Optimal PMU Placement by Teaching-Learning Based Optimization Algorithm

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***Abstract*— In this paper Teaching-Learning-Based Optimization Algorithm (TLBO) is presented for solving the problem of placement of phasor measurement units (PMU) optimally in a power system network for complete observability. The TLBO algorithm enables optimal PMU placement by zero injection measurements and also by not including zero injection measurements. The algorithm has been tested on standard test systems such as IEEE 14-bus, IEEE 30-bus, IEEE 57-bus and the results are contrasted with other optimization algorithms like Genetic Algorithm and Binary PSO.**

Keywords— **Phasor measurement units; Observability; Optimal placement; Teaching-Learning-Based Optimization.**

I. INTRODUCTION

Phasor Measurement Unit (PMU) [1] produces synchronized phasor, frequency and rate of change of frequency approximations from voltage and/ or current indications and a time matching signal which helps in providing a wide area snapshot of the power system. Strategic deployment of the PMUs can help in creating the power system fully observable as well as reduce the cost. The purpose of this paper is to ascertain the least number and optimal locations of PMUs so that the system is topologically detectable.

There are various established and investigative methods to tackle the problem of optimal PMU placement (OPP). Researchers in [2] have suggested an Integer Linear Programming (ILP) based methodology for the problem by including and excluding zero injection measurement cases. Simulated Annealing approach has been recommended in [3]. Several approaches built on Genetic Algorithms (GA) have been suggested in [4]-[5]. A binary particle swarm optimization (BPSO) approach is exercised in [6] to attain the least number of PMUs for thorough topological observability of the power system network.

Teaching-Learning-Based-Optimization (TLBO) is an algorithm which is built on the influence that a teacher has on the learning of students in a class. TLBO has a simple concept and is highly effectual and has been effectively applied to resolve numerous challenges such as optimization of machining processes in [7], mechanical design optimization in [8], site of automatic voltage regulators in distribution systems in [9], etc. TLBO yields superior results when competed to other evolutionary computing techniques like Genetic Algorithm (GA) [9], Particle Swarm Optimization (PSO) [7] and Differential Evolution (DE) [8].

In this paper, the Teaching-Learning-Based Optimization algorithm is exercised for the optimal PMU placement challenge and tested on IEEE 14-bus, 30-bus and 57-bus test systems. For all the test systems, the least number of PMUs needed for full topological observability is computed and the implementation of the algorithm is studied with respect to computation time as well as its capability to provide the same result consistently.

The remainder of the paper is categorized and it is as follows. Section II portrays the PMU placement problem construction and how the constraints are created for thorough observability of the system. A concise summary of GA and PSO is given in section III. The TLBO method is described in section IV. Section V discussed the case study results and the final outcome of the paper has been concluded in section VI.

II. PMU PLACEMENT PROBLEM CONSTRUCTION

For a given n-bus system, the PMU placement problem [10] can be constructed as follows:

$$Min \sum_{i=1}^{n} w_i x_i \qquad (1)$$

$$s.t\ \ f(x) \geq 1$$

where w_i is installation cost of PMU at i^{th} bus and is being supposed to be the same for all buses. The entries of a binary decision variable vector X is defined as:

$$X_i = \begin{cases} 1\ if\ PMU\ is\ placed\ at\ bus\ i \\ 0\ otherwise \end{cases}$$

Consider the IEEE 14-bus test system displayed in the Figure 1 [11]. The black dot near 7^{th} bus signifies that 7^{th} bus is a zero injection bus.

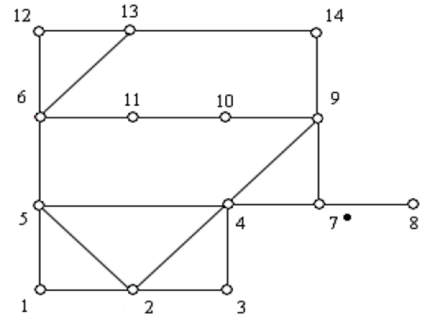

Figure 1: IEEE 14-bus test system graph [11]

Case 1: Excluding zero injection measurements

For forming set of constraints, binary connectivity matrix A is formed first:

$$A_{k,m} = \begin{cases} 1 \; if \; k = m \; or \; k \; and \; m \; are \; connected \\ 0 \; otherwise \end{cases}$$

