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Probabilistic locational marginal price computation in radial distribution system based on active power loss reduction

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Abstract: This study presents a probabilistic approach to calculate locational marginal price (LMP) at distributed generation (DG) buses in an electric power distribution system to ensure that the network operates at reduced active power losses (APLs). The proportional nucleolus theory-based iterative method is proposed as a deterministic approach to compute LMP based on APLs. Uncertainties in load and market price were captured by using a $2m+1$ scheme of the point estimation method. To compute the contribution of the reactive and active power of the DG on the APL of the network, loss sensitivity factors have been developed. The proposed method provides an active power price in terms of LMP and reactive power price to each DG as per its contribution to loss reduction. To verify the performance of the proposed method, it was implemented on the Taiwan Power Company distribution system. The proposed method can be utilised by a distribution company to operate the network at reduced APL and improve the benefit of DG owners.

Nomenclature

CF_i	fuel cost function of distributed generation (DG) 'i' in \$/h
a_i, b_i, c_i	fuel cost parameters for DG unit 'i'
$(\mathrm{PG}^t)_i^j$	real power of DG unit 'i', at epoch 'j' at the 'tth' hour in MW
$(\mathrm{QG}^t)_i^j$	reactive power of DG unit 'i', at epoch 'j' and at the 'tth' hour in MVar
$(\Pi_a^t)_i^j$	real power price of DG unit 'i', at epoch 'j' and at the 'tth' hour in \$/MWh
$(\mathrm{DG}_i^t)^{\mathrm{rev}}$	revenue of the 'ith' DG unit and at the 'tth' hour in \$/h
$(\mathrm{DG}_i^t)^{\mathrm{profit}}$	profit of the 'ith' DG unit and at the 'tth' hour in \$/h
$\mathrm{PG}_i^{\max}$	maximum capacity for the ith DG unit
Ploss_0^t	real power loss of the distribution system without DG at the 'tth' hour
λ^t	market price for real power at the 'tth' hour in \$/MWh
$L(t, D)$	load on the distribution system at the 'tth' hour and the 'Dth' day
Π^c	customer energy price in \$/MWh
$\cos(\Phi_i)$	power factor of the ith DG unit
$\mathrm{benefit}_0^t$	distribution company's (DISCO's) benefit in the absence of the DG unit in \$/h
$\mathrm{benefit}_j^t$	DISCO's benefit in the presence of the DG unit at the 'jth' epoch and at 'tth' hour in \$/h
$\Delta\mathrm{benefit}_j^t$	DISCO's additional benefit in presence of the DG unit at the 'jth' epoch and at the 'tth' hour in \$/h
$(\Pi_r^t)_i^j$	reactive power price of the 'ith' DG unit for the 'tth' hour and epoch 'j' in \$/MVar
$\mathrm{Ploss}_{\mathrm{DG}}^t$	real power loss of the distribution system in the presence of DG at the 'tth' hour
PG_b	real power generation from the DG at the 'bth' bus
QG_b	reactive power generation from the DG at the 'bth' bus
V_b^{R}	real part of voltage at the 'bth' bus
V_b^{I}	imaginary part of voltage at the 'bth' bus
V_b	complex bus voltage at bus 'b'
R_l	'lth' line resistance
$\mathrm{IL}_l^{\mathrm{R}}$	'lth' line current real part
$\mathrm{IL}_l^{\mathrm{I}}$	'lth' line current imaginary part
N_{DG}	total count of DG units in the distribution system
Pshare_i^b	contribution of real power of the 'ith' DG unit at the 'bth' bus on change in real power loss
Qshare_i^b	contribution of reactive power of the 'ith' DG unit at the 'bth' bus on change in real power los
$v(S)$	active power loss reduction (APLR) due to the sub-coalition of DGs
$v(N)$	APLR due to the grand coalition of DGs
y_i	loss imputation for DG 'i'

1 Introduction

The integration of distributed generation (DG) units into the distribution system has been increasing rapidly due to advantages such as the reduction of active power losses (APLs), the stability of the network in terms of voltage, increasing reinforcement horizon, and emission reduction [1]. As distribution network operation has been increasing with DG units, distribution companies (DISCOs) were giving some stimulus to DG owners based on DGs impact on the improvement of network operation. Owing to the merging of DG units, distribution systems are converted from a passive state to an active state as the transmission system [2]. Some of the mechanisms, which were used for efficient operation in the transmission system, can also be applicable for the active distribution system such as nodal pricing. Locational marginal price (LMP) is the best among existing methods of nodal pricing.

DISCOs have not received the desired technological support as in transmission and generation, despite 30–40% of the total investment in the electric power sector goes to electric power distribution systems. In general, 3–6% of active power losses exist in the electric power sector. Total distribution APLs in developed countries are not > 10% of total active power losses in the electric power sector. However, these active power losses are around 20% in developing countries such as India. Owing to this reason, electric power utilities in developing countries are trying to reduce active power losses by effective operating mechanisms such as providing financial incentives [3]. To manage APLs in the distribution network, efficient and effective computation tools have to be developed.

