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LMP computation at DG buses in radial distribution system

Venkataramana Veeramsetty, Venkaiah Chintham
and Vinod Kumar D.M.
*Department of Electrical Engineering, National Institute of Technology,
Warangal, India*

Abstract

Purpose – The purpose of this paper is to estimate the locational marginal price (LMP) at each distributed generation (DG) bus based on DG unit contribution in loss reduction. This LMP value can be used by distribution company (DISCO) to control private DG owners and operate network optimally in terms of active power loss.

Design/methodology/approach – This paper proposes proportional nucleolus game theory (PNGT)-based iterative method to compute LMP at each DG unit. In this algorithm, PNGT has been used to identify the share of each DG unit in loss reduction. New mathematical modeling has been incorporated in the proposed algorithm to compute incentives being given to each DG owner.

Findings – The findings of this paper are that the LMP and reactive power price values for each DG unit were computed by the proposed method for the first time. Network can be operated with less loss and zero DISCO's extra benefit, which is essential in deregulated environment. Fair competition has been maintained among private DG owners using the proposed method.

Originality/value – PNGT has been used for the first time for computation of LMP in distribution system based on loss reduction. Incentives to each DG unit has have been computed based on financial savings of DISCO due to loss reduction. Share of active and reactive power generation of each DG unit on change in active power loss of network due to that DG unit has been computed with new mathematical modeling. The proposed method provides LMP value to each DG unit in such a way that the network will be operated with less loss.

Keywords Artificial intelligence, Simulation, Pricing, Forecasting, Neural networks, Distribution, Cost comparison, Restructuring,

Paper type Research paper

Nomenclature

$(\Delta P_{loss})_b^i$ = Change in active power loss due to change in generation of DG i at bus b;
$(\Pi_a^t)_i^j$ = Active power price of DG i at hour t and iteration j in \$/MWh;
$(\Pi_r^t)_i^j$ = Reactive power price of DG i for hour t and iteration j in \$/MVarh;
$(PG^t)_i^j$ = Active power generation of DG unit i at hour t and iteration j in MW;
$(QG^t)_i^j$ = Reactive power generation of DG unit i at hour t and iteration j in MVar;
$\Delta benefit_j^t$ = Extra benefit of DISCO at hour t and iteration j in \$;
ΔPG_b = Change in active power generation at bus "b" in MW;
$\Delta Ploss$ = Change in active power losses from base case in MW;
ΔQG_b = Change in reactive power generation at bus "b" in MVar;
λ^r = Market price of reactive power at substation bus in \$/MVarh;
λ^t = Market price of active power at hour t in \$/MWh;
$\prod_c$ = Customer price in \$/MWh;
ε_1 = Constraint for checking convergence in terms of DISCO's extra benefit;

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ε_2 = Constraint for checking convergence in terms of DG unit's generation;
a_i, b_i, c_i = Fuel cost coefficients of DG unit i;
$benefit_0^t$ = DISCO's benefit under base case at hour t in \$;
$benefit_j^t$ = DISCO's benefit with DG units at hour t and iteration j in \$;
$cosdg^i$ = Power factor of DG;
DG_{inc}^i = Financial incentive for DG i in \$;
DG_{inc}^P (i) = Financial incentive for DG "i" to generate active power in \$;
DG_{inc}^Q (i) = Financial incentive for DG "i" to generate reactive power in \$;
$e(Y: S)$ = Coalitions excess value of imputation Y;
IL_l^I = Imaginary part of current through line l;
IL_l^R = Real part of current through line l;
$L(t, D)$ = Total demand of the system at hour t and day D in MW;
$MAPE$ = Mean absolute percentage error;
N = Grand coalition of all DG units;
N_{DG} = Number of DG units;
PG_b = Active power generation at bus "b" in MW;
$Ploss_{DG}^t$ = Active power losses when DG units integrated at hour t in MW;
$Ploss_o^t$ = Base case active power losses at hour t in MW;
$Pshare_i^b$ = Share of active power generation of DG i at bus b on change in network loss;
QG_b = Reactive power generation at bus "b" in MVar;
$Qshare_i^b$ = Share of reactive power generation of DG i at bus b on change in network loss;
R_l = Resistance of line "l" in ohms;
$RMSE$ = Root mean square error;
S = Non-empty sub set of coalitions of all DG units;
$v^l(N)$ = Active power loss reduction due to grand coalition N;
$v^l(S)$ = Active power loss reduction due to sub coalition S;
$v^q(N)$ = Reactive power loss reduction due to grand coalition N;
V_b = Complex voltage at bus b;
V_b^I = Imaginary part of complex voltage at bus b;
V_b^R = Real part of complex voltage at bus b;
Y = Set of imputations of all DG units;
y_i = Allocated reduced active power losses to DG unit i; and
yq_i = Allocated reduced reactive power losses to DG unit i.

1. Introduction

The integration of distributed generations (DGs) in to the distribution network has been increased due to benefits like loss reduction, emission reduction, voltage improvement, reliability improvement and reinforcement horizon improvement (Pavani and Singh, 2014; Raghavendra and Gaonkar, 2016). With the integration of DG units, the distribution network has been transformed from passive state to the active state like in transmission network (Yao *et al.*, 2015). Some of the practices applied on transmission network like nodal pricing can also be applied for active distribution network (Sotkiewicz and Vignolo, 2006).

DISCO's decision-maker role is very critical as enabler of the new energy technology in deregulated environment. DISCO's decision-maker role is influenced by social aspects, markets, end-use technology, society and infrastructure (Honkapuro *et al.*, 2014). DISCO's decision-maker has to maintain fair competition among owners of end-use technologies, such as DGs. In addition to this, DISCO's other goals are like controlling private DG owners and operate the network optimally in terms of losses. The above objectives can be fulfilled by DISCO by using nodal pricing. Locational marginal price (LMP) is the most efficient method among other nodal pricing policies (Yao *et al.*, 2015; Orfanogianni and Gross, 2007). In this paper, LMP is used as a financial incentive to each DG unit based on each DG unit's performance in loss reduction.

Various papers were available in literature for computation of LMP in distribution network. The comparison of research contribution by various authors in addressing the different features considered for LMP computation in distribution system is shown in Table I.

All the approaches shown in Table I can be helpful to DISCO to control active power generation of DG units for active power loss reduction. However, some approaches were there for controlling the reactive power generation such that distribution network will operate optimally in terms of active power losses. An analytical method was developed in Naik *et al.*'s study (2014) for optimal sizing and location of DG unit so that the power losses of network were reduced. Here, authors considered both active and reactive components of DG current to reduce line losses. A day ahead coordinating dispatch method called pre-coarse-fine adjustment method for reactive power dispatch was proposed by Zhang *et al.* (2016) to achieve optimal power flow by minimizing power losses and minimizing switching operation of capacitor banks. A decentralized approach was developed by Lin *et al.* (2017) to solve the reactive power optimization problem for integral transmission and distribution network. This method optimizes the distribution network power losses as well as guarantees the voltage security. Reactive power coordinated optimization method using improved harmony search algorithm was proposed by Sheng *et al.* (2016) to minimize power losses and to reduce number of switching device operations while maintaining grid voltage within the allowable range.

