An Overview of Power Harmonic Analysis Based on Triangular Self Convolution Window

Pratishtha Rai, O V S R Varaprasad, *Student Member, IEEE*, D V S S Siva Sarma, *Senior Member,IEEE*
Electrical Engineering Department
National Institute of Technology Warangal
Warangal, INDIA
pratishthar8@gmail.com, varaprasad.oruganti@gmail.com, sivasarma@ieee.org

***Abstract*— Harmonic estimation plays a vital role in maintaining power system, and for reliable measurement of the harmonics, Windowed Interpolated Fast Fourier Transform (WIFFT) has been used. In this paper, supply voltage harmonics are analysed by Triangular Self-Convolution Window (TSCW), Hanning and Hamming WIFFT. TSCW is ameliorated window than previously existing Hanning and Hamming windows. By comparing these three windows it is observed that TSCW reduces the measurement errors and gives more precise results while estimating the harmonics. One more advantage with TSCW is that it is also able to detect the weak harmonics, which is not possible with Hanning and Hamming Window. Narrow main lobe and high side lobe decaying rate make TSCW capable of compressing the spectral leakage. The validity of the proposed comparison was confirmed by simulation and practical experiment of the supply voltage harmonic signal as input.**

Keywords—Hamming window; Harmonics; Hanning window; Triangular Self-Convolution Window (TSCW); Windowed Interpolated Fast Fourier Transform (WIFFT)

I. INTRODUCTION

With the exponential growth of the nonlinear loads in scientific and engineering applications, harmonic estimation becomes a serious issue in the power system. The presence of these harmonics impairs the power quality and affects adversely the economic operation of the power system [1]. Harmonic estimation includes reliable estimation of the frequency, amplitude and phase of each component of the frequency present in the harmonic signal.

In recent years several methods have been introduced for the estimation of the harmonics, among them, the application of zero crossing technique, Kalman filter, Wavelet transform and *S*-transform are discussed in [2]. Nevertheless, FFT is the well-known estimation tool for harmonic analysis. Under synchronous sampling, estimation of the harmonics using FFT can be accurate. But in the case of non-synchronous sampling FFT has certain limitation such as spectral leakage and picket fencing [3].

The unexpected effect of spectral leakage can be reduced by weighting the signal time samples with proper time window [4]. The picket fencing effect can be minimised by interpolation algorithm [5]. Many windows have been introduced to suppress spectral leakage such as Rectangular, Hanning, Hamming, Nuttall, Blackman etc... [6]. Spectral leakage can be reduced by using proper window function. It is illustrated in [7] that narrow main lobe width results to better frequency resolution. Whereas peak side lobe, higher side lobe decaying rate are suggestive of low spectral leakage. Hence, the Triangular window with narrow main lobe width and basic function in both time and frequency domains are considered as the main window. Triangular Self Convolution Window (TSCW) is obtained by convolving the triangular window as presented in [8]. Side lobe behaviour of the TSCW is able to sufficiently reduce the spectral leakage. TSCW with the proper order (number of convolution) of the triangular window exhibits a good deal of lower peak side lobe and higher sidelobe decaying rate as discussed in [8,9].

In this paper the 8th order TSCW with sampling frequencies of 1.5 kHz and 3 kHz are presented for harmonic parameter estimation, especially supply voltage harmonics of the distribution system using National Instruments (NI)-cRIO [10] based data acquisition system and it is compared with 4th order TSCW [9], the widely adopted Hanning and Hamming window based interpolated FFT results. TSCW has reasonable accuracy for frequency and amplitude estimation as well as the ability of the TSCW to detect a weak harmonics, which is not possible with Hanning and Hamming, makes it more suitable for distribution system harmonic estimation. Initially, simulation studies are carried out on the supply voltage harmonic signal using LabVIEW programming to estimate the amplitude and frequency errors of the typical voltage signal. After estimating the error, a real supply voltage signal is considered for the test using NI-cRIO.

II. OUTLINE OF TSCW

TSCW is obtained by convolution of the triangular window.