The significance of notations used in Table 1 for representing different features addressed by researchers is as follows:

P1: reduction of APLs
P2: controllable merchandising surplus (MS)

P3: changing DG financial gain
P4: offering assistance to DG for competing in the reduction of active power loss
P5: calculating LMP at existing circumstances
P6: determining LMP in next running circumstances
P7: DISCO's vital skill for ideal operation
P8: zero MS
P9: impact of real and reactive power production of DG on APLs
P10: financial stimulus offered to DGs from a monetary surplus of DISCO due to active power loss curtailment
P11: reactive power price

As the number of DG units getting merged into the electric power distribution system increases, DISCOs need is felt to control and maintain the impartial clash between DG owners. DISCOs can achieve it by providing financial stimulus in terms of LMP. Very few researchers have computed LMP in the distribution network based on APLs of network and contribution by the researchers in comparison with the proposed method is presented in Table 1. APLs in the distribution system while distributing the power through the feeders in the distribution network are defined as the amount of power loss in the feeders due to the resistance.

Optimal Power Flow (DC-OPF)-based iterative algorithm for LMP calculation by considering marginal losses of the system was proposed in [14]. However, this approach has suffered from power mismatch at reference bus. To avoid the imbalance at the slack bus, it is desirable that the APLs are confined to the transmission lines. The concept of fictitious nodal demand is introduced to describe the transmission line losses. This complete methodology is working well on the transmission system but there is no guarantee for LMP calculation in the distribution system. A new methodology was proposed in [15] to compute continuous LMP based on energy price, congestion, losses, and future limit risk. This methodology smoothens the price curve by avoiding step change due to load variation. The impact of uncertainties that exist in forecasted load on LMP had been investigated in [16, 17]. In this work, the probability density function of random output variable LMP at a particular time has been developed. Two useful curves such as deterministic LMP versus predicted load and estimated LMP versus predicted load are presented. The first curve between deterministic LMP versus predicted load has been developed to help identify the trustworthy regions of the traditional LMP-load curve, whereas the second curve demonstrated to be smooth and, therefore, eliminates the step changes in deterministic LMP simulation.

Distribution LMP (DLMP) as computed in [18] is based on marginal energy cost, marginal loss cost, and marginal congestion cost. This methodology provides information about energy prices at each node in the distribution network to the consumers connected to that node. Decentralised congestion management through DLMP is discussed in [19]. In this work, the authors computed the DLMP using the quadratic programming method. This price information is provided to aggregators for enabling them to make optimal energy plan for flexible loads. DLMP in the distribution system is computed with quadratic programming and loss factors in [20]. A new methodology for computation of the day-ahead market-clearing price for the smart distribution system based on real and reactive power, congestion, voltage support, and loss is proposed in [21]. This approach motivates DG owners to contribute to voltage support and congestion management. DLMPs as computed in [22] are based on congestion, voltage constraints, and APLs using three alternative approaches for decomposing DLMPs such as duality analysis of a second-order conic programming relaxation of optimal power flow; duality analysis of a formulation of optimal power flow with a global power balance constraint and analysis of the contribution of marginal losses on DLMPs. A novel approach for the placement of the DG in the radial distribution system based on DLMP has been proposed in [23]. In this work, the authors have computed LMP using AC-OPF-based marginal energy cost at the reference bus, APLs and congestion. All these works provide a valuable contribution towards LMP computation in the distribution system, however, there is no discussion about the impact of uncertainties that exist in market price and load on DLMP.

Dispatching the proper amount of power from the DG units to the distribution network will lead to a reduction in APLs. The amount of power that needs to be dispatched from the DG units will depend on the active power load on the substation and market price, which are uncertain in nature. Probabilistic methods are required to capture the uncertainties that exist in load and market price to estimate proper active power dispatch from the DG units in the day-ahead operation in such a way that total APLs on the distribution network have been minimised.

DISCO's decision-maker (DM) provides LMPs to DG units, which are less sensitive to the uncertainty that exists in the market and load. To fulfil this requirement of the DM, LMP calculation mechanism, which can handle uncertainty in load and market price, is required. The MCS method has been used in [7, 11] to capture uncertainty in input parameters such as load and market price. Even MCS is a more accurate probabilistic method to capture uncertainty but requires more number of samples and it took more computation time. The 2 m scheme of PEM ($2m$: PEM) has been used in [10] to capture uncertainty but the performance of this method depends on the number of input variables. In the $2m$: PEM method, if the number of input variables increases then the locations of those input variables move away from the probability distribution function (PDF) curve of that input variable. Some times, these locations may move far away from where the probability density function is not known. To avoid these difficulties, the $2m+1$ scheme of the PEM ($2m+1$: PEM) has been used in the proposed method to handle the uncertainty in load and market price so as to ensure that LMP and reactive power price (RPP) values are less sensitive to uncertain input parameters.