The research literature represented in Table I considers active power price for comparison. There exists some literature, which considers reactive power payments for distribution system power loss reduction. Optimal power flow method was developed by Haghighat and Kennedy (2010) to calculate real-time reactive power price (RPP)

Research contribution	Different features researchers addressed										Approach
	A	B	C	D	E	F	G	H	K	L	
Sotkiewicz and Vignolo (2006)	✓		✓		✓	✓					Marginal Loss coefficients
Sotkiewicz and Vignolo (2007)	✓		✓		✓	✓		✓			Reconciliated marginal loss coefficients
Sathyanarayana and Heydt (2013)	✓		✓		✓	✓					Sensitivity factors
Singh and Goswami (2010)	✓				✓						Maximizing DISCO profit
Shaloudegi *et al.* (2012)	✓	✓	✓	✓	✓	✓	✓	✓			Shapley value method
Proposed method	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	Proportional nucleolus game theory

Notes: A: loss reduction; B: controllable merchandising surplus; C: changing DG benefit; D: providing encouragement to DG for participating in loss reduction; E: computing LMP at current operating conditions; F: estimating LMP in next operating conditions; G: DISCO's strategic ability for optimal operation; H: zero merchandising surplus; K: impact of active and reactive power generation of DG on losses; L: incentive provided to DGs from financial savings of DISCO due to loss reduction

Table I.
Comparison of LMP computation features

based on mitigation of power losses by reactive power generation of DG units. Mixed integer non-linear programming-based optimization approach was proposed by Kumar and Gao (2010) for optimal placement of DG unit by considering minimization of fuel cost and power losses of network as objectives. Here, reactive power payments of DG units were computed using Lagrangian equation. An optimal reactive power planning model was proposed by Chen *et al.* (2006) using Monte Carlo simulation, genetic algorithm and Shapely value method to calculate RPP of each DG unit in distribution system. Here, Shapely value method has been used to share reactive power cost of both SVC and DG unit among wind turbines. A mixed integer non-linear programming-based optimization model was developed by Samimi *et al.* (2015) to clear the reactive power market and to obtain the RPP of each DG unit in distribution network. A method for reactive power payments was proposed by Sotkiewicz and Vignolo (2006) based on marginal loss coefficients. Sotkiewicz and Vignolo (2007) proposed reactive power payments based on reconciliation marginal loss coefficients. These coefficients represent the impact of reactive power generation at DG buses on overall power losses of the network. Shapely value-based iterative method was proposed by Shaloudegi *et al.* (2012) for LMP computation that also provides reactive power payments for DG units.

Researchers contribution in Haghighat and Kennedy (2010), Kumar and Gao (2010), Chen *et al.* (2006) and Samimi *et al.* (2015) provided various approaches for RPP computation, which may not provide any guarantee for fair contribution of DG unit's reactive power on power losses, zero merchandising surplus, state estimation of network at next operating condition and maintaining fair competition among DG owners.

The method proposed by Shaloudegi *et al.* (2012) has computed LMP at DG buses based on contribution of DG units in loss reduction. Authors used Shapley value method for allocation of reduced loss among DG units. Shapley value method suffers with some flaws like solution may not lie inside the core (Singh, 1999; Lemaire, 1984). In this method, authors did not consider DISCO's extra benefit in convergence of iterative algorithm and as a consequence the DISCO is incurring financial loss. Incentives have been computed on the basis of wholesale market price that leads to high incremental price (Δ\product\$^{a}$), which may reduce DG units profit. Authors did not consider actual contribution of reactive power of DG units on active power losses while computing reactive power payments

This paper presents a method to compute LMP at DG buses in radial distribution network by allocating reduced loss among DG units using proportional nucleolus game theory (PNGT). PNGT is one of the most efficient solution methods for cooperative game theory problem. The financial savings of DISCO due to loss reduction from base case have been allocated as an incentive to each DG unit based on DG unit's contribution in loss reduction. The allocated incentive of each DG unit again shared among active and reactive power generation of that DG unit based on contribution of these parameters in loss reduction. Impact of active and reactive power of DG on loss reduction has been computed on the basis of sensitivity analysis. RPP has been computed for each DG unit based on actual contribution of generated reactive power on reduction of active power loss of network. Even though reactive power does not provide any effective work on the system, the reactive power cannot be avoided due to its concern in maintaining the network stability and reliability (Parida *et al.*, 2011).

The original contributions of this paper are as follows:

- Fair allocation of reduced losses among DG units using PNGT for the first time.
- Financial incentive to each DG unit has been computed by sharing financial savings of DISCO due to loss reduction.

- Allocated financial incentive to each DG unit is again shared among active and reactive power of that particular DG unit-based sensitivity factors.
- This method enables the DISCO's decision-maker to handle trade off among loss reduction and DISCO's extra benefit.
- Merchandising surplus is controllable.
- A new approach has been developed to compute loss sensitivity factors with respect to active and reactive power generation of DG units for the first time

The proposed method can be used by DISCO to:

- operate the network with low active power loss;
- estimate the state of network in terms of LMP, generation, active power loss and voltage at each bus;
- control private DG owners; and
- maintain fair competition among DG owners.

The remaining part of the paper is organized as follows: Section II presents load forecasting using ANN, computation of loss reduction, allocation of reduced losses using PNGT, calculation of extra benefit, sensitivity factors and iterative algorithm. Section III deals with analytical studies on Taiwan Power Company (TPC) distribution system and Section IV provides conclusions.

2. Problem formulation

An iterative method has been developed to compute LMP at DG buses in radial distribution system based on the following two ideas:

(1) allocation of reduced active power loss of network among DG units using cooperative game theory; and
(2) calculation of LMP at DG bus based on its contribution in loss reduction.

Load at each hour of the day needs to be forecasted to estimate the LMP at each DG bus. A two-layer artificial neural network has been used to forecast the load.

2.1 Load forecasting

Load (L(t,D)) at which hour (t) of day (D) has to be forecasted is considered as output. Whereas load at previous 4 h from the hour (t) where load need to be forecasted and load at same hour (t) for past two days was considered as input. Back propagation algorithm has been used to train the network as it is more flexible, having good learning capabilities and highly suitable for the problems where no mathematical relationship exists between output and input (Li *et al.*, 2015).

2.2 Computation of loss reduction

An iterative distribution load flow algorithm is implemented in two cases based on forecasted load (L(t,D)) and active power price ($\left(\left(\Pi_a^t\right)_i^j\right)$) at hour t of day to compute change in active power loss. In this paper, backward and forward sweep algorithm (Shirmohammadi *et al.*, 1988) has been used to exploit the complete advantage of ladder structure of distribution network, achieve high speed, robust convergence and low memory requirements (Wang *et al.*, 2004; Abdel-Akher, 2013). In this load flow solution, simultaneous control of PQ modeled (Moghaddas-Tafreshi and Mashhour, 2009) DG has been used.

Case 1: Base case – no DG unit is connected in to the system.

Case 2: DG units inject the power in to the system.

Generation of each DG unit based on cost coefficients of that generator and active power price $((\Pi_a^t)_i^j)$ at DG bus has to be computed using equations (1) and (2). The loss reduction has to be computed using equation (3) with reference to the base case.

$$CF_i = a_i\left((PG^t)_i^j\right)^2 + b_i(PG^t)_i^j + c_i \tag{1}$$

$$(PG^t)_i^j = \frac{(\Pi_a^t)_i^j - b_i}{2a_i} \tag{2}$$

$$\Delta Ploss = Ploss_0{}^t - Ploss_{DG}{}^t \tag{3}$$

Now a cooperative game theory is applied to allocate the reduced losses among DG units. In this paper, PNGT is used as a solution method for allocation.

2.3 Proportional nucleolus game theory

The restructured power system is undergoing a continuous challenging issue such as allocation of active power loss of network among players. These players are market participants in transmission system and DG owners in distribution system. As active power loss of either transmission system or distribution system is highly non-linear and non-separable, allocation of active power loss among players is a difficult task. In this paper, cooperative game theory has been used for loss allocation due to the following reasons:

- It is a well-founded economic framework to qualitatively study allocation of active power loss (Lima *et al.*, 2008).
- It provides well-behaved solution with economic features for assessing the interaction of different participants in competitive market for resolving the conflicts among participants (Lakdja *et al.*, 2013).
- It searches the decisions when player's actions directly influence each other. It keeps the equilibrium of these decisions as well (Guanghou *et al.*, 2004).

DISCO has a control over DG owners in distribution network. If all DG units operate in cooperative manner, then network will be operating with less active power loss. Due to DISCO's command over DG owners and to exploit advantages of cooperative operation, DISCOs will enable all players to work as a group. It is assumed that all DG units in distribution system are acting as a group and the DG units are players in this cooperative game problem.

