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# Proportional nucleolus game theory–based locational marginal price computation for loss and emission reduction in a radial distribution system

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**Summary**

In this paper, a proportional nucleolus game theory (PNGT)–based iterative method has been presented to compute locational marginal price (LMP) at buses where distributed generation (DG) units were installed in a distribution network. Proportional nucleolus theory is one of the solution concepts for a cooperative game theory problem. The PNGT-based iterative method provides financial incentives in terms of LMP to DG owners as per their contribution in loss reduction and emission reduction at a particular loading on the distribution system. In this method, LMP values depend on the distribution company's decision maker's preference among loss reduction, emission reduction, and distribution company's additional benefit. This proposed PNGT-based iterative method has been implemented on a Taiwan power company's distribution network consisting of 84 buses with 15 DG units using MATLAB. The computed LMP values have operated the network based on decision-maker priority so as to enable fair competition among DG owners.

**KEYWORDS**

decision making, distribution company (DISCO), distributed generation (DG), emission reduction, locational marginal price (LMP), loss reduction, proportional nucleolus game theory

## 1 | INTRODUCTION

In the last decade, the penetration of distributed generation (DG) resources in distribution networks has increased globally. New local government policies towards reduction in greenhouse gas emission and mitigation of global warming is the main reason for such penetration. Loss reduction, on-peak operating cost reduction, improvement in network reinforcement horizon, service quality, reliability, power quality, and voltage support[1,2] are the benefits seen by the penetration of DG units into the distribution system.

In the restructured electricity market environment, a distribution company's (DISCO's) decision maker needs to provide simultaneous importance to technical decisions like deployment of DG and economical decisions like development of retail competition. The DISCO's decision maker role is very important for efficient operation from technical and economical points of view, as these decisions improve market operations like competition and technical operations like reliability and service quality.[3]

With the integration of DG units, distribution networks have transformed from a passive state to an active state as in transmission networks.[4] A few operating methods such as nodal pricing,[5] which is used in transmission networks in a deregulated environment, are also applicable in active distribution networks. Nodal pricing is one of the mechanisms for financial incentives used by DISCOs to control privately owned DG units and to encourage DG owners to perform technical decisions.[6] Locational marginal price (LMP) is the most effective method to determine nodal price in practice. [3,7]

  

Various papers are available in the literature for the computation of LMP in distribution networks. The comparison of research contribution by various authors in addressing the different features considered for LMP computation in a distribution system are shown in Table 1.

To compute LMP at buses where DG units are integrated into the system, a new method was proposed[12] by considering DG units' share in both loss and emission reduction. The computation of DISCO's extra benefit is not effective, and authors have used nucleolus game theory to allocate loss and emission reduction. However, the nucleolus game theory suffers from some flaws like it is not monotonic and fails in case of an empty core.[13] Authors have considered an equal penalty price for all emissions. In this method, authors calculated incentives using the market price ($\lambda^k$), which provides a high $\Delta\Pi^a$ value. The computation of LMP at DG buses like this may lead to loss of a DG unit's profit.

In this paper, an iterative method, which is an improved version of paper cited by Farsani et al,[12] has been proposed to compute LMP at DG buses by considering active power loss and emission released due to power injection into the system from DG units and substation bus. Here, DISCO provides incentives to DG owners in terms of LMP from financial savings due to loss and emission reduction. The allocated financial incentive of each DG is again shared among the active and reactive power of that particular DG based on the power factor. The proposed method computes nodal prices at DG buses to provide financial incentives to DG owners only, whereas for customers, a uniform price has been considered as in the work of Celebi and Fuller.[14] In this proposed iterative method, there is no change in price for customers. The important part of the iterative algorithm is the allocation of loss reduction and emission reduction among DG units using the proportional nucleolus game theory (PNGT), which is monotonic unlike the nucleolus solution concept. The share of DG in active power loss reduction and emission reduction has been used for calculating LMP in the next iteration. As per the decision maker, with the priority of loss reduction and emission reduction, weight parameters will be changed. After iterative algorithm is converged based on chosen weights, the final LMP values depends on the decision maker's choice in DISCO extra benefit.

The main original contributions of this paper are as follows:

- The PNGT has been used for the first time to compute LMP based on loss and emission reduction.
- Novel mathematical modeling has been developed to compute financial incentives to DGs for their contribution in reduction of active power loss and emission.
- DISCOs were empowered to operate the network optimally.
- A novel tool was developed to enable DISCOs to control private DG owners.

Applications of the proposed method are as shown below:

- The DISCO's decision maker can use this method to maintain fair competition among DG owners.
- The DISCO's decision maker can handle the trade-off among loss, emission, DG benefit, and DISCO's extra benefit.

**TABLE 1** Comparison of locational marginal price (LMP) computation features

| | Different Features Addressed by Researchers | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Research Contribution** | **A** | **B** | **C** | **D** | **E** | **F** | **G** | **H** | **K** | **L** | **M** |
| Sotkiewicz and Vignolo[5] | √ | | | | | | √ | | | | |
| Sotkiewicz and Vignolo[8] | √ | | | | | | √ | | | √ | |
| Singh and Goswami[9] | √ | | | | | | √ | | | | |
| Shaloudegi et al[6] | √ | | | √ | √ | | √ | | √ | √ | √ |
| Sathyanarayana and Heydt[10] | √ | | | | | | √ | | | | |
| Sadeghi Mobarakeh et al[11] | √ | | | √ | | | √ | | | | |
| Farsani et al[12] | √ | √ | √ | √ | √ | √ | √ | √ | √ | √ | √ |
| Proposed method | √ | √ | √ | √ | √ | √ | √ | √ | √ | √ | √ |

A: Loss reduction. B: Emission reduction. C: ] Controllable merchandising surplus. D: Changing distributed generation (DG) benefit. E: Providing encouragement to DG for participating in loss reduction. F: Providing encouragement to DG for participating in emission reduction. G: Computing LMP at current operating conditions. H: Estimating LMP in the next operating conditions. K: Distribution companies' strategic ability for optimal operation. L: Zero merchandising surplus. M: Reactive power price.

- This proposed method can be helpful to the DISCO's decision maker to estimate state of network in terms of LMP, generation, active power loss, and emission in day-ahead operation.

The remainder of the paper has been organized as follows. Section 2 deliberates on the problem formulation to use PNGT for loss reduction and emission reduction. Section 3 presents analytical studies on a Taiwan power company (TPC) distribution network, and conclusions of this paper are mentioned in Section 4.

## 2 | PROBLEM FORMULATION

The proposed PNGT method has been developed based on 2 ideas as appended below:

- Allocation of the share of reduced loss and emission among DG units.
- Financial incentive to DG unit based on its share in loss and emission reduction.

The PNGT has been used for the allocation of reduced loss and emission among DG units, and a new iterative algorithm has been developed to compute the financial incentive to each DG unit based on its contribution in reduction of loss and emission.

### 2.1 | Computation of change in active power loss and emission

An iterative distribution load flow algorithm has been implemented in 2 cases for computing change in loss and emission. In this paper, a backward-and-forward sweep algorithm[15] has been used as this algorithm takes complete advantage of the ladder structure of distribution networks, to achieve high speed, robust convergence, and low memory requirements.[16,17] In this load flow solution, a simultaneously controlled PQ-modeled[18] DG has been used.

Case 1: This is the base case, where no DG was integrated into the system. Total load was supplied from the substation bus.

Case 2: Distributed generation units were integrated into the system. Total load was supplied from the substation bus and DG units.