Constructing matrix A for IEEE 14-bus test system yields:

$$A = \begin{bmatrix} 1&1&0&0&1&0&0&0&0&0&0&0&0&0 \\ 1&1&1&1&1&0&0&0&0&0&0&0&0&0 \\ 0&1&1&1&0&0&0&0&0&0&0&0&0&0 \\ 0&1&1&1&1&0&1&0&1&0&0&0&0&0 \\ 1&1&0&1&1&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&1&0&0&0&0&1&1&1&0 \\ 0&0&0&1&0&0&1&1&1&0&0&0&0&0 \\ 0&0&0&0&0&0&1&1&0&0&0&0&0&0 \\ 0&0&0&1&0&0&1&0&1&1&0&0&0&1 \\ 0&0&0&0&0&0&0&0&1&1&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0&1&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&1&1&0 \\ 0&0&0&0&0&1&0&0&0&0&0&1&1&1 \\ 0&0&0&0&0&0&0&0&1&0&0&0&1&1 \end{bmatrix} \quad (2)$$

Constraints for above test system can be constructed as:

$$f(X) = A.X = \begin{cases} f_1 = x_1 + x_2 + x_5 \geq 1 \\ f_2 = x_1 + x_2 + x_3 + x_4 + x_5 \geq 1 \\ f_3 = x_2 + x_3 + x_4 \geq 1 \\ f_4 = x_2 + x_3 + x_4 + x_5 + x_7 + x_9 \geq 1 \\ f_5 = x_1 + x_2 + x_4 + x_5 + x_6 \geq 1 \\ f_6 = x_5 + x_6 + x_{11} + x_{12} + x_{13} \geq 1 \\ f_7 = x_4 + x_7 + x_8 + x_9 \geq 1 \\ f_8 = x_7 + x_8 \geq 1 \\ f_9 = x_4 + x_7 + x_9 + x_{10} + x_{14} \geq 1 \\ f_{10} = x_9 + x_{10} + x_{11} \geq 1 \\ f_{11} = x_6 + x_{10} + x_{11} \geq 1 \\ f_{12} = x_6 + x_{12} + x_{13} \geq 1 \\ f_{13} = x_6 + x_{12} + x_{13} + x_{14} \geq 1 \\ f_{14} = x_9 + x_{13} + x_{14} \geq 1 \end{cases} \quad (3)$$

The "+" sign serves as the logical "OR" and 1 after " $\geq$ " sign signifies that at least one of the variables appearing in the sum will be non-zero.

Consider constraints for Bus-1 and Bus-2 for illustration:

$$f_1 = x_1 + x_2 + x_5 \geq 1$$
$$f_2 = x_1 + x_2 + x_3 + x_4 + x_5 \geq 1$$

Constraint $f_1 \geq 1$ denotes that at least one PMU should be positioned at any one of the buses 1, 2 and 5 to make Bus-1 observable. Similarly, Constraint $f_2 \geq 1$ denotes that at least one PMU must be positioned at any one of the buses 1, 2, 3, 4, or 5 in order to make Bus-2 observable.

Case 2: Including zero injection measurements

Consider test system as displayed in figure 1 wherein Bus-7 is assumed as zero injection bus. If any three bus voltages, out of the four buses 4, 7, 8 and 9, are known, then the voltage of the fourth bus can easily be estimated by using KCL applied at Bus-7. As such, the net injected current at 7^{th} bus will be known. Based on this concept, the bus which has the injection measurement is merged with any one of its neighbor's.

Supposing Bus-7 to be merged with Bus-8 to form Bus-8' [11], the graph now develops as displayed in figure 2.

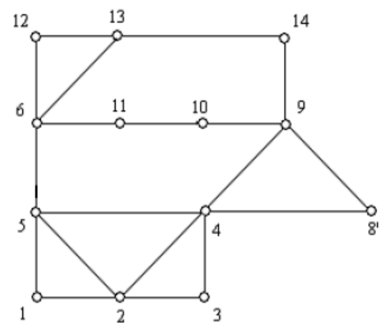

Figure 2: IEEE-14 bus test system graph after merging buses 7 and 8 [11]

The modified constraints are:

$$f(X) = A.X$$
$$= \begin{cases} f_1 = x_1 + x_2 + x_5 \geq 1 \\ f_2 = x_1 + x_2 + x_3 + x_4 + x_5 \geq 1 \\ f_3 = x_2 + x_3 + x_4 \geq 1 \\ f_4 = x_2 + x_3 + x_4 + x_5 + x_7 + x_8 + x_9 \geq 1 \\ f_5 = x_1 + x_2 + x_4 + x_5 + x_6 \geq 1 \\ f_6 = x_5 + x_6 + x_{11} + x_{12} + x_{13} \geq 1 \\ f_{8'} = x_4 + x_7 + x_8 + x_9 \geq 1 \\ f_9 = x_4 + x_7 + x_8 + x_9 + x_{10} + x_{14} \geq 1 \\ f_{10} = x_9 + x_{10} + x_{11} \geq 1 \\ f_{11} = x_6 + x_{10} + x_{11} \geq 1 \\ f_{12} = x_6 + x_{12} + x_{13} \geq 1 \\ f_{13} = x_6 + x_{12} + x_{13} + x_{14} \geq 1 \\ f_{14} = x_9 + x_{13} + x_{14} \geq 1 \end{cases} \quad (4)$$