The number of coalitions existing in a cooperative game problem consisting of n players is equal to $(2^n - 1)$. All players in a coalition inject the power in to the system at a time. The allocation of active power loss among DG units corresponds to the allocation of the payoffs among DG units in the coalition. The problem of active power loss allocation turns into the equilibrium point in the game theory.

Extended core concept has been introduced to compute solution for cooperative games under empty core environment. The main characteristics of extended core are always non-empty and solution concept coincides in cases where core is non-empty. An imputation chooses from extended core in PNGT like nucleolus, which chooses an imputation from the core. The PNGT differs from the nucleolus theory in the formation of definition of excess concerned with coalitions (Satyaramesh and Radhakrishna, 2009), as shown in equation (4).

$$e(Y:S) = \frac{v^J(S) - \sum_{i \epsilon S} y_i}{v^J(S)} \tag{4}$$

The PNGT can grow the core to obtain a unique solution in empty core and large core cases. Thus, PNGT can provide better solution in extended core and core selection problem. This ability of proportional nucleolus to select an imputation is another advantage of extended core solution concept. Solution has been obtained on the basis of the proportional nucleolus game theory concept by solving following linear programming problem as shown in equation (5).

$$\begin{aligned} & \qquad\qquad min\ \varepsilon \\ S.t \qquad & \frac{v^J(S) - \sum_{i \epsilon S} y_i}{v^J(S)} \le \varepsilon \\ & \sum_{i \epsilon N} y_i = v^J(N) \end{aligned} \tag{5}$$

where ε is a small arbitrary real value.

PNGT is explained legibly in the following example. Let us consider that three DG units were integrated in to the network with base case active power loss of 440 kW and assume that the generation of each DG unit is 0.5 kW, 0.75 kW and 1MW. Table II presents losses and reduced losses from base case due to each coalition of DG units.

Objective function:

$$\min 0 * y_1 + 0 * y_2 + 0 * y_3 + 1 * \varepsilon$$

Equality constraint:

$$1 * y_1 + 1 * y_2 + 1 * y_3 + 0 * \varepsilon = 168.15$$

Coalition (S)	Losses	Loss reduction ($v^J(S)$)
S = {DG1 = 0.5 MW}*	420.95	19.05
S = {DG2 = 0.75 MW}*	387.95	52.05
S = {DG3 = 1 MW}*	418.1	21.9
S = {DG1 = 0.5 MW & DG2 = 0.75MW}*	335	105
S = {DG1 = 0.5 MW & DG3 = 1MW}*	378.8	61.2
S = {DG2 = 0.75 MW & DG3 = 1MW}*	354.95	85.05
N = {DG1 = 0.5 MW & DG2 = 0.75 MW & DG3 = 1 MW}#	271.85	168.15

Notes: *Represents sub coalition S; # represents grand coalition N

Table II. Active power loss reduction in kW for different coalitions

Inequality constraints:

$$
\begin{aligned}
&-1 * y_1 + 0 * y_2 + 0 * y_3 - 19.05 * \varepsilon \leq -19.05\\
&0 * y_1 - 1 * y_2 + 0 * y_3 - 52.05 * \varepsilon \leq -52.05\\
&0 * y_1 + 0 * y_2 - 1 * y_3 - 21.9 * \varepsilon \leq -21.9\\
&-1 * y_1 - 1 * y_2 + 0 * y_3 - 105 * \varepsilon \leq -105\\
&-1 * y_1 + 0 * y_2 - 1 * y_3 - 61.2 * \varepsilon \leq -61.2\\
&0 * y_1 - 1 * y_2 - 1 * y_3 - 85.05 * \varepsilon \leq -85.05
\end{aligned}
$$

By solving the above linear programming problem that is formulated using equation (5), the share of each DG unit in loss reduction is y_1 = 53.4753kW, y_2 = 85.6559kW and y_3 = 29.0188kW. Fairness of the above solution is measured in terms of three natural properties, such as individual rationality, coalition rationality and collective rationality.

- Individual rationality:

Share of each DG unit in reduced losses must be greater than or equal to loss reduction when that DG unit is operated alone.

$$
\begin{aligned}
&v^l(1) \leq y_1 \Rightarrow 19.05 \leq 53.4753\\
&v^l(2) \leq y_2 \Rightarrow 52.05 \leq 85.6559\\
&v^l(3) \leq y_3 \Rightarrow 21.90 \leq 29.0188
\end{aligned}
$$

- Coalition rationality:

Loss reduction due to any sub-coalition is less than the sum of allocated loss reduction to each DG unit in that sub-coalition.

$$
\begin{aligned}
&v^l(1,2) \leq y_1 + y_2 \Rightarrow 105 \leq 53.4753 + 85.6559\\
&v^l(1,3) \leq y_1 + y_3 \Rightarrow 61.2 \leq 53.4753 + 29.0188\\
&v^l(2,3) \leq y_2 + y_3 \Rightarrow 85.05 \leq 85.6559 + 29.0188
\end{aligned}
$$

- Collective rationality:

Loss reduction due to coalition of all DG units is equal to the sum of allocated reduced losses to each DG unit.

$$
\begin{aligned}
&v^l(1,2,3) = y_1 + y_2 + y_3\\
&\Rightarrow 168.15 = 53.4753 + 85.6559 + 29.0188
\end{aligned}
$$

2.4 DISCO extra benefit

DISCO extra benefit is defined as difference between DISCO benefit with and without DG units. DISCO benefit without and with DG is calculated using equations (6) and (7), respectively:

$$benefit_0^t = \Pi^c L(t,D) - ((L(t,D) + Ploss_0^t)\lambda^t \tag{6}$$

$$benefit_j^t = \Pi^c L(t,D) - \sum_{i=1}^{N_{DG}} \left((PG^t)_i^j\right)(\Pi_a^t)_i^j - \sum_{i=1}^{N_{DG}} (QG^t)_i^j)(\Pi_r^t)_i^j - L(t,D)$$

$$+ Ploss_{DG}^t - \sum_{i=1}^{N_{DG}} (PG^t)_i^j)\lambda^t \tag{7}$$

The final expression of extra benefit obtained by subtracting equation (6) from equation (7) is as shown in equation (8):

$$\Delta benefit_j^t = \left(Ploss_0^t - Ploss_{DG}^t\right)\lambda^t - \sum_{i=1}^{N_{DG}} (QG^t)_i^j (\Pi_r^t)_i^j - \sum_{i=1}^{N_{DG}} (PG^t)_i^j \left((\Pi_a^t)_i^j - \lambda^t\right) \tag{8}$$

2.5 Impact of active and reactive power generation on active power loss of radial distribution system

The single line diagram of six-bus system shown in Figure 1 has been considered for deriving the expressions for identifying the following:

- sensitivity of generation at any bus on active power loss of radial distribution system; and
- share of active and reactive power generation of any DG unit on change in losses due to injection of that DG unit.

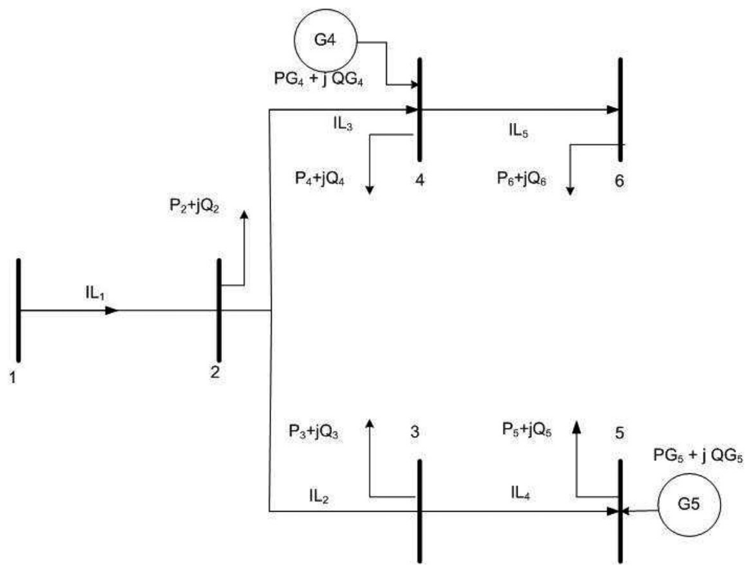

Figure 1.
Single line diagram of six-bus distribution system

It is assumed that the considered system has generators with leading power factor at Buses 4 and 5.