The generation of each unit has been computed based on cost coefficients of that generator and LMP at the DG bus using Equations 1 and 2; the loss reduction from the base case has been computed using Equation 3.

$$CF_i = a_i PG_i^2 + b_i PG_i + c_i \tag{1}$$

$$PG_i = \frac{\left(\Pi_a^t\right)_i^j - b_i}{2a_i} \tag{2}$$

$$\Delta Ploss = Ploss_0{}^t - Ploss_{DG}{}^t \tag{3}$$

Total emissions under case 1 and case 2 have been computed using Equations 4 and 5, respectively, and the change in emission has been computed using Equation 6.

$$Emn_0{}^t = \left(\mathrm{SO_2}^{\mathrm{Sub}} + \mathrm{CO_2}^{\mathrm{Sub}} + CO^{\mathrm{Sub}} + \mathrm{NO}_x{}^{\mathrm{Sub}}\right)\mathrm{P_{Sub}}{}^t \tag{4}$$

Compute emission cost using Equations 7 and 8 under case 1 and case 2, respectively, based on the penalty price of the respective greenhouse gas emission.

A cooperative game theory is required to identify the contribution of each DG on the change in system loss and emission. In this paper, PNGT has been used for the allocation of change in loss and emission among DG units.

$$Emn_{\mathrm{DG}}{}^{t} = \sum_{i=1}^{N_{\mathrm{DG}}} \left(\mathrm{SO_2}^{\mathrm{DG}_i} + \mathrm{CO_2}^{\mathrm{DG}_i} + \mathrm{CO}^{\mathrm{DG}_i} + \mathrm{NO}_x{}^{\mathrm{DG}_i}\right)\left(PG^t\right)_i^j + \left(\mathrm{SO_2}^{\mathrm{Sub}} + \mathrm{CO_2}^{\mathrm{Sub}} + \mathrm{CO}^{\mathrm{Sub}} + \mathrm{NO}_x{}^{\mathrm{Sub}}\right) \left(P_{\mathrm{Load}}{}^{t} + Ploss_j{}^{t} - \sum_{i=1}^{N_{\mathrm{DG}}} \left(PG^t\right)_i^j\right) \quad (5)$$

$$\Delta Emn = Emn_0{}^{t} - Emn_{\mathrm{DG}}{}^{t} \quad (6)$$

$$EC_0{}^{t} = \left(\mathrm{SO_2}^{\mathrm{Sub}} P_{\mathrm{SO_2}} + \mathrm{CO_2}^{\mathrm{Sub}} P_{\mathrm{CO_2}} + \mathrm{CO}^{\mathrm{Sub}} P_{\mathrm{CO}} + \mathrm{NO}_x{}^{\mathrm{Sub}} P_{\mathrm{NO}_x}\right) P_{\mathrm{Sub}}{}^{t} \quad (7)$$

$$EC_{\mathrm{DG}}{}^{t} = \sum_{i=1}^{N_{\mathrm{DG}}} \left(\mathrm{SO_2}^{\mathrm{DG}_i} P_{\mathrm{SO_2}} + \mathrm{CO_2}^{\mathrm{DG}_i} P_{\mathrm{CO_2}} + \mathrm{CO}^{\mathrm{DG}_i} P_{\mathrm{CO}} + \mathrm{NO}_x{}^{DG_i} P_{\mathrm{NO}_x}\right)\left(PG^t\right)_i^j + \left(\mathrm{SO_2}^{\mathrm{Sub}} P_{\mathrm{SO_2}} + \mathrm{CO_2}^{\mathrm{Sub}} P_{CO_2} + CO^{\mathrm{Sub}} P_{\mathrm{CO}} + \mathrm{NO}_x{}^{\mathrm{Sub}} P_{\mathrm{NO}_x}\right) * \left(P_{\mathrm{Load}}{}^{t} + Ploss_j{}^{t} - \sum_{i=1}^{N_{\mathrm{DG}}} \left(PG^t\right)_i^j\right) \quad (8)$$

## 2.2 | Proportional nucleolus game theory

The players in cooperative game theory are DG units in a distribution network. A game consisting of $n$ players has ($2^n-1$) coalitions. All DG units in a coalition inject power into the network simultaneously. The allocation of active power loss and emission among DG units corresponds to the allocation of the payoffs among DG units in the coalition. The problem of active power loss allocation and emission allocation then turns into the equilibrium point in cooperative game theory. In this paper, the PNGT method has been used as a solution concept for cooperative game theory.

An extended core concept has been introduced to compute the solution for cooperative games under an empty-core environment. The main characteristic of an extended core is being always nonempty, and the solution concept coincides in cases where the core is nonempty. An imputation chooses from an extended core in proportional nucleolus theory, like a nucleolus that chooses an imputation from the core. The proportional nucleolus differs from the nucleolus in the formation of definition of excess concerned with coalitions[13] as shown in the following:

$$e(Y:S) = \frac{v(S) - \sum_{i \epsilon S} y_i}{v(S)}. \quad (9)$$

An imputation set represented by $Y$ is shown in Equation 10 for a cooperative game consisting of $n$ players. Elements in this set represent imputation of each DG.

$$Y = [y_1, y_2, \cdots, y_n] \quad (10)$$

The PNGT can grow the core to obtain a unique solution in empty-core and large-core cases. Thus, the proportional nucleolus can provide a better solution in extended core and core selection problems. This ability of the proportional nucleolus to select an imputation is another advantage of the extended core solution concept. The solution has been obtained based on the PNGT by solving the following linear programming problem:

$$\begin{aligned} &\min \varepsilon \\ &\sum_{i \epsilon N} y_i = v^k(N) \\ &\frac{v^k(S) - \sum_{i \epsilon S} y_i}{v^k(S)} \leq \varepsilon, \end{aligned} \quad (11)$$

where $\varepsilon$ is a small arbitrary real value.

Now the proportional nucleolus solution concept for cooperative game theory has been explained using a sample calculation. Consider a distribution system with 3 DG units. The distribution system has a base case loss of 220 kW. Active power loss due to each coalition of DG units is shown in Table 2

**TABLE 2** Loss reduction for different coalitions

| Coalition($S$) | Loss Due to Coalition, kW | Loss Reduction Due to Coalition, kW |
|---|---|---|
| 1 | 207.3 | 12.7 |
| 2 | 185.3 | 34.7 |
| 3 | 205.4 | 14.6 |
| 1, 2 | 150.0 | 70.0 |
| 1, 3 | 179.2 | 40.8 |
| 2, 3 | 163.3 | 56.7 |
| 1, 2, 3 | 107.9 | 112.1 |

A linear programming problem has been formulated using Equation 11, and the objective function is shown in

$$\min \quad 0*y_1 + 0*y_2 + 0*y_3 + 1*\varepsilon \tag{12}$$

such that the equality and inequality constraints shown in Equations 13 and 14, respectively, have to be satisfied.

$$1*y_1 + 1*y_2 + 1*y_3 + 0*\varepsilon \quad = \quad 112.1 \tag{13}$$

$$\begin{aligned} -1*y_1 + 0*y_2 + 0*y_3 - 12.7*\varepsilon &\leq -12.7 \\ 0*y_1 - 1*y_2 + 0*y_3 - 34.7*\varepsilon &\leq -34.7 \\ 0*y_1 + 0*y_2 - 1*y_3 - 14.6*\varepsilon &\leq -14.6 \\ -1*y_1 - 1*y_2 + 0*y_3 - 70*\varepsilon &\leq -70 \\ -1*y_1 + 0*y_2 - 1*y_3 - 40.8*\varepsilon &\leq -40.8 \\ 0*y_1 - 1*y_2 - 1*y_3 - 56.7*\varepsilon &\leq -56.7 \end{aligned} \tag{14}$$

By solving the above linear programming problems, the share of each DG unit in total loss reduction has been computed and is shown in Table 3.

The monotonic property states that if the value of a grand coalition increases and all other subcoalition values remain constant, then everyone's (each DG participant) payoff ($xLoss(i)$ or $y_i$) should increase.[19] The PNGT is monotonic, and this can be explained as below.

