# Probabilistic Optimal Power Flow with Wind Energy Penetration and Integration of Storage System

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*Abstract* — **This paper Proposes Probabilistic optimal power flow to analyse the variation of Locational Marginal Prices (LMPs) with penetration of wind energy. The uncertainty of wind speed and load is modelled using probability distribution functions based on their historical data. Integration of storage unit with wind is proposed to assess economic advantages. In this paper Two Point Estimation Method (2PEM) is used to develop Probabilistic Optimal Power Flow. IEEE-24 bus reliable test system is used to carry out the analysis with uncertainty of wind and load using MATPOWER. The output of 2PEM is compared with Monte Carlo Simulation (MCS) results.**

***Keywords—Probabilistic Optimal Power Flow, Two Point Estimation Method, Wind Energy, Compressed Air Energy Storage (CAES), Locational Marginal Prices (LMPs)***

## I. INTRODUCTION

Fossil fuels based conventional power generation occupied majority of electrical generation sector. Fossil fuels are costly and spoil the environment. Renewable sources are inexpensive and eco- friendly but these have its own limitations. Wind energy is one of the widely used renewable sources. Currently, the penetration of wind energy has increased steadily across India with an installed capacity of 31,692 MW of overall capacity of 258701.45MW as of January 31, 2015 due to its environmental and economic advantages [1]. The principal drawback of wind speed is its intermittent nature and uncertainty in continuous availability Hence, the continuous power supply is not assured by wind energy. To overcome this problem, storage system is integrated to ensure the continuous power supply at the grid.

In this paper, load is also considered as uncertain. The uncertainty of wind speed and load variation has been addressed by developing Probabilistic Density Function (PDF) based on the statistics obtained from the historical data. With known PDFs of uncertain variables as input to Probability Optimal Power Flow (P-OPF), the output variable's (Locational Marginal Price's) Probability Density (PD) can be determined. With uncertain wind variation, the variations of Locational Marginal Prices are determined.

Several methods have been proposed for analysis of engineering systems with uncertain input variables. Basically analysis can be carried out in three ways to deal with the uncertain variables in any system. They are Analytical methods, Simulation methods and approximate methods. Analytical methods involve complex mathematics and laborious computations. Approximate methods are simple in formulation and computation whereas simulation methods are exact studies. Two Point Estimation Method (2PEM) [4] is an example of approximate method and Monte Carlo Simulation (MCS) technique is an example of simulation method. A very few publications have addressed Probabilistic optimal power flow using Point Estimation Method [2]-[6].

The paper cited in reference [2] presents P-OPF with 2m+1 point estimation method by considering correlated wind speeds. 2PEM is used and compared with MCS to handle correlated uncertain variables in renewable energy sources in [3], [4]. The uncertainty in load variation is considered in P-OPF with First Order Second moment (FOSM) method in [5].

The OPF problem with energy storage integration attempted in few publications [7], [8]. Storage system is simulated as charge/discharge dynamic model approach in and integrated in restructure power system for cost minimisation [7]. Compressed Air Energy Storage (CAES) is used as time shifting energy storage system to improve the performance of wind energy [9]-[11]. Dynamic model and control design of CAES is proposed in [9]. Linear regression method of estimation of Weibull parameters based on stochastic model is presented in [12]

In this paper Two Point Estimation Method (2PEM) based Probabilistic Optimal Power Flow is proposed to evaluate the variation of Locational Marginal Prices (LMPs) due to uncertainty of wind speed and load variation. To ensure continuity and to reduce the cost, the power system is integrated with storage system. The rest of the paper is organized as follows: Section II presents mathematical modelling such as the probabilistic model of wind and load based on their historical hourly data and modelling of storage system. Two Point Estimation Method based POPF has been employed for analysis of the LMPs variation. Section III discusses the results on the case study and Section IV presents the findings on the study as conclusions.