A. TSCW function

Triangular Window function is defined by [8]

$$w_t(m) = \begin{cases} \dfrac{2m}{L-1} & \Big| m = 0,1,2 \ldots L/2 - 1 \\ \dfrac{2L-4-2m}{L-2} & \Big| m = L/2, \ldots\ldots, L-1 \\ 0 & \Big| m = L. \end{cases} \tag{1}$$

978-1-4799-5141-3/14/$31.00 ©2016 IEEE

Authorized licensed use limited to: NATIONAL INSTITUTE OF TECHNOLOGY WARANGAL. Downloaded on December 01,2025 at 06:18:28 UTC from IEEE Xplore. Restrictions apply.

Here L is the length of the window, which is equal to the set of 2^i where i is a natural number. By taking the convolution of Triangular window, TSCW can be obtained. p^{th} order self-convolution can be obtained by the p-1 self-convolution of p instances of TSCW as follows

$$w_{T\text{-}p}(n)=w_t(m) * w_t(m) \ldots\ldots\ldots * w_t(m) \qquad (2)$$

where $m= 0,1 \ldots. L\text{-}1$, for p^{th}-order TSCW.

B. TSCW in frequency domain

The Triangular window function derived from Discrete-Time Fourier Transform (DTFT) can be represented by

$$W_T(w) = \frac{2e^{jwLT/2}}{L}\left(\frac{\sin(Lw/4)}{\sin(w/2)}\right)^2 \qquad (3)$$

where w is continuous angular frequency and is given by w=2π/T. The DTFT of the TSCW is given by

$$W_{T-P}(w) = \frac{2^p e^{\frac{-jpLw}{2}}}{L^p}\left(\frac{\sin(Lw/4)}{\sin(w/2)}\right)^{2p} \qquad (4)$$

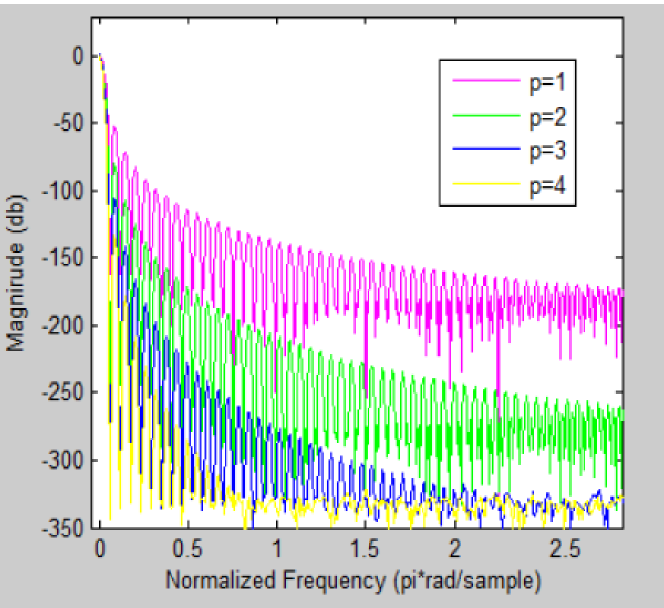

Fig. 1. Frequency response of TSCW

From the Fig. 1, it can be seen that side lobe decaying rate, the peak of the side lobe level is proportion to the order of TSCW. It means that higher will be the order than accuracy also will be higher.

III. Estimation of Harmonic Parameters

A. Interpolation Methodology

Harmonic parameter estimation through WIFFT includes discrete signal truncating weighting of the signal by the suitable window, interpolation for correction and then signal parameter estimation [11]. A time sampled multi-frequency voltage harmonic signal $x(n)$ in distribution system composed of different harmonics, can be expressed as

$$x(n) = \sum_{k=1}^{K} A_k \sin((2\pi f_k n / f_s) + \phi_k) \qquad (5)$$

where $n=0,1,2,\ldots..N\text{-}1$ and N are acquisition length; K is the frequency components number; f_s is the sampling frequency; A_k, f_k, Φ_k are amplitude, frequency, and phase of k^{th} harmonics respectively. FFT of the windowed sample signal $x(n)*w(n)$ on N sample points can be given by

