies 2, 5, 4, 4, 2, 3 & 3. From table 3.2, solution 1 and solution 2 both have equal SORI. The individual bus redundancies of solution 2 are unevenly distributed. In case of solution 1, the distribution of individual redundancies have more uniformity compared with solution 2. Therefore, solution 1 is the best between them. This is not displayed in SORI index but clearly visible from NSORI. NSORI is maximum for a solution which has at most uniformity in NBOI's of all buses. In other words, the solution with maximum NSORI exibits cumulative redundancy superiority as well as individual bus redundancy superiority. SORI however failed to detect this feature.

Table 3.2: NBOIs and NSORI of the solutions having equal SORI

Solution	Bus 1 BOI(NBOI)	Bus 2 BOI(NBOI)	Bus 3 BOI(NBOI)	Bus 4 BOI(NBOI)	Bus 5 BOI(NBOI)	Bus 6 BOI(NBOI)	Bus 7 BOI(NBOI)	SORI(NSORI)
1-4-5-6	1(0.5)	2(0.4)	2(0.5)	2(0.5)	2(1)	1(0.33)	1(0.33)	11(3.56)
1-3-5-7	1(0.5)	3(0.6)	1(0.25)	3(0.75)	1(0.5)	1(0.33)	1(0.33)	11(3.26)

3.3 Results and discussions

The proposed method was tested on IEEE 14- bus and 30-bus systems and studied for four cases.

1. Base case
2. Single line contingency
3. Loss of PMU
4. Considering channel limits (3, 4 and 5)

The minimum number of PMUs required and the corresponding number of multiple solutions available for various cases are reported in table. 3.3 for IEEE 14 & 30- bus systems. In case of IEEE 14-Bus system, a whole set of 6181 feasible solutions are obtained from the proposed algorithm. Solutions reported in table.3.4 are part of the feasible solution set. Similarly, for IEEE 30-bus system the whole set of 126786561 feasible solutions are obtained.

3.3.1 IEEE 14-Bus system

Table.3.4 reports multiple optimal solutions for base case, line contingency and loss of PMU while maintaining observability and maximizing redundancy for IEEE 14-bus systems. In order to showcase the effectiveness of multiple solutions, other objectives like direct monitoring of generator buses and weak buses are added in remarks for the solutions presented.

Table 3.3: Number of solutions obtained for various cases

Test cases	IEEE 14		IEEE 30	
	No of PMUs	No of solutions	No of PMUs	No of solutions
Base case	4	5	10	858
Line contingency	7	6	15	72
Loss of PMU	9	8	21	1080
Channel limt 3	6	14	11	99
Channel limit 4	4	1	10	252
Channel limit 5	4	4	10	528

1, 2, 3, 6 and 8 are generator buses of IEEE 14- bus system, 12, 13, 14 and 9 are weak buses [10], while the complete set of multiple optimal solutions for 3 cases is presented in Table.3.4.

Base case

A total of 5 multiple optimal solutions are obtained. Solution 2,6,7,9(S_1)has highest SORI & NSORI, monitoring directly two generator buses and also one weak bus. While solution 2,6,8,9(S_2) is monitoring three generator buses and one weak bus. If direct monitoring of generator buses and weak buses is the priority while installing the PMUs, the solution 2,6,8,9(S_3) is the optimal one.

Line contingency

A total of 6 multiple optimal solutions are obtained. Solution 1,3,6,7,9,11,13(S_1) has the highest SORI & NSORI, monitoring two weak buses and also one generator bus directly. While solution 1,3,6,8,9,11,13(S_3) is monitoring two generator and two weak buses.

Loss of PMU

A total of 8 multiple optimal solutions are obtained. Solution 2,4,5,6,7,8,9,11,13(S_1) has highest SORI & NSORI and is monitoring three generator and two weak buses. While solution 1,2,3,6,7,8,9,11,13(S_7) is monitoring all generator buses and two weak buses.

Table 3.4: Optimal locations of PMUs for the IEEE 14 - Bus system while maintaining observability and maximizing redundancy

Test case	No of solutions	Optimal Locations	SORI	NSORI	Remarks
Base case	5	2,6,7,9(S_1)	19	4.95	Maximum SORI
		2,6,8,9(S_2)	17	4.58	Generator buses monitoring priority
		2,7,11,13(S_3)	16	4.33	monitoring weak bus and generator monitoring
		2,7,10,13(S_4)	16	4.33	weak bus monitoring
		2,8,10,13(S_5)	14	3.96	monitoring weak bus and generator monitoring minimum SORI
Line contingency	6	1,3,6,7 ,9,11,13(S_1)	27	7.13	Maximum SORI generator monitoring
		1,3,6,7,9,10,13(S_2)	27	7.13	Maximum SORI
		1,3,6,8,9,11,13(S_3)	25	6.76	Monitoring generator buses & weak buses
		1,3,6,8,9,10,13(S_4)	25	6.76	-
		1,3,6,7,10,13,14(S_5)	25	6.63	-
		1,3,6,7,10,12,14(S_6)	24	6.3	-
Loss of PMU	8	2,4,5 6,7,8,9,11,13(S_1)	39	10.13	three generator & two weak buses
		2,4,5,6,7,8 ,9,10,13(S_2)	39	10.13	-
		2,3,5,6,7,8 ,9,11,13(S_3)	36	9.48	-
		2,3,5,6,7,8,9,10,13(S_4)	36	9.48	-
		1,2,4,6,7,8,9,11,13(S_5)	37	9.76	-
		1,2,4,6,7,8,9,10,13(S_6)	37	9.76	-
		1,2,3,6,7,8,9,11,13(S_7)	34	9.11	All generator buses 9 & 13 are weak buses
		1,2,3,6,7,8,9,10,13(S_8)	34	9.11	-

3.3.2 IEEE 30-Bus system

Table.3.5 reports multiple solutions for base case, line contingency and loss of PMU while maintaining observability and maximizing redundancy for IEEE 30-bus system. In order to showcase the effectiveness of multiple solutions, other minor objectives like direct monitoring of generator buses and weak buses are added as remarks for the solutions presented.

Table 3.5: Optimal locations of PMUs for the IEEE 30 - bus system while maintaining observability and maximizing redundancy

Test case	No of solutions out of total solutions	Optimal Locations	SORI	NSORI	Remarks
Base case	4(858)[1]	2,4,6,9,10,12,15,20,25,27(S_1)	52	13.3	-
		2,4,6,9,10,12,15,19,25,27(S_2)	52	13.5	Maximum NSORI
		2,4,6,9,10,12,15,18,25,27(S_3)	52	13.3	-
		1,5 ,8,10,11,12,15,20,25,27(S_4)	41	11.43	1,5,8 and11 are generator buses
line contingency	4(72)[2]	2,3,7,8,9,10 ,12,15,17,19,22,24,25,27,30(S_1)	62	16.74	-
		2,3,7,8,9,10,12,15,17,19,22,24,25,27,29(S_2)	62	16.74	-
		2,3,7,8,9,10,12,15,16,19,22,24,25,27,30(S_3)	62	16.77	-
		2,3,7,8,9,10 ,12,15,16,19,22,24,25,27,29(S_4)	62	16.77	-
loss of PMU	5(48(1080))[3]	2,3,4,6,7,9,10,11,12,13,15,17,18,20,22,24,25,26,27,28,30(S_1)	85	22.64	-
		2,3,4,5,6,9,10,11,12,13,15,16,18,19,22,24,25,26,27,28,30(S_2)	85	22.93	5 is the generator bus
		2,3,4,5,6,9,10,11,12,13,15,16,18,19,22,24,25,26,27,28,30(S_3)	85	22.93	5 is the generator bus
		1,2,4,5 , 6,9,10,11,12,13,15,16,18,19,22,24,25,26,27,28,30(S_4)	85	22.93	1,5,11 and 13 are generator buses
		1,2,4,5 , 6,9,10,11,12,13,15,16,18,19,22,24,25,26,27,28,29(S_5)	85	22.93	1,5,11 and 13 are generator buses

[1] 4 out of 858 solutions are presented

[2] 4 solutions have maximum SORI out of 72 solutions

[3] Out of 1080 solutions 48 solutions have maximum SORI and only 5 solutions are reported

1, 2, 5, 8, 11 and 13 are generator buses of IEEE 30-bus system.

Base case

A total of 858 optimal solutions are obtained. Only four solutions are presented in table.3.5. 3 solutions have maximum SORI. But only one solution has maximum NSORI. This is where the advantage of NSORI lies compared to SORI. The solutions having maximum SORI fail to monitor generator buses. But the 4^{th} optimal solution is monitoring 4 generator buses though it has less NSORI.

Line contingency

A total of 72 optimal solutions are obtained. 4 solutions are presented in the table.3.5 having maximum SORI. Out of 4 only 2 solutions have maximum NSORI.

Loss of PMU

A total of 1080 solutions are obtained. 48 solutions have maximum SORI, out of them only 4 solutions have maximum NSORI. The 4^{th} and 5^{th} solutions have maximum SORI and NSORI and they also monitor 4 generator buses directly.

3.3.3 Considering Channel limits

Table 3.6: Optimal locations of PMUs for the IEEE 14 & 30 - Bus system considering channel limits

Test system	channel limit	Optimal Locations
IEEE 14	3	2,5,7,9,11,13
		2,5,6,7,9,14
		2,5,6,7,9,13
		2,5,6,7,9,12
	4	2,6,7,9
	5	2,7,11,13
		2,7,10,13
		2,6,8,9
		2,6,7,9
IEEE 30	3	2 ,4,6,9,10,12 ,15,20,24,25,27
		2,4 ,6,9,10,12,15,20,22,25,27
		2,4,6,9,10,12,15,19,24,25,27
	4	2,4,6,9,10,12,15,20,25,27
		2,4,6,9,10,12,15,19,25,27
		2,4,6,9,10,12,15,18,25,27
	5	2,4,6,9,10,12,15,20,25,27
		2,4,6,9,10,12,15,19,25,27
		2,4,6,9,10,12,15,18,25,27

Table.3.6 presented some of the multiple optimal solutions obtained considering PMU channel limits 3,4 and 5 for base case. The total number of multiple optimal solutions obtained for the considered channel limits is given in Table.3.3

3.3.4 Validation of the proposed method

Table 3.7: Comparison of results with various methods for base case

Methodology	IEEE 14		IEEE 30	
	#[4]PMUs	#[4]solutions	#[4]PMUs	#[4]solutions
Exhaustive search [19]	4	1	10	1
ILP based [6]	4	1	10	1
BPSO [7]	4	1	10	1
BSP [14]	4	1	10	1
MBPSO [9]	4	1	10	1
EBPSO [10]	4	4	-	-
TBBA [28]	4	3	10	4
MO-BPSO [29]	4	5	10	11
MINLPBB [30]	4	5	10	10
GA-MST [31]	4	5	10	24
Proposed Method	**4**	**5**	**10**	**858**

[4] # number of

Table.3.7 is presented for validating the proposed method with conventional mathematical optimization and Meta-heuristic techniques. From table.3.7 it is evident that, other than the proposed method no method has claimed to achieve complete set of multiple optimal solutions. Hence, the proposed method has the ability to obtain complete set of multiple solutions. Meta-heuristic algorithms ensure one global optimal solution at a time. They are good if the objective function has only one global optimal solution. The OPP problem is a multi global optimal problem under single objective case(minimization of number of PMUs) and also under two objective cases(minimization of number of PMUs and maximization of redundancy). Meta- heuristic algorithms will provide one global optimal point in one run. There is no guarantee of getting all multiple global optima points with meta-heuristic algorithms by running them several times. The proposed method will provide all global optimal points in single run.

Once the multiple optimal solution set is available, one solution can be easily chosen based on the required objectives. In this thesis, only generator monitoring and weak bus monitoring has been taken as a other minor objectives to explain the significance of multiple optimal solutions. Only one run of the proposed algorithm come up with complete set of feasible solutions. From that set, one can pick the optimal solution which meets the required objectives. It is also observed that, optimal PMU placement problem is a multiple global optima problem even while considering two objectives. OPP is a specialized optimization problem having multiple global optimal solutions.

3.4 Summary and Comments

In this chapter, a novel Topology based Optimal PMU placement method-II is proposed to obtain the whole set of the multiple optimal solutions. The proposed method works only on binary connectivity matrix of the system without using any classical or meta-heuristic optimization technique. Once multiple optimal solution set is available, the solution which best fits the utility sub - ordinate objectives like direct monitoring of generator and weak buses ,etc can be selected. The proposed method ensures global optima. From the results, it is observed that OPP problem is a special optimization problem having multiple global optimal solutions. In addition, this chapter also proposed normalized BOI(NBOI) and SORI(NSORI). Unlike SORI, the solution which has maximum NSORI has maximum individual bus redundancy uniformity. NSORI also has the ability to further prune multiple solutions. As the algorithm progresses, the solution matrix length increases. Due to this, personal computers having medium RAM capacity are not sufficient to run the algorithm for large power systems. This method is robust for placement of PMU's in multi-area power systems. The advantage of obtaining the biopsy of solution space in one run cannot be overlooked.

Chapter 4

Synchrophasor assisted Power System State Estimation with a Quadratically-decaying Exponential Criterion

4.1 Introduction

In recent times, power systems have been populating with PMUs. Under the assumption that, PMUs provide complete observability of the power system, the relation between the measurement set and state variables change from non-linear to linear. Because of this linearity, the iterative nature of WLSE changed to non-iterative, further decreasing the computational time. In a similar manner, computational time of the robust state estimators also decreased, interestingly competeing with WLSE . This made the researchers to have a re-look on robust estimators.

In this chapter, Quadratic-constant estimator is studied. An improved version is presented to make it superior and suitable for PMU assisted state estimation. The Quadratic-Constant(QC) estimator had been proposed in [76]. Section. 4.2 describes the objective function modeling. The objective function is given by equation(4.2). The QC characteristics are shown in figure.4.1. Break even point(τ_i) is the point at which characteristics of the QC estimator will change from quadratic to constant. The probable values of break even point such as 2.5, 3, ..5 are given based on engineering judgment. It showed that QC estimator has excellent ability to suppress bad data and ensure robust convergence characteristics though it has non-convex objective function.

In [77] the concept of variable break even point is introduced by fixing initial and final break even point with constant step length. It enhanced the bad data suppression ability. In [78] the problem of misidentification and unidentification is observed. To overcome this a variable break even point with variable step lengths at different convergence levels is proposed. It is observed that bad data had been suppressed even with moderate redundancy. The above discussed methods are suitable for SCADA assisted power system state estimation because of the iterative nature of the solution process. Except fixed break-even point QC, it is not possible to implement these QC variants for a measurement set having linear relation with state variables.

This chapter proposed a Quadratically-decaying Exponential criterion by changing the constant part of the QC into decaying exponential. This has been achieved by giving exponentially decaying weightage to the corresponding normalized residuals after break-even point. Before the break-even point the objective function is similar to quadratic and after the break-even point it is like a decaying exponential.

