Improved Hanning Window based Interpolated FFT for Power Harmonic Analysis

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***Abstract*—Power system harmonics are inevitable in the presence of non-linear loads. The presence of harmonics has a greater impact on the terminal apparatus. Accurate analysis of Power system harmonics plays an important role to have a better mitigation. Windowed Interpolated FFT (WIFFT) based algorithms are gaining importance due to their accurate analysis. The major advantage of WIFFT based algorithms includes reduction in error due to spectral leakage and picket-fence effect. This paper represents the application of improved Hanning window based Interpolated FFT (IFFT) algorithm for harmonic analysis. The proposed algorithm is used to analyze a static simulated signal in MATLAB. The simulation results shows the effectiveness of Hanning window based IFFT compared to Hamming window based IFFT. The algorithm is also used to analyze a dynamic signal simulated in LabVIEW and its potential towards online tracking of a signal.**

Keywords— Hanning Window, Picket-fence effect, Power system harmonics, Spectral leakage, Windowed Interpolated FFT (WIFFT).

I. INTRODUCTION

The increased usage of non-linear loads led to the importance of the quality of power that has been delivered to the premises of the customers. These non-linear loads are the causers and also the victims of various power quality problems. Power system harmonics form a significant part of the power quality problems that are to be analyzed and mitigated.

Power system harmonics indicates the presence of various frequency components along with fundamental frequency. As indicated in [1], the consequences of the presence of power system harmonics in the system include damage to the sensitive equipment, over heating of the equipment, interference with neighboring communication network, increase in system losses which indicates an economic impact as well. Harmonic analysis forms a major part to have an effective mitigation. FFT based analysis are widely used due to the simplicity of the algorithm and a faster computation time which makes it suitable for real time applications [2]-[4]. FFT based analysis is accurate when strict synchronous sampling is maintained [5]. But, maintaining strict synchronous sampling is difficult even with the usage of discrete phase locked loop technique because of time varying harmonics, fundamental frequency instability and fundamental variations [6]. There are limitations such as spectral leakage, picket-fence effect which occur when synchronous sampling is not maintained. Hence, Windowed Interpolated FFT based algorithms are widely used to eliminate these limitations [7]. The accuracy of analysis depends on the choice of window that is used for harmonic analysis.

To reduce the spectral leakage a suitable windowed FFT is used for harmonic analysis. The width of the major lobe of the window indicates frequency resolution and roll-off rate of the side lobe indicates leakage effect. Higher roll-off rate of the side lobe indicates reduced leakage effect i.e. good accuracy [8]. The error due to picket-fence effect is reduced using windowed interpolated FFT algorithms.

The organization of the paper is as follows. Section II introduces the harmonic analysis of a signal in the presence of asynchronous sampling. The characteristics of Hanning window, the basic formulas for interpolated FFT and its implementation are given in Section III. Simulation results of a static signal and a dynamic signal are given in Section IV. Finally, Section V gives the conclusions that are drawn.

II. HARMONIC ANALYSIS BASED ON ASYNCHRONOUS SAMPLING

A sampled signal of length N $(n = 0, 1, 2, \ldots, N-1)$ in time domain can be represented as

$$x(t) = \sum_{h=1}^{H} A_h \sin(2\pi f_h n T_s + \varphi_h) \quad (1)$$

Where h represents the harmonic order, H represents the total number of harmonic components. f_h, A_h, φ_h represents frequency, amplitude, phase of h^{th} harmonic. T_s is the sampling interval. The FFT of the windowed sampled signal $x(n)w(n)$ on N sample points can be given as

$$X(k) = \sum_{h=1}^{H} \frac{A_h}{2j}\left[e^{j\varphi_h} W_f(k-k_h) - e^{-j\varphi_h} W_f(k+k_h)\right] \quad (2)$$

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where $k = 0, 1, \ldots, N-1$, W_f represents the FFT of the window function that is used for harmonic analysis, k_h represents the signal frequency and is given as

$$k_h = \frac{f_h N}{f_s} = l_h + \xi_h \quad (3)$$

Where l_h represents an integer value and $\xi_h (0 \le \xi_h \le 1)$ is the fractional part which caused by the non-synchronous sampling. From (3), the spectral line that represents the h^{th} harmonic may lie between the spectral lines, l_h^{th} and the $(l_{h+1})^{th}$, or l_h^{th} and the $(l_{h-1})^{th}$. The fractional part ξ_h can be determined from interpolation techniques from which amplitude, phase, and frequency of h^{th} harmonic can be accurately determined.