$$X_{2p}(\lambda) = \sum_{k=1}^{K} \frac{A_k}{2j}(e^{j\phi_k} W_{T-p}(\lambda-\lambda_k) - e^{-j\phi_k} W_{T-p}(\lambda+\lambda_k)) \qquad (6)$$

where $\lambda \in (1{:}K)$ and W_{2p} represents the FFT of the window function. Ignoring the effect of the component part of the negative frequency, the DFT $X_{2p}(\lambda)$ is modified as

$$X_p(\lambda) = \sum_{k=1}^{K} \frac{A_k}{2j} e^{j\phi_k} W_{T-p}(\lambda-\lambda_k) \qquad (7)$$

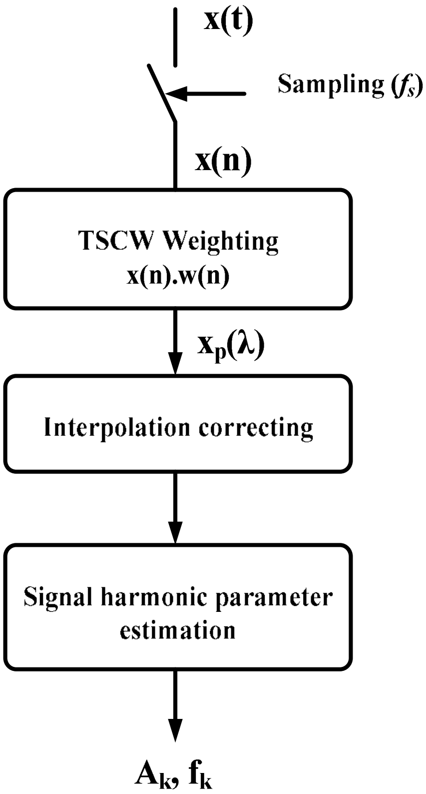

Fig. 2. Flow Chart of TSCW based interpolated FFT method

Authorized licensed use limited to: NATIONAL INSTITUTE OF TECHNOLOGY WARANGAL. Downloaded on December 01,2025 at 06:18:28 UTC from IEEE Xplore. Restrictions apply.

The spectral line related to the k^{th} order harmonic should recline between the two highest amplitude spectral lines. [11] If two maximum points are obtained at l_{k1} *and* $l_{k2}(=l_{k1}+1)$ and magnitudes $y_1=|X_p(l_{k1})|$ and $y_{k2}=|X_p(l_{k2})|$. Consider β derived from [11].

$$\beta = \frac{y_2 - y_1}{y_2 + y_1} \tag{8}$$

From the value of β value of α can be calculated by polynomial curve fitting technique as discussed below. Algorithm for evaluation of coefficients of β for α calculation [11]

a) Take a group of random values of α between -0.5 to 0.5. Evaluate the value of β for different α using:

$$\beta = \frac{y_2 - y_1}{y_2 + y_1} \cong \frac{|W_{T-p}(-\alpha - 0.5)| - |W_{T-p}(-\alpha + 0.5)|}{|W_{T-p}(-\alpha - 0.5)| + |W_{T-p}(-\alpha + 0.5)|} \tag{9}$$

b) Use *polyfit (β, α, J),* for inverse curve fitting and the coefficients of β, can be obtained.

$$\alpha = r_0\beta + r_1\beta^2 + r_2\beta^3 + \dots\dots\dots\dots + r_{J-1}\beta^J \tag{10}$$

where J is the order of the fitting polynomial approximation and its value is selected on the basis of accuracy. In this paper, J equals to 7 is considered.

c) Here α value is derived for 4th and 8th order as follows.

$$\alpha_{4th} = 0.1538\beta^7 + 0.2377\beta^5 + 0.4273\beta^3 + 4.8582\beta \tag{11}$$

$$\alpha_{8th} = 0.3024\beta^7 + 0.4782\beta^5 + 0.7486\beta^3 + 9.9090\beta \tag{12}$$