## II. METHODOLOGY

### A. *Probabilistic Modelling of Wind*

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Uncertainty of Wind speed is modelled based on historical hourly data. Wind speed data is obtained from the Iowa Environmental Mesonet [11]. Wind speed uncertainty is characterized using Probability Density Function (PDF) using Weibull distribution function [12]. The general form of the Weibull distribution function, which is a two-parameter function, for wind speed is given by

$$f(v) = \left(\frac{k}{\lambda}\right)\left(\frac{v}{\lambda}\right)^{k-1} e^{-\left(\frac{v}{\lambda}\right)^{k}} \quad (1)$$

Where $f(v)$ is Probability distribution function, $v$ is wind velocity, k, λ are shape and scale parameter respectively. Shape and scale parameters are determined using Newton Raphson - Maximum Likelihood estimation method. The parameters are given as

$$\bar{\lambda} = \left(\frac{\sum_{i=1}^{n} v_i^{\bar{k}}}{n}\right)^{1/\bar{k}} \quad (2)$$

$$\overline{k_{r+1}} = \overline{k_r} + \frac{A + (1/\bar{k}_r) - C_r/B_r}{\left(\frac{1}{\overline{\overline{k_r^2}}}\right) + (B_r H_r - C_r^2)/B_r^2} \quad (3)$$

Where A, $C_r$, $B_r$ and $H_r$ are given by

$$A = \frac{\sum_{i=1}^{n} \ln x_i}{n}\ ; \quad B_r = \sum_{i=1}^{n} x_i^{\beta_r} \quad and$$

$$C_r = \sum_{i=1}^{n} v_i^{k_r} \ln\ v_i\ ; \quad H_r = \sum_{i=1}^{n} v_i^{k_r} (\ln\ v_i)^2$$

*B. Probabilistic Modelling of Load*

Unpredictability of Load is modelled using Normal distribution with curve fitting technique [5],[13] and the Load data has been taken from IEEE-24 bus reliable test system [14] for the analysis. The Normal distribution function is as follows:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{(x-\mu)^2}{2\sigma^2}} \quad (4)$$

Where x is load, μ is its mean value and **σ** is the standard deviation.

C. *Electrical Storage Energy*

Compressed Air Energy Storage (CAES) system is the technology popularly used to store the energy. The energy is stored in the form of compressed air in underground structures or above ground units of vessels. To compress the air the excess wind power is used. When needed compressed air is mixed with natural gas, and used to run the turbines to reproduce the electrical energy [9]. The process cycle of CAES is as shown in figure 1. It has long lifetime, large capacity and low capital, suitable for large scale renewable energy integration [10].

The energy storage system is modelled as load when excess wind power is available with available transmission capacity [8]. This occurs when the storage unit in charging mode with excess wind and available transmission capacity ($W_P$ - $L_t$ > 0). When the wind power generation is absent, the storage system will act as generator, it will discharge in peak load hours when it is most required ($W_p$ - $L_t$ < 0).

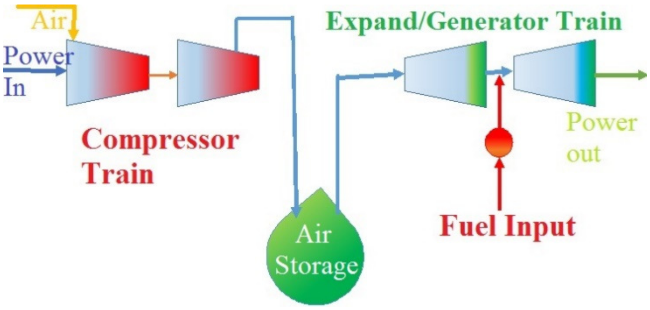


Figure 1: Process cycle of Compressed Air Energy Storage System

The equations for charging and discharging is given as follows

For Charging: when $(W_p - L_t) > 0$

$$S_t = (1 - d_s)S_{t-1} + \eta_s(W_p - L_t) \quad (5)$$

For Discharge: when $(W_p - L_t) < 0$

$$S_t = (1 - d_s)S_{t-1} + \eta_d(W_p - L_t) \quad (6)$$

Where $S_t$ is storage, $d_s$ is discharge rate, $\eta_s$, $\eta_d$ are charging and discharging efficiencies respectively.