A matrix called bus incident beyond line (BIBL) has been developed in such a way that if node "b" connected is beyond line "l," then set BIBL(l,b) = 1 otherwise set BIBL(l, b) = 0. BIBL matrix for the above six-bus system is represented in Table III.

Current drawn at each bus has been computed based on net withdrawal complex power and voltage at that bus. The final expression for current at each bus is shown in equation (9):

$$\begin{aligned} I_2 &= \frac{P_2 + jQ_2}{V_2^*} \\ I_3 &= \frac{P_3 + jQ_3}{V_3^*} \\ I_4 &= \frac{(P_4 - PG_4) + j(Q_4 - QG_4)}{V_4^*} \\ I_5 &= \frac{(P_5 - PG_5) + j(Q_5 - QG_5)}{V_5^*} \\ I_6 &= \frac{P_6 + jQ_6}{V_6^*} \end{aligned} \tag{9}$$

Current through each line in the six-bus distribution network has been computed by applying Kirchhoff's Current Law at each bus and the final expression of current through each line is shown in equation (10):

$$\begin{aligned} IL_1 &= I_2 + IL_2 + IL_3 = \frac{P_2 + jQ_2}{V_2^*} + IL_2 + IL_3 \\ IL_2 &= I_3 + IL_4 = \frac{P_3 + jQ_3}{V_3^*} + IL_4 \\ IL_3 &= I_4 + IL_5 = \frac{(P_4 - PG_4) + j(Q_4 - QG_4)}{V_4^*} + IL_5 \\ IL_4 &= I_5 = \frac{(P_5 - PG_5) + j(Q_5 - QG_5)}{V_5^*} \\ IL_5 &= I_6 = \frac{P_6 + jQ_6}{V_6^*} \end{aligned} \tag{10}$$

	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5	Bus shap6
Line 1	0	1	1	1	1	1
Line 2	0	0	1	0	1	0
Line 3	0	0	0	1	0	1
Line 4	0	0	0	0	1	0
Line 5	0	0	0	0	0	1

Table III. Bus incident beyond line (BIBL) matrix

All line currents shown above are complex quantities. After rearranging real and imaginary parts, complex line currents are shown in equation (11).

$$\begin{aligned} IL_1 &= IL_1^R + jIL_1^I \\ IL_2 &= IL_2^R + jIL_2^I \\ IL_3 &= IL_3^R + jIL_3^I \\ IL_4 &= IL_4^R + jIL_4^I \\ IL_5 &= IL_5^R + jIL_5^I \end{aligned} \tag{11}$$

where real and imaginary parts of each line current are shown in equations (12) and (13), respectively.

$$\begin{aligned} IL_1^R &= \frac{P_2V_2^R + Q_2V_2^I}{|V_2|^2} + \frac{P_3V_3^R + Q_3V_3^I}{|V_3|^2} + \frac{(P_4 - PG_4)V_4^R + (Q_4 - QG_4)V_4^I}{|V_4|^2} \\ &\quad + \frac{(P_5 - PG_5)V_5^R + (Q_5 - QG_5)V_5^I}{|V_5|^2} + \frac{P_6V_6^R + Q_6V_6^I}{|V_6|^2} \\ IL_2^R &= \frac{P_3V_3^R + Q_3V_3^I}{|V_3|^2} + \frac{(P_5 - PG_5)V_5^R + (Q_5 - QG_5)V_5^I}{|V_5|^2} \\ IL_3^R &= \frac{(P_4 - PG_4)V_4^R + (Q_4 - QG_4)V_4^I}{|V_4|^2} + \frac{P_6V_6^R + Q_6V_6^I}{|V_6|^2} \\ IL_4^R &= \frac{(P_5 - PG_5)V_5^R + (Q_5 - QG_5)V_5^I}{|V_5|^2} IL_5^R = \frac{P_6V_6^R + Q_6V_6^I}{|V_6|^2} \end{aligned} \tag{12}$$

$$\begin{aligned} IL_1^I &= \frac{P_2V_2^I + Q_2V_2^R}{|V_2|^2} + \frac{P_3V_3^I + Q_3V_3^R}{|V_3|^2} + \frac{(P_4 - PG_4)V_4^I + (Q_4 - QG_4)V_4^R}{|V_4|^2} \\ &\quad + \frac{(P_5 - PG_5)V_5^I + (Q_5 - QG_5)V_5^R}{|V_5|^2} + \frac{P_6V_6^I + Q_6V_6^R}{|V_6|^2} \\ IL_2^I &= \frac{P_3V_3^I + Q_3V_3^R}{|V_3|^2} + \frac{(P_5 - PG_5)V_5^I + (Q_5 - QG_5)V_5^R}{|V_5|^2} \\ IL_3^I &= \frac{(P_4 - PG_4)V_4^I + (Q_4 - QG_4)V_4^R}{|V_4|^2} + \frac{P_6V_6^I + Q_6V_6^R}{|V_6|^2} \\ IL_4^I &= \frac{(P_5 - PG_5)V_5^I + (Q_5 - QG_5)V_5^R}{|V_5|^2} IL_5^I = \frac{P_6V_6^I + Q_6V_6^R}{|V_6|^2} \end{aligned} \tag{13}$$

Active power loss of system can be computed using equation (14). Change in active power losses with respect to change in active power generation at Bus 4 is nothing but sensitivity of losses with respect to active power generation at Bus 4 is shown in equation (15):

$$P_{loss} = |IL_1|^2R_1 + |IL_2|^2R_2 + |IL_3|^2R_3 + |IL_4|^2R_4 + |IL_5|^2R_5 \tag{14}$$

$$\begin{aligned}\frac{\partial P_{loss}}{\partial PG_4} &= -2IL_1^RR_1\frac{V_4^R}{|V_4|^2} - 2IL_1^IR_1\frac{V_4^I}{|V_4|^2} - 2IL_3^RR_3\frac{V_4^R}{|V_4|^2} - 2IL_3^IR_3\frac{V_4^I}{|V_4|^2} \\ &= -2\frac{[BIBL(:,4)]^T\left(IR^{Real}V_4^R + IR^{Imag}V_4^I\right)}{|V_4|^2}\end{aligned} \tag{15}$$

Similarly, sensitivity of losses with respect to reactive power generation at Bus 4 with the DG having lagging power factor is shown in equation (16) and with the DG having leading power factor is shown in equation (17).