Let us assume that loss reduction due to the grand coalition by considering all DG units is increased from 112.1 to 150 kW and there is no change in loss reduction due to each subcoalition. A new payoff or share in loss reduction for each DG can be obtained by solving the linear programming problem using the objective function shown in 12

**TABLE 3** Payoffs for different grand coalition values

| | Payoff for Each DG, kW Value of Grand Coalition | |
|---|---|---|
| DG Units | 112.1 kW | 150 kW |
| 1 | 34.9761 | 46.7685 |
| 2 | 57.7780 | 77.3450 |
| 3 | 19.3459 | 25.8865 |

Abbreviation: DG, distributed generation.

subjected to the inequality constraints shown in Equation 14 and a new equality constraint shown in Equation 15. Inequality constraints are not changed as the values of subcoalitions were kept constant.

$$1{*}y_1 + 1{*}y_2 + 1{*}y_3 + 0{*}\varepsilon \quad = \quad 150 \tag{15}$$

The obtained new payoffs or share of each DG unit in loss reduction is shown Table 3.

On the basis of the definition of monotonic, it can be concluded that PNGT exhibits monotonic property as payoffs of each DG increased while increasing the value of the grand coalition.

## 2.3 | Load forecasting for day-ahead operation

The system operator must forecast the system load in each hour of the next day to compute the LMP at each DG bus. Artificial neural network (ANN)–based load forecasting has been used in this paper because of their efficient performance in forecasting.[20] The back-propagation algorithm has been used to train the given ANN because of its flexibility and learning capabilities and is highly suitable for problems where no relationship is found between the output and input. [21]

The ANN has been trained by considering $D$ day $t$ hour load $L(t,D)$ as output and $(t-1)$, $(t-2)$, $(t-3)$, and $(t-4)$ hour loads of $D$ day and $t$ hour load of $(D-1)$ and $(D-2)$ days as input data. Because of this, ANN can predict the next hour load based on the last 4 hours' load and the load of the same hour of the previous 2 days. After training, the given ANN has been tested by 24 hours of load data as testing data. The ANN has been trained and tested by taking 9 months of practical load data online.[22] The testing performance of the network was measured by using the mean absolute percentage error and root mean square error. [23]

## 2.4 | DISCO's extra benefit

The DISCO's benefit is defined as the difference between the total revenue collected from the customers and the sum of total amount paid to purchase power from DG buses and the substation bus and the penalty due to emission, which is shown in Equations 16 and 17.

$$benefit_0{}^t = \pi^c D - \left(\left(PLoad^t + Ploss_0{}^t\right)\lambda^t + EC_0{}^t\omega_e\right) \tag{16}$$

The DISCO's extra benefit is defined as the difference between the base case benefit and benefit after DGs inject power into the distribution system. As per the definition, the extra benefit is obtained by subtracting Equation 16 from Equation 17, and the final expression is as shown in Equation 18.

$$\text{benefit}_j{}^t = \pi^c D - \left(\sum_{i=1}^{N_{\text{DG}}} \left(PG^t\right)_i^j \left(\pi_a^t\right)_i^j + \sum_{i=1}^{N_{\text{DG}}} \left(QG^t\right)_i^j \left(\pi_r^t\right)_i^j + \left(PLoad^t + Ploss_j{}^t - \sum_{i=1}^{N_{\text{DG}}} \left(PG^t\right)_i^j\right)\lambda^t + EC_j{}^t\omega_e\right) \tag{17}$$

$$\Delta\text{benefit}_j{}^t = \left(Ploss_0^t - Ploss_j^t\right)\lambda^t - \sum_{i=1}^{N_{\text{DG}}} \left(PG^t\right)_i^j {*} \left(\left(\pi_a^t\right)_i^j - \lambda^t\right) - \sum_{i=1}^{N_{\text{DG}}} \left(QG^t\right)_i^j \left(\pi_r^t\right)_i^j + \left(EC_0{}^t - EC_j{}^t\right)\omega_e \tag{18}$$

The DISCO's extra benefit has been increased because of loss and emission reduction in the presence of DG units. The DISCO's extra benefit is nothing but merchandising surplus obtained from loss and emission reduction. In general, this merchandising surplus is greater than 0. Minimization of this merchandising surplus is required for fair competition.[6] Expression for a DISCO's extra benefit in terms of merchandising surplus as shown in Equation 19 has been derived using loss reduction in Equation 20 and emission reduction in Equation 21.

$$\Delta\text{benefit}_j{}^t = \left(MS^t\right)_{\text{loss}}^j + \left(MS^t\right)_{\text{Emn}}^j, \tag{19}$$

where

$$\left(MS^t\right)_{\text{loss}}^j = \left(Ploss_0^t - Ploss_j^t\right)\lambda^t - \sum_{i=1}^{N_{\text{DG}}} \left(PG^t\right)_i^j \left(\left(\pi_a^t\right)_i^j - \lambda^t\right) - \sum_{i=1}^{N_{\text{DG}}} \left(QG^t\right)_i^j \left(\pi_r^t\right)_i^j, \tag{20}$$

$$(MS^t)^j_{\text{Emn}} = \left(EC_0{}^t - EC_j{}^t\right)\omega_e. \tag{21}$$

In the work of Khatib,[24] penalty has been allocated to DISCOs for emission while serving the customers. The main aim of this strategy is to reduce emission. As per this proposed strategy, DISCOs that deliver electricity from high-emission–releasing generators receive a high penalty price. As the penalty is more, the customer price leads to a higher value. Whereas DISCOs that deliver electricity from low-emission–releasing generators receive a low penalty price. Since penalty is less, the customer price will be low. The parameter $\omega_e$ shown in Equations 18 and 21 represents the share of the DISCO and generators in the total system penalty price for emission. From this discussion, the $\omega_e$ value is set to 0.5 in this paper, but the final value depends on the decision maker's choice.

## 2.5 | PNGT-based iterative algorithm

A PNGT-based iterative algorithm to compute LMP at each DG bus and then estimate the generation based on its contribution for reduction in loss and emission is explained in Algorithm 1. The DISCO predicted generation as an economic signal for DG units to achieve optimal operation of the network.

In Algorithm 1, convergence has been checked in step 9 by using $\Delta benefit^t_j$ and $\Delta Pmax$ values. The condition based on $\Delta Pmax$ was satisfied if there is no considerable change in loss and emission because of an increase in generation. In such a case, incremental price related to loss and emission reduction is very small so that there was no significant change in LMP values and generation. The incentive provided to each generator from the extra benefit of the DISCO until $\Delta benefit^t_j$ is less than the small value $\varepsilon_1$.