Once α is calculated, and then by using its value, harmonic parameter f_k and A_k can be estimated by using following formula

$$f_k = (l_{k1} + \alpha + 0.5) f_s / N \tag{13}$$

$$A_l = \frac{2|X_p(l_{k1})|}{W_{T-p}(-\alpha - 0.5)} \tag{14}$$

IV. SIMULATION RESULTS

In this section performance of the 4th order, 8th order TSCW are compared with Hanning and Hamming Windows. A multi-harmonic voltage signal is simulated in LabVIEW, which is given by

$$x(t) = \sum_{k=1}^{11} A_k \sin(2\pi kft + \phi) \tag{15}$$

where k is the order of harmonics, A_k is the amplitude of the k^{th} order of harmonics and f is fundamental frequency. Here the sampling frequency (f_s) =1.5 kHz and 3 kHz and number of sample N=1024 are considered.

TABLE I. AMPLITUDE OF HARMONICS

Harmonics Order k	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**9**	**11**
Amplitude A_k in volts	240	0.6	9	0.08	3.2	0.06	1.9	0.09	0.1
Phase in degrees	0	10	20	30	40	50	60	70	80

The 4th and 8th order TSCW based interpolated FFT results are compared with Hanning and Hamming window based interpolated FFT. It illustrates that frequency estimation with 8th order TSCW is more accurate than 4th order TSCW, Hanning and Hamming windowed interpolated FFT. In the case of amplitude estimation, the performance of the TSCW is identical with Hanning and Hamming WIFFT based on the tabulation of Table II and Table III. But for weak harmonics, the performance of the TSCW is better.

The simulation studies are performed for 1.5 kHz and 3 kHz sampling frequencies. For 1.5 kHz sampling frequency mean of the relative error in frequency estimation with 8th order TSCW is 1.5938E-4 while with 3 kHz, it is 2.746E-4. The mean of the relative frequency estimation error with the Hanning window is equals to 0.0092 for 1.5 kHz and 0.0187 for 3 kHz. The mean of the relative frequency estimation error with the Hamming window is equals to 0.01 for 1.5 kHz and 0.022 for 3 kHz.

In the case of amplitude, the mean of relative error with 8th order TSCW is 0.05 for 1.5 kHz and 0.0375 for 3 kHz, whereas with Hanning window mean of relative error is 0.0372 for 1.5 kHz and 0.0392 for 3 kHz. With Hamming window mean of relative error is 0.022 for 1.5 kHz and 0.35 for 3 kHz. Table II and Table III shows that 8th order TSCW has reasonable improvement in measurement of harmonics when compared to the Hanning and Hamming windowed interpolated FFT. Further, the TSCW based interpolated FFT is implemented in an NI-cRIO based data acquisition and estimation system.

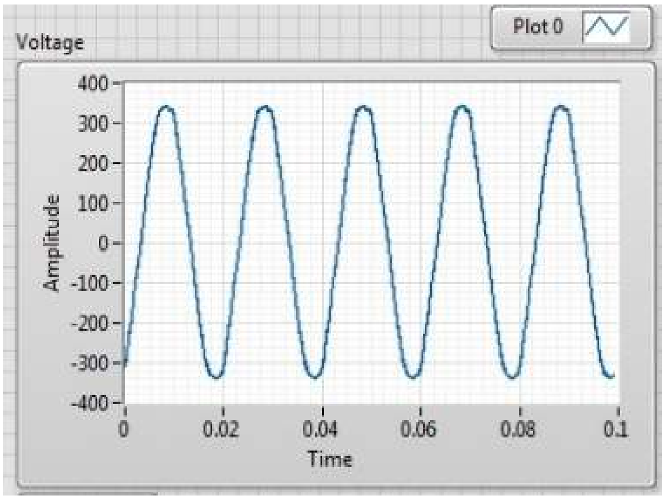

Fig. 3. Snapshot of supply voltage signal captured from NI-cRIO

Authorized licensed use limited to: NATIONAL INSTITUTE OF TECHNOLOGY WARANGAL. Downloaded on December 01,2025 at 06:18:28 UTC from IEEE Xplore. Restrictions apply.