In the deterministic approach, the DG dispatch has been computed by assuming that the distribution network will be operated at a particular load and market price. In this case, uncertainties that exist in load and market prices were not

Table 1 Comparison of research contributions on LMP calculation in the radial distribution system

Research contribution	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	Deterministic approach	Uncertainty capture
	Different features researchers addressed												
[4]	✓				✓							genetic algorithm	No
[5]	✓		✓		✓							bi-level optimisation	no
[6]	✓		✓		✓							sensitivity factors	no
[7]	✓		✓	✓	✓							average marginal loss cost	Monte Carlo simulation (MCS)
[8]	✓		✓		✓						✓	marginal loss coefficients	no
[9]	✓		✓		✓			✓			✓	reconciliated marginal loss coefficients	no
[10]	✓	✓	✓	✓	✓	✓	✓	✓				Honey bee mating optimisation	2 m point estimation method (PEM)
[11]	✓	✓	✓	✓	✓	✓	✓	✓			✓	Shapley value method	MCS
[12]	✓		✓	✓	✓	✓	✓					no	no
	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	proportional nucleolus theory (PNT)	no
[13]	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	PNT	no
proposed method	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	PNT	$2m+1$ PEM

considered. In the deterministic approach, DG units dispatch has been computed in such a way that the distribution network will be operated optimally in terms on APLs at the given load and market price, whereas in the probabilistic approach, the DG dispatch has been computed in such a way that the distribution network has been operated optimally by considering uncertainties that exist in load on the electric power distribution substation and market price. The distribution company (DISCO) will send the information on how much active power needs to be dispatched from each DG unit to each DG owner. DG owners will make necessary arrangements in the speed governing system of the DG unit to dispatch estimated active power. The distribution system operator can use this proposed approach to inform the DG owners about the generation (which is less sensitive to uncertainties that exist in load and market price) of their DG units in day-ahead operation for optimal operation of the distribution system in terms of APLs.

In this study, LMP and RPP of the individual DG unit have been computed based on the DG units impact on APL reduction (APLR). PNT is a cooperative game theory and it has been employed to allocate the change in APL among DG units. The proposed method computes LMP and RPP at a particular load on the distribution system and market price of active power. Both load and market price for active power have uncertain behaviour.

The allocation of change in APL among DG units in [11] has been done using the Shapley value method. The main drawback of this cooperative game theory is that the solution may not exist within the core. To avoid this difficulty, PNT has been used for the first time for the allocation of change in APL among DG units.

The original contribution of this study is as follows:

- Loss sensitivity factors were developed to measure the actual impact of real and reactive power generation of the individual DG unit on the APL of the network.
- LMP and RPP have been computed using loss sensitivity factors.
- PNT has been used for the allocation of change in the APL of the network among DG units.
- The PEM scheme, i.e. the $2m+1$ scheme was utilised to capture the uncertainty.
- Monetary incentives were granted to DG owners from the monetary savings of the DISCO due to the APLR.

DISCO can use the proposed method in the following ways:

- To maintain fair competition between DG owners.
- To engage the network optimally based on the APL.
- To predict the network state in current and next operating conditions.
- To encourage DG owner's participation in reducing the APL of the distribution network.
- To estimate the total load on the substation.
- To estimate the market price for real power generation.
- To estimate LMP and corresponding real power of the DG units.
- To estimate the APLs on the system.

The security-constrained economic dispatch method used by North American Transmission System Operators (TSOs) focusing on the transmission network with two stages called the unit commitment stage in day-ahead operation and economic load dispatch in real-time by considering transmission constraints and security constraints. Unit commitment is focusing on identifying the units, which need to be committed based on forecasted conditions, whereas economic dispatch is focusing on the determination of the generation of committed units in real-time for the lowest production cost [24]. However, the proposed method is mainly focusing on the distribution of network operation. It will determine the generation of DG units in real-time by considering the uncertainties that exist in input parameters such as load and market price. The main goal of the proposed method is to determine the generation of DG units in real-time such that the distribution network will operate at minimum APLs.

The organisation of the remaining part of the paper is as follows: Section 2 describes deterministic and probabilistic approaches for computing LMP based on APLR. Section 3 provides the analysis of simulation results and Section 4 presents conclusions of this paper.

2 Deterministic and probabilistic locational marginal price computation based on APLR

2.1 DG generation corresponding to maximum DG profit

DG profit is stated as the mismatch among total generation cost and total revenue collected from DISCO in terms of financial incentives. The total cost of generation is computed using (1), total revenue collected from DISCO is computed using (2) and DG profit is calculated using (3)

$$\mathrm{CF}_i = a_i((\mathrm{PG}^t)_i^j)^2 + b_i(\mathrm{PG}^t)_i^j + c_i \tag{1}$$

$$(\mathrm{DG}_i^t)^{\mathrm{rev}} = (\mathrm{PG}^t)_i^j(\Pi_a^t)_i^j + (\mathrm{QG}^t)_i^j(\Pi_r^t)_i^j \tag{2}$$

$$(\mathrm{DG}_i^t)^{\mathrm{profit}} = (\mathrm{DG}_i^t)^{\mathrm{rev}} - \mathrm{CF}_i \tag{3}$$

The condition for maximising the DG profit is presented in (4). The real and reactive power of DG corresponding to maximum DG profit is shown in (6) and (8), respectively. In case $(\mathrm{PG}^t)_i^j$ is more than $\mathrm{PG}_i^{\max}$ then set the value of $(\mathrm{PG}^t)_i^j$ is equal to $\mathrm{PG}_i^{\max}$

$$\frac{\partial(\mathrm{DG}_i^t)^{\mathrm{profit}}}{\partial(\mathrm{PG}^t)_i^j} = 0 \tag{4}$$

$$(\mathrm{PG}^t)_i^j = \frac{(\Pi_a^t)_i^j + (\Pi_r^t)_i^j * \tan(\Phi) - b_i}{2a_i} \tag{5}$$