4.2 Objective function modeling

It is assumed that, the power system is completely observable by PMUs. The relationship between measurements and state variables is given in equation(4.1)

$$Z_1 = HX + \epsilon \tag{4.1}$$

Where,

Z_1	2mx1 PMU measurement vector includes
	real and imaginary parts of voltage and line current phasors
X	2nx1 state vector
$H : R^n \to R^m$	Linear vector function maps
	state vector to measurement vector(design matrix)
ϵ	2mx1 error vector

$$C_i(r_i) = \begin{cases} (1/2){r_i}^2 & |r_i| \leq \tau_i \\ (1/2)\tau_i^2 & |r_i| \geq \tau_i \end{cases} \tag{4.2}$$

$$J(x) = \sum_{i=1}^{N} W_i C_i(r_i) \tag{4.3}$$

The Quadratic-Constant estimator objective function for one measurement is given in equation(4.2) and the characteristics are shown in figure.4.1. In equation(4.2) r_i is the normalized residue of i^{th} measurement and τ_i is the break-even point. Objective function of QC is given in (4.3). From the discussion in section 4..1, it is evident that only fixed point QC is suitable for PMU assisted state estimation. Giving zero weight to measurements after break even point is working well for iterative algorithms. But for non iterative algorithms, it makes the system occasionally unobservable because of misidentification or divergence in the solution.

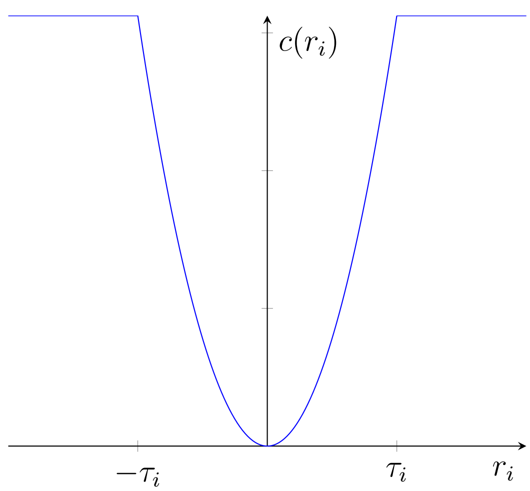

Figure 4.1: Quadratic- Constant characteristics

The idea of the proposed formulation is, if the formulated objective function has the ability to give less weightage to measurements which are having high residue and more weightage to measurements which are having low residue. The influence of measurements which have high residue on the estimation will be less. i.e suppressing bad data ability is incorporated. Therefore, to achieve the same, constant weighting to the measurements is transformed into a function of normalized residues of the measurements. The weight function chosen is the exponentially decaying function given by equation(4.4). Here $4 * \tau$ is the breakeven point. For removing the leverage measurements from the measurement set, scaling concept has been used [50]

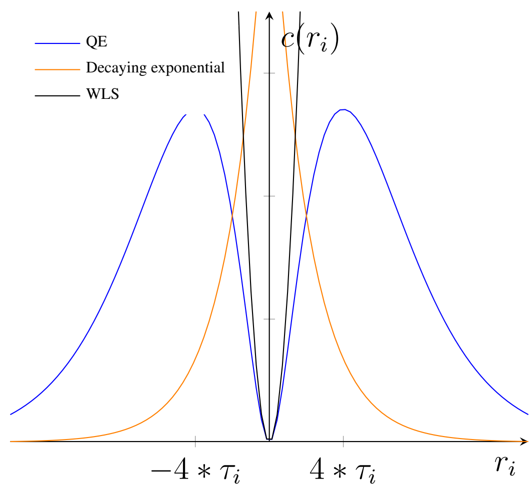

Figure 4.2: Quadratic- decaying Exponential criterion characteristics

$$W(r_i) = \begin{cases} 1 & |r_i| \leq 4 * \tau_i \\ e^{-|r_i|/\tau} & |r_i| > 4 * \tau_i \end{cases} \tag{4.4}$$

The formulated objective function for one measurement is the product of the square of residual and the weight function given in equation(4.4). This is given in equation(4.5). Characteristics of WLS, weight function and the resultant characteristics of the proposed estimator are shown in figure.4.2 . The proposed criterion is named as Quadratically-decaying Exponential criterion(QE). The characteristics of the QE estimator are non-convex in nature. The objective function of the proposed estimator is given in (4.6). The flowchart of the proposed estimator is shown in figure.4.3

$$C_i(r_i) = (1/2)W(r_i){r_i}^2 \quad (4.5)$$

$$J(x) = \sum_{i=1}^{N} C_i(r_i) \quad (4.6)$$

4.2.1 Relationship with WLS estimator

In the absence of corrupt measurements in the measurement set, normalized residues of the measurements are less than the break-even point. The additional weight function value becomes unity.

$$J(x) = \sum_{i=1}^{N} W_i {r_i}^2 \quad (4.7)$$

From (4.7), it can be concluded that the proposed method in the absence of corrupt data is nothing but WLS estimator.

4.2.2 Comparison with Quadratic-Constant Criterion

The proposed Quadratically decaying exponential criterion and QC estimator both will work on break even point. The QC estimator is not suitable for PMU based state estimation where as QE is modelled for PMU based state estimation.

The inclusion of measurements which violate break-even point thersold and assigning exponentially decaying weightage, make the proposed QE estimator giving scope to the suspicious measurements also, when participating in the state estimation. This enhances the noise filtering ability of the proposed estimator. It is observed that, application of QC to PMU based state estimation provides less accurate results under the presence of Gaussian noise and diverged solution under the presence of corrupt data.

The compared methods are presented in the following subsections to make the chapter self sufficient.

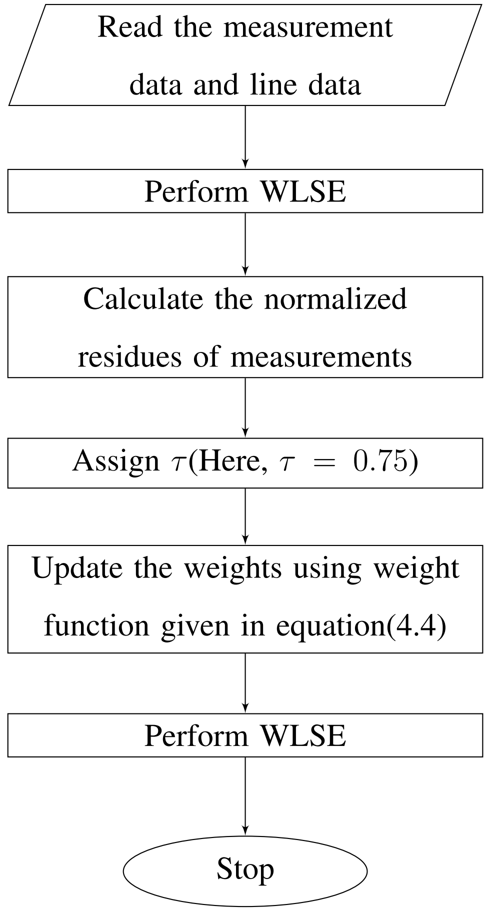

Figure 4.3: Flow chart of the proposed estimator

4.2.3 PMU assisted linear WLS state estimator

WLS state estimator is the most widely used method for processing measurements [1]. It is the best estimator for processing mesurement set which contains Gaussian noise. With respect to conventional measurements the solution process of WLS is non- iterative. In case of purely PMU measurement assisted state estimation, solution process of WLS is non-iterative and is given below. The relationship between the state vector and PMU measurement set is linear and is given in equation(4.8). Where H_1 is called as a design matrix. The column entities of H_1 for different measurements such as real and imaginary parts of voltage and current measurements are given from equation(4.10) to equation(4.13)

$$Z_1 = H_1 X + r \tag{4.8}$$

$$\begin{bmatrix} z_1 \\ . \\ z_i \\ . \\ z_{2m} \end{bmatrix} = \begin{bmatrix} H_{11} & . & H_{1p} & . & H_{1N} & H_{1N+1} & . & H_{1N+p} & . & H_{12N} \\ . & . & . & . & . & . & . & . & . & . \\ H_{i1} & . & H_{ip} & . & H_{iN} & H_{iN+1} & . & H_{iN+p} & . & H_{i2N} \\ . & . & . & . & . & . & . & . & . & . \\ H_{2m1} & . & H_{2mp} & . & H_{2mN} & H_{2mN+1} & . & H_{2mN+p} & . & H_{2m2N} \end{bmatrix} \begin{bmatrix} e_1 \\ . \\ e_p \\ . \\ e_N \\ f_1 \\ . \\ f_p \\ . \\ f_N \end{bmatrix} \tag{4.9}$$

$$H_{i,p} = \begin{cases} 1 & Z_i = V_{re} \\ 0 & Z_i = V_{im} \\ gline_k & Z_i = I_{re} \\ bline_k + y_{cp} & Z_i = I_{im} \end{cases} \tag{4.10}$$

$$H_{i,q} = \begin{cases} 0 & Z_i = V_{re} \\ 0 & Z_i = V_{im} \\ -gline_k & Z_i = I_{re} \\ -bline_k & Z_i = I_{im} \end{cases} \tag{4.11}$$

$$H_{i,p+N} = \begin{cases} 0 & Z_i = V_{re} \\ 1 & Z_i = V_{im} \\ -bline_k - y_{cp} & Z_i = I_{re} \\ gline_k & Z_i = I_{im} \end{cases} \tag{4.12}$$

$$H_{i,q+N} = \begin{cases} 0 & Z_i = V_{re} \\ 0 & Z_i = V_{im} \\ bline_k & Z_i = I_{re} \\ -gline_k & Z_i = I_{im} \end{cases} \tag{4.13}$$

$$Z_1 = [Z^r \quad Z^i]^T \tag{4.14}$$

$$X^T = \left[\begin{array}{cccccccccc} e_1 & .. & e_p & .. & e_N & f_1 & .. & f_p & .. & f_N \end{array} \right] \tag{4.15}$$

The non- iterative solution is computed using equation(4.16) as is given below

$$X = (H_1^T W H_1)^{-1} H_1^T W Z_1 \tag{4.16}$$

Where

- Z^r real part of measurement set
- Z^i imaginary part of measurement set
- H design matrix
- e real part of state vector
- f imaginary part of state vector
- r residual vector
- W weight matrix or inverse of covariance matrix

4.2.4 PMU assisted LAV based state estimator

LAV based state estimator [50] minimizes the sum of absolute residuals of the measurement set while obtaining solution vector. The objective function of LAV based state estimator is given below:

$$f(r_i) = \sum_{i=1}^{2m} |r_i| \tag{4.17}$$

With the help of equation(4.8) and equation(4.17) the objecive function of LAV based state estimator is reformulated and is given below

$$min \quad c^T|r_i|$$

$$Z_1 - HX = r \tag{4.18}$$

By rearranging the above equations and defining some strictly positive variables, the LAV problem can be expressed as a linear programming problem. The following equations are rewritten with respect to rearranged objective function

$$min \quad cy \tag{4.19}$$

$$My = b \tag{4.20}$$

$$y \geq 0 \tag{4.21}$$

$$c = [Z_n \quad O_m] \tag{4.22}$$

$$M = [H \quad -H \quad I \quad -I]^T \tag{4.23}$$

$$y = [x_a \quad x_b \quad u \quad v]^T \tag{4.24}$$

$$b = z \tag{4.25}$$

$$x = x_a - x_b \tag{4.26}$$

$$r = u - v \tag{4.27}$$

Where

Z_n	1x2n vector, zeros are the entities of the vector
O_m	1x2m vector, ones are the entities of the vector
x_a	1xn vector
x_b	1xn vector
u	1xm vector
v	1xm vector

4.3 Case Study and Results

In this section, case study is conducted on IEEE 14, 30, 57 & 118-bus test systems. Simulations were carried out using PC with 4GB RAM and Windows 10 operating system. The algorithm is implemented in PYTHON language, SPYDER IDE platform.

Simulation results for two cases viz, measurement data having only Gaussian noise and corrupted measurement data is tested on IEEE 14, 30, 57 & 118 - bus systems.. However, IEEE 14 and 30-bus system are used to show the estimation accuracy of the QE estimator and IEEE 57 and 118-bus system are used to show the computational performance of the QE estimator.

True values of the measurement set are obtained by running load flow study using MATPOWER software [4]. 100 sets of measurement data are simulated by adding Gaussian errors having standard deviation of 0.001. Voltage magnitude and phase angle mean square error for each simulation is computed using equation(4.28). The proposed method used τ=0.75 to fix the break event point at 3. In case of QC estimator τ itself is a break even point and it is set to 3. In case of proposed method however $4 * \tau$ is the break even point.

$$MSE = \sqrt{(1/N)\sum_{i=1}^{N}(x_i^{estimated} - x_i^{true})^2} \tag{4.28}$$

4.3.1 IEEE 14 and 30-bus system

For comparing estimation accuracy of the QE estimator with WLSE and LAV IEEE 14 & 30-bus systems are used considering two cases such as measurement data having only Gaussian noise and corrupt measurement data. PMU locations are taken like in [1] for IEEE 30-Bus system and for IEEE 14-Bus system are considered like in [5]. For IEEE 30-Bus system, line current measurements 1-2, 2-4 and 15-18 are corrupted by setting them to zero and for IEEE 14-Bus system, line current measurements 1-2, 2-5, 7-9 are corrupted by setting them to zero.

Table.4.1 presents the estimation accuracy comparison of QE estimator with WLSE &

LAV for IEEE 14-bus system considering measurement data having only Gaussian noise. True values of the states are given for ascertaining closeness of estimated values with the actual values. From the table.4.2 it is observed that considering corrupt data, the estimation accuracy of QE estimator closely follows true values. WLSE & LAV estimated values have largely deviated from true values i.e they are unable to suppress the bad data.