TABLE II. RELATIVE ERROR IN FREQUENCY ESTIMATION

Sampling frequency =1.5kHz					Sampling frequency =3kHz			
Order of Harmonic (k)	***4th order TSCW WIFFT***	***8th order TSCW WIFFT***	***Hanning WIFFT***	***Hamming WIFFT***	***4th order TSCW WIFFT***	***8th order TSCW WIFFT***	***Hanning WIFFT***	***Hamming WIFFT***
1	0.001958	0.000453	0.029318	0.026981	0.004629	0.001071	0.058644	0.053245
2	0.000623	0.000144	0.015138	0.029223	0.001071	0.000453	0.033491	0.062699
3	0.000178	0.000041	0.009768	0.009483	0.001068	0.000247	0.019543	0.018234
4	0.001292	0.000300	0.007337	0.009390	0.000623	0.000144	0.014672	0.027485
5	0.000891	0.000207	0.005868	0.005316	0.000356	0.000082	0.011723	0.010829
6	0.000623	0.000144	0.004889	0.006752	0.000178	0.000041	0.009845	0.015582
7	0.000432	0.000100	0.004190	0.003593	0.000051	0.000012	0.008371	0.008232
9	0.000178	0.000041	0.003257	0.002762	0.001069	0.000248	0.006521	0.003977
11	0.000016	0.000004	0.002668	0.002324	0.000745	0.000173	0.005334	0.004057

TABLE III. RELATIVE ERROR IN AMPLITUDE ESTIMATION

Sampling frequency =1.5kHz					Sampling frequency =3kHz			
Order of Harmonic (k)	***4th order TSCW WIFFT***	***8th order TSCW WIFFT***	***Hanning WIFFT***	***Hamming WIFFT***	***4th order TSCW WIFFT***	***8th order TSCW WIFFT***	***Hanning WIFFT***	***Hamming WIFFT***
1	0.020945	0.022243	0.023152	0.091352	0.023236	0.023560	0.023082	0.101250
2	0.011830	0.016990	0.021820	1.683920	0.020930	0.022240	0.176260	1.605350
3	0.003178	0.008290	0.023319	0.067339	0.017140	0.020050	0.023214	0.083012
4	0.168470	0.104500	0.022260	0.651800	0.011830	0.016990	0.023380	0.399020
5	0.189630	0.115941	0.022485	0.118401	0.005052	0.013065	0.023285	0.079420
6	0.203940	0.123640	0.022720	0.086380	0.003180	0.008280	0.015280	0.100720
7	0.023236	0.127502	0.022910	0.147529	0.012821	0.002675	0.023330	0.065262
9	0.017140	0.020050	0.023207	0.129493	0.179887	0.110680	0.022293	0.632930
11	0.012822	0.002675	0.022090	0.187308	0.197658	0.120263	0.022565	0.288545

V. EXPERIMENTAL RESULTS

The harmonic analysis of a supply voltage feeding different linear and nonlinear loads, such as lamp load and computer loads are performed using the 8th order TSCW in the LabVIEW real-time environment. The supply voltage driving the linear and non-linear loads are sensed with the help of NI c-RIO analogue I/O module and processed to the 8th order TSCW based interpolated FFT algorithm. The obtained results have been compared with the results of power quality analyser measured values.

The relative errors of the amplitude and frequency of the supply voltage with the help of 8th order TSCW are tabulated in Table IV. The measurement accuracy of the harmonic frequency is improved in the case of 8th order TSCW when compared to 4th order TSCW, Hanning and Hamming window. From the Tabulated values, it can be observed that the 8th order TSCW based Interpolated FFT is accurately measured the harmonic components that are present in the supply voltage that is being analysed. The snapshot of the front panel indicating the analysis of supply voltage is shown in Fig.3.