In (5), the value of $(\Pi_r^t)_i^j * \tan(\Phi)$ is very small compared to $((\Pi_a^t)_i^j - b_i)$. Hence, (5) is deduced to (6) by neglecting $(\Pi_r^t)_i^j * \tan(\Phi)$. Lower and upper boundaries of the DG unit's active power generation are shown in (7)

$$(\mathrm{PG}^t)_i^j = \frac{(\Pi_a^t)_i^j - b_i}{2a_i} \tag{6}$$

$$0 \le (\mathrm{PG}^t)_i^j \le \mathrm{PG}_i^{\max} \tag{7}$$

If $(\Pi_a^t)_i^j \le b_i$, then $(\mathrm{PG}^t)_i^j = 0$. For a given LMP $= ((\Pi_a^t)_i^j - b_i)/2a_i$, if $(\mathrm{PG}^t)_i^j \ge \mathrm{PG}_i^{\max}$ then set $(\mathrm{PG}^t)_i^j = \mathrm{PG}_i^{\max}$ as the DG cannot generate beyond its capacity

$$(\mathrm{QG}^t)_i^j = (\mathrm{PG}^t)_i^j * \tan(\Phi) \tag{8}$$

2.2 DISCO's extra benefit

DISCO's extra benefit is stated as the mismatch among DISCO's monetary gain with and without DG units power injection into the distribution system. DISCO's benefit in the absence and presence of DGs power injection is calculated as shown in (9) and (10), respectively. DISCO's extra benefit is computed using (11)

$$\mathrm{benefit}_0^t = \Pi^c L(t,D) - ((L(t,D) + \mathrm{Ploss}_0^t)\lambda^t \tag{9}$$

$$\begin{aligned}\mathrm{benefit}_j^t = \Pi^c L(t,D) - \sum_{i=1}^{N_{\mathrm{DG}}}((\mathrm{PG}^t)_i^j)(\Pi_a^t)_i^j \\ - \sum_{i=1}^{N_{\mathrm{DG}}}(\mathrm{QG}^t)_i^j(\Pi_r^t)_i^j - (L(t,D) \\ + \mathrm{Ploss}_{\mathrm{DG}}^t - \sum_{i=1}^{N_{\mathrm{DG}}}(\mathrm{PG}^t)_i^j)\lambda^t\end{aligned} \tag{10}$$

$$\Delta\text{benefit}_j^t = (\text{Ploss}_0^t - \text{Ploss}_{\text{DG}}^t)\lambda^t - \sum_{i=1}^{N_{\text{DG}}} (\text{QG}^t)_i^j (\Pi_r^t)_i^j - \sum_{i=1}^{N_{\text{DG}}} (\text{PG}^t)_i^j ((\Pi_a^t)_i^j - \lambda^t) \tag{11}$$

2.3 Loss sensitivity factors

In this study, loss sensitivity factors were used to measure the actual impact of real and reactive power generation at any bus in the distribution system on APL. The sensitivity of the APL of the radial distribution system with respect to real power generation at bus 'b' is computed using (12). Similarly sensitivity of DG units reactive power generation at bus 'b' on the APL has been computed using (13) and (14) for lagging and leading power factor DGs, respectively. The loss sensitivity factors, which were utilised for computing the probabilistic LMP at each DG bus have been validated in [25]

$$\frac{\partial P_{\text{loss}}}{\partial \text{PG}_b} = -2\frac{[\text{BIBL}(:,b)]^{\text{T}}(\mathbf{IR}^{\text{Real}}V_b^{\text{R}} + \mathbf{IR}^{\text{Imag}}V_b^{\text{I}})}{|V_b|^2} \tag{12}$$

For lagging power factor generator

$$\frac{\partial P_{\text{loss}}}{\partial \text{QG}_b} = -2\frac{[\text{BIBL}(:,b)]^{\text{T}}(\mathbf{IR}^{\text{Real}}V_b^{\text{I}} - \mathbf{IR}^{\text{Imag}}V_b^{\text{R}})}{|V_b|^2} \tag{13}$$

For leading power factor generator

$$\frac{\partial P_{\text{loss}}}{\partial QG_b} = -2\frac{[\text{BIBL}(:,b)]^{\text{T}}(\mathbf{IR}^{\text{Imag}}V_b^{\text{R}} - \mathbf{IR}^{\text{Real}}V_b^{\text{I}})}{|V_b|^2} \tag{14}$$

where $\mathbf{IR}^{\text{Real}}$ matrix is shown in (15) and $\mathbf{IR}^{\text{Imag}}$ matrix is shown in (16) and bus incident beyond line (BIBL) matrix was developed such that if bus 'b' connected beyond line 'l' then set the value of BIBL(l,b) to '1' otherwise set it to '0'. The number of rows and columns of the BIBL matrix is equal to the number of lines and buses in the distribution network, respectively

$$\mathbf{IR}^{\text{Real}} = \left[\text{IL}_1^{\text{R}}R_1 \quad \text{IL}_2^{\text{R}}R_2 \quad \text{IL}_3^{\text{R}}R_3 \quad \ldots \quad \text{IL}_{\text{nline}}^{\text{R}}R_{\text{nline}}\right]^{\text{T}} \tag{15}$$

$$\mathbf{IR}^{\text{Imag}} = \left[\text{IL}_1^{\text{I}}R_1 \quad \text{IL}_2^{\text{I}}R_2 \quad \text{IL}_3^{\text{I}}R_3 \quad \ldots \quad \text{IL}_{\text{nline}}^{\text{I}}R_{\text{nline}}\right]^{\text{T}} \tag{16}$$