*D. Market Based Optimal Power Flow*

The bids are taken as inputs to the Optimal Power Flow (OPF) which determines optimal values of the supplies and locational marginal prices (LMPs) which in turn minimize the Hourly Social Cost (HSC). Wind generation and Storage system are incorporated into OPF. The Optimal Power Flow (OPF) is formulated as a nonlinear optimization problem that minimizes the power system quadratic operating cost. In this paper, the Primal-Dual Interior Point Method (PDIPM) was used to solve the optimization problem [6]. The objective function (Obj Fun) and its constraints are as follows:

$$Obj\ Fun = min\left\{\sum_{i=1}^{N_G} P_{Gi,t}\left(a_i P_{Gi,t} + b_i\right)\right\} = \min(\text{HSC}) = \min f(z) \quad (7)$$

*S.t.* G (Z) = 0 and Ll < H (Z) < Ul are given as below

a. Power flow equations:

$$P_{Gi,t}(V,\delta) = P_{Gi,t} - P_{Di,t} = \sum_{j=1}^{N_G} |V_{i,t}||V_{i,t}|[G_{ij}\cos(\theta_{i,t} - \delta_{i,t}) + B_{ij}\sin(\theta_{i,t} - \delta_{i,t})] \quad (8)$$

$$Q_{Gi,t}(V,\delta) = Q_{Gi,t} - Q_{Di,t} = \sum_{j=1}^{N_G} |V_{i,t}||V_{i,t}|[G_{ij}\sin(\theta_{i,t} - \delta_{i,t}) - B_{ij}\cos(\theta_{i,t} - \delta_{i,t})] \quad (9)$$

b. Power output limit of thermal generating units:

$$P_i^{min} \le P_{i,t} \le P_i^{max} \quad (10)$$
$$Q_i^{min} \le Q_{i,t} \le Q_i^{max} \quad (11)$$

c. Bus voltage limit:

$$|V_i^{min}| \le |V_{i,t}| \le |V_i^{max}| \quad (12)$$

d. Power flow limits of line from bus *i* to bus *j*:

$$PL_{ij}, t \le PL_{ij}, \mathrm{t} \quad (13)$$

The solution of this problem by Newton's method requires the Lagrangian form as shown below.

$$L(z) = f(x) + \mu^T h(x) + \lambda^T g(x) \quad (14)$$

where $Z = [x\ \mu\ \lambda]^T$ , $\mu\ and\ \lambda$ vectors of the Lagrangian multiplier and g(x) includes the active (or binding) inequality constraints.

$$Gradient = \nabla L(Z) = \left[\frac{\partial L(Z)}{\partial Z_i}\right] = a\ vector\ of\ first\ partial\ derivatives\ of\ Lagrangian \quad (15)$$

$$Hessian = \nabla^2 L(Z) = H = \left[\frac{\partial^2 L(Z)}{\partial z_i \partial z_j}\right] = a\ matrix\ of\ the\ second\ partial\ derivative\ of\ Lagrangian \quad (16)$$

The Market based OPF algorithm is as follows:

[Step 1] Initialize of the OPF
  a. Initial guess values at which inequalities are violated.
  b. Initial guess values of z vector (bus voltages and angles, generator output power, Transformer tap ratios and phase shifts, all Lagrange multipliers).

[Step 2] Solve the inequalities that have to be added or removed using the information from Lagrangian multipliers for hard constraints and direct evaluation for soft constraints

[Step 3] Determine viability of the OPF solution. This ensures that no generator is not at a limit.

[Step 4] Calculate the gradient and Hessian of the Lagrangian.

[Step 5] Solve the equation [H] Δz = ∇L (z).

[Step 6] Update solution $z_{new} = z_{old} - \Delta z$.

[Step 7] Check whether ‖Δz‖ < e. If not, go to Step 4, otherwise continue.

[Step 8] Check whether inequalities have been enforced correctly or not. If not go to Step 2. If so, problem is solved.

### *E. Probabilistic Optimal Power Flow*

Point Estimation Method (PEM) is developed to calculate the moments of a nonlinear function. These moments are function of one or more random variables. The input variables to this function are random and whose Probability Density (PD) is known. The output variables are affected by random inputs. These moments are mainly mean, standard deviation and skewness. Depending up on the random input variables order, higher order PEM is chosen. But only two moments are sufficient to know the PDF of output random variable. 2PEM is calculating two moments i.e. mean and standard deviation [6].