Table 4.1: Comparison of estimation accuracy of QE estimator with WLSE & LAV for IEEE 14-bus system considering only Gaussian noise

Bus number	True Values		WLSE		LAV		QE	
	V(pu)	δ(degrees)	V(pu)	δ(degrees)	V(pu)	δ(degrees)	V(pu)	δ(degrees)
1	1.06	-1.20819e-18	1.0606	-0.000528455	1.06069	-0.00193394	1.06045	0.00655356
2	1.04	-4.9261	1.04037	-4.92362	1.04039	-4.92384	1.04043	-4.91931
3	1.0002	-12.7264	1.00062	-12.7156	1.00057	-12.7154	1.0006	-12.705
4	1.00708	-10.2472	1.00755	-10.2482	1.00743	-10.2473	1.00747	-10.2433
5	1.01098	-8.71647	1.0118	-8.71485	1.01196	-8.71624	1.01139	-8.71359
6	1.05882	-14.2635	1.05953	-14.2578	1.05959	-14.258	1.05929	-14.2647
7	1.05006	-13.3631	1.05048	-13.3682	1.05042	-13.368	1.05042	-13.3746
8	1.07848	-13.3631	1.07927	-13.3659	1.07923	-13.3657	1.0788	-13.3743
9	1.04422	-14.9789	1.04441	-14.9774	1.04436	-14.9773	1.04457	-14.987
10	1.03928	-15.1442	1.03933	-15.1459	1.03931	-15.1459	1.03934	-15.1526
11	1.04542	-14.8378	1.04588	-14.8315	1.0459	-14.8315	1.04579	-14.831
12	1.04381	-15.1357	1.04429	-15.139	1.04433	-15.1391	1.044	-15.1555
13	1.03893	-15.2166	1.03952	-15.2126	1.03956	-15.2127	1.0394	-15.2161
14	1.02372	-16.1054	1.0243	-16.0917	1.02429	-16.0917	1.02431	-16.1

Table 4.2: Comparison of estimation accuracy of QE estimator with WLSE & LAV for IEEE 14-bus system considering corrupt measurement data

Bus number	True Values		WLSE		LAV		QE	
	V(pu)	δ(degrees)	V(pu)	δ(degrees)	V(pu)	δ(degrees)	V(pu)	δ(degrees)
1	1.06	-1.20819e-18	1.04706	-2.45518	1.05546	-1.13358	1.05991	0.00981172
2	1.04	-4.9261	1.03807	-5.10833	1.04026	-4.92614	1.04026	-4.93138
3	1.0002	-12.7264	0.9996	-12.8286	1.00049	-12.7098	1.00063	-12.7189
4	1.00708	-10.2472	1.00719	-10.2787	1.00735	-10.245	1.00763	-10.2517
5	1.01098	-8.71647	1.01244	-8.56052	1.01125	-8.71715	1.01081	-8.71929
6	1.05882	-14.2635	1.05991	-14.1601	1.05908	-14.2611	1.05923	-14.2596
7	1.05006	-13.3631	1.04947	-13.5935	1.05028	-13.3629	1.0505	-13.3636
8	1.07848	-13.3631	1.07832	-13.5124	1.07883	-13.3629	1.07901	-13.3632
9	1.04422	-14.9789	1.04515	-14.7989	1.0444	-14.9798	1.04463	-14.9816
10	1.03928	-15.1442	1.03995	-15.0207	1.03949	-15.1427	1.03955	-15.1444
11	1.04542	-14.8378	1.04613	-14.7318	1.04549	-14.8436	1.04559	-14.8436
12	1.04381	-15.1357	1.04466	-15.0416	1.04388	-15.1535	1.0441	-15.1368
13	1.03893	-15.2166	1.03983	-15.1173	1.03926	-15.21	1.03933	-15.2086
14	1.02372	-16.1054	1.02445	-15.9715	1.02368	-16.0896	1.02399	-16.1111

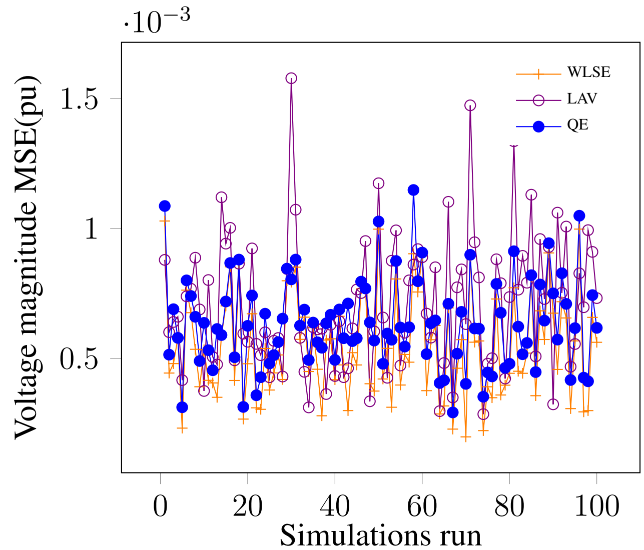

Figure 4.4: Comparison of Voltage magnitude MSE of QE with WLSE & LAV for IEEE 30-bus system considering only Gaussian noise

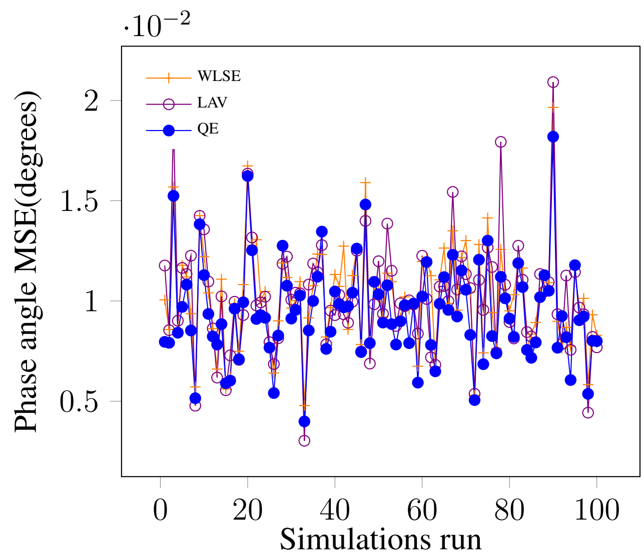

Figure 4.5: Comparison of Phase angle MSE of QE with WLSE & LAV for IEEE 30-bus system considering only Gaussian noise

The voltage magnitude and phase angle MSE for IEEE 30-Bus system considering Gaussian noise are presented in figures.4.4 & 4.5. Voltage magnitude MSE of WLSE is found to be less compared with QE & LAV . Phase angle MSE of QE is less compared with WLSE. i.e even under normal case also, QE estimator is competing with WLSE. This feature of QE competing with WLSE under the presence of Gaussian noise is because of using WLSE in the proposed method. The proposed method is tuning the weights of measurements in the WLSE in such a way as to suppress bad data.

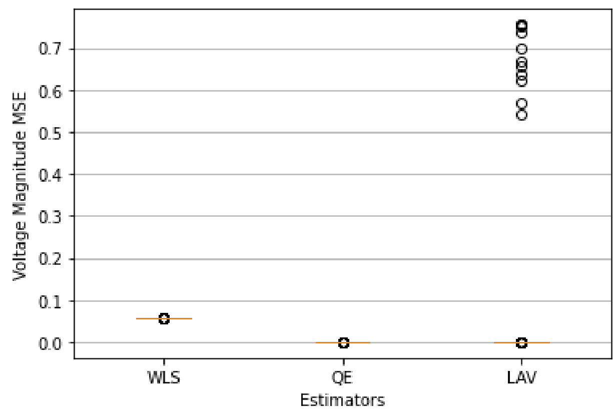

Figure 4.6: Comparison of Voltage magnitude MSE of QE with WLSE & LAV for IEEE 118-bus system considering corrupt measurement data

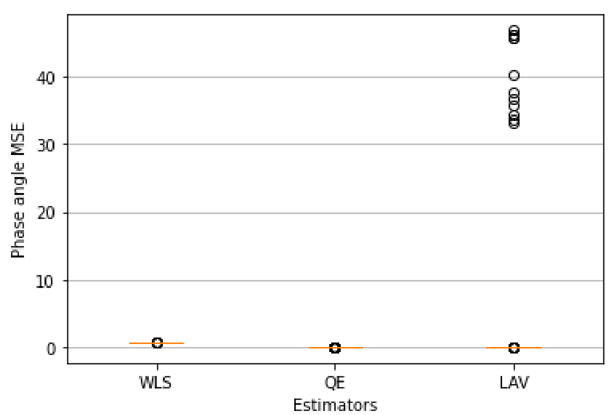

Figure 4.7: Comparison of Phase angle MSE of QE with WLSE & LAV for IEEE 118-bus system considering corrupt measurement data

4.3.2 IEEE 57 and 118-bus system

For comparing computational performance of the proposed estimator with WLSE & LAV IEEE 57 and 118-bus systems are used considering two cases such as measurement data having only Gaussian noise and corrupt measurement data. PMU placement for loss of PMU case [5] is taken as a measurement set for both systems. Line current measurements 2-12, 3-5, 32-114, 42-49, 49-66, 70-75, 87-86, 100-106 and voltage measurements at buses 61, 84 and 106 and line current measurements 1-2, 1-17 and 9-12 are corrupted for generating bad data for IEEE 118 and 57-Bus systems.

Table 4.3: Comparison of Voltage magnitude and phase angle MSE of QE with WLS and LAV

Test System	Estimator	considering Gaussian noise			considering Corrupt data		
		Magnitude MSE(pu)	Phase angle MSE(degrees)	CPU time(sec)	Magnitude MSE(pu)	Phase angle MSE(degrees)	CPU time(sec)
IEEE14	QE	0.000372	0.006145	0.000455sec	0.00038	0.006228	0.000432sec
	WLSE	0.0002818	0.006336	0.000405sec	0.00672	1.26	0.000347sec
	LAV	0.000348	0.00796	0.022sec	0.00135	0.303	0.027sec
IEEE 30	QE	0.000613	0.00964	0.00071sec	0.00066	0.00994	0.00069sec
	WLSE	0.000549	0.0106	0.00058sec	0.00364	0.2304	0.00059sec
	LAV	0.000669	0.0106	0.034sec	0.001213	0.018688	0.0334sec
IEEE57	QE	0.00039735	0.00975	0.0028535sec	0.0003932	0.009963	0.0022sec
	WLSE	0.0002414	0.008039	0.00066sec	0.002733	0.5	0.0011sec
	LAV	0.00033677	0.01306	0.25sec	0.00122284	0.192	0.2745sec
IEEE118	QE	0.00034	0.0035	0.0146sec	0.00045864	0.00716	0.0126sec
	WLSE	0.000114	0.0049	0.0025sec	0.057	0.62	0.0023sec
	LAV	0.00022	0.0059	5.36sec	0.019	1.14	5.4sec

From the table.4.3, computational time of WLSE further shoots up when used in coordination with any bad data detection and re estimation techniques like largest normalized residues test. Computational time of WLSE depend on the number of bad measurements. The more the bad measurements, the more will be the time. QE estimator computational performance is independent of the number of corrupt measurements in the measurement set. It is evident that, LAV estimator is computationally inefficient compared with QE. Voltage magnitude MSE and phase angle MSE of QE under the presence of corrupt data is very less compared to LAV and WLS estimators.

The box plot of QE, WLS & LAV Voltage magnitude MSE and Phase angle MSE error for 1000 simulations considering bad data is shown in figures.4.6 & 4.7. Though LAV estimator is performing on par with QE in many instances, Voltage magnitude and Phase angle MSE box plot of LAV contain some outliers with large deviations. The above discussion further illustrates the usefulness of Quadratically-decaying Exponential Criterion(QE) based estimator for systems with only PMU measurements.

Figure.4.8 shows the relationship between computational time and order of system. It is evident that, the proposed method is computationally efficient and superior than LAV estimator.

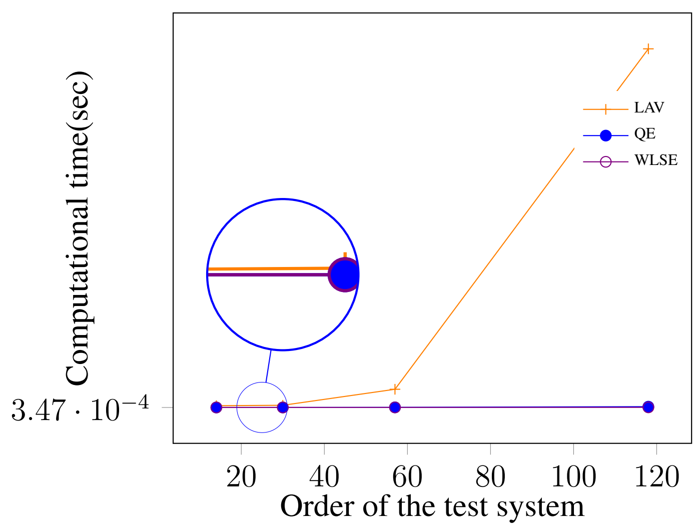

Figure 4.8: Comparison of computational time of QE with WLSE & LAV while increasing order of the system

4.4 Summary and Comments

This chapter explored the ability of robust estimators under assumption that entire power system is observable by PMUs. The Quadratic-Constant criterion based estimator is studied and improved to make it suitable for PMU assisted state estimation. The proposed criterion is termed Quadratic-decaying Exponential criterion. The estimation accuracy of QE estimator is competing with the WLSE under the presence of Gaussian noise and its performance is superior under the presence of corrupt data. The computational performance of the QE estimator is little lower compared with WLSE under the presence of Gaussian noise and superior under the presence of corrupt data. The computational performance of the QE estimator is independent of the number of bad measurements in the measurement set. The estimation accuracy of LAV compared with QE had some outliers with large deviation from median and the computational performance increases greatly with the order of the system compared with QE. Therefore, the proposed QE estimator exhibits superior performance compared with LAV estimator.

Chapter 5

Linear tracking sequential hybrid power system state estimator

5.1 Introduction

The PMUs deployed until now have not been able to provide complete observability of the system, as both SCADA and PMUs will coexist up to a certain period. Therefore, in order to exploit the best features associated with PMU measurements while doing state estimation with SCADA measurements, hybrid state estimation techniques are required. Many hybrid state estimation techniques are proposed in the literature to incorporate PMU measurements with traditional SCADA measurements.

This chapter proposes a linear tracking Sequential hybrid power system state estimator. In stage 1, the state of the power system is obtained using only SCADA measurements, with the help of the proposed linear model. In stage2, the state of the power system is estimated using both PMU measurements and state vector obtained in the first stage. During the instances when only PMU measurements are available, SCADA pseudo measurements are computed with the help of previous instant state vector.

5.2 Classical SCADA based state estimation

In classical SCADA based SE(SCADASE), the measurements and state vector follow non-linear relation reported in [22] and equation(5.1) shows the relationship

$$Z_s = h(X) + e \tag{5.1}$$

Where Z_s is the SCADA measurement vector consisting of real and reactive power injections, real and reactive power flows and voltage magnitude measurements. X is the state vector and e is the error vector. It is assumed that, the errors of SCADA measurements follow Gaussian distribution and are independent of each other. The real power and reactive power injections, real power and reactive power flow injections and voltage magnitude equations are given from equation(5.2)-(5.6)

$$P_p = (e_p^2 + f_p^2)G_{pp} + \sum_{i=1}^{N} G_{pq}(e_p e_q + f_p f_q) + B_{pq}(f_p e_q - e_p f_q) \tag{5.2}$$

$$Q_p = -(e_p^2 + f_p^2)B_{pp} + \sum_{i=1}^{N} G_{pq}(f_p e_q - e_p f_q) - B_{pq}(e_p e_q + f_p f_q) \tag{5.3}$$

$$P_{pq} = (e_p^2 + f_p^2)gline_k - gline_k(e_p e_q + f_p f_q) - bline_k(f_p e_q - e_p f_q) \tag{5.4}$$

$$Q_{pq} = -(e_p^2 + f_p^2)(Y_{cp} + bline_k) - gline_k(f_p e_q - e_p f_q) + bline_k(e_p e_q + f_p f_q) \tag{5.5}$$

$$V_p = \sqrt{e_p^2 + f_p^2} \tag{5.6}$$

$$X^T = \begin{bmatrix} e_1 & .. & e_p & .. & e_N & f_1 & .. & f_p & .. & f_N \end{bmatrix} \tag{5.7}$$

By minimizing the following objective function, the optimal estimates of the system are obtained and is given in equation(5.8)

$$J(X) = [Z_s - h(X)]^T R^{-1} [Z_s - h(X)] \tag{5.8}$$

Where R is the error covariance matrix. First order optimality condition is used for obtaining the iterative solution vector and is given in equation(5.9) for k^{th} iteration.

$$\Delta X_k = [H^T W H]^{-1} H^T W \Delta Z_k \tag{5.9}$$

Where H is the mx2N jacobian matrix and the iterative process will be terminated after attaining the pre-specified convergence limit.

5.3 Sequential hybrid state estimation

A sequential synchrophasor assisted hybrid state estimator is proposed in [51,52,55] by combining both PMU and SCADA measurements. It is also called two stage state estimator. In the first stage, SCADA measurement based WLS state estimator is used to obtain the state vector. In stage 2 the states obtained in the previous stage and PMU measurements are used in the linear state estimation to get the final states. Stage2 estimator model is given [55]. Equation(5.10) is used for obtaining the final states of the system in a non-iterative way.

$$X = [H_1^T W H_1]^{-1} H_1^T W Z_1 \tag{5.10}$$

Where H_1 is the design matrix. Z_1 is the measuremet vector which includes stage1 state vector and PMU measurements.