The contribution of real power generation of DG at 'b' on change in APL due to the DG at bus 'b' has been computed using (17). Similarly contribution of reactive power generation of DG at bus 'b' on change in APL of the network has been computed using (18)

$$P\text{share}_i^b = \frac{\frac{\partial P_{\text{loss}}}{\partial PG_b}}{\frac{\partial P_{\text{loss}}}{\partial PG_b} + \frac{\partial P_{\text{loss}}}{\partial QG_b}\tan(\Phi)} \tag{17}$$

$$Q\text{share}_i^b = \frac{\frac{\partial P_{\text{loss}}}{\partial QG_b}}{\frac{\partial P_{\text{loss}}}{\partial PG_b}\cot(\Phi) + \frac{\partial P_{\text{loss}}}{\partial QG_b}} \tag{18}$$

2.4 Proportional nucleolus theory (PNT)

PNT is working efficiently for loss allocation problems due to its extended core feature [13, 26]. In this study, PNT has been used to allocate the change in APL of the network from the base case among DG units. The base case is considered as the distribution network without any injection from the DG units. In this loss allocation problem, DG units are acting as players. The change in APL of the network allocated among DG units by solving a linear programming problem is as shown in (19)

$$\begin{aligned} &\min \ \varepsilon \\ &\sum_{i \epsilon N} y_i = v(N) \\ &\frac{v(S) - \sum_{i \epsilon S} y_i}{v(S)} \le \varepsilon \end{aligned} \tag{19}$$

where ε is a small arbitrary real value.

Core: The core of a Transferable Utility (TU) game (N, v) is the set of all payoff allocations that are individually, coalitionally, and collectively rational. In other words, the core is the set of all imputations that are coalitionally rational as shown below

$\text{Core}(N, v) = \{(y_1, \ldots, y_n)\ R^n\text{: } \sum_{i=1}^{N} y_i = v(N); \sum_{i \epsilon S} y_i \ge v(S) \forall\ S \subseteq N\}$

An allocation x is said to be in the core of (N, v) if y is feasible for N and no coalition can improve upon it. This implies that y is in the core of (N, v) if $\sum_{i \epsilon N} y_i = v(N)$ and $\sum_{i \epsilon S} y_i \ge v(S)$.

Coalition: Coalition is a player's group in the game. There are two types of coalitions such as grand coalition and sub-coalition. Grand coalition is the group of all players, and sub-coalition is the sub-group of all players in the cooperative game. For example, if game contains three players such as X, Y, and Z. Then $\{X, Y, Z\}$ is called grand coalition and subsets such as $\{X\}, \{Y\}, \{Z\}, \{X, Y\}, \{X, Z\}, \{Y, Z\}$ are called sub-coalitions.

Imputation: A payoff vector is called as imputation. Imputation is both group and individual rational. The imputations set can be written as $\{y = (y_1, \ldots, y_n)$ is $v(N)$ and $y_i \ge v(i)$ for all $i \in N\}$.

Proportional nucleolus game theory is explained legibly in the following example. Let us consider that three DG units were integrated into the network with base case active power loss of 440 kW and assume that the generation of each DG unit is 0.5 kW, 0.75 kW, and 1 MW. Table 2 presents losses and reduced losses from the base case due to each coalition of DG units.

Objective function

$$\min\ 0 * y_1 + 0 * y_2 + 0 * y_3 + 1 * \varepsilon$$

Equality constraint

$$1 * y_1 + 1 * y_2 + 1 * y_3 + 0 * \varepsilon = 168.15$$

Inequality constraints

$$\begin{aligned} -1 * y_1 + 0 * y_2 + 0 * y_3 - 19.05 * \varepsilon &\le -19.05 \\ 0 * y_1 - 1 * y_2 + 0 * y_3 - 52.05 * \varepsilon &\le -52.05 \\ 0 * y_1 + 0 * y_2 - 1 * y_3 - 21.9 * \varepsilon &\le -21.9 \\ -1 * y_1 - 1 * y_2 + 0 * y_3 - 105 * \varepsilon &\le -105 \\ -1 * y_1 + 0 * y_2 - 1 * y_3 - 61.2 * \varepsilon &\le -61.2 \\ 0 * y_1 - 1 * y_2 - 1 * y_3 - 85.05 * \varepsilon &\le -85.05 \end{aligned}$$

By solving the above linear programming problem that is formulated using (14), the share of each DG unit in loss reduction are $y_1 = 53.4753\,\text{kW}$, $y_2 = 85.6559\,\text{kW}$, and $y_3 = 29.0188\,\text{kW}$. As discussed previously, the main feature of PNT is extended core. The extended core is the inter-sectional area formed in a triangle by all equality and inequality constraints. This is done by pretending that the plane of the plot is the plane $y_1 + y_2 + y_3 = 168.15$, and giving each point on the plane three coordinates, which add to 168.15. The equilateral triangle with an extended core, which is the region where all the points satisfy all the equality and inequality constraints as shown in Fig. 1.