Let Z be randomly varying input with known PDF for function g (Z), with n random variables. **Z= [$\mu_{X1}$, ………, $x_{ki}$ , …………….. $\mu_{Xn}$]** every time the random variable is replaced with concentration of particular random variable, remaining variables are replaced with its mean values. So that in 2PEM for each random variable OPF is executed twice. For n random variables OPF is executed 2n times. The 2PEM algorithm is given below

[Step 1] Assign appropriate pdf to each probabilistic variable.

[Step 2] Set E(Y) = 0; E (Y2) = 0

[Step 3] Set k = 1.

[Step 4] Determine the necessary parameters for the 2PEM
  $\zeta k, 1 = \sqrt{n}$
  $\xi k, 2 = -\sqrt{n}$
  $P_{k,i} = \frac{1}{2n}$ *i=1, 2*

where *n* denotes the number of probabilistic variables.

[Step 5] Determine the two concentrations using the input vector X
  $x_{k,i} = \mu X_k + \xi_{k,i}\sigma_{Xk} \qquad i = 1,2$

[Step 6] Calculate **Z= [$\mu_{X1}$,……., $x_{ki}$ , …….. $\mu_{X2n}$] i=1, 2**. Note that $\mu_{Xk}$ is replaced by $x_{ki}$ (*i*=1, 2.) at each iteration.

[Step 7] Run the deterministic market-based OPF for both concentrations

[Step 8] Calculate HSC

[Step 9] Update the mean E(Y) and mean square E ($Y^2$)

$$E(Y) \cong \sum_{k=1}^{n}\sum_{i=1}^{2}\left(P_{k,i} X\, h([\mu_{X1} \ldots\ldots x_{k,i} \ldots\ldots \mu_{Xn}])\right)$$

$$E(Y^2) \cong \sum_{k=1}^{n}\sum_{i=1}^{2}\left(P_{k,i} X\, h^2([\mu_{X1} \ldots\ldots x_{k,i} \ldots\ldots \mu_{Xn}])\right)$$

[Step 10] Set k = k + 1 and repeat steps 6–13 for all input variables.

[Step 11] Calculate the expected value and standard deviation

$$\mu_Y = E(Y)$$

$$\sigma_Y = \sqrt{E(Y) - \mu_Y^2}$$

[Step 12] Repeat from Step 4 to Step 11 for k = k+1 until the list of uncertain variables is exhausted.

## III. RESULTS

### A. Test System

In order to provide a basis for comparison of results obtained from different methods, it is desirable to have a reference or "test" system which incorporates the basic data needed in reliability evaluation. The IEEE-24 Reliability Test System which provides such "reliability test system" [14] has been utilized for testing.

### B. Load Data:

The load model gives hourly loads for one year on a per unit basis, expressed in chronological fashion so that daily, weekly, and seasonal patterns can be modelled. The generating system contains 32 units, ranging from 12 to 400 MW. Data is given on both reliability and operating costs of generating units [14].

The transmission system contains 24 load/generation buses connected by 38 lines or autotransformers at two voltages, 138 and 230 kV. The transmission system includes cables, lines on a common right of way, and lines on a common tower. Transmission system data includes line length, impedance, ratings, and reliability data. The Load variation is calculated from IEEE-24 bus system with the following specific considerations

- Spring 17th Week with 75.4% peak, afternoon peak load
- Annual peak of 2850 MW

The plot of the Load curve for 24 hours is shown in figure 2, and load curve has been captured on a particular day (17th week 4th day of week i.e. Wednesday).

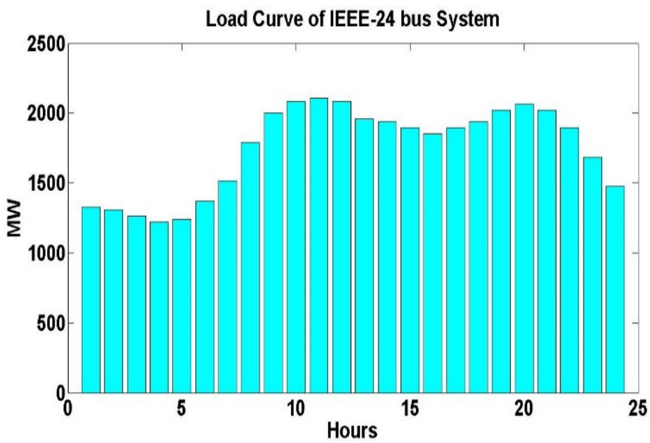


Figure 2: Plot of the daily load curve

### C. Wind Data

The Wind data from January 2014 to January 2015 has been taken from Iowa Environmental Mesonet [11]. Wind speed distribution is calculated using Weibull distribution function. The PDF and Cumulative Distribution Function (CDF) are calculated using graphical and maximum likelihood methods and are as shown in figure 3.