**Algorithm 1** PNGT-based iterative algorithm

**Inputs**

1: Hour ($t$) of the day ($D$)
2: Forecasted load $L(t, D)$
3: Market price $\lambda^t$

**Steps**

1: Run the load flow and compute base case loss with forecasted load $L(t, D)$.
2: Set iteration $j = 1$, $(\pi^i_a)^j_t = \lambda^t$, and $PG^0_i = 0$, where $i = 1, 2, \ldots, N_{\text{DG}}$.
3: $i = 1$ ▷ $i$ represents the DG number
4: **while** $i \neq N_{\text{DG}} + 1$ **do**
  Compute generation using

$$PG^j_i = \frac{\left(\pi^i_a\right)^j_t - b_i}{2a_i}. \tag{22}$$

5:   $i \leftarrow i + 1$
6: **end while**
7: Run the load flow and compute loss ($\nu^l(N)$) and emission ($\nu^e(N)$) due to coalition ($N$) of all DG units based on generation computed in step 4.
8: Run the load flow and compute loss ($\nu^l(S)$) and emission ($\nu^e(S)$) due to each subcoalition ($S$) of all DG units based on generation computed in step 4.
9: Compute the DISCO extra benefit ($\Delta benefit_j{}^t$) using Equation 18 and set $\Delta Pmax = \max(PG^j_i - PG^{j-1}_i)$.
10: **if** $\Delta benefit_j{}^t \leq \epsilon_1$ OR $\Delta Pmax \leq \epsilon_2$ **then** ▷ $\epsilon_1$ and $\epsilon_2$ are small values for checking convergence
11:   *GoTo* ⇝ step 20
12: **else**
13:   *GoTo* ⇝ step 15
14: **end if**
15: Compute the share of each DG unit in loss reduction $xLoss(i)$ and in emission reduction $xEmn(i)$ using PNGT as shown in Section 2.2.
16: Compute the financial incentive of each DG unit for its contribution in loss reduction and emission reduction using respectively

$$DGgain^i_{\mathrm{loss}} = \omega_1 \frac{\left(\left(Ploss^t_0 - Ploss^t_j\right)\lambda^t + \left(MS^t\right)^j_{\mathrm{Emn}}\right) xLoss(i)}{Ploss^t_0 - Ploss^t_j} \tag{23}$$

and

$$DGgain^i_{\mathrm{Emn}} = \omega_2 \frac{\left(\left(Ploss^t_0 - Ploss^t_j\right)\lambda^t + \left(MS^t\right)^j_{\mathrm{Emn}}\right) xEmn(i)}{Emn^t_0 - Emn^t_j}. \tag{24}$$

17: Distribute the incentive of each DG unit among active and reactive power generation as shown in

$$\begin{aligned} DGgain^i_P &= DGgain^i_{\mathrm{loss}} * \left(cosdg^i\right)^2 + DGgain^i_{\mathrm{Emn}}, \\ DGgain^i_Q &= DGgain^i_{\mathrm{loss}} * \left(1 - \left(cosdg^i\right)^2\right). \end{aligned} \tag{25}$$

18: Compute active and reactive power price for next iteration using respectively

$$\left(\left(\pi^i_a\right)^{j+1}_t - \lambda^t\right) \frac{\left(\pi^i_a\right)^{j+1}_t - b_i}{2a_i} = DGgain^i_P \tag{26}$$

and

$$\left(\pi^i_r\right)^{j+1}_t = \lambda^r + \frac{DGgain^i_Q}{QG^j_i}. \tag{27}$$

The reactive power price at the substation bus is less than 1% of the active power price[25]; hence, the $\lambda^r$ value is considered as 0.

19: Increment iteration $j = j + 1$ and go to step 3.

20: Stop iterative algorithm for hour $t$ and take print out of required data.

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$\omega_1$ and $\omega_2$ represent weight for loss and emission reduction, respectively. These values depend on the DISCO's decision maker priority among loss reduction and emission reduction. If the decision maker decides to give top priority to loss reduction, then assign $\omega_1$=1 and $\omega_2$=0. Similarly, if the decision maker decides to operate the network with a lower emission, then assign $\omega_1$=0 and $\omega_2$=1. The values of $\omega_1$ and $\omega_2$ vary between 0 and 1 based on the decision maker choice.

## 3 | ANALYTICAL STUDIES

The proposed PNGT-based iterative algorithm has been implemented on a TPC distribution network. The TPC distribution network consists of 84 buses and 11 feeders. Complete information of TPC distribution system has been considered from Su and Lee.[26] Assuming that 15 DG units with a 0.9 lagging power factor have been connected to the TPC distribution network, the type and locations of 1-MW DG units are shown in Table 4.

Results shown in this section were obtained based on a simulation in the MATLAB[27] environment on a 3.4-GHz and 4-GB RAM machine with an i7 processor. Complete information about emission coefficients of DG units and the substation bus and cost coefficients of DG units are taken from Farsani et al.[12] All the simulation results shown in this section are based on realistic price data as available in another work,[22] and the forecasted load is shown in Figure 1.

### 3.1 | Load forecasting using ANN

Loads on the test systems have been forecasted by training and testing the ANN as explained in Section 2.3. The values of mean absolute percentage error and root mean square error after testing are found to be 1.6945 and 0.5289, respectively, for the TPC distribution system. The forecasted load for a 24-hour predicted zone for the TPC distribution network is shown in Figure 1.

### 3.2 | Impact of $\omega_1$ and $\omega_2$ on the DG unit's generation, active power prices, reactive power price, loss, and emission

Tables 5 and 6 present results obtained by the proposed PNGT-based iterative method at a market price of $\lambda^t$=\$25.34/MW and $\lambda^t$=\$13.31 /MW using different values of $\omega_1$ and $\omega_2$ for the TPC distribution system. As shown in Table 5, when

**TABLE 4** Distributed generation unit type and location in a Taiwan power company distribution system

| Unit | Type | Location |
|---|---|---|
| 1 | 1 | 4 |
| 2 | 1 | 65 |
| 3 | 1 | 25 |
| 4 | 1 | 35 |
| 5 | 1 | 84 |
| 6 | 2 | 55 |
| 7 | 2 | 12 |
| 8 | 2 | 72 |
| 9 | 2 | 20 |
| 10 | 2 | 47 |
| 11 | 3 | 11 |
| 12 | 3 | 60 |
| 13 | 3 | 41 |
| 14 | 3 | 30 |
| 15 | 3 | 76 |

Type 1: Combined cycle gas turbine. Type 2: Gas internal combustion engine. Type 3: Diesel internal combustion engine.

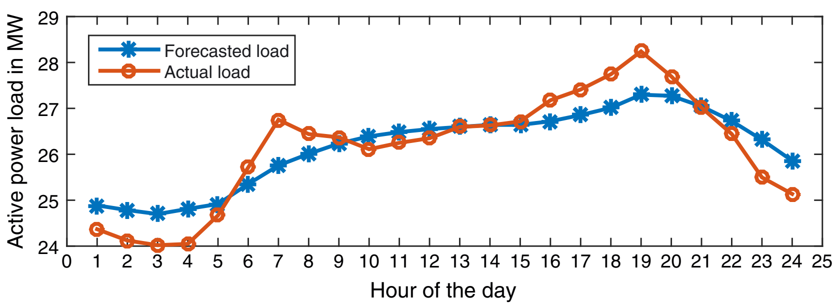


**FIGURE 1** Forecasted load for a Taiwan power company distribution system

the market price is $\lambda^t$=\$25.34/MW, the TPC distribution system power loss increases continuously as $\omega_2$ increases, which means that the DISCO wants to encourage DG units that have a high impact on emission reduction. For example, DG6, DG7, DG8, DG9, and DG10 with smaller emission coefficients compared with other DG units have increased their generation as $\omega_2$ increases. Similarly, emission increases as $\omega_1$ increases, which means that the DISCO wants to encourage DG units that have a high impact on loss reduction like DG11. When the market price is \$13.31/MW, all DG units in TPC distribution systems are off because the market price is less than the *b* coefficient in the cost function of DG units. In such a case, active power loss and emission are the same as those in the base case.

As shown in Table 6, when the market price is \$25.34/MW, LMP values of all type 2 units in the TPC distribution system increase with an increase in the value of $\omega_2$ because these units have more impact on emission with low emission coefficients. Similarly, LMP values of the remaining DG units vary based on their contribution in loss and emission reduction. For example, the LMP of DG11 increases consistently with $\omega_1$, which means it has much impact on loss of the TPC distribution system. When the market price is \$13.31/MW, all DG units are inactive because the market price is less than the *b* coefficient for all DG units and their LMP values are equal to market price.