For the DG having lagging power factor:

$$\begin{aligned}\frac{\partial P_{loss}}{\partial QG_4} &= -2IL_1^RR_1\frac{V_4^I}{|V_4|^2} + 2IL_1^IR_1\frac{V_4^R}{|V_4|^2} - 2IL_3^RR_3\frac{V_4^I}{|V_4|^2} + 2IL_3^IR_3\frac{V_4^R}{|V_4|^2} \\ &= -2\frac{[BIBL(:,4)]^T\left(IR^{Real}V_4^I - IR^{Imag}V_4^R\right)}{|V_4|^2}\end{aligned} \tag{16}$$

For the DG having leading power factor:

$$\begin{aligned}\frac{\partial P_{loss}}{\partial QG_4} &= 2IL_1^RR_1\frac{V_4^I}{|V_4|^2} - 2IL_1^IR_1\frac{V_4^R}{|V_4|^2} + 2IL_3^RR_3\frac{V_4^I}{|V_4|^2} - 2IL_3^IR_3\frac{V_4^R}{|V_4|^2} \\ &= -2\frac{[BIBL(:,4)]^T\left(IR^{Imag}V_4^R - IR^{Real}V_4^I\right)}{|V_4|^2}\end{aligned} \tag{17}$$

In general, the sensitivity of active power losses with active power generation is shown in equation (18) and the sensitivity of active power loss with reactive power generation for the DGs having lagging and leading power factor is shown in equations (19) and (20), respectively.

$$\frac{\partial P_{loss}}{\partial PG_b} = -2\frac{[BIBL(:,4)]^T\left(IR^{Real}V_b^R - IR^{Imag}V_b^I\right)}{|V_b|^2} \tag{18}$$

For generator having lagging power factor:

$$\frac{\partial P_{loss}}{\partial QG_b} = -2\frac{[BIBL(:,b)]^T\left(IR^{Real}V_b^I - IR^{Imag}V_b^R\right)}{|V_b|^2} \tag{19}$$

For generator having leading power factor:

$$\frac{\partial P_{loss}}{\partial QG_b} = -2\frac{[BIBL(:,b)]^T\left(IR^{Imag}V_b^R - IR^{Real}V_b^I\right)}{|V_b|^2} \tag{20}$$

where IR^{Real} matrix is shown in equation (21) and IR^{Imag} matrix is shown in equation (22):

$$IR^{Real} = \left[IL_1^R R_1 \; IL_2^R R_2 \; IL_3^R R_3 \; IL_4^I R_4 \; \ldots . \; IL_{nline}^R R_{nline}\right] T \tag{21}$$

$$IR^{Imag} = \left[IL_1^I R_1 \; IL_2^I R_2 \; IL_3^I R_3 \; IL_{nline}^I R_4 \; \ldots . \; IL_{nline}^I R_{nline}\right] T \tag{22}$$

Total change in active power loss of radial distribution system due to change in active and reactive power generation of DG i at bus b is shown in equation (23). Share of active and reactive power generations of DG unit i at bus b on system active power loss is shown in equations (24) and (25), respectively, where Φ is phase angle corresponding to power factor of DG i.

$$(\Delta P_{loss})_b^i = \frac{\partial P_{loss}}{\partial PG_b} \Delta PG_b + \frac{\partial P_{loss}}{\partial QG_b} \Delta QG_b \tag{23}$$

$$Pshare_i^b = \frac{\frac{\partial P_{loss}}{\partial PG_b}}{\frac{\partial P_{loss}}{\partial PG_b} + \frac{\partial P_{loss}}{\partial QG_b} \tan(\Phi)} \tag{24}$$

$$Qshare_i^b = \frac{\frac{\partial P_{loss}}{\partial QG_b}}{\frac{\partial P_{loss}}{\partial PG_b} cot(\Phi) + \frac{\partial P_{loss}}{\partial QG_b}} \tag{25}$$

2.6 Proportional nucleolus game theory-based iterative algorithm

The PNGT-based iterative algorithm to compute LMP values at each DG bus is as shown in Algorithm 1. PNGT has been used in this algorithm for fair allocation of reduced losses among DG units.

Algorithm 1 PNGT-based iterative algorithm

Inputs
1: Read hour (t) of the day (D)
2: Read Forecasted load L(t,D)
3: Read λ^t

Steps
1: Run the load flow and compute base case losses with forecasted load L(t,D)
2: Set iteration $j = 1$, $\left(\Pi_a^t\right)_i^j = \lambda^t$, and $\left(PG^t\right)_i^0 = 0$ where $i = 1, 2, [\ldots] N_{DG}$
3: $i = 1$ ▷ i represents DG number
4: **while** $i \neq N_{DG} + 1$ **do**
Compute Generation using equations (2) and (26)

$$\left(QG^t\right)_i^j = \left(PG^t\right)_i^j * \frac{\sqrt{1 - \left(cosdg^i\right)^2}}{cosdg^i} \tag{26}$$

▷If calculated generation using equation (2) exceeds maximum capacity of DG unit then DG generation is set to generation upper limit. The constraint considered for DG generation is as shown in equation (27)

$$0 \leq \left(PG^t\right)_i^j \leq PG_i^{\max} \tag{27}$$

5: $i \leftarrow i + 1$
6: **end while**
7: Run the load flow and compute change in active power losses due to coalition of all DG units ($v^1(N)$) based on generation computed using equations (2) and (26).
8: Run the load flow and compute losses due to each sub coalition of DG units ($v^1(S)$) based on generation computed using equations (2) and (26).
9: Compute DISCO extra benefit ($\Delta benefit_j^t$) using equation (8) and set error = max(($(PG^t)_i^j$ - $(PG^t)_i^{j-1}$) where i = 1, 2, [...] N_{DG}
10: **if** $\Delta benefit_j^t \leq \epsilon_1$ OR $\Delta Pmax \leq \epsilon_2$ **then**
11: *GoTo* $\rightsquigarrow$ *Step*20
12: **else**
13: *GoTo* $\rightsquigarrow$ Step 15
14: **end if**

▷ Where ϵ_1 and ϵ_2 are small values

15: Compute share of each DG unit in loss reduction (y_i) using proportional nucleolus game theory as shown in Section 2.3
16: Compute incentive provided to each DG unit as shown in equation (28).

$$DG_{inc}^i = \frac{y_i}{v^l(N)} * \lambda^t v^l(N) = y_i * \lambda^t \tag{28}$$

17: Distribute incentive of each DG unit among active and reactive power generation as shown in equation (29).

$$\begin{aligned} DG_{inc}^P(i) &= DG_{inc}^i * Pshare_i^b \\ DG_{inc}^Q(i) &= DG_{inc}^i * Qshare_i^b \end{aligned} \tag{29}$$

18: Compute active and reactive power price for next iteration using equations (30) and (31) respectively.

$$\left(\left(\Pi_a^i\right)_t^{j+1} - \lambda^t\right) \frac{\left(\Pi_a^i\right)_t^{j+1} - b_i}{2a_i} = DG_{inc}^P(i) \tag{30}$$

$$\left(\Pi_r^i\right)_t^{j+1} = \lambda^r + \frac{DG_{inc}^Q(i)}{(QG^t)_i^j} \tag{31}$$

◁ Reactive power price at substation bus is less than 1per cent of active power price (Rider and Paucar, 2004), and so the value of λ^r is set to zero.

19: Increment iteration j = j + 1 and go to Step 3.
20: Stop iterative algorithm for hour 't' and take print out of required data.

3. Analytical studies

The proposed method was implemented on 84-bus TPC distribution network. Table IV represents the location of 15 DG units of various types operating at 0.9 lagging power factor with 1 MW capacity. The cost coefficients of each type of DG are represented in Table V. The proposed method has been simulated under MATLAB (Release, 2013) environment on realistic price data drawn from IESO (2015) and the TPC distribution system data captured from Su and Lee's study (2003).

3.1 Load forecasting

TPC distribution system's load for next day has been forecasted using historical data and by employing ANN as explained in Section 2.1. Mean absolute percentage error and root mean square error based on testing of network are found to be 1.3424 and 0.0094, respectively. Predicted load for next 24 h predicted zone for TPC distribution network is as shown in Figure 2.

3.2 Impact of market price (λ^t) on locational marginal price of distributed generation units

Table VI presents LMP values for each DG bus at market price of 19.23 \$/MWh, 21.59 \$/MWh and 25.07 \$/MWh. When market price is 19.23 \$/MWh, all DG units are off as market price is less than "b" coefficient value of generator. Hence, there is reduction in active power loss of network and no incentives to DG units. LMP value at each DG bus is equal to market price only. Whereas LMP values at DG buses for market prices of 21.59 \$/MWh and 25.07 \$/MWh are based on DG's contribution in loss reduction. As DG11 has high impact on loss reduction, LMP of DG11 unit is more when compared with remaining DG units.