5.4 Linear Sequential hybrid state estimation model

This section presents a new linear sequential hybrid state estimation model for estimating the states of the power system. SCADA measurements consist of voltage magnitude measurements, real and reactive power flows and real and reactive power injections. PMU measurements include voltage and branch current phasors. The refresh rate of PMU measurements is 1-2 cycles where as SCADA measurements it is 2-5seconds. i.e PMU measurements update rate(p) is 100 times faster than SCADA measurements update rate(S) for the above given minimum update rate of SCADA measurements shown in Figure.5.1.

The proposed method is estimating states of the system by adapting two stage proce-

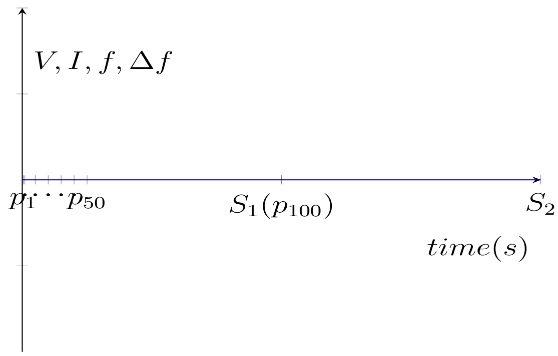

Figure 5.1: Reporting time of SCADA and PMU data

dure. Stage1 processes only SCADA measurements and in stage2 PMU measurements along with processed stage1 state vector is used for estimating the final state of the system.

5.4.1 Stage1: SCADA measurements processing

In stage 1, SCADA measurements are processed to get the intermediate state vector. This is based on the assumption that load variations occuring in the power system in between the two PMU reporting rates are following linearity. Therefore, by expanding the equation(5.1) using Taylor's series will get equation(5.11). The column entities of jacobian matrix for different measurements such as real and reactive power injections, real and reactive power flows and voltage magnitude are given from equation(5.13) to equation(5.16)

$$\Delta Z = H\Delta X + e \tag{5.11}$$

$$\begin{bmatrix} \Delta Z_1 \\ . \\ \Delta Z_i \\ . \\ \Delta Z_m \end{bmatrix} = \begin{bmatrix} \frac{\partial h_1}{\partial e_1} & . & \frac{\partial h_1}{\partial e_p} & . & \frac{\partial h_1}{\partial e_N} & \frac{\partial h_1}{\partial f_1} & . & \frac{\partial h_1}{\partial f_p} & . & \frac{\partial h_1}{\partial f_N} \\ . & . & . & . & . & . & . & . & . & . \\ \frac{\partial h_i}{\partial e_1} & . & \frac{\partial h_i}{\partial e_p} & . & \frac{\partial h_i}{\partial e_N} & \frac{\partial h_i}{\partial f_1} & . & \frac{\partial h_i}{\partial f_p} & . & \frac{\partial h_i}{\partial f_N} \\ . & . & . & . & . & . & . & . & . & . \\ \frac{\partial h_m}{\partial e_1} & . & \frac{\partial h_m}{\partial e_p} & . & \frac{\partial h_m}{\partial e_N} & \frac{\partial h_m}{\partial f_1} & . & \frac{\partial h_m}{\partial f_p} & . & \frac{\partial h_m}{\partial f_N} \end{bmatrix} \begin{bmatrix} \Delta e_1 \\ . \\ \Delta e_p \\ . \\ \Delta e_N \\ \Delta f_1 \\ . \\ \Delta f_p \\ . \\ \Delta f_N \end{bmatrix} + \begin{bmatrix} e \\ . \\ e \\ . \\ e \\ e \\ . \\ e \\ . \\ e \end{bmatrix} \tag{5.12}$$

$$\frac{\partial h_i}{\partial e_p} = \begin{cases} 2G_{pp}e_p + \sum_{q \neq p}^{N} G_{pq}e_q - B_{pq}f_q & Z_i = P_{inj} \\ -2B_{pp}e_p - \sum_{q \neq p}^{N} G_{pq}f_q - B_{pq}e_q & Z_i = Q_{inj} \\ 2gline_k e_p - gline_k e_q + bline_k f_q & Z_i = P_{flow} \\ -2(Y_{cp} + bline_k)e_p + gline_k f_q + bline_k e_q & Z_i = Q_{flow} \\ e_p/\sqrt{e_p^2 + f_p^2} & Z_i = V_{mag} \end{cases} \tag{5.13}$$

$$\frac{\partial h_i}{\partial f_p} = \begin{cases} 2G_{pp}f_p + \sum_{q \neq p}^{N} G_{pq}f_q + B_{pq}e_q & Z_i = P_{inj} \\ -2B_{pp}f_p + \sum_{q \neq p}^{N} G_{pq}e_q - B_{pq}f_q & Z_i = Q_{inj} \\ 2gline_k f_p - gline_k f_q - bline_k e_q & Z_i = P_{flow} \\ -2(Y_{cp} + bline_k)f_p - gline_k f_q + bline_k f_q & Z_i = Q_{flow} \\ f_p/\sqrt{e_p^2 + f_p^2} & Z_i = V_{mag} \end{cases} \tag{5.14}$$

$$\frac{\partial h_i}{\partial e_q} = \begin{cases} G_{pq}e_p + B_{pq}f_p & Z_i = P_{inj} \\ G_{pq}f_p - B_{pq}e_p & Z_i = Q_{inj} \\ -gline_k e_p - bline_k f_p & Z_i = P_{flow} \\ -gline_k f_p + bline_k e_p & Z_i = Q_{flow} \\ 0 & Z_i = V_{mag} \end{cases} \tag{5.15}$$

$$\frac{\partial h_i}{\partial f_q} = \begin{cases} G_{pq}f_p - B_{pq}e_p & Z_i = P_{inj} \\ -G_{pq}e_p - B_{pq}f_p & Z_i = Q_{inj} \\ -gline_k f_p + bline_k e_p & Z_i = P_{flow} \\ gline_k e_p + bline_k f_p & Z_i = Q_{flow} \\ 0 & Z_i = V_{mag} \end{cases} \tag{5.16}$$

$$\Delta Z = Z - Z_{cal/X_0} \tag{5.17}$$

In equation.5.11 ΔX is the difference between the present state and previous state.

$$\Delta Z = H[X - X_0] + e \tag{5.18}$$

Take HX_0 to the left hand side of equation.5.18

$$\Delta Z + HX_0 = HX + e \tag{5.19}$$

$\Delta Z + HX_0$ given in equation.5.19 is termed as Z_s^{new}

$$Z_s^{new} = HX + e \tag{5.20}$$

Therefore, Z_s^{new} is

$$Z_s^{new} = \Delta Z + HX_0 \tag{5.21}$$

The relationship between Z_s^{new} and state vector becomes linear. As equation(5.21) is linear, the solution vector can be obtained in a single step and the process becomes non-iterative. The intermediate state vector X_{int} is computed using equation(5.22)

$$X_{int} = (H^T W^{-1} H)^{-1} H^T W^{-1} Z_s^{new} \tag{5.22}$$

5.4.2 Stage2: PMU measurements processing

In stage2, for processing both PMU measurements and intermediate state vector, linear state estimation model proposed in [50] and reproduced in subsection.4.2.3 is used for obtaining the final states of the system.

5.4.3 Pseudo SCADA measurements generation

As PMUs are only available in limited number, PMU measurements alone can't give complete observability of the system. Therefore, for tracking the system state in all PMU measurement reporting arrivals, pseudo measurements are required to be generated during two successive SCADA reporting intervals. Previous time instant estimated state vector is used for obtaining the pseudo SCADA measurements. The flowchart of the proposed linear sequential hybrid state estimator is shown in figure.5.2

5.4.4 Linear sequential hybrid estimator step by step procedure

- Stage 1

    1. Get the SCADA measurements and previous instant state vector
    2. Process the measurements using proposed linear model
    3. Check for presence of bad data using largest normalized residue test
    4. If presence of bad data is found repeat step2 by eliminating the detected bad measurements

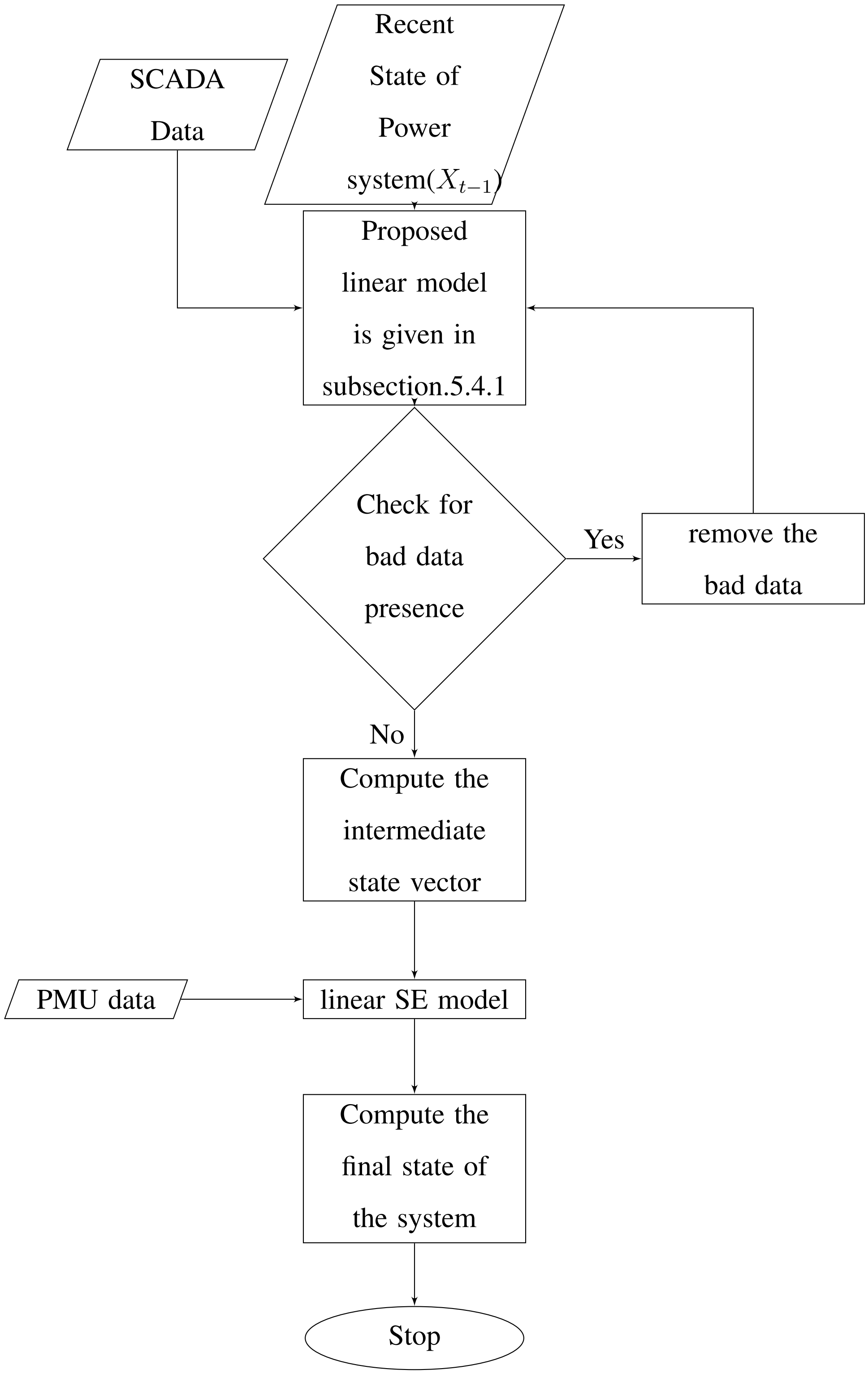

Figure 5.2: Flow chart of the linear sequential hybrid state estimator

5. Evaluate the intermediate state vector

- Stage 2

1. Take PMU measurements and intermediate state vector as a input.

2. Process the measurements using linear state estimation model

3. Estimate the final states of the power system

5.5 Case study and results

The effectiveness of the proposed linear sequential hybrid state estimator is tested on IEEE 14, 30, 57 and 118-Bus test systems. Newton-Raphson power flow algorithm is used for generating true values of the measurement set. Gaussian errors having standard deviations $\sigma_{SCADA} = 0.01$ & $\sigma_{PMU} = 0.001$ are correspondingly used. It is assumed that SCADA measurements arrival rate is 25 times slower than PMU measurements. Python programming with SPYDER IDE environment is used for coding the proposed algorithm. Mean square errors(MSE) of voltage and phase angle for every simulation are computed using equation(5.23).

$$MSE = \sqrt{(1/N)\sum_{i=1}^{N}(x_i^{estimated} - x_i^{true})^2} \tag{5.23}$$

All bus voltage magnitude measurements, real and reactive power flow measurements and real and reactive power injection measurements are considered as SCADA measurements for all test systems.

For comparing the proposed method with ANN based method [59], the training of the radial basis function neural network(RBFNN) is done with 200 load variation patterns of real time load curve data taken from PJM market [79] . For every load variation corresponding set of PMU measurements and estimated state vector are recorded. These are used for training the ANN network. The input to RBFNN is measurement set and output is state vector. MATLAB nntool box is used for implementing RBFNN based power system state estimation using PMU measurements.

5.5.1 IEEE 14- Bus system & 30- Bus system

For IEEE 14- Bus system, 4 PMUs are required to make the system completely observable. For checking the efficacy of the proposed linear sequential hybrid state estimator(LSHSE) only two PMU locations are considered in this study.

100 simulations are carried out by considering one time instant load change from previous to present instant. MSE variations of the voltage for all runs are plotted in figure.5.3. From the plots, it is observed that estimation accuracy of the ANN based method is very inferior compared with other techniques including proposed method for the considered PMU locations.

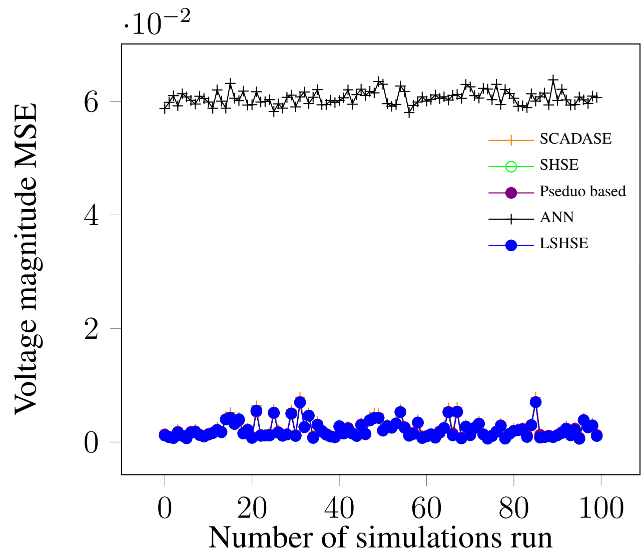

Figure 5.3: Comparison of voltage magnitude MSE of proposed LSHSE with SCADASE, ANN, Pseduo based & SHSE for IEEE 14-bus system

For IEEE 30- Bus system, ten PMUs are needed for making system completely observable. For checking the effectiveness of the proposed LSHSE only five PMU locations are considered in this study.