2.5 $2m + 1$ scheme of PEM ($2m + 1$ PEM)

Some techniques were available in the literature to capture the uncertainties in random input variables. These techniques are mainly categorised as follows [27, 28]:

- MCS
- Analytical methods

• Approximation methods

MCS is widely used in power system applications. It randomly generates multiple samples of uncertain input variables. The deterministic approach was executed for each sample. The main drawback of this approach is that it requires more number of samples and takes more computation time. Computationally analytical methods were more effective but these methods consider some mathematical assumptions. Approximation methods describes the probabilistic properties of random output variables. The PEM was one of the approximation methods and the main advantages of the PEM are as follows:

• The PEM method provides a solution that is almost near to the MCS.
• Computation time is less in comparison with MCS.
• Less probabilistic information of random input variables is sufficient.

Owing to the merits of the PEM, the proposed method utilises it for capturing the uncertainty in random input variables. If a random variable of interest can be expressed in an equation as the result of a mathematical operation of other random variables, then the PEM developed by Rosenblueth provides a direct computational procedure to obtain moment such as mean-variance estimates for that random variable [29].

In this study, the authors have estimate random variable location marginal price (LMP) at each DG bus based on the contribution of the DG on that bus for reduction of APLs. APLs in the distribution network depends on the total load of the distribution system, injection of active power from the DG units into the network and how much power is drawn from the grid. This means that APLs in the system depend on random variables such as load and market price. From this discussion, one can conclude that the random variable location marginal price (LMP) at each DG bus is depending on other input random variables such as active power load on substation and market price for active power at energy exchange.

Point estimate methods focusing on the probabilistic enlightenment furnished by concentrations of each random input variable on K points. The first few central moments such as mean, variance, skewness, and kurtosis of a random problem input variable are called concentrations. By using these points and the function, which describe input and output variables, enlightenment about the uncertainty accompanying with problem output random variables can be obtained.

The kth concentration $(\rho_{l,k}, \omega_{l,k})$ of a random variable ρ_l is defined as a pair composed of location $\rho_{l,k}$ and weight $\omega_{l,k}$. The location $\rho_{l,k}$ is the kth value of variable ρ_l at which the function F is evaluated. The weight $\omega_{l,k}$ is a weighting factor, which accounts for the relative importance of this evaluation in the output random variables.

By using Hongs point estimate methods, the function F has to be evaluated only K times for each input random variable ω_l at the K points made up of the kth location $\rho_{l,k}$ of the input random

Table 2 Active power loss reduction in kW for different coalitions

Coalition, S	Losses	Loss reduction ($v^l(S)$)
S ={DG1=0.5 MW}[a]	420.95	19.05
S ={DG2=0.75 MW}[a]	387.95	52.05
S ={DG3=1 MW}[a]	418.1	21.9
S ={DG1=0.5 MW & DG2=0.75 MW }[a]	335	105
S ={DG1=0.5 MW & DG3=1 MW }[a]	378.8	61.2
S ={DG2=0.75 MW & DG3=1 MW }[a]	354.95	85.05
N ={DG1=0.5 MW & DG2=0.75 MW & DG3=1 MW}[b]	271.85	168.15

[a]Represents sub-coalition S.
[b]Represents grand coalition N.