III. IMPROVED HANNING WINDOW BASED INTERPOLATED FFT ALGORITHM

A. Improved Hanning Window

The discrete time representation of a Hanning window with order M is given as

$$w_H(n) = \sum_{h=0}^{M-1} (-1)^h a_h \cos(\frac{2\pi hn}{N}) \quad (4)$$

Where $n=0, 1, 2, \ldots, N-1$.

N represents the total number of samples, a_h represents the window coefficients. The coefficients a_h follow the conditions given as

$$\sum_{h=0}^{M-1} (-1)^h a_h = 0, \sum_{h=0}^{M-1} a_h = 1 \quad (5)$$

For Hanning window $M=2$, $a_0=0.5$, $a_1=0.5$ [9].

The FFT of the Hanning window is given as

$$W_H(\omega) = \sum_{h=0}^{M-1} (-1)^h \frac{a_h}{2} [W_R(\omega - h) + W_R(\omega + h)] \quad (6)$$

Where

$$W_R(\omega) = \frac{\sin(\omega\pi)}{\sin(\omega\pi/N)} e^{-j(N-1)\omega\pi/N} \quad (7)$$

$W_R(\omega)$ represents FFT of rectangular window.

The performance parameter of the window function includes Equivalent Noise Band Width (ENBW) which indicates the capacity of a window function to extract the amplitude of the signal from the background noise that is present. Lower value of ENBW indicates a better extraction of the signal from the background noise. ENBW is given as

$$ENBW = N \sum_n w_H{}^2(nT) / [\sum_n w_H(nT)]^2 \quad (8)$$

B. Basic Formulas for Interpolated FFT

From (2) the contribution from the negative frequencies can be neglected. The spectrum corresponding to l_h^{th} spectral line which represents the h^{th} harmonic can be given as

$$X(\xi_h) = \frac{A_h}{2j} [e^{j\varphi_h} W_f(\xi_h - k_h)] \quad \text{, for } \xi_h = 0,1, \ldots, N-1 \quad (9)$$

$$y_1 = |X(l_{h1})| = |A_h| . |W_f(2\pi(l_{h1} - K_h)/N| \quad (10)$$

$$y_2 = |X(l_{h2})| = |A_h| . |W_f(2\pi(l_{h2} - K_h)/N| \quad (11)$$

The ratio of y_1 and y_2 is used for interpolated FFT algorithm and ξ_h, the fractional part which is present due to nonsynchronous sampling can be obtained. A polynomial interpolation based on the least-square method is adopted as it resulted in simple polynomial expression that contains odd terms of an independent variable α

Since $0 \le k_h - l_{h1} \le 1$, α is given as

$$\alpha = k_h - l_{h1} - 0.5 \quad \text{for } -0.5 \le \alpha \le 0.5 \quad (12)$$

Then y_1 and y_2 which are even and symmetrical on α are given as

$$y_1 = |X(l_{h1})| = |A_h| . |W_H(2\pi(-\alpha + 0.5)/N| \quad (13)$$

$$y_2 = |X(l_{h2})| = |A_h| . |W_H(2\pi(-\alpha - 0.5)/N| \quad (14)$$

The dependent variable β which is a function of independent variable α can be defined as

$$\beta = g(\alpha) = \frac{(y_2 - y_1)}{(y_2 + y_1)} \quad (15)$$

From (13) and(14), β can be written as

$$\beta = \frac{|W_H(2\pi(-\alpha+0.5)/N| - |W_H(2\pi(-\alpha-0.5)/N|}{|W_H(2\pi(-\alpha+0.5)/N| + |W_H(2\pi(-\alpha-0.5)/N|} \quad (16)$$

Inverse of (16) gives α in terms of β which is obtained from polynomial interpolation based on least-square technique. For Hanning window the relation between α and β obtained from polynomial interpolation is given as

$$\alpha = 1.5\,\beta \quad (17)$$

The parameters f_h, A_h, φ_h can be obtained as

$$k_h = \alpha + l_{h1} + 0.5 \quad (18)$$

$$f_h = K_h f_s / N \quad (19)$$

$$A_h = \frac{2y_1}{|W_f(2\pi(l_{h1} - K_h)/N|} \quad (20)$$

$$\varphi_h = Arg\big(X(\xi_h)\big) - \pi\alpha \tag{21}$$

Where $Arg\big(X(\xi_h)\big)$ indicates phase of the spectral line corresponding to h^{th} harmonic.