100 simulations are carried out by considering one time instant load change from previous to present instant. MSE variations of the voltage and current for all runs are plotted in figure.5.4 and 5.5. From the plots, it is observed that estimation accuracy of the proposed method is competing

with sequential hybrid state estimator. MSE variations of the proposed method are coinciding with the sequential hybrid state estimator for all simulation runs and is superior than traditional estimator and pseduo measurement based method [57].

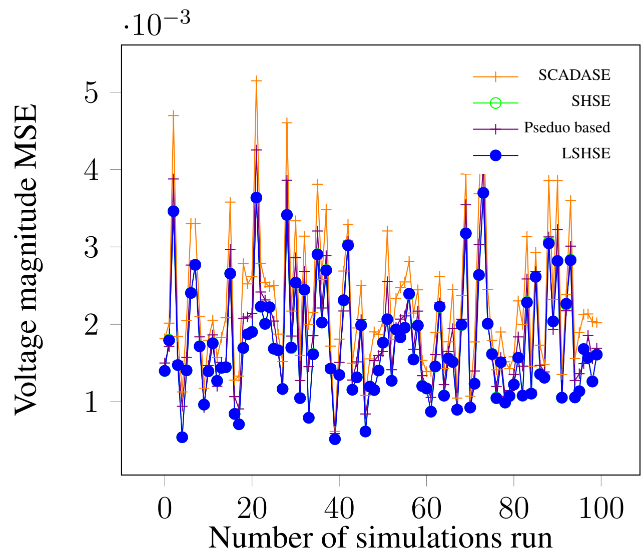

Figure 5.4: Comparison of voltage magnitude MSE of proposed LSHSE with SCADASE, Pseudo based & SHSE for IEEE 30-bus system

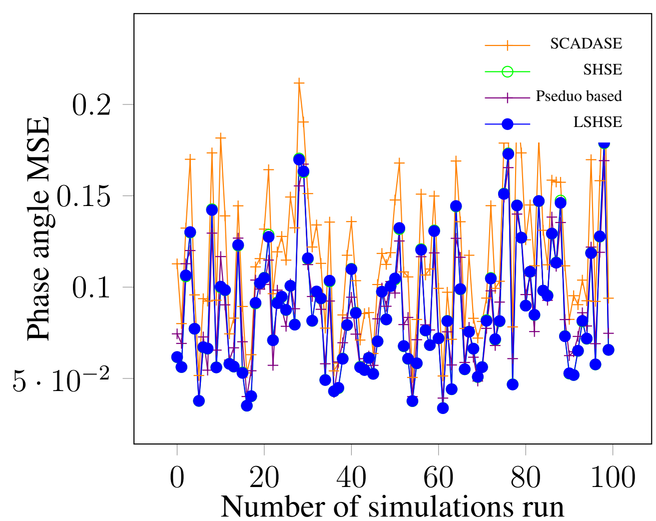

Figure 5.5: Comparison of Phase angle MSE of proposed LSHSE with SCADASE, Pseudo based & SHSE for IEEE 30-bus system

5.5.2 IEEE 57-Bus & IEEE 118-Bus system

For IEEE 57-Bus system, 17 PMUs are needed for making system completely observable. 8 PMU locations are considered for testing the effectiveness of the proposed linear sequential hybrid state estimator.

For IEEE 118-Bus system, 32 PMUs are needed for making the system completely observable. 17 PMU locations are considered for testing the effectiveness of the proposed linear sequential hybrid state estimator(LSHSE).

100 simulations are carried by considering one time instant load change from previous to present instant. MSE variations of the voltage and current for all runs are plotted in figure.5.6 and 5.7. From the plots, it is observed that estimation accuracy of the proposed method is competing with sequential hybrid state estimator. The same kind of responses are reported for IEEE 57-Bus system. MSE variations of the proposed method are either better or coinciding with the sequential hybrid state estimator for all simulation runs and is superior than the traditional estimator and pseudo measurement based method.

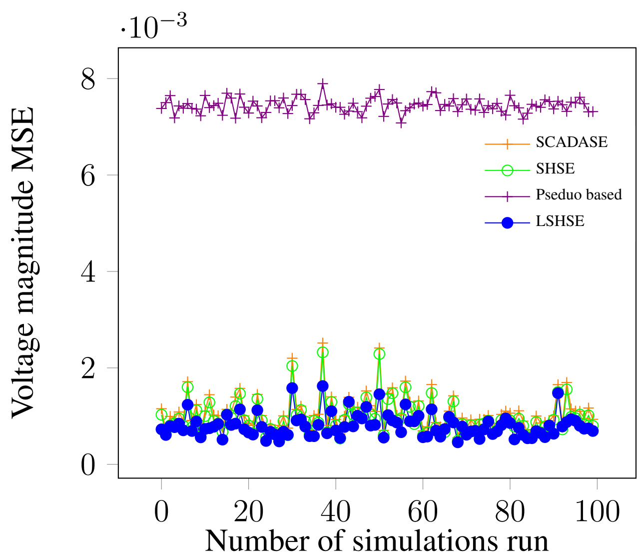

Figure 5.6: Comparison of voltage magnitude MSE of proposed LSHSE with SCADASE, Pseudo based & SHSE for IEEE 118-bus system

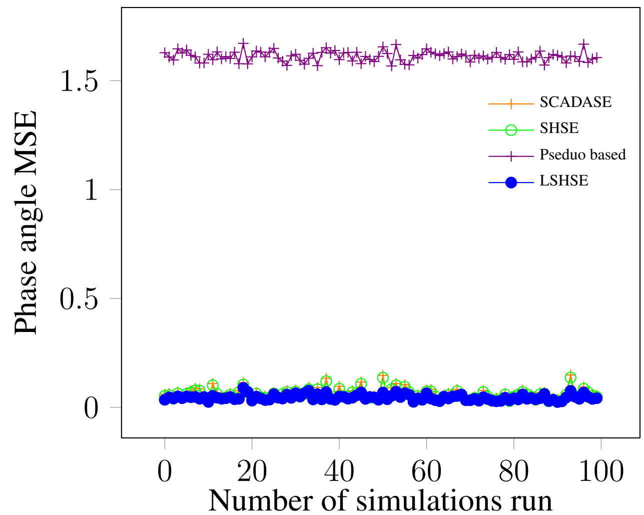

Figure 5.7: Comparison of Phase angle MSE of proposed LSHSE with SCADASE, Pseudo based & SHSE for IEEE 118-bus system

Table 5.1: Comparison of Voltage magnitude and phase angle MSE of proposed LSHSE with SHSE , Pseudo measurement based method, SCADASE

Test system	SCADASE			SHSE			Pseduo based method			LHSE		
	VMSE	PMSE	Time	VMSE	PMSE	Time	VMSE	PMSE	Time	VMSE	PMSE	Time
IEEE14	0.00251889	0.0933072	0.007619sec	0.00216176	0.079611	0.0101sec	0.00222505	0.0819069	0.007619sec	0.00216207	0.0795978	0.0048sec
IEEE 30	0.00228392	0.117233	0.0199sec	0.0017368	0.0876459	0.0196sec	0.00189804	0.0892479	0.0199sec	0.00173643	0.0876563	0.0061sec
IEEE 57	0.00226102	0.137517	0.0389sec	0.000963133	0.0496114	0.0718sec	0.00199334	0.435973	0.0389sec	0.000940506	0.0487705	0.0146sec
IEEE 118	0.00107783	0.0657681	0.1786sec	0.000958556	0.0580434	0.2218sec	0.00743882	1.6109	0.1786sec	0.000803559	0.0458205	0.0405sec

From the table.5.1 , The proposed linear sequential hybrid state estimator(LSHSE) estimation accuracy is better than the traditional non-linear method and competing with sequential hybrid state estimator. The estimation accuracy of the pseudo measurement based method is deteriorating as the order of the system is increasing. Due to the non-iterative nature of the proposed method, computational time is drastically reduced compared with other three methods.

5.5.3 Sensitivity analysis

Sensitivity analysis [80] is done on the proposed method for different PMU measurements and SCADA measurements standard deviation variations. State variance obtained for different combinations of $\pm 50\%$ variation of standard deviation of PMU and SCADA measurements is presented in table.5.2. It is observed that, the state variance of proposed LSHSE is less compared with SCADASE. The influence of variation of standard deviation of SCADA measurements is high compared with standard deviation of PMU measurements on the state variance.

Table 5.2: Sensitivity analysis comparison of proposed LHSE with SCADASE & SHSE

σ_{SCADA}	σ_{PMU}	SCADASE	SHSE	LHSE
0.005	0.0005	0.0010972	0.000546229	0.000546229
0.01	0.0005	0.00219501	0.00102612	0.00102612
0.015	0.0005	0.00329338	0.00149456	0.00149456
0.005	0.001	0.00109726	0.000588619	0.000588619
0.01	0.001	0.00219411	0.00109246	0.00109246
0.015	0.001	0.00329125	0.00157717	0.00157717
0.005	0.0015	0.00109726	0.000617386	0.000617386
0.01	0.0015	0.00219434	0.00113965	0.00113965
0.015	0.0015	0.00329114	0.00163869	0.00163869

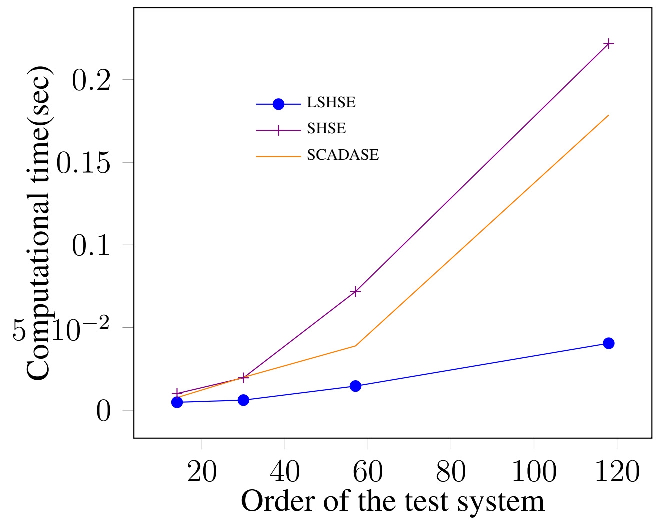

Figure 5.8: Comparison of computational time of proposed LSHSE with SCADASE and SHSE with increasing order of the system

Comparison of computational time of the proposed method with traditional estimator and sequential hybrid state estimator is plotted in figure.5.8. It is observed that, the proposed method is computationally efficient and superior than SCADASE estimator and sequential hybrid state estimator.

5.6 Summary and comments

In this chapter, a linear sequential hybrid state estimator(LSHSE) is proposed to efficiently utilize the limited number of PMUs deployed into the system along with existing SCADA measurements. This paper proposed a linear mathematical model by assuming that, in between the two PMU reporting rates the load variations are following linearity. In stage1, proposed linear model is used for processing SCADA measurements to compute the intermediate state vector. In stage 2, linear SE model is used for processing both PMU measurements and the intermediate states obtained in stage 1 to get the final states of the power system. The estimation accuracy of the proposed method is better compared with the traditional estimator, pseudo measurement based method, ANN based method and is competing with sequential hybrid state estimator. The proposed method is non-iterative and computationally very efficient compared with the traditional estimator, pseudo measurement based method and sequential hybrid state estimator. Finally, the proposed method is exhibiting better performance than sequential hybrid state estimator or two stage hybrid state estimator.

Chapter 6

Linear tracking single stage hybrid power system state estimator

6.1 Introduction

This chapter explored the idea of single stage linear hybrid state estimator by combining the transformed SCADA measurements in chapter 5 with PMU measurements.

6.2 Modeling of Linear hybrid state estimator

This section presents a linear single stage hybrid state estimation model(LHSE) for estimating the states of power system considering both SCADA and PMU measurements. From the discussion in chapter 5, it is evident that the transformed SCADA measurements are also having linear relation with state vector. These transformed measurements are combined with PMU measurements to get a single stage hybrid model. The transformed SCADA measurements are given in equation(6.1)

$$Z_{SCADA}^{new} = HX + e \tag{6.1}$$

The relationship between PMU measurements and state vector is linear and is given in equation.6.2

$$Z_1 = H_1 X + e \tag{6.2}$$

Combining the modeled new SCADA measurement set with PMU measurement set gives

$$Z_{new} = \begin{bmatrix} Z_{SCADA}^{new} \\ Z_1 \end{bmatrix} \tag{6.3}$$

Therefore, the new design matrix is

$$H_{new} = \begin{bmatrix} H \\ H_1 \end{bmatrix} \tag{6.4}$$

Z_{new} is the new measurement set including both transformed SCADA and PMU measurements. It has a linear relation with state vector and is given in equation(6.5)

$$Z_{new} = H_{new} X + e \tag{6.5}$$

By applying weighted least square technique, the state vector solution is given in equation.6.6. It is non- iterative.

$$X = (H_{new}^T W^{-1} H_{new})^{-1} H_{new}^T W^{-1} Z_{new} \tag{6.6}$$

$$W = \begin{bmatrix} W_1 & 0 \\ 0 & W_2 \end{bmatrix} \tag{6.7}$$

Where W is the weight matrix or inverse co-variance matrix. W_1 and W_2 are inverse co-variance matrices for SCADA and PMU measurements.

6.2.1 Pseudo SCADA measurement generation

It is known that, PMU measurements alone can't give complete observability of the system. For tracking the state of the power system in the instances of SCADA not reporting intervals, SCADA measurements are calculated using previous state vector of the power system. The calculated SCADA measurements are used as pseudo measurements in SCADA not reporting intervals.