TABLE IV. RELATIVE ERROR SUMMARY

Order of Harmonic (k)	Amplitude	Frequency
1	3.474E-3	4.379E-3
5	1.18431E-1	2.92E-4
7	2.0576E-1	7.81E-5
8	2.1255E-1	9.63E-5
9	1.68254E-1	9.15E-5
10	9.101E-2	8.09E-5
11	2.228E-1	1.81E-5
12	1.271E-2	4.18E-5
13	4.43E-3	1.09E-5
15	1.4382E-1	6.3E-5
16	4.6216E-1	6.97E-5

VI. CONCLUSIONS

The major problems with the harmonic analysis using window function based interpolated FFT is spectral leakage and picket fencing. This paper has chosen a TSCW with narrow main lobe and high side lobe decaying rate for

Authorized licensed use limited to: NATIONAL INSTITUTE OF TECHNOLOGY WARANGAL. Downloaded on December 01,2025 at 06:18:28 UTC from IEEE Xplore. Restrictions apply.

minimization of error due to spectral leakage and picket fencing. Results of simulation show that 8^{th} order TSCW is accurate than 4^{th} order TSCW, Hanning and Hamming based interpolated FFT for harmonic frequency and amplitude estimation. The performance of the 8^{th} order TSCW based interpolated FFT for distribution system supply voltage harmonic estimation is presented.

REFERENCES

[1] A. Medina, J. Segundo-Ramirez, P. Ribeiro, W. Xu, K. L. Lian, G. W. Chang, V. Dinavahi, and N. R. Watson, "Harmonic Analysis in Frequency and Time Domain", IEEE Trans. on Power Delivery, vol. 28, no. 3, pp. 1813-1821, July 2013.

[2] Julio Barros, Ramón I. Diego, "Applications of wavelet transform for analysis of harmonic distortion in power systems: A review", IEEE Trans. on Inst. Meas., vol. 61, no. 10, pp.2604-2611, Oct. 2012.

[3] O.V.S.R.Varaprasad, R.K. Panda, and D.V.S.S.Siva Sarma, "A novel synchronous sampling algorithm for power system harmonic analysis". Proc. In 2013 Annual IEEE India Conference (INDICON), IIT Bombay,India, pp. 1-5. Dec. 2013.

[4] Thomas Grandke, "Interpolation Algorithms for Discrete Fourier Transforms of Weighted Signals", IEEE Trans. on Inst. Meas., vol. 32, no. 2, pp.350-355, June 1983.

[5] Ruipeng Diao and Qingfeng Meng, "An Interpolation Algorithm for Discrete Fourier Transforms of Weighted Damped Sinusoidal Signals" IEEE Trans. on Inst. Meas., vol. 63, no. 6, pp.1505-1513, June 2014.

[6] F.J.Harris, On the use of windows for harmonic analysis with the discrete Fourier transform, Proc IEEE 66-1 (1978) 51–83.

[7] Irini S. Reljin, Branimir D. Reljin, and Veljko D. Papic, "Extremely Flat-Top Windows for Harmonic Analysis", IEEE Trans. on Inst. Meas.,vol. 56, no. 3, pp.1025-1041, June 2007.

[8] He Wen, Zhaosheng Teng, and Siyu Guo, "Triangular Self-Convolution Window With Desirable Sidelobe Behaviors for Harmonic Analysis of Power System", IEEE Trans. on Inst. Meas.,vol. 59, no. 3, pp.543-552, March 2010.

[9] He Wen, Junhao Zhang, Zhuo Meng, Siyu Guo, Fuhai Li, and Yuxiang Yang, "Harmonic Estimation Using Symmetrical Interpolation FFT Based on Triangular Self-Convolution Window" IEEE Trans. on Ind.Infor.,vol.11, no. 1, pp.16-26, Feb. 2015.

[10] http://www.ni.com/compactrio/whatis/

[11] O.V.S.R.Varaprasad, D.V.S.S.Siva Sarma, Rakesh Kumar Panda, "Advanced windowed interpolated FFT algorithms for harmonic analysis of electrical power system", Proc. In Eighteenth National Power Systems Conference (NPSC), IEEE, IIT Guwahati, India, pp. 1-6, Dec. 2014.

Authorized licensed use limited to: NATIONAL INSTITUTE OF TECHNOLOGY WARANGAL. Downloaded on December 01,2025 at 06:18:28 UTC from IEEE Xplore. Restrictions apply.