C. Implementation

The harmonic analysis of a signal using Windowed Interpolated FFT can be obtained using the following steps.

i) Read the sampled signal, window function, No. of samples, sampling frequency.
ii) The sampled signal is converted into weighted signal using window function.
iii) Calculate FFT of the obtained signal.
iv) Calculate FFT of the window function.
v) Take epsilon=0.01 in order to detect harmonics whose amplitudes are greater than or equal to 0.01.
vi) Consider a vector random which is used to store the information of harmonic orders having amplitude greater than 0.01.
vii) If random is not equal to zero for a particular index then continue towards interpolation.
viii) Express α in terms of β obtained from polynomial interpolation.
ix) Calculate the corresponding frequencies, amplitudes, phases using the relation that is mentioned in (19), (20), (21).
x) Calculate other parameters such as THD, ENBW.
xi) Plot the frequency spectrum and display THD, ENBW.

The proposed algorithm is used to estimate the amplitude, frequency, phase of the power signal that is to be analyzed.

IV. Simulation Results

This section shows the simulation results of Hanning window based Interpolated FFT algorithm. A mathematical signal is simulated in MATLAB which can be represented as

$$x(t) = \sum_{h=1}^{11} A_h \sin(2\pi h f t + \varphi_h) \tag{22}$$

Where h represents the order of the harmonic, A_h represents the amplitude of h^{th} harmonic, f indicates the fundamental frequency, φ_h represents the phase of h^{th} harmonic.

A sampling frequency of 3 kHz and N=1024 is considered for the analysis using Hanning window based interpolated FFT analysis.

A. Analysis of a Static Signal

As indicated in (22) a static signal is simulated in MATLAB environment. The amplitudes and the corresponding phases of the simulated signal are as given in Table I. The simulated signal is analyzed using Hanning window based interpolated FFT. The results are compared with Hamming window based interpolated FFT. Table II indicates the comparison of amplitude accuracies in terms of absolute errors. Table III indicates the comparison of accuracy in frequency estimation and Table IV indicates the comparison of accuracy in phase estimation.

TABLE I. Simulated signal Amplitude and phase information

Parameters	Harmonic orders					
	1	3	5	7	9	11
Amplitude (pu)	1	0.2	0.2	0.1	0.1	0.05
Phase (deg)	0	60	90	0	30	45

TABLE II. Amplitude estimation using different algorithms

Harmonic Order (h)	Absolute Error of Amplitude Estimation (%)	
	Hamming WIFFT	Hanning WIFFT
1	3.7280E-1	2.8620E-1
3	5.8780E-1	5.1059E-1
5	1.6759E0	1.3920E0
7	2.0536E0	1.7049E0
9	1.1941E0	0.9022E-1
11	2.5802E-1	2.2522E-1

TABLE III. Frequency estimation using different algorithms

Harmonic Order (h)	Absolute Error of Frequency Estimation (%)	
	Hamming WIFFT	Hanning WIFFT
1	2.6999E-1	3.0750E-4
3	2.0653E-1	7.6244E-4
5	1.1776E-1	6.9891E-4
7	3.8854E-1	4.4505E-3
9	4.1399E-1	4.5310E-3
11	4.2032E-1	1.1510E-2

TABLE IV. Phase estimation using different algorithms

Harmonic Order (h)	Absolute Error of Phase Estimation (%)	
	Hamming WIFFT	Hanning WIFFT
1	5.9751E-1	2.3080E-2
3	7.9030E-1	7.2303E-2
5	2.3169E-1	1.1672E-1
7	2.3770E-1	1.8848E-1
9	4.9970E-1	1.3812E-1
11	9.6958E-1	8.9237E-2

From the above Tables, it is clear that the accuracy of amplitude, frequency and phase estimation is more using Hanning Window based interpolated FFT algorithm compared to Hamming Window based interpolated FFT algorithm.

B. Analysis of a Dynamic Signal

The harmonic analysis of a dynamic signal is done using the proposed Hanning window based interpolated FFT algorithm is done in the LabVIEW environment. The simulated signal is a dynamic signal which imitates a real time signal. The simulated dynamic signal is given as

$$x(t) = 15\sin(2\pi f t) + 8\sin(6\pi f t) \tag{23}$$

The signal indicated by (23) contains a fundamental and third harmonic component with amplitudes 15, 8 respectively. A sampling frequency of 3 kHz, and N=1024 is considered for analysis using WIFFT based on Hanning window.

The snapshot of the front panel indicating the analysis of dynamic signal is as shown in Fig.1. The analysis shows the frequency spectrum and its logarithmic plot. The performance index ENBW is also indicated and THD estimated using the proposed algorithm is almost near to the actual value i.e. 53.33% for the signal as indicated in (23).

The time elapsed for its execution is also indicated and is in the range of 2-8 msec which is very small. This indicates its potential application for a real time signal to have online tracking of the signal.