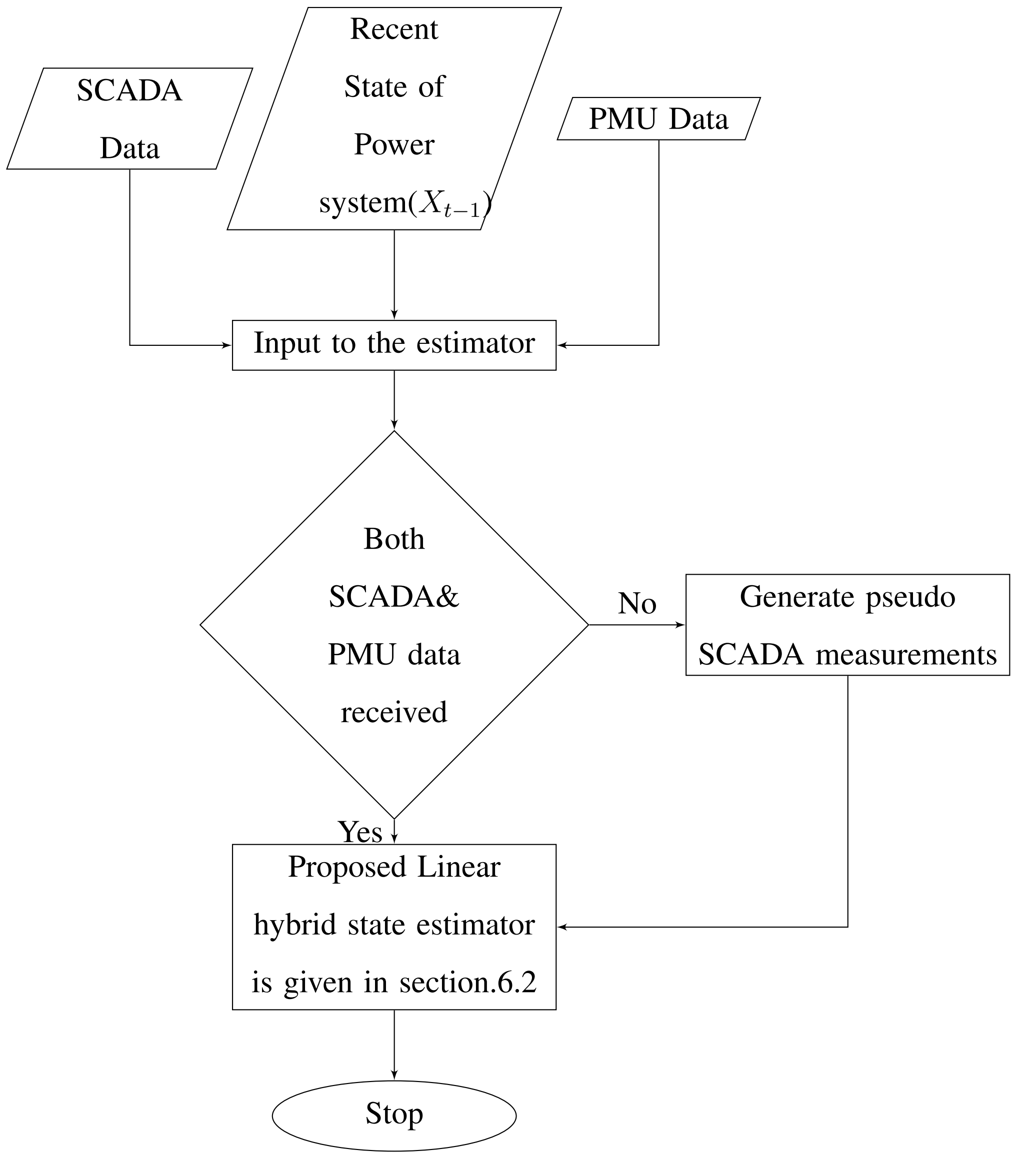

Figure 6.1: Flow chart of the linear hybrid state estimator

The flow chart of the proposed linear hybrid estimator is shown in figure.6.1. When both SCADA and PMU data are available, state vector is obtained using the proposed linear technique. If only PMU data is available, SCADA measurements are predicted with the help of previous instance state vector. Obtain the state vector using proposed linear hybrid state estimator using PMU measurements and predicted pseudo SCADA measurements

6.3 Results and discussions

The efficacy of the proposed linear hybrid state estimator is evaluated on IEEE 14 ,30, 57 and 118-Bus test systems. True values of the measurements are generated using N-R power flow algorithm. 100 sets of measurement data are simulated by adding Gaussian errors having standard deviations $\sigma_{PMU} = 0.001$ & $\sigma_{SCADA} = 0.01$. It is assumed that PMU measurements reporting rate is 25 times faster than SCADA measurements reporting rate. For programming the proposed algorithm in PC, Python SPYDER IDE platform is used. Voltage magnitude and phase angle mean square error for each simulation is computed using equation(6.8)

$$MSE = \sqrt{(1/N)\sum_{i=1}^{N}(x_i^{estimated} - x_i^{true})^2} \tag{6.8}$$

6.3.1 IEEE 14- Bus system & 30- Bus system

For IEEE 14-Bus system, 13 injections and 24 line flows are taken as SCADA measurements [5] these being P_3, P_{12}, P_{15}, P_6, P_9, P_{10}, P_{13}, Q_{15}, Q_3, Q_9, Q_6, Q_{10}, Q_{13}, P_{23}, P_{52}, P_{47}, P_{49}, P_{612}, P_{613}, P_{611}, P_{78}, P_{87}, P_{910}, P_{914}, P_{1213}, Q_{23}, Q_{52}, Q_{47}, Q_{49}, Q_{612}, Q_{613}, Q_{611}, Q_{78}, Q_{87}, Q_{910}, Q_{914}, Q_{1213}. 4 PMU measurements are required for making the system completely observable(2,7,11,13) through PMUs. only 2 PMUs(2, 13) are considered for testing the proposed linear hybrid state estimator.

For IEEE 30- Bus system, all real and reactive power injections, all real and reactive power flows and all bus voltage magnitude measurements are taken as SCADA measurements. 10 PMUs are required for making system completely observable. Only 5 PMUs(2, 6, 10, 15 ,25) are considered for testing the proposed linear hybrid state estimator.

100 simulations are run by considering one time instance load change from previous to present instant . The proposed method Voltage magnitude MSE and Phase angle MSE variations for all runs are plotted in figure.6.2 and 6.3 for IEEE 14-bus system. It is observed that, the proposed method is performing better than the traditional estimator and two stage hybrid state estimator.

Similar responses are observed for IEEE 30-Bus system.

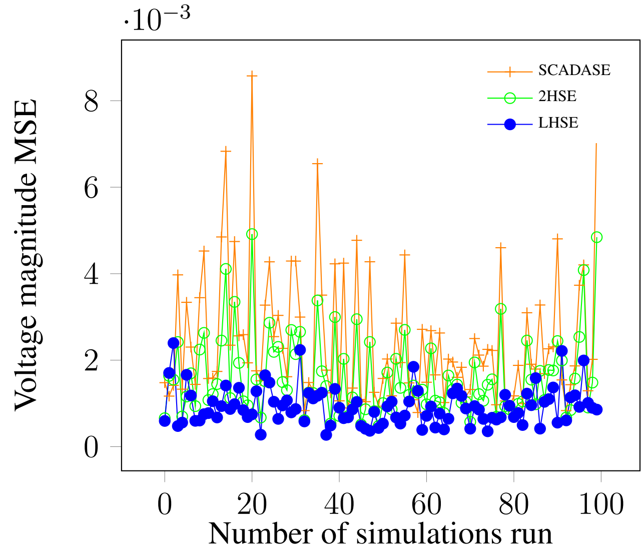

Figure 6.2: Comparison of voltage magnitude MSE of proposed LHSE with SCADASE & 2HSE for IEEE 14-bus system

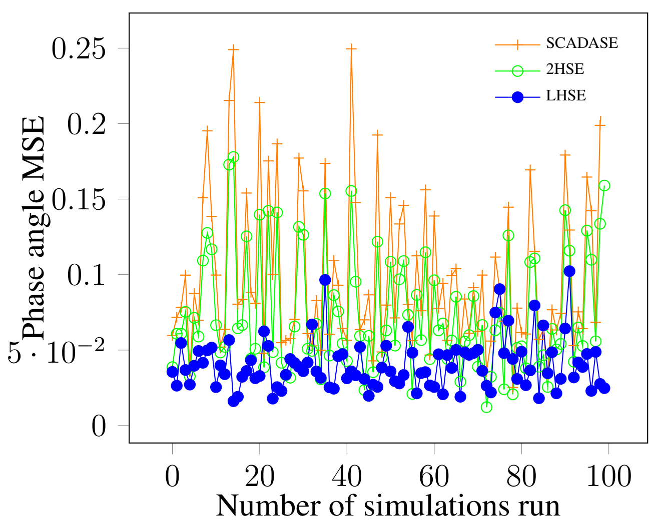

Figure 6.3: Comparison of Phase angle MSE of proposed LHSE with SCADASE & 2HSE for IEEE 14-bus system

6.3.2 IEEE 57- Bus system & IEEE 118-Bus system

For IEEE 57- Bus system, all real power flows(80), reactive power flows(80), voltage magnitude(57) measurements are taken as SCADA measurements. 17 PMU measurements are required for making system completely observable(1,4,6, 9,15,20,24,28,30,32,36,38,41,47,51,53,57). But, only 8 PMUs (1, 6, 24, 38, 57, 41, 51, 53) are taken for testing the proposed linear hybrid state estimator.

For IEEE 118-Bus system, all real and reactive power injections, all real and reactive power flows and all voltage magnitude measurements are taken as SCADA measurements. 32 PMUs are required for making the system completely observable. Only 17 PMUs are considered for testing the proposed linear hybrid state estimator.

100 simulations are run by considering one time instance load change from previous to present instant . The proposed method Voltage magnitude MSE and Phase angle MSE variations for all runs are plotted in figures.6.4 and 6.5. It is observed that, the proposed method is performing better than the traditional estimator and two stage hybrid state estimator. Similar responses are observed for IEEE 118-Bus system.

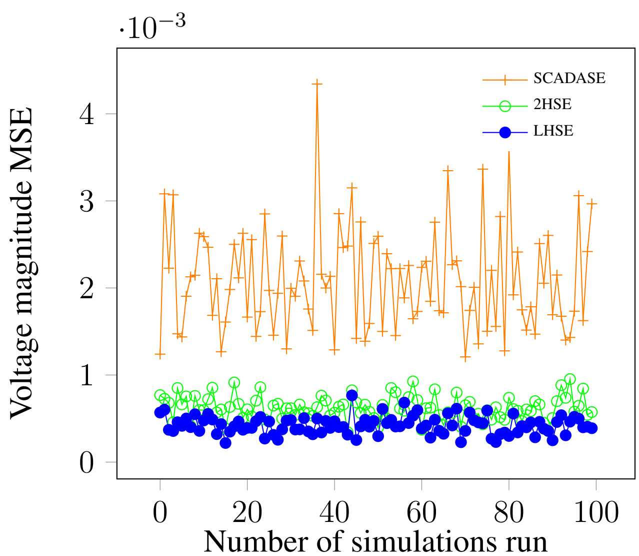

Figure 6.4: Comparison of voltage magnitude MSE of proposed LHSE with SCADASE & 2HSE for IEEE 57-bus system

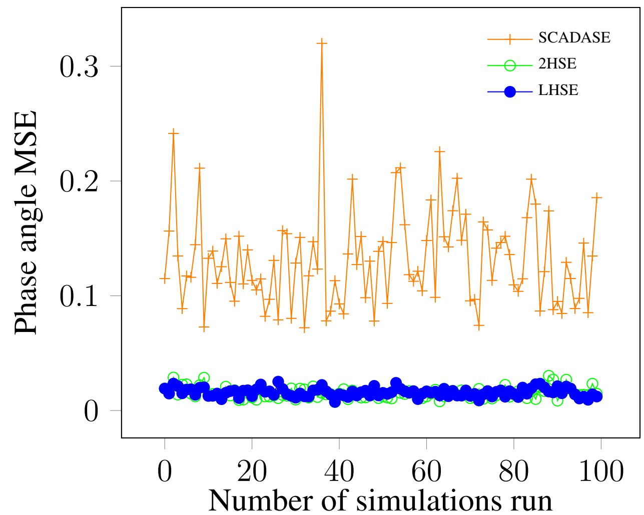

Figure 6.5: Comparison of Phase angle MSE of proposed LHSE with SCADASE & 2HSE for IEEE 57-bus system

Table 6.1: Comparison of Voltage magnitude and phase angle MSE of proposed LHSE with 2HSE and SCADASE

Test System	SCADASE			2HSE			LHSE		
	VMSE	PMSE	Computational time	VMSE	PMSE	Computational time	VMSE	PMSE	Computational time
IEEE 14	0.00246782	0.0916694	0.007619sec	0.00160932	0.0670478	0.0101sec	0.00088168	0.0397719	0.00298sec
IEEE 30	0.00234384	0.114783	0.0199sec	0.00140954	0.0703057	0.0196sec	0.000828491	0.041867	0.00936sec
IEEE 57	0.00208864	0.130758	0.0389sec	0.000602425	0.0157629	0.0718sec	0.000418998	0.016595	0.0208sec
IEEE 118	0.00100078	0.0640778	0.1236sec	0.000917665	0.0385209	0.1198sec	0.000424498	0.0219884	0.046sec

From table.6.1 , it is apparent that the estimation accuracy of the proposed linear hybrid state estimator(LHSE) is better than the traditional non-linear method and two stage hybrid state estimator. Due to the non-iterative nature of the proposed method, computational time is drastically reduced.

6.3.3 Tracking ability of the proposed algorithm

The proposed method is tested for load variations of upto 60 seconds interval of the load curve. The load curve data is taken from the PJM market [28] and is given in figure.6.6. Normalized load curve data is used for determining the actual load curve data of the IEEE 118-Bus test system.

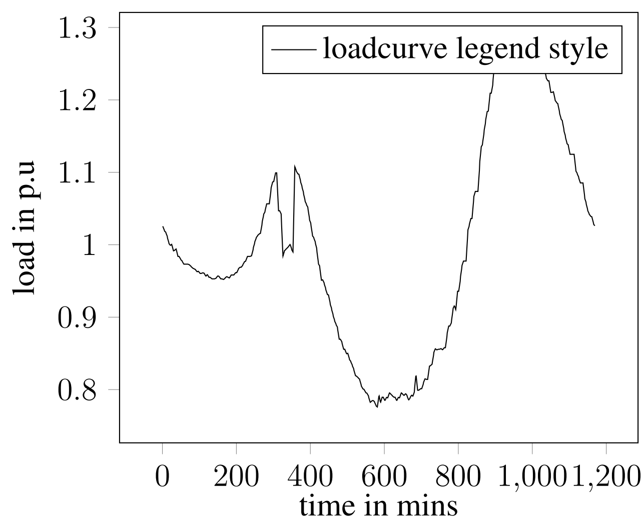

Figure 6.6: Daily load curve of IEEE 118 - Bus system

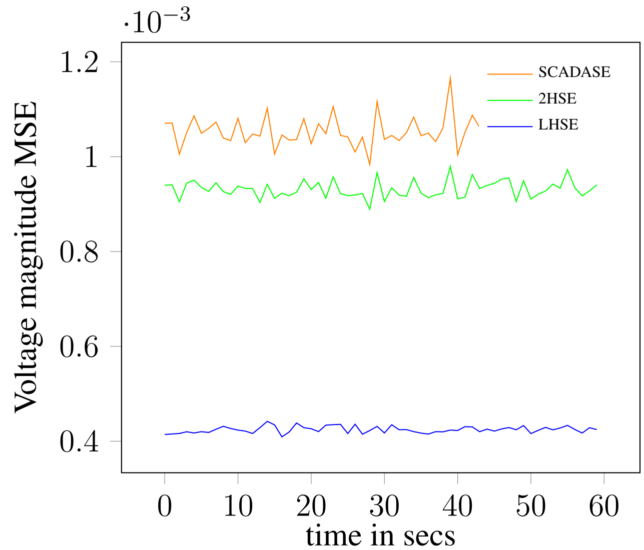

Figure 6.7: Comparison of voltage magnitude MSE of proposed LHSE with SCADASE & 2HSE for a duration of 60 seconds for IEEE 118 - Bus system

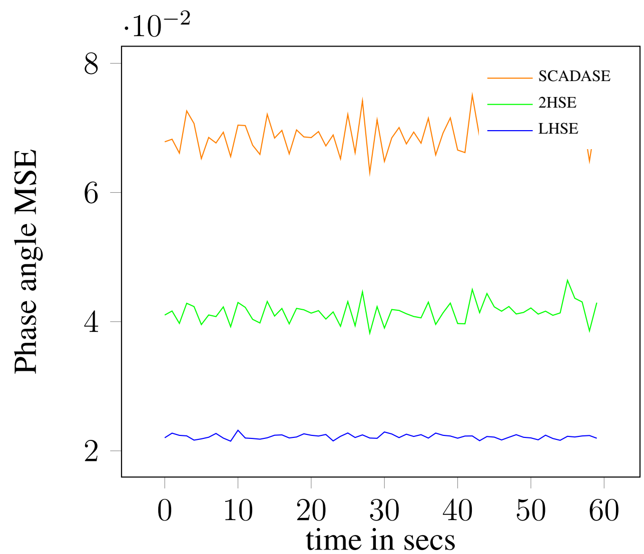

Figure 6.8: Comparison of phase angle MSE of proposed LHSE with SCADASE & 2HSE for a duration of 60 seconds for IEEE 118 - Bus system