III. Genetic Algorithm and Particle Swarm Optimization algorithm

A. Genetic Algorithm

Genetic Algorithm (GA) [12] is an optimization technique based on Darwin's philosophy of "Survival of the Fittest" and "Principle of Nature". GA uses a population of individuals and each individual denotes a solution. The solution is evaluated by each individual's fitness, which is calculated by the fitness function.

To arrive at the optimal value, GA uses the selection, crossover, and mutation operators on the population of individuals. At the start of the search, the population is randomly initialized. Based on the fitness value, individuals are selected to advance from current to next generation. The crossover and mutation operations are executed on these individuals to develop the next generation. This process is continued till some stopping criteria are met.

B. Particle Swarm Optimization Algorithm

Particle Swarm Optimization (PSO) was initially developed by Eberhart and Kennedy in 1995 [13]. It is also an evolutionary computing technique which is built on the movement and intelligence of a bunch of particles or swarms. Each location of a particle denotes an answer to the problem. In PSO, each individual particle moves in the search space, by constantly updating its velocity based on its personal best location (pbest) as well as the best location among the others (gbest) in the population (swarm) using the equations

$$V_{id} = V_{id} + \varphi * (pbest - X_{id}) + \varphi * (gbest - X_{id}) \quad (5)$$

$$X_{id} = X_{id} + V_{id} \quad (6)$$

where i is the ith particle, φ is an arbitrary positive number produced for each *id* and X_{id} is the location coordinate and the velocity is V_{id} in a D-dimensional space.

Kennedy and Eberhart established a binary version of PSO (BPSO) for binary problems [14]. They projected a model in which the possibility of binary decision taken by a particle is a function of personal and social factors. The velocity is revised as in PSO and it is used as a limit to make one of the two decisions (0 or 1). If the velocity is higher, the individual is more likely to choose 1, and if velocity is lower, then 0 is chosen. The limit should be in the span of [0, 1]. The sigmoidal function maps the velocity to the span of [0, 1].

$$s(V_{id}) = 1/1 + \exp(-V_{id}) \quad (7)$$

IV. Teaching-Learning-Based Optimization Algorithm

Teaching-Learning-Based-Optimization is a teaching-learning process enthused algorithm which is built on the influence that a teacher has on the learning of students in a class. The algorithm contemplates two modes of learning:

(i) through a teacher known as the teacher phase and the other (ii) interacting with the other learners known as the learner phase. In this optimization algorithm, the students are considered to be the population and each subject they learn is considered as a variable and the learner's result is similar to the fitness value of the optimization problem. The best solution in the entire population is deemed to be the teacher. The best solution is the best fitness value of the objective function. The TLBO algorithm is divided into two parts, 'Teacher phase' and 'Learner phase'.

A. Teacher phase

A good teacher helps to increase the average marks of a class and tries to bring the learners up to teacher's level in terms of knowledge. In practice a teacher can only help to improve the average of a class up to some limit depending on the capability of the class.

Let M_i be the mean and T_i which gives the best fitness among all students, be the teacher at any iteration i. The teacher T_i will try to move the class mean M_i towards its own level, and let the new mean be M_{new}. The solution is revised using the difference between the existing mean and the new mean given by

$$Difference\_Mean_i = r_i * (M_{new} - T_F * M_i) \quad (8)$$

where T_F is known as the teaching factor that decides the value of mean to be altered, and r_i is a random number in the range [0, 1]. The value of T_F can be either 1 or 2.

The difference mean modifies the existing solution as

$$X_{new,i} = X_{old,i} + Difference\_Mean_i \quad (9)$$

B. Learner phase

Learners will enhance their knowledge by interacting with the teacher and also by interacting among themselves. A learner can interact arbitrarily with other learners with the help of group deliberations, demonstrations, formal interactions, etc. A learner can learn something new from other learner who has more knowledge. For a minimization problem, the learner's modification is conveyed as follows:

Arbitrarily choose two learners X_i and X_j where i ≠ j

$$X_{new,i} = X_{old,i} + r_i(X_i - X_j) \quad if\ fitness(X_i) < fitness(X_j) \quad (10)$$

$$X_{new,i} = X_{old,i} + r_i(X_j - X_i) \quad if\ fitness(X_j) < fitness(X_i) \quad (11)$$

Accept X_{new} if it contributes a better function value.