Table 7 shows the reactive power prices of each DG unit by implementing the proposed PNGT-based iterative method on the TPC distribution system with different weight parameter combinations when the market prices are \$25.34/MW and \$13.31/MW, respectively. When the market price is \$13.31/MW, all DG units are off as the market price

**TABLE 5** Impact of $\omega_1$ and $\omega_2$ on the distributed generation (DG) unit's generation (kW), loss (kW), and emission (kg)

| | | $\lambda^t$=$25.34/MW | | | $\lambda^t$=$13.31/MW | | |
|---|---|---|---|---|---|---|---|
| Type | $\omega_1$ | 0.75 | 0.5 | 0.25 | 0.75 | 0.5 | 0.25 |
| | $\omega_2$ | 0.25 | 0.5 | 0.75 | 0.25 | 0.5 | 0.75 |
| 1 | DG1 | 513 | 497 | 481 | 0 | 0 | 0 |
| | DG2 | 461 | 460 | 459 | 0 | 0 | 0 |
| | DG3 | 467 | 465 | 462 | 0 | 0 | 0 |
| | DG4 | 478 | 473 | 466 | 0 | 0 | 0 |
| | DG5 | 493 | 484 | 473 | 0 | 0 | 0 |
| 2 | DG6 | 619 | 635 | 650 | 0 | 0 | 0 |
| | DG7 | 554 | 589 | 623 | 0 | 0 | 0 |
| | DG8 | 648 | 655 | 662 | 0 | 0 | 0 |
| | DG9 | 613 | 631 | 647 | 0 | 0 | 0 |
| | DG10 | 552 | 587 | 622 | 0 | 0 | 0 |
| 3 | DG11 | 755 | 728 | 699 | 0 | 0 | 0 |
| | DG12 | 624 | 634 | 644 | 0 | 0 | 0 |
| | DG13 | 656 | 657 | 657 | 0 | 0 | 0 |
| | DG14 | 614 | 628 | 640 | 0 | 0 | 0 |
| | DG15 | 594 | 613 | 632 | 0 | 0 | 0 |
| Loss, kW | | 243.24 | 243.91 | 244.99 | 398.39 | 398.39 | 398.39 |
| Emission, kg | | 22 732 | 22 680 | 22 634 | 24 404 | 24 404 | 24 404 |

**TABLE 6** Impact of $\omega_1$ and $\omega_2$ on the distributed generation (DG) unit's locational marginal price ($/MW), loss (kW), and emission (kg)

| | | $\lambda^t$=$25.34/MW | | | $\lambda^t$=$13.31/MW | | |
|---|---|---|---|---|---|---|---|
| Type | $\omega_1$ | 0.75 | 0.5 | 0.25 | 0.75 | 0.5 | 0.25 |
| | $\omega_2$ | 0.25 | 0.5 | 0.75 | 0.25 | 0.5 | 0.75 |
| 1 | DG1 | 26.95 | 26.77 | 26.58 | 13.31 | 13.31 | 13.31 |
| | DG2 | 26.35 | 26.34 | 26.33 | 13.31 | 13.31 | 13.31 |
| | DG3 | 26.42 | 26.39 | 26.36 | 13.31 | 13.31 | 13.31 |
| | DG4 | 26.55 | 26.49 | 26.41 | 13.31 | 13.31 | 13.31 |
| | DG5 | 26.72 | 26.61 | 26.48 | 13.31 | 13.31 | 13.31 |
| 2 | DG6 | 26.56 | 26.73 | 26.89 | 13.31 | 13.31 | 13.31 |
| | DG7 | 25.87 | 26.24 | 26.61 | 13.31 | 13.31 | 13.31 |
| | DG8 | 26.87 | 26.95 | 27.01 | 13.31 | 13.31 | 13.31 |
| | DG9 | 26.50 | 26.68 | 26.86 | 13.31 | 13.31 | 13.31 |
| | DG10 | 25.85 | 26.22 | 26.59 | 13.31 | 13.31 | 13.31 |
| 3 | DG11 | 27.55 | 27.28 | 26.99 | 13.31 | 13.31 | 13.31 |
| | DG12 | 26.56 | 26.57 | 26.57 | 13.31 | 13.31 | 13.31 |
| | DG13 | 26.56 | 26.57 | 26.57 | 13.31 | 13.31 | 13.31 |
| | DG14 | 26.14 | 26.28 | 26.40 | 13.31 | 13.31 | 13.31 |
| | DG15 | 25.94 | 26.13 | 26.32 | 13.31 | 13.31 | 13.31 |
| Loss, kW | | 243.24 | 243.91 | 244.99 | 398.39 | 398.39 | 398.39 |
| Emission, kg | | 22 732 | 22 680 | 22 634 | 24 404 | 24 404 | 24 404 |

**TABLE 7** Impact of $\omega_1$ and $\omega_2$ on the distributed generation (DG) unit's reactive power price ($/MVar), loss, and emission

| | | Taiwan Power Company Distribution System | | | | | |
|---|---|---|---|---|---|---|---|
| | | $\lambda^t$=$25.34/MW | | | $\lambda^t$=$13.31/MW | | |
| Type | $\omega_1$ | 0.75 | 0.5 | 0.25 | 0.75 | 0.5 | 0.25 |
| | $\omega_2$ | 0.25 | 0.5 | 0.75 | 0.25 | 0.5 | 0.75 |
| 1 | DG1 | 0.651 | 0.440 | 0.223 | 0 | 0 | 0 |
| | DG2 | 0.368 | 0.248 | 0.125 | 0 | 0 | 0 |
| | DG3 | 0.402 | 0.271 | 0.137 | 0 | 0 | 0 |
| | DG4 | 0.464 | 0.313 | 0.159 | 0 | 0 | 0 |
| | DG5 | 0.546 | 0.369 | 0.187 | 0 | 0 | 0 |
| 2 | DG6 | 0.380 | 0.255 | 0.128 | 0 | 0 | 0 |
| | DG7 | 0.056 | 0.037 | 0.019 | 0 | 0 | 0 |
| | DG8 | 0.526 | 0.354 | 0.179 | 0 | 0 | 0 |
| | DG9 | 0.351 | 0.236 | 0.119 | 0 | 0 | 0 |
| | DG10 | 0.044 | 0.028 | 0.014 | 0 | 0 | 0 |
| 3 | DG11 | 0.906 | 0.614 | 0.313 | 0 | 0 | 0 |
| | DG12 | 0.289 | 0.194 | 0.097 | 0 | 0 | 0 |
| | DG13 | 0.438 | 0.295 | 0.149 | 0 | 0 | 0 |
| | DG14 | 0.242 | 0.162 | 0.082 | 0 | 0 | 0 |
| | DG15 | 0.148 | 0.099 | 0.050 | 0 | 0 | 0 |
| Loss, kW | | 243.24 | 243.91 | 244.99 | 398.39 | 398.39 | 398.39 |
| Emission, kg | | 22 732 | 22 680 | 22 634 | 24 404 | 24 404 | 24 404 |

is less than the *b* coefficient of DG units. In such a case, the reactive power price is equal to the market price of the reactive power, which is 0, as mentioned in Section 2.5. When the market price is $25.34/MW, the reactive power price of each DG unit is based on its contribution in loss reduction, as shown in Table 7. The reactive power price of each DG unit decreases with a decrease in $\omega_1$.

## 3.3 | Impact of $\omega_1$ and $\omega_2$ on DG units' LMP

Figure 2 presents the computed LMP values of each DG unit by implementing the PNGT-based iterative algorithm on a specified test system with weights $\omega_1$=0 and $\omega_2$=1, when the spot price of the substation bus is $24.95/MW. Since more weight is provided by the decision maker to emission reduction, DG units that release low emission would get more price for generation. DG6, DG7, DG8, DG9, and DG10 received more incentive compared with other DGs because of their low emission coefficients.