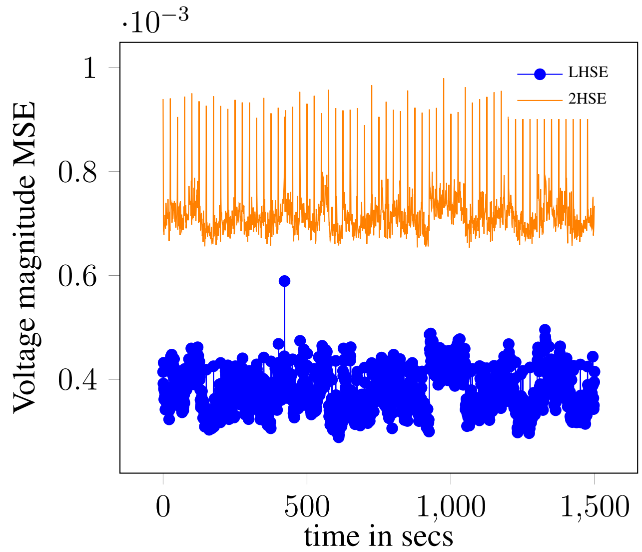

Figure 6.9: Comparison of voltage magnitude MSE of proposed LHSE and 2HSE for a duration of 60 seconds in all PMU reporting intervals for IEEE 118 - Bus system

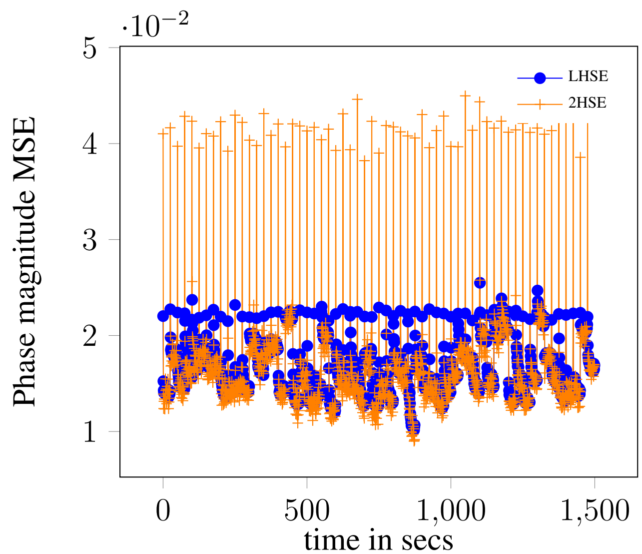

Figure 6.10: Comparison of phase angle MSE of proposed LHSE and 2HSE for a duration of 60 seconds in all PMU reporting intervals for IEEE 118 - Bus system

Table 6.2: Salient features of the LHSE and SCADASE

Feature	LHSE	SCADASE
Relation with measurements	Linear	Non-linear
Measurements considered	PMU & SCADA	Only SCADA
Estimation accuracy	More	Less
Computational time	Less	More
Solution process	Non-iterative	Iterative

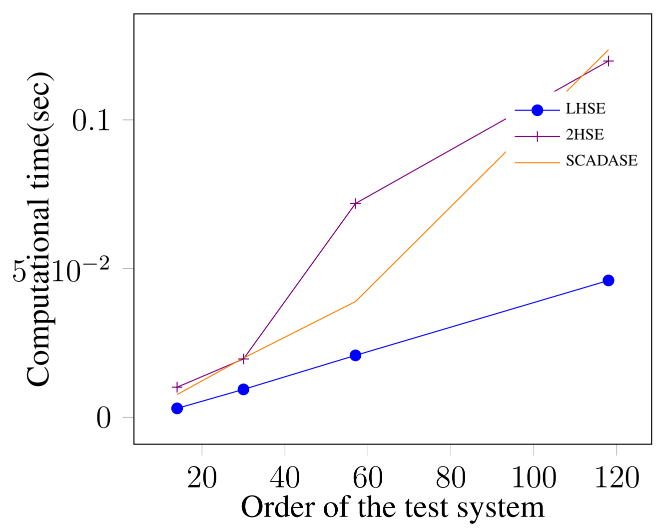

Figure 6.11: Comparison of computational time of LHSE with SCADASE while increasing order of the system

100 simulations are run for every instant of load change during 60 seconds interval of the load curve. The proposed method voltage magnitude MSE and phase angle MSE variation during 60 second interval for all SCADA measurement reporting rates are plotted in figures.6.7 & 6.8 and for all PMU measurements the reporting rates are plotted in figure.6.9 & 6.10. The comparison of MSEs with traditional estimator and two stage hybrid state estimator for all SCADA reporting rates are shown in figures.6.7 and 6.8 and for all PMU reporting rates in figures.6.9 & 6.10.

The voltage magnitude MSE and phase angle MSE of the proposed method per 100 simulations per every instance during 60 seconds interval of the load curve are given in figure.6.7 & 6.8. The voltage magnitude MSE and phase angle MSE of the proposed method per 100 simulations at every instance during the 60 second interval of the load curve in all PMU reporting rates are given in figures.6.9 & 6.10. It is observed that the proposed method has the ability to track the system in all PMU reporting rates. The salient features of the proposed method are compared with the traditional estimator and given in table.6.2. The computational time comparison plot of the proposed method versus traditional estimator and two stage hybrid state estimator is given in figure.6.11. It is observed that the proposed method is computationally efficient and superior than SCADASE estimator and two stage hybrid state estimator.

6.4 Summary and Comments

This chapter proposed a linear hybrid state estimator by integrating PMU and SCADA measurements. A linear single stage state estimation model is proposed by assuming variations in the power system in between the two successive PMU reporting rates are following linearity. The estimation accuracy of the proposed method is better than traditional non-linear SCADA based estimator and two stage hybrid state estimator. The proposed method is non-iterative, as it is having linear relationship between the states and measurements. The computational time of the proposed method is 4 to 5 times lower than that of the traditional state estimator and two stage hybrid state estimator. Therefore, the proposed method not only shows better estimation accuracy and computational superiority, it also has the ability to track in all PMU reporting rates.

Chapter 7

Conclusions and Future Scope

7.1 Conclusions

The present day power system demands a more effective WAMPAC system for it's monitoring in a better manner than the existing one. Since, SCADA measurements reporting rates are much slower, with SCADA alone utilities can't meet effective monitoring requirement. Recently introduced PMUs reporting rates are much faster than SCADA measurements and are also more accurate. The objective of the present study is i) to provide better PMU deployment models into the power system state estimation, ii) to provide effective models for PMU only assisted state estimation techniques and iii) to provide hybrid state estimation models for utilizing PMU and SCADA measurements in the power system state estimation. The following contributions are made.

In chapter 2, a topology based OPP method- I is proposed. The proposed method provides multiple optimal PMU locations without using any conventional or heuristic optimization techniques by working only on the binary connectivity matrix. The drawback of the proposed method is that it failed to get multiple solutions for different cases such as line contingency, loss of PMU and considering channel limit.

In chapter 3, a novel topology based Optimal PMU placement method-II is proposed to obtain a whole set of multiple optimal solutions. The proposed method works only on binary connectivity matrix of the system without using any classical or meta-heuristic optimization technique. Once multiple optimal solution set is available, the solution which best fits the utility sub - ordinate objectives like direct monitoring of generator and weak buses ,etc can be selected. The proposed method assures global optima. From the results, It is observed that OPP problem is a special optimization problem having multiple global optimal solutions. In addition, this chapter also proposed normalized BOI(NBOI) and SORI(NSORI). Unlike SORI, the solution which has maximum NSORI has maximum individual bus redundancy uniformity. NSORI also has the ability to further prune multiple solutions. As the algorithm progresses, the solution matrix length increases. Due to this, personal computers having medium RAM capacity are not sufficient to run the algorithm for large power systems. But, this method is robust for placement of PMUs in multi-area power systems. The advantage of obtaining the biopsy of solution space in one run cannot be overlooked.

Chapter 4 explored the ability of robust estimators under assumption that entire power system is observable by PMUs. The Quadratic-Constant criterion based estimator is studied and improved to make it suitable for PMU assisted state estimation. The proposed criterion is termed as Quadratic-decaying Exponential criterion. The estimation accuracy of the QE estimator is competing with WLSE under the presence of Gaussian noise and its performance is superior under the presence of corrupt data. The computational performance of the QE estimator is little low compared with WLSE under the presence of Gaussian noise and superior under the presence of corrupt data. The computational performance of the QE estimator is independent of the number of bad measurements in the measurement set. The estimation accuracy of LAV compared with QE had some outliers with a large deviation from median and the computational performance increases greatly with the order of the system compared with QE. Therefore, the proposed estimator exhibits superior performance compared with LAV estimator.

In chapter 5, a linear sequential hybrid state estimator is proposed to effectively utilize the limited number of PMUs deployed in the power system along with existing SCADA measurements. It is assumed that, in between the two PMU reporting rates the load variations are following linearity. In stage1, a linear model is proposed for processing SCADA measurements to compute the intermediate states. In stage 2, linear SE model is used for processing both PMU measurements and the intermediate states obtained in stage 1 to get the final states of the power system. The estimation accuracy of the proposed method is better compared with traditional estimator and is competing with sequential hybrid state estimator. The proposed method is both non-iterative and computationally very efficient. The proposed method is exhibiting better performace than the Sequential hybrid state estimator or two stage hybrid state estimator

Chapter 6 proposed a linear hybrid state estimator by integrating PMU and SCADA measurements. The linear single stage state estimation model is proposed by assuming that, the variations in the power system in between the two successive PMU reporting rates are following linearity. The estimation accuracy of the proposed method is better than the traditional non-linear SCADA based estimator and two stage hybrid state estimator. The proposed method is non-iterative, as it is having a linear relationship between the states and measurements. The computational time of the proposed method is 4 to 5 times less than the traditional state estimator and two stage hybrid state estimator. Therefore, the proposed method not only shows better estimation accuracy and computational superiority, it also has the ability to track all PMU reporting rates.

7.2 Future Scope

This thesis can be further extended in the following areas

- The topology based optimal PMU placement strategies will generate very large matrix as the order of the system increases. The mechanism to reduce the size or eliminate some unimportant nodes is very essential.
- A method or mechanism is required for appropriate selection of break even point τ with varying system sizes.
- Application of the proposed topology based methods for deployment of PMUs for providing security to cyber attacks and for multi- area state estimation.
- Application of the proposed quadratically decaying exponential criterion for integrating both

PMU and SCADA measurements and for PMU only assisted multi-area state estimation.

- The feasibility testing of the proposed methods with respect to distribution system state estimation and micro grid state estimation has to be further examined.

Bibliography

[1] T. L. Baldwin, L. Mili, M. B. Boisen, and R. Adapa, "Power system observability with minimal phasor measurement placement," *IEEE Transactions on Power Systems*, vol. 8, no. 2, pp. 707–715, May 1993.

[2] B. D. Milosevic and M. Begovic, "Nondominated sorting genetic algorithm for optimal phasor maesurement placement," *IEEE Power Engineering Review*, vol. 22, no. 12, pp. 61–61, Dec 2002.

[3] B. Xu and A. Abur, "Observability analysis and measurement placement for systems with pmus," in *IEEE PES Power Systems Conference and Exposition, 2004.*, Oct 2004, pp. 943–946 vol.2.

[4] B. Gou, "Optimal placement of pmus by integer linear programming," *IEEE Transactions on Power Systems*, vol. 23, no. 3, pp. 1525–1526, Aug 2008.

[5] M. Korkali and A. Abur, "Placement of pmus with channel limits," in *2009 IEEE Power Energy Society General Meeting*, July 2009, pp. 1–4.

[6] S. Chakrabarti, E. Kyriakides, and D. G. Eliades, "Placement of synchronized measurements for power system observability," *IEEE Transactions on Power Delivery*, vol. 24, no. 1, pp. 12–19, Jan 2009.

[7] A. Ahmadi, Y. Alinejad-Beromi, and M. Moradi, "Optimal pmu placement for power system observability using binary particle swarm optimization and considering measurement redundancy," *Expert Systems with Applications*, vol. 38, no. 6, pp. 7263 – 7269, 2011. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0957417410013886

[8] A. Pal, G. A. Sanchez-Ayala, V. A. Centeno, and J. S. Thorp, “A pmu placement scheme ensuring real-time monitoring of critical buses of the network,” *IEEE Transactions on Power Delivery*, vol. 29, no. 2, pp. 510–517, April 2014.

[9] N. H. A. Rahman and A. F. Zobaa, “Integrated mutation strategy with modified binary pso algorithm for optimal pmus placement,” *IEEE Transactions on Industrial Informatics*, vol. 13, no. 6, pp. 3124–3133, Dec 2017.

[10] T. K. Maji and P. Acharjee, “Multiple solutions of optimal pmu placement using exponential binary pso algorithm for smart grid applications,” *IEEE Transactions on Industry Applications*, vol. 53, no. 3, pp. 2550–2559, May 2017.

[11] N. C. Koutsoukis, N. M. Manousakis, P. S. Georgilakis, and G. N. Korres, “Numerical observability method for optimal phasor measurement units placement using recursive tabu search method,” *IET Generation, Transmission Distribution*, vol. 7, no. 4, pp. 347–356, April 2013.

[12] K. Jamuna and K. Swarup, “Multi-objective biogeography based optimization for optimal pmu placement,” *Applied Soft Computing*, vol. 12, no. 5, pp. 1503 – 1510, 2012. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1568494612000075

[13] H. H. Müller and C. A. Castro, “Genetic algorithm-based phasor measurement unit placement method considering observability and security criteria,” *IET Generation, Transmission Distribution*, vol. 10, no. 1, pp. 270–280, 2016.

[14] G. N. Korres, N. M. Manousakis, T. C. Xygkis, and J. Löfberg, “Optimal phasor measurement unit placement for numerical observability in the presence of conventional measurements using semidefinite programming,” *IET Generation, Transmission Distribution*, vol. 9, no. 15, pp. 2427–2436, 2015.

[15] N. M. Manousakis and G. N. Korres, “Optimal pmu placement for numerical observability considering fixed channel capacity :a semidefinite programming approach,” *IEEE Transactions on Power Systems*, vol. 31, no. 4, pp. 3328–3329, July 2016.

[16] R. F. Nuqui and A. G. Phadke, "Phasor measurement unit placement techniques for complete and incomplete observability," *IEEE Transactions on Power Delivery*, vol. 20, no. 4, pp. 2381–2388, Oct 2005.

[17] D. Dua, S. Dambhare, R. K. Gajbhiye, and S. A. Soman, "Optimal multistage scheduling of pmu placement: An ilp approach," *IEEE Transactions on Power Delivery*, vol. 23, no. 4, pp. 1812–1820, Oct 2008.

[18] R. Sodhi and M. I. Sharieff, "Phasor measurement unit placement framework for enhanced wide-area situational awareness," *IET Generation, Transmission Distribution*, vol. 9, no. 2, pp. 172–182, 2015.

[19] S. Chakrabarti and E. Kyriakides, "Optimal placement of phasor measurement units for power system observability," *IEEE Transactions on Power Systems*, vol. 23, no. 3, pp. 1433–1440, Aug 2008.