Table IV. Type and location of 1 MW capacity DG units

Unit	Type	Location	Unit	Type	Location
1	1	4	9	2	20
2	1	65	10	2	47
3	1	25	11	3	11
4	1	35	12	3	60
5	1	84	13	3	41
6	2	55	14	3	30
7	2	12	15	3	76
8	2	72			

Notes: Type 1: combined cycle gas turbine; Type 2: gas internal combustion engine; Type 3: diesel internal combustion engine

Table V. DG units cost coefficients

Type	a (\$/MW^2h)	b (\$/MWh)	c (\$/h)
1	5.8	21	0
2	5.3	20	0
3	5.0	20	0

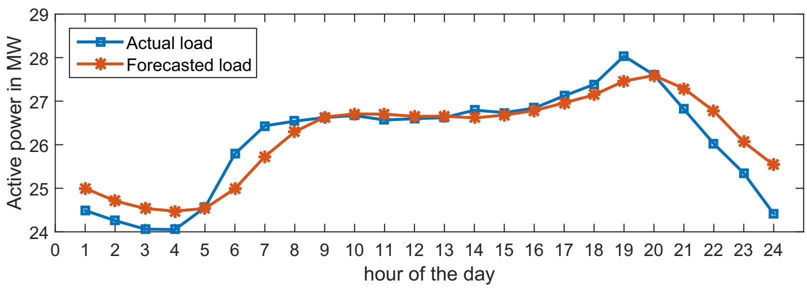

Figure 2. Forecasted load for TPC distribution system

DG unit	$\lambda^t = 19.23$ (\$/MWh)	$\lambda^t = 21.59$ (\$/MWh)	$\lambda^t = 25.07$ (\$/MWh)
DG1	19.23	22.30	25.82
DG2	19.23	22.04	25.44
DG3	19.23	22.06	25.54
DG4	19.23	22.10	25.58
DG5	19.23	22.22	25.80
DG6	19.23	22.04	25.59
DG7	19.23	21.65	25.14
DG8	19.23	22.18	25.76
DG9	19.23	21.97	25.49
DG10	19.23	21.66	25.10
DG11	19.23	22.62	26.21
DG12	19.23	21.94	25.36
DG13	19.23	22.13	25.55
DG14	19.23	21.88	25.39
DG15	19.23	21.77	25.26

Table VI. LMP in \$/MWh at DG buses for different market prices

3.3 Impact of market price (λ^t) on generation of distributed generation units

Table VII shows generation of DG units for different market prices. When market price is 19.23 \$/MWh, all DG units are not able to generate power as wholesale market price is less than "b" coefficient of DG units. However, DG unit's generation at market prices 21.59 \$/MWh and 25.07 \$/MWh depends on incentive provided by DISCO. As DG11 receives more incentive from DISCO due to its huge contribution in loss reduction, it has more generation when market prices are either 21.59 \$/MWh or 25.07 \$/MWh.

3.4 Impact of market price (λ^t) on reactive power price of distributed generation units

Impact of market price on RPP of DG units is shown in Table VIII. When market price is 19.23 \$/MWh, DG units cannot generate power as market price is less than "b" coefficient. Hence, RPP is shown in Table VIII as zero, which is equal to RPP at substation bus. When market price is 21.59 \$/MWh and 25.07 \$/MWh, RPP of each DG unit is based on DG unit's

DG unit	$\lambda^t = 19.23$ (\$/MWh)	$\lambda^t = 21.59$ (\$/MWh)	$\lambda^t = 25.07$ (\$/MWh)
DG1	0	112	416
DG2	0	90	383
DG3	0	92	391
DG4	0	94	395
DG5	0	105	413
DG6	0	192	527
DG7	0	156	485
DG8	0	205	543
DG9	0	186	518
DG10	0	156	482
DG11	0	262	621
DG12	0	194	536
DG13	0	213	556
DG14	0	188	539
DG15	0	177	526

Table VII. DG unit's generation in kW for different market prices

DG unit	λ^t = 19.23 (\$/MWh)	λ^t = 21.59 (\$/MWh)	λ^t = 25.07 (\$/MWh)
DG1	0	0.5387	0.6333
DG2	0	0.3435	0.4100
DG3	0	0.3726	0.4495
DG4	0	0.4086	0.4850
DG5	0	0.4680	0.5806
DG6	0	0.2986	0.3670
DG7	0	0.0454	0.0567
DG8	0	0.4645	0.5818
DG9	0	0.2996	0.3645
DG10	0	0.0643	0.1059
DG11	0	0.7774	0.9501
DG12	0	0.2683	0.3302
DG13	0	0.4506	0.5742
DG14	0	0.2118	0.2607
DG15	0	0.1390	0.1724

Table VIII.
DG unit's reactive power price in \$/$MV ar\ h$ for different market prices

reactive power contribution in loss reduction. DG11 has high RPP due to the reactive power contribution in loss reduction.

3.5 Variation in active power loss of network during iterative algorithm for different market prices

Figure 3 shows variation in active power loss of network as iterations increases in the proposed method at different market prices. Active power loss of network decreases as iterations increases. This is due to increasing of incentive to DG units thereby increasing the generation that have positive impact of loss reduction.

3.6 Variation in extra benefit of DISCO during iterative algorithm for different market prices

Variation in extra benefit of DISCO as iterations progress in the proposed method is shown in Figure 4. In deregulated environment, zero extra benefit is nothing but zero merchandising surplus is essential. This can be achieved by the proposed method and is as

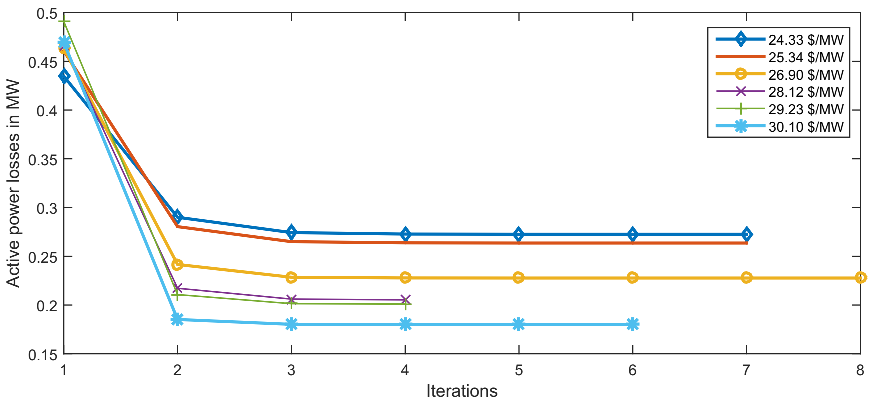

Figure 3.
Variation in active power loss for different market prices

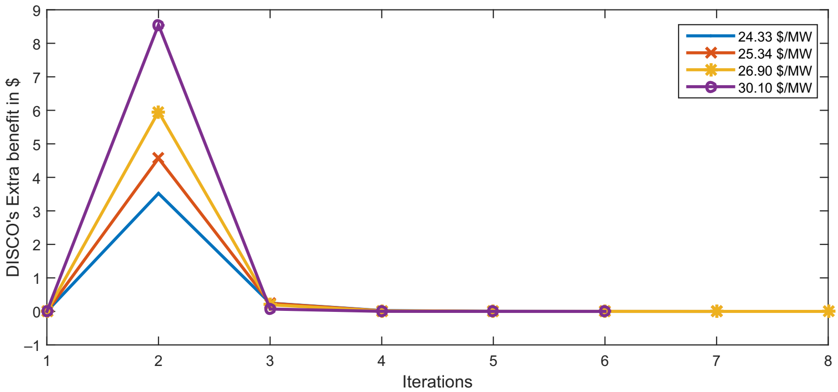

Figure 4.
DISCO's extra benefit variation for different market prices

shown in Figure 4. Proposed method provides incentive to each DG unit based on unit's contribution in loss reduction. These incentives are given from financial savings of DISCO due to loss reduction until DISCO's extra benefit reaches to zero.