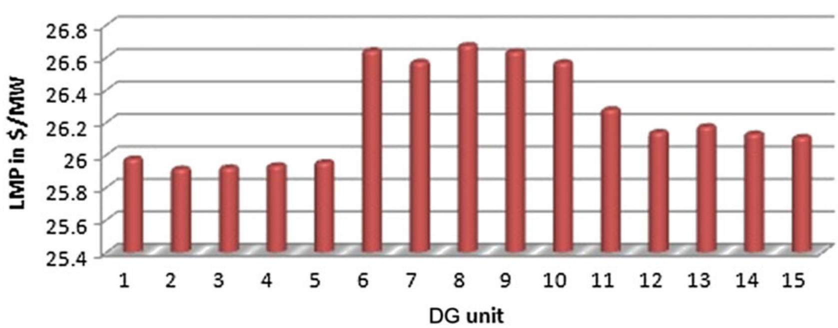


**FIGURE 2** Locational marginal price (LMP) values of each distributed generation (DG) unit at $\omega_1$=0 and $\omega_2$=1

Figure 3 represents variation in LMP values of all type 2 (low emission coefficients) DG units with various $\omega_1$ and $\omega_2$ values when the spot price of the substation bus is $24.95/MW for the TPC distribution system. The LMP values of all type 2 DG units increase with $\omega_2$ as these are low–emission coefficient generators.

## 3.4 | Variation in extra benefit of DISCO

Figure 4 shows variation in the DISCO's extra benefit with the proposed PNGT-based iterative algorithm at different market prices when $\omega_1$=0.5 and $\omega_2$=0.5 for the TPC distribution system. The proposed iterative algorithm will provide 0 extra benefit at all market prices. This happens because the DISCO operator provides some financial incentives to the DG unit based on its contribution in loss and emission reduction from the DISCO's extra benefit. As iterations progress in the proposed algorithm, the DISCO's extra benefit decreases and reaches 0.

## 3.5 | Variation in active power loss of the distribution network during iterative algorithm

Figure 5 shows variations in active power loss of the distribution network as iterations progress in the proposed method at $\omega_1$=0.5 and $\omega_2$=0.5 for different market prices. As iterations progress, LMP and generation of DGs that have a positive impact on loss reduction increase. This results in the reduction of active power loss in the distribution network.

## 3.6 | Variation in emission of the distribution network during iterative algorithm

Variations in emission from the distribution network as iterations progress in the proposed method at $\omega_1$=0.5 and $\omega_2$=0.5 for different market prices are shown in Figure 6. As iterations progress, incentives and generation of low–emission coefficient generators increase, which results in a decrease in emission.

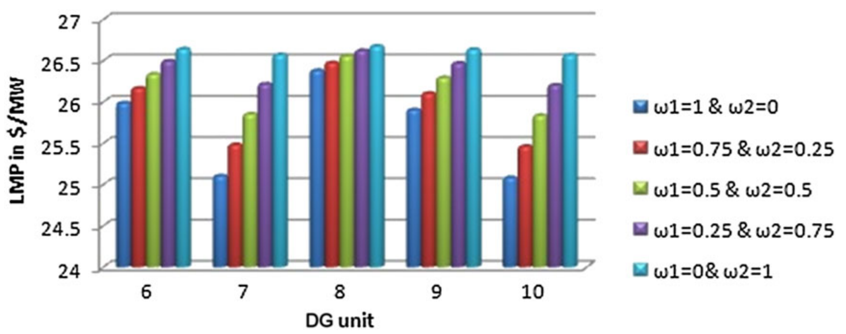


**FIGURE 3** Variation in locational marginal price (LMP) values of type 2 distributed generation (DG) units

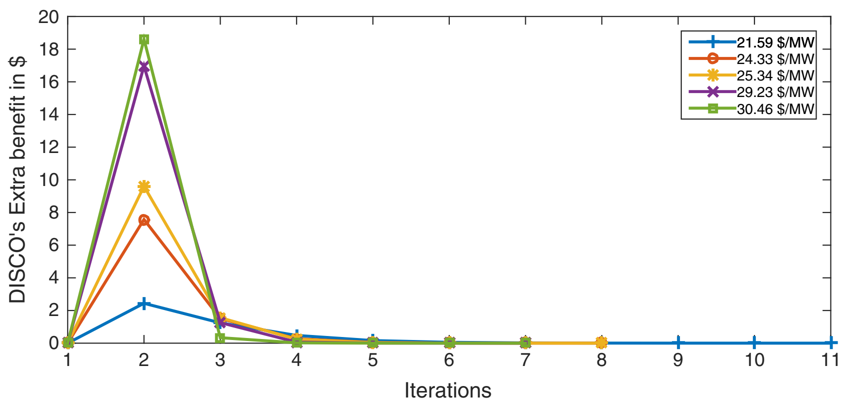


**FIGURE 4** Distribution company's (DISCO's) extra benefit variation

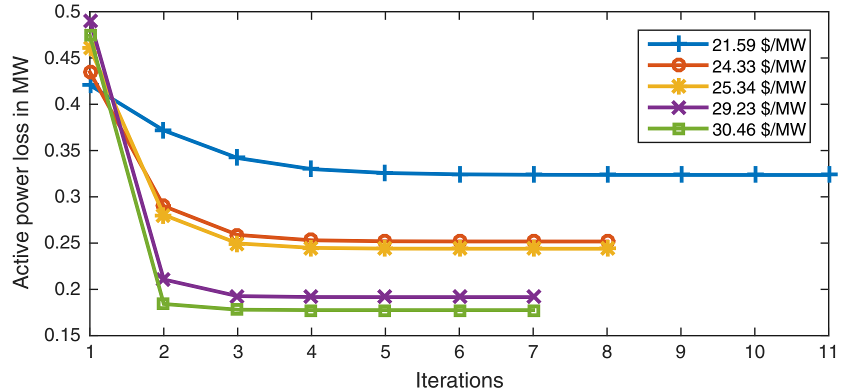


**FIGURE 5** Distribution company's active power loss variation

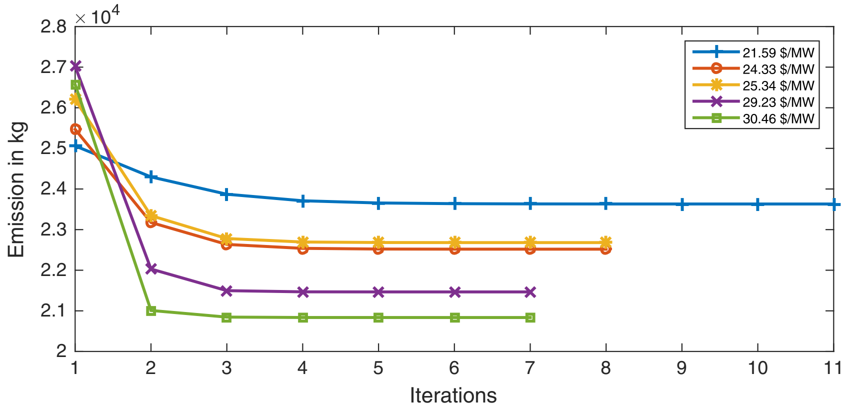


**FIGURE 6** Distribution company's emission variation

## 3.7 | Comparisons in terms of emission, active power loss, and DISCO's extra benefit

The proposed method has been compared in terms of emission, loss, and DISCO's extra benefit with some published techniques on LMP computation like the iterative nucleolus method,[12] iterative Shapley method,[6] marginal loss method,[5] and uniform price method[6,12] to demonstrate accuracy and validity.