The flow-chart of the TLBO algorithm from [8] is given below in Figure 3:

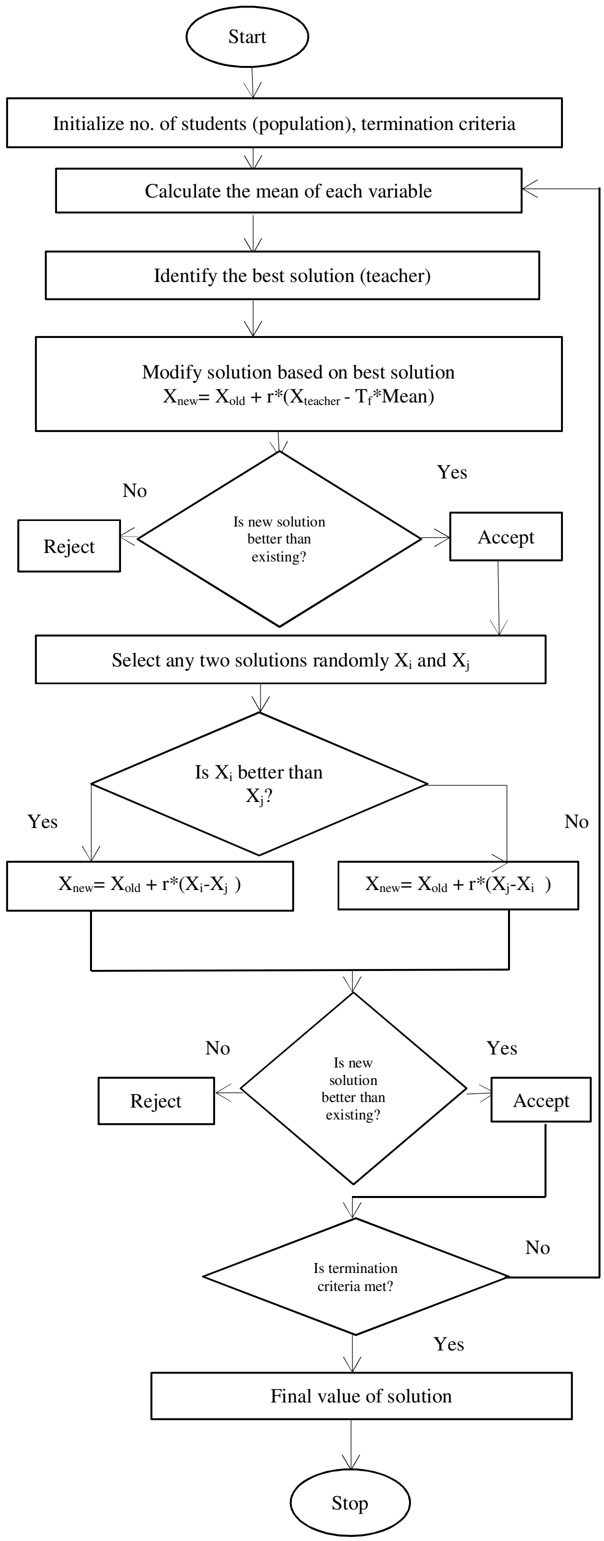

Figure 3: Flow-chart of TLBO [8] algorithm

V. BINARY TLBO

In teacher phase and learner phase, the velocity (v_{id}) of each learner can be computed as follows:

Teacher phase: $r_i * (M_{new} - T_F * M_i)$
Learner phase: $r_i * \left(X_j - X_i\right)$

In Binary TLBO [16], while updating the position of particles, the velocity equations are used to determine the transition from 0 to 1 or from 1 to 0 so as to limit the velocity in the region of [0,1]. A '*Tanh*' transformation is employed to the component of velocity as:

$$Tanh(v_{id}^k) = \frac{\exp\left(\left|2v_{id}^k\right|\right) - 1}{\exp\left(\left|2v_{id}^k\right|\right) + 1} \qquad (12)$$

The equation for updating locations is then substituted with:

$$X_{id}^k = \begin{cases} 1\ if\ rand\ < Tanh(v_{id}^k) \\ 0\ otherwise \end{cases}$$