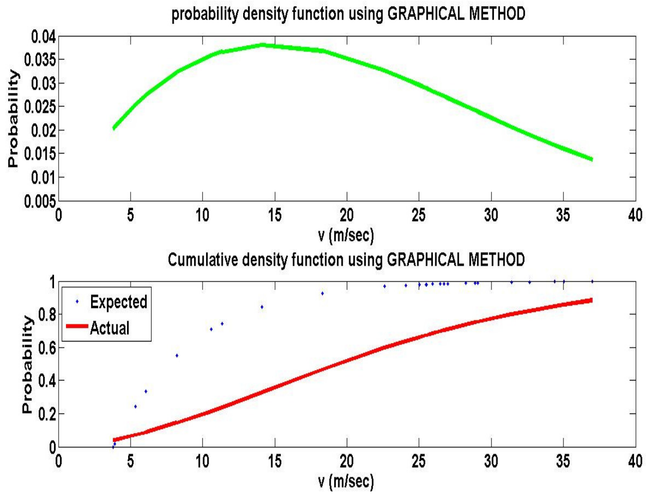


Figure 3: PDF and CDF using Maximum Likelihood method

### D. Load Probability distribution function

The Load is modelled as normal distribution function and the PDF of load at 10th bus is shown in figure 4.

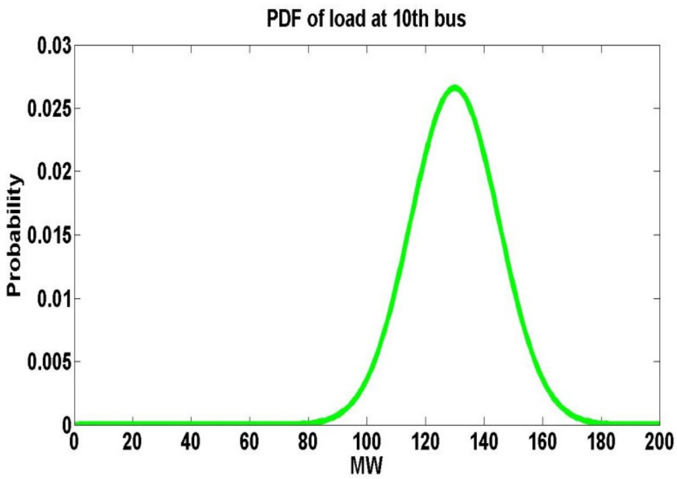


Figure 4: Load Probability Distribution Function

### F. LMPs calculation with integration of wind and storage unit

In IEEE-24 RTS system, wind generation is integrated at Bus-14. The penetration level of wind is taken as 40% of installed capacity of conventional generation. Storage unit is integrated at same bus of wind. Whenever the excess amount of wind generation is available storage unit acts as load (charging) and acts as generator when the wind generation is absent. The comparison of Load without wind, with wind and with storage integration is shown as a pictorial representation in figure 5.

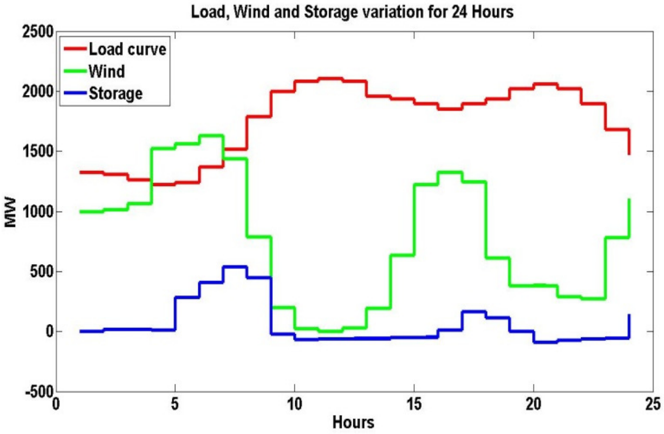


Figure 5: Comparison of Load without wind, with wind and with storage integration for 24 hours