Figures 7 and 8 show emission and active power loss of the TPC distribution system at each hour of the day, respectively, based on the proposed method and iterative nucleolus method[12] with forecasted load. In case the market price is less than the *b* coefficient value of all DG units, then no DG unit can inject the power into the network. In such a case, the network operated as a passive network and total load was supplied only from the substation. Because of this, both methods operate the network with the same amount of greenhouse gas emission. The amount of emission released from the network depends on emission coefficients at the substation bus. Similarly, as no DG injects the power into the network, the network looks like the base case network and both methods operate the network with the same amount of active power loss, which is equal to the base case loss.

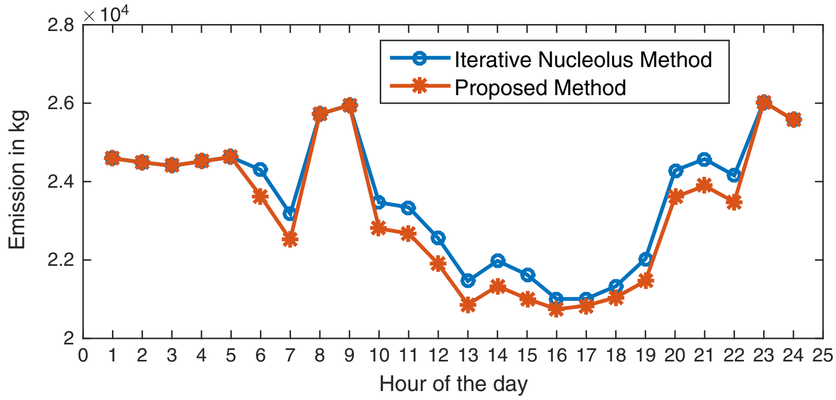


**FIGURE 7** Comparison in terms of network emission with the iterative nucleolus method[12]

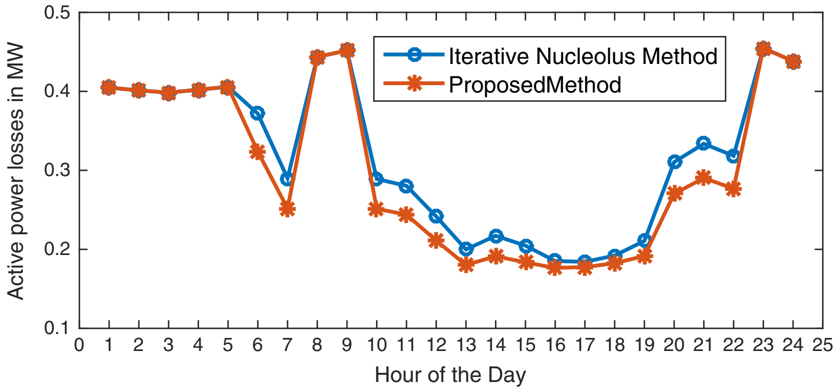


**FIGURE 8** Comparison in terms of network loss with the iterative nucleolus method[12]

The iterative nucleolus method proposed by Farsani et al[12] provides incentives to DG owners based on market price and contribution of DGs in loss and emission reduction. This type of computation leads to more incentive in each iteration, but it also results in quickly reaching a negative DISCO extra benefit. To avoid this drawback, in the proposed method, incentives were computed based on the DISCO's financial savings due to loss and emission reduction and contribution of DGs in loss and emission reduction. This type of computation provides less incentive in each iteration and more LMP at the time of convergence as this method reaches a negative DISCO extra benefit slowly.

In comparison with the iterative nucleolus method,[12] the proposed method provides more LMP to DG units that have a positive impact on emission reduction. Hence, the proposed method enables more generation from low–emission coefficient DG units. At all hours of the day, except where the market price is less than *b* coefficient of all DG units, the proposed method can operate the network with less emission as shown in Figure 7.

In comparison with the iterative nucleolus method,[12] the proposed method provides more LMP to DG units that have a positive impact on loss reduction. Hence, the proposed method enables more generation from DG units having a positive impact on loss reduction. At all hours of the day, except where the market price is less than the *b* coefficient of all DG units, the proposed method can operate the network with less active power loss as shown in Figure 8.

The proposed method has been compared with the iterative Shapley method[6] using forecasted load at each hour of the day in terms of loss at $\omega_1$=1 and $\omega_2$=0. This combination of $\omega_1$ and $\omega_2$ has been used as the iterative Shapley method developed based on loss only. The results thus obtained have been presented in Figure 9. The proposed method operates the network with less active power loss by remunerating more to DG units based on their contribution in loss reduction.

The proposed method's performance has been compared with the iterative nucleolus method[12] and conventional methods like the uniform price method[6,12] and marginal loss method[5] in terms of active power loss and reduced loss amount at different market prices. All these methods have been implemented on the TPC distribution system with load data as presented by Su and Lee.[26] However, while implementing the proposed method and iterative nucleolus method, $\omega_1$ and $\omega_2$ values were considered as 0.5 and 0.5, respectively. As per the results presented in Table 8, it has been observed that the proposed method drives the complete test system towards less loss and more reduced loss amount. This was due to more remuneration provided by the proposed method to DG owners in terms of LMP. Operating the network with less loss is the DISCO's decision maker requirement as it releases the line capacity and improves voltage profile and maximum load that can be supplied by the system.

The proposed method has been validated with the iterative nucleolus method[12] and conventional methods like uniform price method[6,12] and marginal loss method[5] in terms of the DISCO's extra benefit at $\omega_1$=0.5 and $\omega_2$=0.5. All these

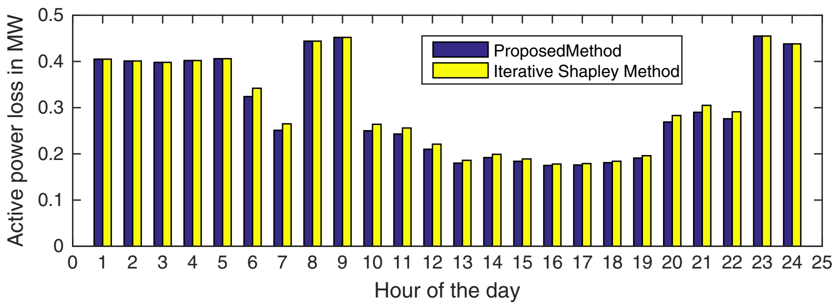


**FIGURE 9** Comparison in terms of loss with the iterative Shapley method[6]

**TABLE 8** Comparison of the proposed method in terms of active power loss and reduced loss amount

| Market Price $\lambda^t$, \$/MW | Reduced Loss Amount, \$ | | | | Active Power Loss, kW | | | |
|---|---|---|---|---|---|---|---|---|
| | Proposed Method | Iterative Nucleolus[12] | Marginal Loss[5] | Uniform Price[6,12] | Proposed Method | Iterative Nucleolus[12] | Marginal Loss[5] | Uniform Price[6,12] |
| 26 | 6.708 | 6.302 | 5.332 | 4.802 | 274.0 | 289.6 | 326.9 | 347.3 |
| 27 | 7.619 | 6.917 | 6.069 | 5.608 | 249.8 | 275.8 | 307.2 | 324.3 |
| 28 | 8.484 | 7.201 | 6.773 | 6.300 | 229.0 | 274.8 | 290.1 | 307.0 |
| 29 | 9.187 | 7.830 | 7.435 | 7.088 | 215.2 | 262.0 | 275.6 | 287.6 |
| 30 | 9.690 | 8.202 | 7.923 | 7.800 | 209.0 | 258.6 | 267.9 | 272.0 |
| 31 | 10.168 | 8.503 | 8.345 | 8.212 | 204.0 | 257.7 | 262.8 | 267.1 |

**TABLE 9** Comparison of proposed method in terms of the distribution company's (DISCO's) extra benefit

| Market Price $\lambda^t$, \$/MW | DISCO's Extra Benefit, \$ | | | |
|---|---|---|---|---|
| | Proposed Method | Iterative Nucleolus[12] | Marginal Loss[5] | Uniform Price[6,12] |
| 20 | 0 | 0 | 0 | 0 |
| 22 | 0 | 1.1 | 4.35 | 6.3 |
| 24 | 0 | 0.73 | 6.94 | 13.6 |
| 26 | 0 | 1.9 | 9.14 | 20.8 |
| 28 | 0 | 0.46 | 10.72 | 27.7 |
| 30 | 0 | 1.4 | 11.65 | 34.6 |

methods have been implemented on the TPC distribution system with loads as presented by Su and Lee.[26] The obtained DISCO extra benefit values at different market prices are presented in Table 9. In a deregulated environment, to maintain fair competition among DG owners, nonzero positive DISCO extra benefit needs to be minimized. This requirement has been fulfilled by the proposed method by reducing the DISCO's extra benefit to 0. However, the 3 remaining methods have nonzero positive extra benefit.