3.7 Comparison of proposed method in terms of active power losses

The proposed method is compared with Shapley value-based iterative method as men tioned by Shaloudegi *et al.* (2012) in terms of losses during 24 h of the day. As shown in Figure 5, both proposed method and existing method provide equal loss, which is equal to base case losses at hours of the day where market price is less than "b" coefficient of all DG units. Whereas at the remaining hours of the day, proposed method enables the DISCO to operate the distribution system with less active power loss in comparison with the existing method.

3.8 Comparison of proposed method in terms of distributed generation profit

Figure 6 shows comparison of the proposed method with Shapley value-based iterative method (Shaloudegi *et al.*, 2012) in terms of profit of DG units at market price 24.95 $/MWh. Proposed method provides more profit to DG owners as incentives provided from DISCO's financial savings due to loss reduction. DG profit is computed as difference between total

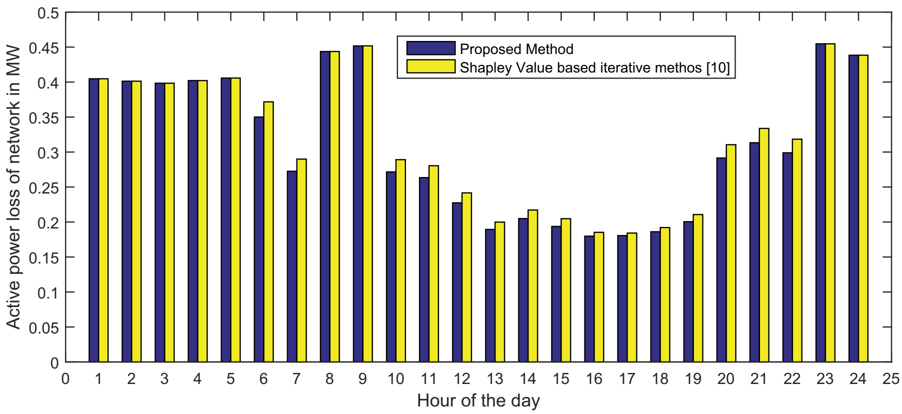

Figure 5.
Comparison in terms of active power loss of network

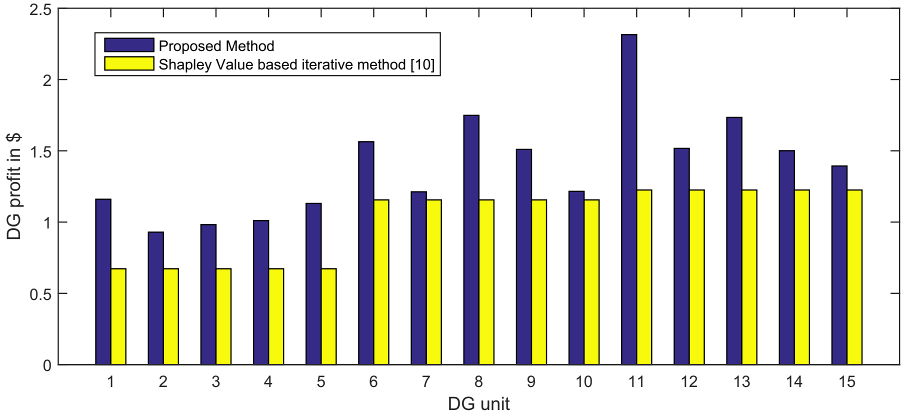

Figure 6.
Comparison in terms of DG units profit

amount paid by DISCO for both active and reactive power generation and total generation cost. As DG11 has high positive impact on loss reduction, DG11 gets more profit in comparison with the remaining DG units.

4. Conclusions

The PNGT-based iterative method has been developed to compute LMP at DG buses based on loss reduction. DG units which have high positive impact of loss reduction received more incentive. PNGT was used for the first time for computing LMP at DG buses based on loss reduction. Sensitivity factors were developed to identify share of active and reactive power generation of each DG unit on change in losses of network due to injection of that DG. Incentives were provided to DG units from DISCO's financial savings due to loss reduction.

This method provides controllable MS, opportunity to DISCO to handle trade off among losses and extra benefit. This method can be helpful to DISCO decision-maker to control private DG owners, to operate network optimally in terms of losses and to estimate state of network. Fair allocation of loss reduction among DG units was achieved by PNGT and the proposed method is also helpful to society/customers for getting quality power.

As there will be an increase of DG penetration into the network in future, this method may resolve some problems related to distribution system planning and operation. This work can be extended by considering emission, reliability and service quality for the benefit of the utilities. Furthermore, this work can also be extended by considering the reactive power losses in the RDS as elaborated in the Appendix of the paper.

References

Abdel-Akher, M. (2013), "Voltage stability analysis of unbalanced distribution systems using backward/forward sweep load-flow analysis method with secant predictor", *IET Generation, Transmission & Distribution*, Vol. 7 No. 3, pp. 309-317.

Chen, L., Zhong, J. and Gan, D. (2006), "Reactive power planning and its cost allocation for distribution systems with distributed generation", IEEE Power Engineering Society General Meeting, PES.

Guanghou, J., Gengyin, L. and Ming, Z. (2004), "Application of game theory in power quality market", *Power System Technology, PowerCon 2004, 2004 International Conference on, vol. 2*, pp. 1386-1390.

Haghighat, H. and Kennedy, S. (2010), "A model for reactive power pricing and dispatch of distributed generation", Power and Energy Society General Meeting, IEEE, pp. 1-10.

Honkapuro, S., Tuunanen, J., Valtonen, P. and Partanen, J. (2014), "DSO tariff structures: development options from stakeholders' viewpoint", *International Journal of Energy Sector Management*, Vol. 8 No. 3, pp. 263-282.

IESO (2015), available at: www.ieso.ca (accessed 18 December 2015).

Kumar, A. and Gao, W. (2010), "Optimal distributed generation location using mixed integer non-linear programming in hybrid electricity markets", *IET Generation, Transmission & Distribution*, Vol. 4 No. 2, pp. 281-298.

Lakdja, F., Gherbi, F.Z. and Kratima, F.E. (2013), "Applications of cooperative game theory in power system allocation problems", *Leonardo Journal of Sciences*, Vol. 12 No. 23, pp. 125-136.

Lemaire, J. (1984), "An application of game theory: cost allocation", *Astin Bulletin*, Vol. 14 No. 1, pp. 61-81.

Li, C., Li, Y., Cao, Y., Ma, J., Kuang, Y., Zhang, Z., Li, L. and Wei, J. (2015), "Credibility forecasting in short-term load forecasting and its application", *IET Generation, Transmission & Distribution*, Vol. 9 No. 13, pp. 1564-1571.

Lima, D.A., Contreras, J. and Padilha-Feltrin, A. (2008), "A cooperative game theory analysis for transmission loss allocation", *Electric Power Systems Research*, Vol. 78 No. 2, pp. 264-275.

Lin, C., Wu, W., Zhang, B., Wang, B., Zheng, W. and Li, Z. (2017), "Decentralized reactive power optimization method for transmission and distribution networks accommodating large-scale dg integration", *IEEE Transactions on Sustainable Energy*, Vol. 8 No. 1, pp. 363-373.

Moghaddas-Tafreshi, S. and Mashhour, E. (2009), "Distributed generation modeling for power flow studies and a three-phase unbalanced power flow solution for radial distribution systems considering distributed generation", *Electric Power Systems Re-Search*, Vol. 79 No. 4, pp. 680-686.

Naik, S.N.G., Khatod, D.K. and Sharma, M.P. (2014), "Analytical approach for optimal siting and sizing of distributed generation in radial distribution networks", *IET Generation, Transmission & Distribution*, Vol. 9 No. 3, pp. 209-220.