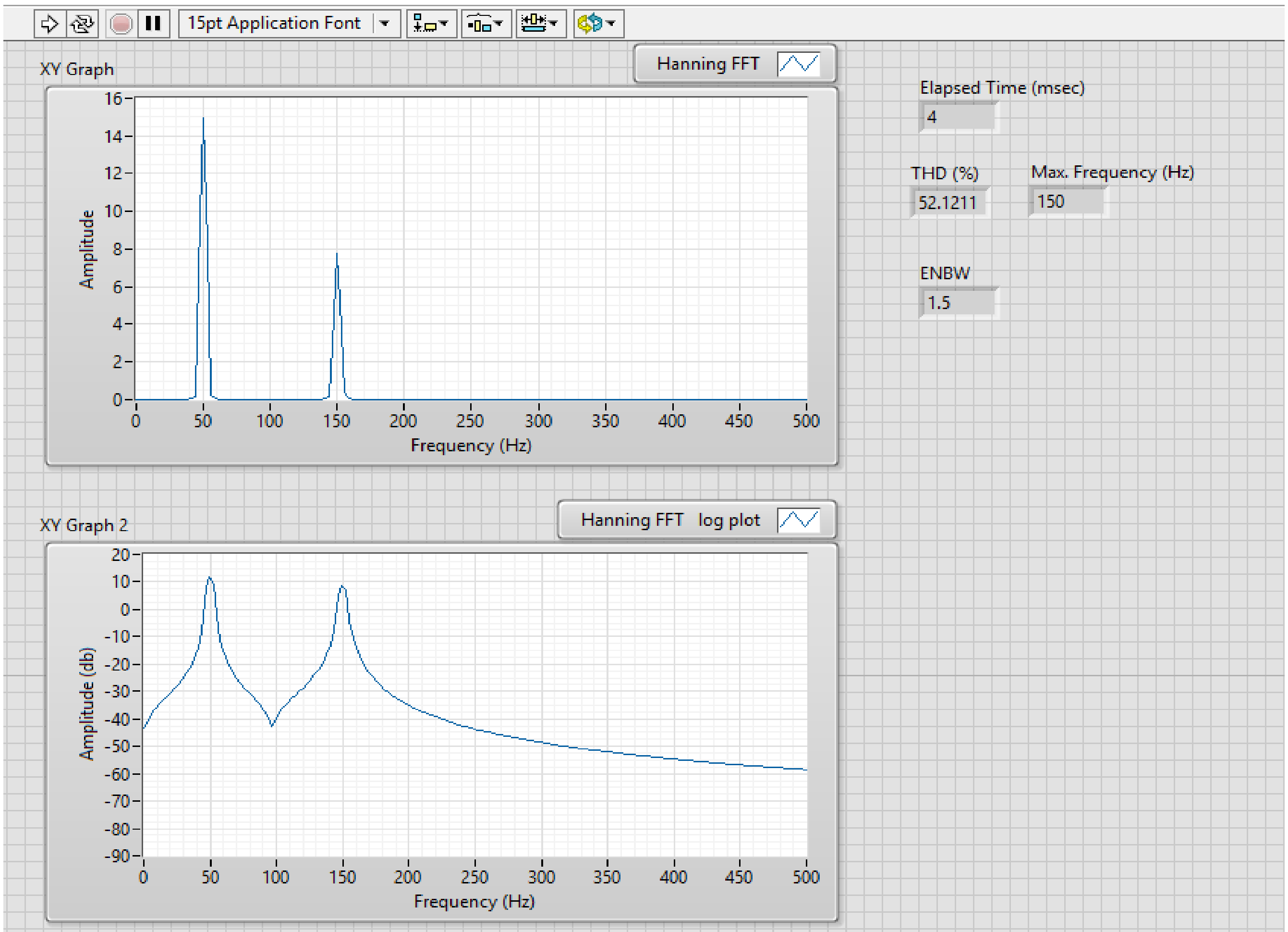

Fig. 1. Snapshot of Front panel of a Dynamic signal Analysis using Hanning Window based Interpolated FFT in LabVIEW

V. Conclusion

The Hanning window based Interpolated FFT algorithm is effective and accurate in harmonic analysis of a signal. The accuracy of Hanning window based algorithm for amplitude, frequency, phase estimation is more compared to that of Hamming window based algorithm. The frequency estimation is very much accurate. The major feature of the algorithm includes analysis of a dynamic signal in LabVIEW. The time taken for one time execution is very small i.e. 2 to 8 msec. Hence, the proposed algorithm can be used for analysis of a signal in real time environment which can be used for online tracking.

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