## 4 | CONCLUSION

This paper proposes a PNGT-based iterative method to compute LMP at DG buses in such a way that active power loss and emission have been reduced. In this method, financial incentives have been provided to DG owners from reduced loss and reduced emission cost from the base case. The DISCO's extra benefit at any iteration has been computed effectively based on LMP and active power generation of DG units at that iteration. As the system load is probabilistically variable, 2 layers of ANN have been implemented to forecast load on the system for the next 24 hours.

Nucleolus game theory is not monotonic, and hence, to overcome this drawback, for the first time, PNGT, which is monotonic, has been used to compute LMPs at DG buses.

As the integration of DG units into the distribution network is expected to grow in the future, the proposed PNGT-based iterative method can be helpful to DISCOs to maintain fair competition among private DG owners. Distribution companies can use this work to operate the network optimally in terms of loss and emission. This work is also helpful to DISCOs in estimating the state of the network in terms of DG units' generation with controllable DISCO extra benefit in day-ahead operations. The proposed method computes LMP to only DG owners based on the DISCO's decision maker priority among the DISCO's extra benefit, loss reduction, and emission reduction. The proposed method will not have any impact on customer prices. As all the countries are trying to reduce greenhouse gas emission, this work can help DISCOs to reduce emission.

This proposed PNGT-based iterative method can be extended by considering technical objectives like reliability improvement and service quality.

## NOMENCLATURE

| | |
|---|---|
| $(\Delta\Pi_a^t)_i^j$ | Incremental active power price for DG unit *i*, for iteration *j* and hour *t* |
| $(\Delta\Pi_r^t)_i^j$ | Incremental reactive power price for DG unit *i*, for iteration *j* and hour *t* |
| $(\Pi_a^t)_i^j$ | Active power price of DG unit *i*, for iteration *j* and hour *t*($/MW) |
| $(\Pi_r^t)_i^j$ | Reactive power price of DG unit *i*, for iteration *j* and hour *t*($/MVar) |
| $(MS^t)_{\text{Emn}}^j$ | Merchandising surplus due to emission reduction in iteration *j* and hour *t*($) |
| $(MS^t)_{\text{Loss}}^j$ | Merchandising surplus due to loss reduction in iteration *j* and hour *t*($) |
| $(PG^t)_i^j$ | Active power generation of DG unit *i*, for iteration *j* and hour *t*(MW) |
| $(QG^t)_i^j$ | Reactive power generation of DG unit *i*, for iteration *j* and hour *t*(MVar) |
| $\Delta benefit_j^t$ | DISCO's extra benefit under iteration *j* and hour *t*($) |
| $\Delta Ploss,\Delta Emn$ | Change in active power loss and emission from the base case, respectively |
| $\Delta Pmax$ | Maximum change in generation among all DG units (MW) |
| $\lambda^t,\lambda^r$ | Market price of active and reactive power, respectively, at hour *t* |
| $\omega_1,\omega_2$ | Weight corresponding to loss reduction and emission reduction |
| $\omega_e$ | Emission cost sharing factor between DISCO and generators |
| $\Pi^c$ | Customer price ($/MW) |
| $a_i,b_i,c_i$ | Cost coefficients of DG unit *i* |
| $benefit_j^t$ | DISCO's benefit under iteration *j* at hour *t*($) |
| $benefit_o^t$ | DISCO's benefit under base case at hour *t*($) |
| $cosdg^i$ | Power factor of DG unit *i* |
| $DGgain_P^i$ | Financial incentive to DG unit *i* to generate active power ($) |
| $DGgain_Q^i$ | Financial incentive to DG unit *i* to generate reactive power ($) |
| $DGgain_{\text{loss}}^i$ | Financial incentive to DG unit *i* for its contribution in loss reduction ($) |
| $DGgain_{\text{Emn}}^i$ | Financial incentive to DG unit *i* for its contribution in emission reduction ($) |
| $e(Y{:}S)$ | Coalitions excess value of imputation *Y* |
| $EC_{\text{DG}}^t$ | Emission cost when DG units integrated at hour *t*($) |
| $EC_j^t$ | Emission cost at hour *t*, iteration *j*($) |
| $EC_o^t$ | Emission cost at hour *t* in base case ($) |
| $Emn_{\text{DG}}^t$ | Emission when DG units integrated at hour *t*(kg) |
| $Emn_j^t$ | Emission under iteration *j*, at hour *t*(kg) |
| $Emn_o^t$ | Emission in base case at hour *t*(kg) |
| $L(t,D)$ | System load at hour *i* of day *D* |
| $N$ | Coalition of all DG units |
| $N_{\text{DG}}$ | Number of DG units |
| $P_{CO_2}$ | Penalty for $CO_2$ emission ($/kg) |
| $P_{CO}$ | Penalty for CO emission ($/kg) |
| $P_{\text{Load}}^t$ | Total demand of the system at hour *t*(MW) |
| $P_{NO_x}$ | Penalty for $NO_x$ emission ($/kg) |
| $P_{SO_2}$ | Penalty for $SO_2$ emission ($/kg) |
| $P_{\text{Sub}}^t$ | Active power supplied from substation bus at hour *t*(MW) |
| $PG_i$ | Active power generation of DG unit *i* |
| $Ploss_{\text{DG}}^t$ | Active power loss when DG units were integrated at hour *t*(MW) |
| $Ploss_j^t$ | Active power loss under iteration *j*, at hour *t*(MW) |
| $Ploss_o^t$ | Active power loss in the base case at hour *t*(MW) |
| $S$ | Nonempty subset of coalitions of all DG units |
| $v^k(N)$ | *Loss*(*k*=*l*)/*Emission*(*k*=*e*) reduction due to coalition *N* |
| $v^k(S)$ | *Loss*(*k*=*l*)/*Emission*(*k*=*e*) reduction due to coalition *S* |
| $xLoss(i), xEmn(i)$ | Share of DG unit *i* in loss reduction and emission reduction, respectively |
| $Y$ | Set of imputations of all DG units |
| $y_i$ | Loss/emission reduction allocation imputation of DG unit *i* |
| $CO_2^{DG_i}$ | $CO_2$ released by DG unit *i*(kg/MW) |
| $CO_2^{\text{Sub}}$ | $CO_2$ released based on load at the substation bus (kg/MW) |

| | |
|---|---|
| $CO^{DG_i}$ | CO released by DG unit $i$(kg/MW) |
| $CO^{Sub}$ | CO released based on load at the substation bus (kg/MW) |
| ${NO_x}^{DG_i}$ | $NO_x$ released by DG unit $i$(kg/MW) |
| ${NO_x}^{Sub}$ | $NO_x$ released based on load at the substation bus (kg/MW) |
| ${SO_2}^{DG_i}$ | $SO_2$ released by DG unit $i$(kg/MW) |
| ${SO_2}^{Sub}$ | $SO_2$ released based on load at the substation bus (kg/MW) |

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