The Locational Marginal Prices of IEEE 24-bus RTS has been computed importing the OPF algorithm of MATPOWER package on MATLAB environment. After integrating the wind generation data into the Reliability Test System data at Bus-14, the OPF algorithm of MATPOWER package on MATLAB environment has been run to compute the Locational Marginal Prices. The results thus obtained are compared with the LMPs of original RTS. Subsequently, by integrating the Storage system data at Bus-14 of RTS, the OPF algorithm of MATPOWER package [15] on MATLAB [16] environment has been run to compute the LMPs.

In figure 6 at $10^{th}$ Bus LMPs for all 24 hours are compared without wind, with wind and with storage unit integration. It is observed that the LMPS at charging hour going very high compared to wind alone.

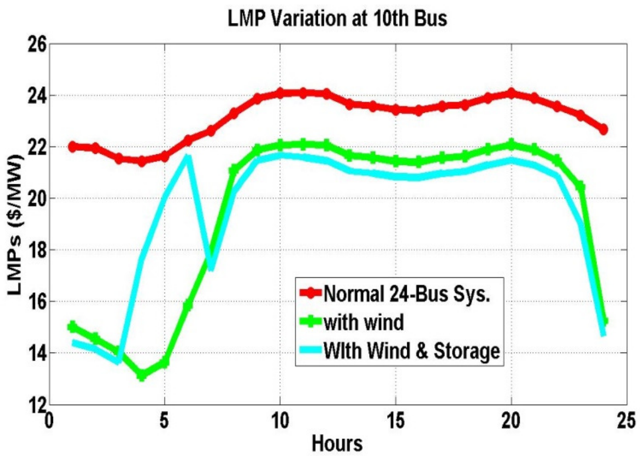


Figure 6: Comparison of LMPs of each hour at Bus-10

In figure 7 at $10^{th}$ hour LMPs of all buses are compared without wind, with wind and with storage unit integration. It is observed that LMPs with wind and Storage unit are giving low values.

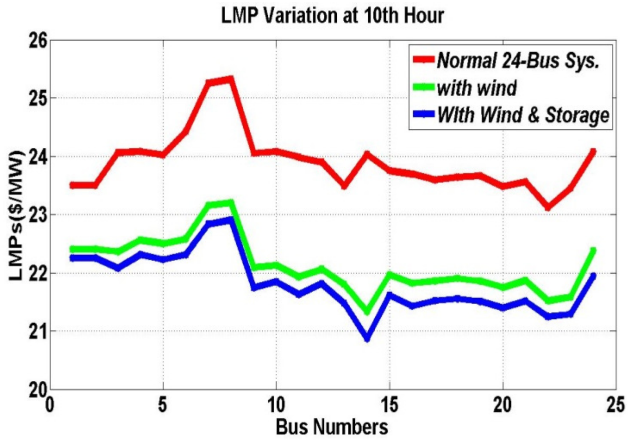


Figure 7: Comparison of LMPs for $10^{th}$ hour of all buses

*F. Probabilistic optimal power flow and Monte Carlo simulation with wind penetration*

Probability of LMP at each bus is calculated using two point estimation (2PEM) method based on Probabilistic optimal power flow. Results are compared with Monte Carlo Simulation (MCS) techniques. 1000 wind samples are randomly taken to simulate Monte Carlo. Both method's mean and standard deviation were compared and are as shown in Table 1.