VI. CASE STUDIES

The case studies were implemented in MATLAB on a 4GB RAM, 2.53 GHz, Intel i3 computer. The parameters used for executing the GA, BPSO, TLBO algorithms are displayed in Table I, Table II and Table III respectively. Optimal PMU placement with BPSO, GA and TLBO for both the cases i.e. by including and excluding zero injection measurements has been verified. For all the test systems [17] considered, optimal number of PMUs and their optimal locations have been presented in Table IV and Table V respectively. To validate efficiency of the TLBO method, a comparative analysis with GA and BPSO is shown in Figure 4. For an IEEE 14 bus system excluding zero injection measurements, the TLBO algorithm takes only 5 iterations whereas GA takes 9 iterations and BPSO takes 20 iterations. So TLBO converges faster than GA and BPSO as it doesn't need tuning of parameters and hence takes less time compared to these methods. Even though Optimal Placement of PMU is an offline process, the convergence time still remains a significant criterion particularly for large scale power systems.

TABLE I
PARAMETERS USED FOR GA

No. of chromosomes (population)	40
Max. No. of iterations	200
Crossover Probability	0.9
Mutation Probability	0.001

TABLE II
PARAMETERS USED FOR BPSO

No. of particles (population)	100
Max. No. of iterations	1000
C_1, C_2	2
V_{max}	6

TABLE III
PARAMETERS USED FOR TLBO

No. of students (population)	70
Max. No. of iterations	200
T_F	2

Case 1: Excluding zero-injection measurements

TABLE IV:
EXCLUDING ZERO INJECTION MEASUREMENTS

Test System	Optimum No. of PMUs			Optimal PMU Locations (Buses) by TLBO
	BPSO [15]	GA	TLBO	
IEEE 14 BUS	4	4	4	2, 6, 7, 9
IEEE 30 BUS	10	10	10	2, 4, 6, 9, 10, 12, 19, 23, 26, 30
IEEE 57 BUS	17	17	17	1, 4, 9, 20, 22, 25, 27, 29, 32, 36, 41, 45, 46, 48, 51, 53, 57

Case 2: Including zero-injection measurements

TABLE V:
INCLUDING ZERO INJECTION MEASUREMENTS

Test System	Optimum No. of PMUs			Optimal PMU Locations (Buses) by TLBO
	BPSO [6]	GA [4]	TLBO	
IEEE 14 BUS	3	3	3	2, 6, 9
IEEE 30 BUS	7	7	7	3, 7, 10, 12, 18, 24, 29
IEEE 57 BUS	11	11	11	1, 13, 18, 19, 25, 29, 32, 38, 51, 54, 56

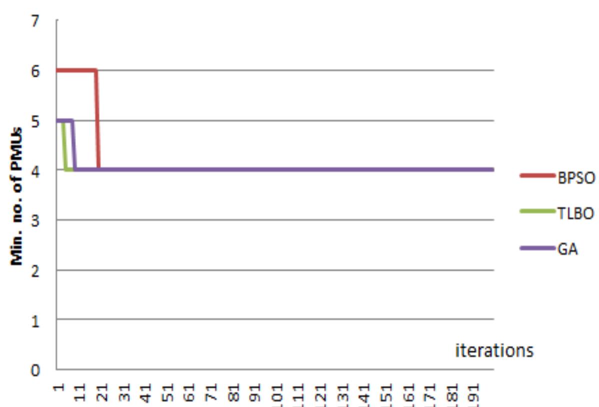

Figure 4: Distinction of convergence characteristics of various methods for IEEE 14-bus optimal PMU placement excluding zero injection measurements

VII. Conclusion

This paper presents a Teaching-Learning-Based Optimization Algorithm (TLBO) for a new application in solving the optimal phasor measurement unit (PMU) placement challenge in a power system network for thorough topological observability. The TLBO method employed here ensures optimal PMU placement by including and excluding zero injection measurements. This method has been validated on standard test systems and its efficiency has been demonstrated in comparison with GA and BPSO. Many optimization methods need different parameter settings which will influence the performance of the algorithm. GA requires probability of crossover, mutation rate, and type of selection method; BPSO requires the variation of weight and the maximum value of velocity. In comparison with other optimization techniques TLBO does not require any tuning of parameters. Hence, the implementation of TLBO algorithm is simpler and it has been found from the verified results that TLBO converges faster than Genetic Algorithm and BPSO.

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BIOGRAPHIES

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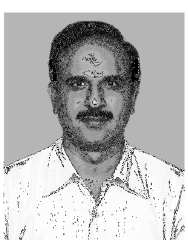

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