Orfanogianni, T. and Gross, G. (2007), "A general formulation for lmp evaluation", *IEEE Transactions on Power Systems*, Vol. 22 No. 3, pp. 1163-1173.

Parida, S.K., Srivastava, S.C. and Singh, S.N. (2011), "A review on reactive power management in electricity markets", *International Journal of Energy Sector Management*, Vol. 5 No. 2, pp. 201-214.

Pavani, P. and Singh, S. (2014), "Placement of dg for reliability improvement and loss minimization with reconfiguration of radial distribution systems", *International Journal of Energy Sector Management*, Vol. 8 No. 3, pp. 312-329.

Raghavendra, P. and Gaonkar, D. (2016), "Online voltage estimation and control for smart distribution networks", *Journal of Modern Power Systems and Clean Energy*, Vol. 4 No. 1, pp. 40-46.

Release, M. (2013), The mathworks Inc., Natick, Massachusetts, Vol. 488.

Rider, M. and Paucar, V. (2004), "Application of a nonlinear reactive power pricing model for competitive electric markets", *IEE Proceedings – Generation, Transmission and Distribution*, Vol. 151 No. 3, pp. 407-414.

Samimi, A., Kazemi, A. and Siano, P. (2015), "Economic-environmental active and reactive power scheduling of modern distribution systems in presence of wind generations: a distribution market-based approach", *Energy Conversion and Management*, Vol. 106, pp. 495-509.

Sathyanarayana, B.R. and Heydt, G.T. (2013), "Sensitivity-based pricing and optimal storage utilization in distribution systems", *IEEE Transactions on Power Delivery*, Vol. 28 No. 2, pp. 1073-1082.

Satyaramesh, P. and Radhakrishna, C. (2009), "Use of cooperative game theory concepts for loss allocation in bilateral electricity markets", *International Journal of Power and Energy Systems*, Vol. 29 No. 4, p. 272.

Shaloudegi, K., Madinehi, N., Hosseinian, S. and Abyaneh, H.A. (2012), "A novel policy for locational marginal price calculation in distribution systems based on loss reduction allocation using game theory", *IEEE Transactions on Power Systems*, Vol. 27 No. 2, pp. 811-820.

Sheng, W., Liu, K.Y., Liu, Y., Ye, X. and He, K. (2016), "Reactive power coordinated optimisation method with renewable distributed generation based on improved harmony search", *IET Generation, Transmission & Distribution*, Vol. 10 No. 13, pp. 3152-3162.

Shirmohammadi, D., Hong, H., Semlyen, A. and Luo, G. (1988), "A compensation-based power flow method for weakly meshed distribution and transmission networks", *IEEE Transactions on Power Systems*, Vol. 3 No. 2, pp. 753-762.

Singh, H. (1999), "Introduction to game theory and its application in electric power markets", *IEEE Computer Applications in Power*, Vol. 12 No. 4, pp. 18-20.

Singh, R.K. and Goswami, S. (2010), "Optimum allocation of distributed generations based on nodal pricing for profit, loss reduction, and voltage improvement including voltage rise issue", *International Journal of Electrical Power & Energy Systems*, Vol. 32 No. 6, pp. 637-644.

Sotkiewicz, P.M. and Vignolo, J.M. (2006), "Nodal pricing for distribution networks: efficient pricing for efficiency enhancing dg", *IEEE Transactions on Power Systems*, Vol. 21 No. 2, p. 1013.

Sotkiewicz, P.M. and Vignolo, J.M. (2007), "Towards a cost causation-based tariff for distribution networks with dg", *IEEE Transactions on Power Systems*, Vol. 22 No. 3, pp. 1051-1060.

Su, C.T. and Lee, C.S. (2003), "Network reconfiguration of distribution systems using im- proved mixed-integer hybrid differential evolution", *IEEE Transactions on Power Delivery*, Vol. 18 No. 3, pp. 1022-1027.

Wang, Z., Chen, F. and Li, J. (2004), "Implementing transformer nodal admittance matrices into backward/forward sweep-based power flow analysis for unbalanced radial distribution systems", *IEEE Transactions on Power Systems*, Vol. 19 No. 4, pp. 1831-1836.

Yao, J., Zhengyu, W., Jiang, C. and Zhang, Y. (2015), "Dispatch and bidding strategy of active distribution network in energy and ancillary services market", *Journal of Modern Power Systems and Clean Energy*, Vol. 3 No. 4, pp. 565-572.

Zhang, L., Tang, W., Liang, J., Cong, P. and Cai, Y. (2016), "Coordinated day-ahead reactive power dispatch in distribution network based on real power forecast errors", *IEEE Transactions on Power Systems*, Vol. 31 No. 3, pp. 2472-2480.

Appendix. Future work for computing LMP in radial distribution system based on active and reactive power losses

In deregulated environment, all active stakeholders of electric power market will place the bids in energy exchange either to inject the active power into the grid or withdraw the active power from the grid. DISCO/Aggregator will place the bid in energy exchange for the amount of active power required at substation bus that depends on active power losses of distribution network as shown in equation (32). Most of the financial transactions between stakeholders in electric power market are based on active power. Due to these reasons, LMP at DG buses have been computed on the basis of active power losses.

$$P_{sub}^{t} = L(t, D) + Ploss_{DG}^{t} - \sum_{i=1}^{N_{DG}} (PG^{t})_{i}^{j} \tag{32}$$

However, this work can also be further extended by considering the reactive power losses. PNGT-based iterative method can be modified by considering two cooperative games among DGs. First game is for fair allocation of reduced active power losses from base case among DG units, and second game is for fair allocation of reduced reactive power losses from base case among DG units. The financial incentives given to each DG unit based on its contribution in active and reactive power loss reduction have to be computed using equation (33). The values of ω_a and ω_r represent DISCO's priority for reduction of active power losses and reactive power losses, respectively. DISCO's decision-maker selects the values of ω_a and ω_r in such a way that the sum of ω_a and ω_r is equal to one.

$$DG_{inc}^{i} = \left(\lambda^{t} v^{l}(N) + \lambda^{t} v^{q}(N)\right) * \left(\frac{\omega_a y_i}{v^{l}(N)} + \frac{\omega_r yq_i}{v^{q}(N)}\right) \tag{33}$$

Total financial incentive received by each DG unit i located at bus b is again shared among active and reactive power generation as shown in equations (34) and (35).

$$DG_{inc}^{P}(i) = DG_{inc}^{i} * \frac{Ploss\frac{\partial Ploss}{\partial PG_b} + Qloss\frac{\partial Qloss}{\partial PG_b}}{Ploss\frac{\partial Ploss}{\partial PG_b} + Qloss\frac{\partial Qloss}{\partial PG_b} + tan(\Phi) * \left(Ploss\frac{\partial Ploss}{\partial QG_b} + Qloss\frac{\partial Qloss}{\partial QG_b}\right)} \tag{34}$$

$$DG_{inc}^{Q}(i) = DG_{inc}^{i} * \frac{Ploss\frac{\partial Ploss}{\partial QG_b} + Qloss\frac{\partial Qloss}{\partial QG_b}}{Ploss\frac{\partial Ploss}{\partial QG_b} + Qloss\frac{\partial Qloss}{\partial QG_b} + cot(\Phi) * \left(Ploss\frac{\partial Ploss}{\partial PG_b} + Qloss\frac{\partial Qloss}{\partial PG_b}\right)} \tag{35}$$

LMP value of each DG i at bus b has to be computed by using PNGT-based iterative method using equations (30) and (31).

If the value of ω_a is more than ω_r, then DG units, which have more positive impact on active power loss reduction, will receive more incentive. If DISCO's decision-maker is willing to reduce reactive power loss, then the value of ω_r raises over ω_a value and DG units, which have more positive impact on reactive power loss reduction, will receive more incentive as LMP.

Corresponding author
Venkaiah Chintham can be contacted at: ch.venkaiah@ieee.org