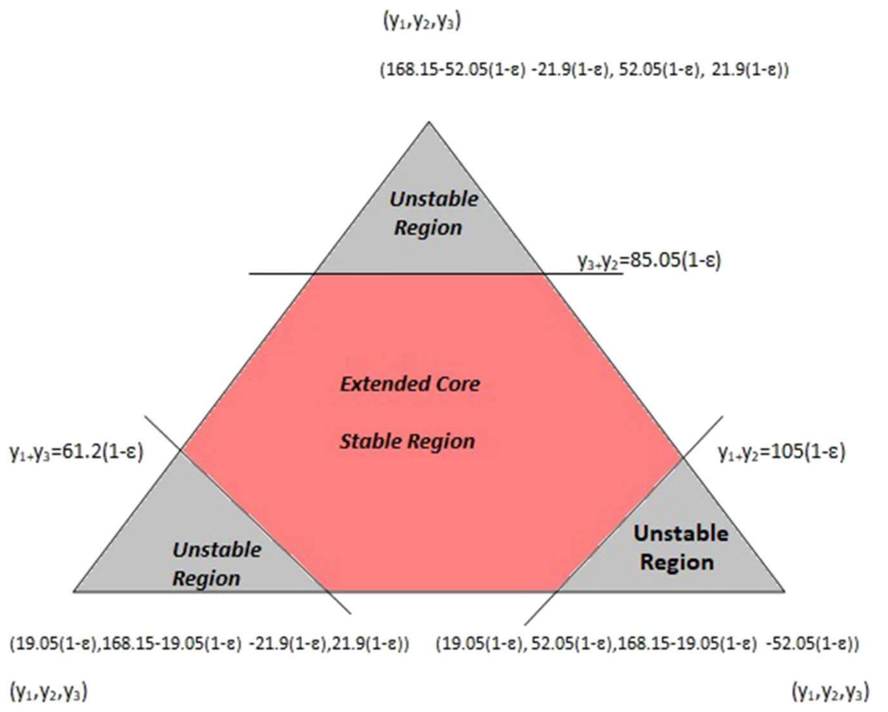

Fig. 1 *Core existence for PNT solution*

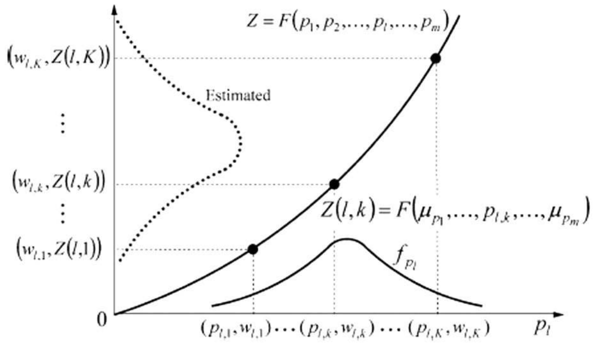

Fig. 2 *Hong's point estimate methods [28]*

Table 3 DG type and location information in the TPC distribution system

Unit	Type	Location	Unit	Type	Location
1	1	4	9	2	20
2	1	65	10	2	47
3	1	25	11	3	11
4	1	35	12	3	60
5	1	84	13	3	41
6	2	55	14	3	30
7	2	12	15	3	76
8	2	72			

Table 4 Fuel cost parameters of each type of DG unit

Type	a_i, \$/MW^{2}h	b_i, \$/MWh	c_i, \$/h
1	5.8	21	0
2	5.3	20	0
3	5.0	20	0

variable ρ_l and the mean (μ) of the $m-1$ remaining input variables, i.e. at the K points (μ_{ρ_1}, $\mu_{\rho_2}, \ldots$, $\mu_{\rho_{1,k}}, \ldots$, μ_{ρ_m}). In other words, the deterministic problem has to be solved K times for each input random variable ρ_1, and the difference among these problems is the deterministic value $\rho_{1,k}$ assigned to ρ_1, while the remaining input random variables are fixed to their corresponding mean. The number of K evaluations to carry out depends on the scheme used. Therefore, the total number of evaluations of F is $K \times m$.

Specific variants or schemes of Hongs point estimate method take into account one more evaluation of function F at the point made up of the m input random variables means (μ_{ρ_1}, $\mu_{\rho_2}, \ldots$, $\mu_{\rho_{1,k}}$, $\ldots\mu_{\rho_m}$). Hence, for these schemes, the total number of evaluations of F is $K \times m + 1$. Hongs point estimate method is described with the aid of figure as shown in Fig. 2.

The K concentrations ($\rho_{l,k}$, $\omega_{l,k}$) of the m input random variables ρ_l are obtained from the probabilistic input data (e.g. the probability density function f_l).

3 Analytical studies

The proposed method as presented in Appendices 1 and 2 has been implemented on the 84 bus Taiwan Power Company (TPC) distribution system. Line and bus data of the TPC distribution system were drawn from [30]. It is assumed that 15 DG units of different types with a capacity of 1 MW having 0.9 lagging power factor is connected to the TPC distribution system as represented in Table 3. Fuel cost parameters of each type of DG unit is presented in Table 4. Realistic average load and market price data at each hour of the day has been drawn from [31] and represented in Fig. 3.

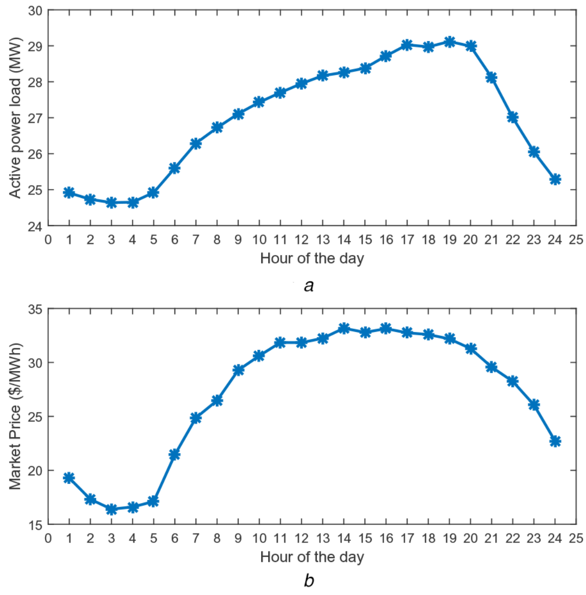

Fig. 3 *Average load and market price curves*
(a) Average load curve [31], ***(b)*** Average market price curve [31]

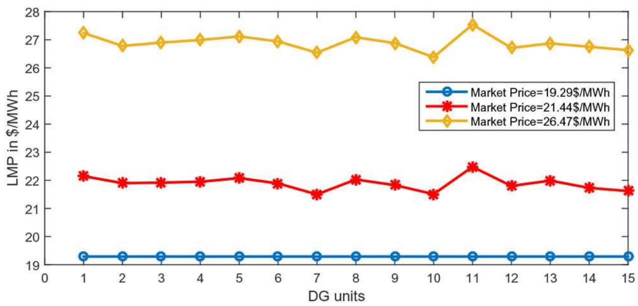

Fig. 4 *DLMP of each DG unit at different market prices*

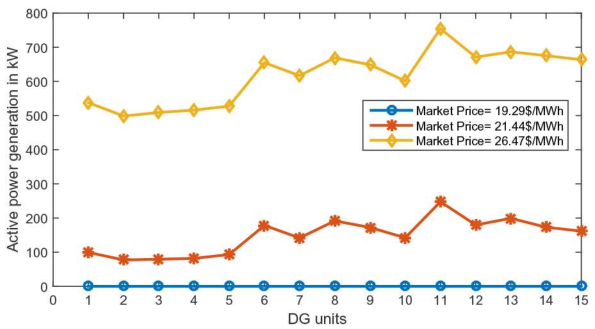

Fig. 5 *Deterministic active power generation (DGEN) of each DG unit at different market prices*