[20] R. Sodhi, S. Srivastava, and S. Singh, "Optimal pmu placement method for complete topological and numerical observability of power system," *Electric Power Systems Research*, vol. 80, no. 9, pp. 1154 – 1159, 2010. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0378779610000647

[21] M. H. F. Wen, J. Xu, and V. O. K. Li, "Optimal multistage pmu placement for wide-area monitoring," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4134–4143, Nov 2013.

[22] S. Li and Z. Meng, "Optimal pmu placement based on improved binary artificial bee colony algorithm," in *2017 IEEE Transportation Electrification Conference and Expo, Asia-Pacific (ITEC Asia-Pacific)*, 2017, pp. 1–6.

[23] S. P. Singh and S. P. Singh, "A novel multi-objective pmu placement method for power system state estimation," in *2018 International Electrical Engineering Congress (iEECON)*, 2018, pp. 1–4.

[24] M. Shafiullah, M. I. Hossain, M. Abido, T. Abdel-Fattah, and A. Mantawy, "A modified optimal pmu placement problem formulation considering channel limits under various

contingencies," *Measurement*, vol. 135, pp. 875 – 885, 2019. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0263224118311862

[25] A. A. Laouid, M. Mounir Rezaoui, A. Kouzou, and R. D. Mohammedi, "Optimal pmus placement using hybrid pso-gsa algorithm," in *2019 4th International Conference on Power Electronics and their Applications (ICPEA)*, 2019, pp. 1–5.

[26] L. Sun, T. Chen, X. Chen, W. K. Ho, K. Ling, K. Tseng, and G. A. J. Amaratunga, "Optimum placement of phasor measurement units in power systems," *IEEE Transactions on Instrumentation and Measurement*, vol. 68, no. 2, pp. 421–429, 2019.

[27] N. M. Manousakis and G. N. Korres, "Optimal allocation of phasor measurement units considering various contingencies and measurement redundancy," *IEEE Transactions on Instrumentation and Measurement*, vol. 69, no. 6, pp. 3403–3411, 2020.

[28] V. Basetti and A. K. Chandel, "Optimal pmu placement for power system observability using taguchi binary bat algorithm," *Measurement*, vol. 95, pp. 8 – 20, 2017. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0263224116305334

[29] L. Abdelkader Azzeddine, M. Ridh Djamel, K. Abdellah, and R. Mohamed Mounir, "Optimal pmu placement in power system based on multi-objective particle swarm optimization," in *2018 15th International Multi-Conference on Systems, Signals Devices (SSD)*, 2018, pp. 941–946.

[30] R. Babu and B. Bhattacharyya, "Strategic placements of pmus for power network observability considering redundancy measurement," *Measurement*, vol. 134, pp. 606 – 623, 2019. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0263224118310558

[31] M. Meenakshi Devi and M. Geethanjali, "Hybrid of genetic algorithm and minimum spanning tree method for optimal pmu placements," *Measurement*, vol. 154, p. 107476, 2020. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0263224120300130

[32] F. C. Schweppe and J. Wildes, "Power system static-state estimation, part i: Exact model," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-89, no. 1, pp. 120–125, 1970.

[33] A. Monticelli, F. F. Wu, and M. Yen, "Multiple bad data identification for state estimation by combinatorial optimization," *IEEE Power Engineering Review*, vol. PER-6, no. 7, pp. 73–74, 1986.

[34] A. Monticelli, "Reliable bad data processing for real-time state estimation," *IEEE Power Engineering Review*, vol. PER-3, no. 5, pp. 31–32, 1983.

[35] B. M. Zhang, S. Y. Wang, and N. D. Xiang, "A linear recursive bad data identification method with real-time application to power system state estimation," *IEEE Transactions on Power Systems*, vol. 7, no. 3, pp. 1378–1385, 1992.

[36] B. M. Zhang and K. L. Lo, "A recursive measurement error estimation identification method for bad data analysis in power system state estimation," *IEEE Transactions on Power Systems*, vol. 6, no. 1, pp. 191–198, 1991.

[37] E. Handschin, F. C. Schweppe, J. Kohlas, and A. Fiechter, "Bad data analysis for power system state estimation," *IEEE Transactions on Power Apparatus and Systems*, vol. 94, no. 2, pp. 329–337, 1975.

[38] M. R. Irving, R. C. Owen, and M. J. H. Sterling, "Power-system state estimation using linear programming," *Proceedings of the Institution of Electrical Engineers*, vol. 125, no. 9, pp. 879–885, 1978.

[39] L. Mili, V. Phaniraj, and P. J. Rousseeuw, "Least median of squares estimation in power systems," *IEEE Transactions on Power Systems*, vol. 6, no. 2, pp. 511–523, 1991.

[40] E. Caro and A. J. Conejo, "State estimation via mathematical programming: a comparison of different estimation algorithms," *IET Generation, Transmission Distribution*, vol. 6, no. 6, pp. 545–553, June 2012.

[41] S. Gastoni, G. Granelli, and M. Montagna, "Robust state-estimation procedure based on the maximum agreement between measurements," *IEEE Transactions on Power Systems*, vol. 19, no. 4, pp. 2038–2043, 2004.

[42] A. Al-Othman and M. Irving, "Robust state estimator based on maximum constraints satisfaction of uncertain measurements," *Measurement*, vol. 40, no. 3, pp. 347 – 359, 2007. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0263224106002442

[43] W. Wu, Y. Guo, B. Zhang, A. Bose, and S. Hongbin, "Robust state estimation method based on maximum exponential square," *IET Generation, Transmission Distribution*, vol. 5, no. 11, pp. 1165–1172, 2011.

[44] G. He, S. Dong, J. Qi, and Y. Wang, "Robust state estimator based on maximum normal measurement rate," *IEEE Transactions on Power Systems*, vol. 26, no. 4, pp. 2058–2065, 2011.

[45] Y. Chen, J. Ma, P. Zhang, F. Liu, and S. Mei, "Robust state estimator based on maximum exponential absolute value," *IEEE Transactions on Smart Grid*, vol. 8, no. 4, pp. 1537–1544, 2017.

[46] B. Wang, G. He, and S. Dong, "An improved algorithm for state estimator based on maximum normal measurement rate," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4879–4880, 2013.

[47] Y. Chen, J. Ma, and J. Zhang, "Robust state estimator based on hyperbolic cosine function," in *2017 IEEE Power Energy Society General Meeting*, 2017, pp. 1–2.

[48] M. Huang, Z. Wei, G. Sun, T. Ding, H. Zang, and Y. Zhu, "A multi-objective robust state estimator for systems measured by phasor measurement units," *IEEE Access*, vol. 6, pp. 14 620–14 628, 2018.

[49] Y. Chen, R. Chen, Y. Shen, P. Zhang, Z. Liu, Y. Lang, and X. Yang, "A novel robust state estimator based on rectangular pulse function," *IEEE Access*, vol. 7, pp. 113 525–113 533, 2019.

[50] M. Göl and A. Abur, "Lav based robust state estimation for systems measured by pmus," *IEEE Transactions on Smart Grid*, vol. 5, no. 4, pp. 1808–1814, July 2014.

[51] M. Zhou, V. A. Centeno, J. S. Thorp, and A. G. Phadke, "An alternative for including phasor measurements in state estimators," *IEEE Transactions on Power Systems*, vol. 21, no. 4, pp. 1930–1937, 2006.

[52] R. F. Nuqui and A. G. Phadke, "Hybrid linear state estimation utilizing synchronized phasor measurements," in *2007 IEEE Lausanne Power Tech*, 2007, pp. 1665–1669.

[53] T. Bi, X. Qin, and Q. Yang, "A novel hybrid state estimator for including synchronized phasor measurements," *Electric Power Systems Research*, vol. 78, no. 8, pp. 1343 – 1352, 2008. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0378779607002374

[54] R. Sodhi, S. C. Srivastava, and S. N. Singh, "An improved phasor assisted state estimator," in *2009 IEEE Power Energy Society General Meeting*, 2009, pp. 1–7.

[55] R. Sodhi, S. C. Srivastava, and S. N. Singh, "Phasor-assisted hybrid state estimator," *Electric Power Components and Systems*, vol. 38, no. 5, pp. 533–544, 2010. [Online]. Available: https://doi.org/10.1080/15325000903376925

[56] S. Chakrabarti, E. Kyriakides, G. Ledwich, and A. Ghosh, "A comparative study of the methods of inclusion of pmu current phasor measurements in a hybrid state estimator," in *IEEE PES General Meeting*, 2010, pp. 1–7.

[57] M. Asprou and E. Kyriakides, "A constrained hybrid state estimator including pseudo flow measurements," in *7th Mediterranean Conference and Exhibition on Power Generation, Transmission, Distribution and Energy Conversion (MedPower 2010)*, 2010, pp. 1–6.

[58] G. Valverde, S. Chakrabarti, E. Kyriakides, and V. Terzija, "A constrained formulation for hybrid state estimation," *IEEE Transactions on Power Systems*, vol. 26, no. 3, pp. 1102–1109, 2011.

[59] A. Kumar and S. Chakrabarti, "Ann-based hybrid state estimation and enhanced visualization of power systems," in *ISGT2011-India*, 2011, pp. 78–83.

[60] G. N. Korres and N. M. Manousakis, "State estimation and bad data processing for systems including pmu and scada measurements," *Electric Power Sys-*

tems Research, vol. 81, no. 7, pp. 1514 – 1524, 2011. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0378779611000769

[61] A. Simões Costa, A. Albuquerque, and D. Bez, "An estimation fusion method for including phasor measurements into power system real-time modeling," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 1910–1920, 2013.

[62] S. K. Mallik, S. Chakrabarti, and S. N. Singh, "A robust regularized hybrid state estimator for power systems," *Electric Power Components and Systems*, vol. 42, no. 7, pp. 671–681, 2014. [Online]. Available: https://doi.org/10.1080/15325008.2014.890968

[63] V. Kirincic, S. Skok, and V. Terzija, "A two-step hybrid power system state estimator," *International Transactions on Electrical Energy Systems*, vol. 25, no. 7, pp. 1158–1172, 2015. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1002/etep.1894

[64] M. Göl and A. Abur, "A hybrid state estimator for systems with limited number of pmus," *IEEE Transactions on Power Systems*, vol. 30, no. 3, pp. 1511–1517, 2015.

[65] A. Dubey and S. Chakrabarti, "An unscented kalman filter based hybrid state estimator considering conventional and pmu measurements," in *2016 IEEE 6th International Conference on Power Systems (ICPS)*, 2016, pp. 1–6.

[66] A. M. Stanković, V. Švenda, A. T. Sarić, and M. K. Transtrum, "Hybrid power system state estimation with irregular sampling," in *2017 IEEE Power Energy Society General Meeting*, 2017, pp. 1–5.

[67] B. Özsoy and M. Göl, "A hybrid state estimation strategy with optimal use of pseudo-measurements," in *2018 IEEE PES Innovative Smart Grid Technologies Conference Europe (ISGT-Europe)*, 2018, pp. 1–6.

[68] J. Zhao, G. Zhang, K. Das, G. N. Korres, N. M. Manousakis, A. K. Sinha, and Z. He, "Power system real-time monitoring by using pmu-based robust state estimation method," *IEEE Transactions on Smart Grid*, vol. 7, no. 1, pp. 300–309, 2016.

[69] M. Khosravi, M. Banejad, and H. T. Shandiz, "Forecast aided measurements data synchronisation in robust power system state estimation," *IET Generation, Transmission & Distribution*, vol. 10, no. 10, pp. 2379–2388, 2016. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1049/iet-gtd.2015.1138

[70] T. Wu, C. Y. Chung, and I. Kamwa, "A fast state estimator for systems including limited number of pmus," *IEEE Transactions on Power Systems*, vol. 32, no. 6, pp. 4329–4339, 2017.

[71] S. Goleijani and M. T. Ameli, "Neural network-based power system dynamic state estimation using hybrid data from scada and phasor measurement units," *International Transactions on Electrical Energy Systems*, vol. 28, no. 2, p. e2481, 2018, e2481 ITEES-17-0575.R1. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1002/etep.2481

[72] N. M. Manousakis and G. N. Korres, "A hybrid power system state estimator using synchronized and unsynchronized sensors," *International Transactions on Electrical Energy Systems*, vol. 28, no. 8, p. e2580, 2018, e2580 ITEES-17-0372.R2. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1002/etep.2580

[73] A. S. Dobakhshari, S. Azizi, M. Abdolmaleki, and V. Terzija, "Linear lav-based state estimation integrating hybrid scada/pmu measurements," *IET Generation, Transmission & Distribution*, vol. 14, no. 8, pp. 1583–1590, 2020. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1049/iet-gtd.2019.1850

[74] A. Dubey, S. Chakrabarti, A. Sharma, and V. Terzija, "Optimal utilisation of pmu measurements in power system hybrid state estimators," *IET Generation, Transmission & Distribution*, vol. 13, no. 21, pp. 4978–4986, 2019. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1049/iet-gtd.2019.0010

[75] A. S. Dobakhshari, M. Abdolmaleki, V. Terzija, and S. Azizi, "Robust hybrid linear state estimator utilizing scada and pmu measurements," *IEEE Transactions on Power Systems*, vol. 36, no. 2, pp. 1264–1273, 2021.

[76] E. Handschin, F. C. Schweppe, J. Kohlas, and A. Fiechter, "Bad data analysis for power system state estimation," *IEEE Transactions on Power Apparatus and Systems*, vol. 94, no. 2, pp. 329–337, March 1975.

[77] K. l. Lo, P. s. Ong, R. D. McColl, A. M. Moffatt, and J. L. Sulley, "Development of a static state estimator part i: Estimation and bad data suppression," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-102, no. 8, pp. 2486–2491, Aug 1983.

[78] F. Zhuang and R. Balasubramanian, "Bad data suppression in power system state estimation with a variable quadratic-constant criterion," *IEEE Power Engineering Review*, vol. PER-5, no. 4, pp. 42–42, April 1985.

[79] "Real time load curve from pjm market." [Online]. Available: https://dataviewer.pjm.com/dataviewer/pages/public/load.jsf

[80] T. A. Stuart and C. J. Herczet, "A sensitivity analysis of weighted least squares state estimation for power systems," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-92, no. 5, pp. 1696–1701, 1973.

Publications

Journals:

- Chejarla Madhu Kishore, Matam Sailaja Kumari"***Multiple Solutions for Optimal PMU Placement Using a Topology-Based Method***". J. Inst. Eng. India Ser. B 102, 249–259 (2021). https://doi.org/10.1007/s40031-020-00532-y[**Scopus, UGC-CARE List (India) indexed**]

- Chejarla Madhu Kishore, Matam Sailaja Kumari "***Synchrophasor assisted power system state estimation with a quadratically-decaying exponential criterion.***" Int Trans Electr Energ Syst. 2021;e13051. https://doi.org/10.1002/2050-7038.13051 [**SCIE indexed**]

- Chejarla Madhu Kishore, Matam Sailaja Kumari "***Linear sequential hybrid state estimator***" Int Trans Electr Energ Syst. 2021;e13193. doi:10.1002/2050-7038.13193 [**SCIE indexed**]

- Chejarla Madhu Kishore, Matam Sailaja Kumari "***Linear tracking hybrid power system state estimator***" Electric Power Systems Research. [**SCI indexed**] is under review.

Conferences:

- Chejarla Madhu Kishore, Matam Sailaja Kumari "***Multiple solutions for optimal PMU placement using graph theory***" had been presented in ICSTACE2021 conference held at SVNIT Surat.