Table 1: Comparison of 2PEM and MCS with wind penetration

| Methods | Mean value | Standard deviation | Execution Time |
|---|---|---|---|
| 2PEM (with wind) | 18.3730 | 17.4981 | 22.483308 seconds |
| MCS (with wind) | 18.4697 | 17.5902 | 34 minutes |

## IV. CONCLUSION

This paper develops a probabilistic framework that incorporates the wind generation and along with energy storage system into the POPF model. The proposed POPF uses the two point-estimation scheme to characterize the system uncertainties. Historical hourly data are used to stochastically model the wind speed and load based on their distribution parameters. The proposed method was tested on the IEEE 24-bus RTS system and the performance was evaluated and compared with MCS. The LMPs of IEEE 24 -bus RTS have been computed using a standard and robust MATPOWER package on MATLAB environment. Subsequently the wind and storage is incorporated into the system using programming. Cost (LMPs) analysis of the IEEE 24-bus RTS system shows the decreased cost of the system when the energy storage is utilized with wind generation. This is due to the increased utilization of wind power which can be stored and discharged whenever it is required. This provides the system operator with an economic advantage with renewable generation.

In this paper, LMP are considered uncertain and their first two moments of PDF are calculated. The proposed method is computationally faster than MCS approach.

REFERENCES

[1] Ministry of Power, Central Electricity Authority Report-2015, Government of India http://www.cea.nic.in/, 2015

[2] Yiming Li, Wenyuan Li, Wei Yan, Juan Yu, Xia Zhao, “Probabilistic Optimal Power Flow Considering Correlations of Wind Speeds Following Different Distributions” IEEE Transactions on Power Systems, Volume 29, Issue- 4 Pages1847-1854, 2014

[3] Aien, M., Fotuhi-Firuzabad, M., Rashidinejad, M.,"Probabilistic Optimal Power Flow in Correlated Hybrid Wind–Photovoltaic Power Systems",IEEE Transactions on Smart Grid, Volume- 5, Issue- 1, Pages- 130 –138, 2014.

[4] Saunders C.S.,"Point Estimate Method Addressing Correlated Wind Power for Probabilistic Optimal Power Flow "IEEE Transactions on Power Systems, Volume- 29, Issue- 3 Pages- 1045 – 1054, 2014.

[5] Xue Li, Yuzeng Li, Shaohua Zhang ,"Analysis of Probabilistic Optimal Power Flow Taking Account of the Variation of Load Power", IEEE Transactions on Power Systems, Volume- 23, Issue- 3 Pages- 992 – 999, 2008.

[6] G. Verbic and C. A. Canizares, “Probabilistic optimal power flow in electricity markets based on two-point estimate method,” IEEE Transaction on Power System, Vol. 21, No. 4, PP. 1883–1893, Nov. 2006.

[7] Gayme, D., Topcu, U. "Optimal power flow with large-scale storage integration",IEEE Transactions on Power Systems, Volume- 28, Issue- 2, Pages- 709 – 717, 2013.

[8] Y. Levron, J. M. Guerrero, and Y. Beck, “Optimal power flow in microgrids with energy storage,” IEEE Transaction on Power Systems, Vol. 28, No. 3, PP. 3226-3234, Aug. 2013.

[9] Martinez, M.; Molina, M.G.; Frack, P.F.; Mercado, P.E.,"Dynamic Modeling, Simulation and Control of Hybrid Energy Storage System Based on Compressed Air and Supercapacitors", IEEE (Revista IEEE America Latina) Latin America Transactions, Volume: 11, Issue: 1, Pages: 466 – 472, 2013.

[10] EPRI-DOE Handbook of Energy Storage for Transmission & Distribution Applications, EPRI, Palo Alto, CA, and U.S. Department of Energy, Washington, DC, 1001834.

[11] The Iowa Environmental Mesonet https://mesonet.agron.iastate.edu/, 2014-2015.

[12] Fernandez, A., Vazquez, M ,”Improved Estimation of Weibull Parameters Considering Unreliability Uncertainties”, IEEE Transactions on Reliability, Volume-61, Issue- 1, Pages- 32 – 40, 2012.

[13] Daniel Villanueva, Andres E. Feijoo, Josel. Pazos, “An analytical method to solve the probabilistic load flow considering load demand correlation using the DC load flow”, Electric Power Systems Research, Vol.110, PP-1–8, 2014.

[14] IEEE committee report, “A reliability test system, “IEEE Transaction on Power Apparatus and Systems, Vol. 4, No. 3, PP. 1238-1244, 1989.

[15] http: //www.pserc.cornell.edu//matpower/

[16] http: //in.mathworks.com/products/matlab/

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