3.1 Deterministic approach

Fig. 4 represents LMP values of each DG unit using the proposed deterministic approach. When market prices are 21.44 and 26.47 \$/MWh then DG owners receive incentives as LMP based on DGs contribution in APLR. DG11 has LMPs 22.48 and 27.54 \$/MWh when market prices are 21.44 and 26.47 \$/MWh, respectively. DG11 receives more incentive as LMP over remaining DG units due to its huge contribution to APLR. In this way, DISCO can maintain fair competition among DG owners by providing proper financial incentives using the proposed method based on unit performance in network operation.

Fig. 5 presents the active power generation of each DG unit at different market prices. If market prices are 21.44 and 26.47 \$/MWh, then active power generation of DG units depends on LMP values the DG units received from the DISCO. The DG units generate the active power such that the DG owners will receive maximum profit at the given LMP. DG11 receives more incentives in terms of LMP due to its huge contribution to APLR. This unit

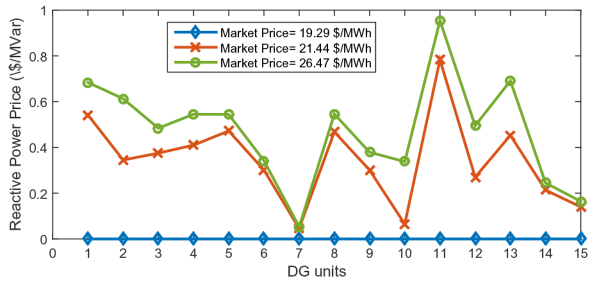

Fig. 6 *Deterministic RPP (DRPP) of each DG unit at different market prices*

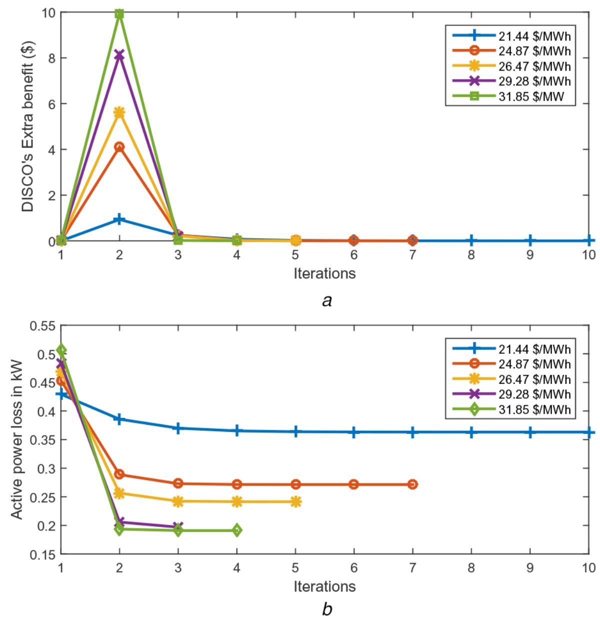

Fig. 7 *Variation of DISCO's extra benefit and APL of the network with respect to epochs in the deterministic approach*
(a) Variation of DISCO's extra benefit with respect to epochs in the deterministic approach, ***(b)*** Variation of the APL of the network with respect to epochs in the deterministic approach

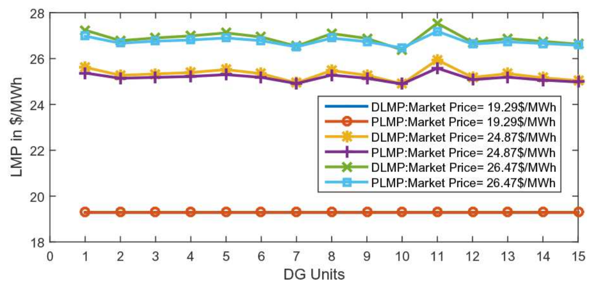

Fig. 8 *PLMP and DLMP of each DG unit at different market prices*

will inject more active power into the network in comparison with the remaining DG units.

Fig. 6 presents RPP of each DG unit at different market prices. The values of RPP of each DG unit at the market price of 21.44 and 26.47 $/MWh depends on the contribution of injected reactive power of that DG unit on APLR. As DG11 has more contribution in APLR, it receives more RPP in comparison with the remaining DG units.

When the market price is 19.29 $/MWh, no incentive has been provided to DGs over market price as shown in Fig. 4. This is due to the inactive state of DG units. The DG units are inactive since '*b*' coefficients of all DG units are more than the market price. Owing to the same reason, no generation from DGs as shown in Fig. 5 and no incentive has been provided to DGs over the market price for reactive power generation as shown in Fig. 6

Fig. 7*a* presents variation of DISCO's extra benefit with respect to epochs. The proposed method provides zero extra benefits at

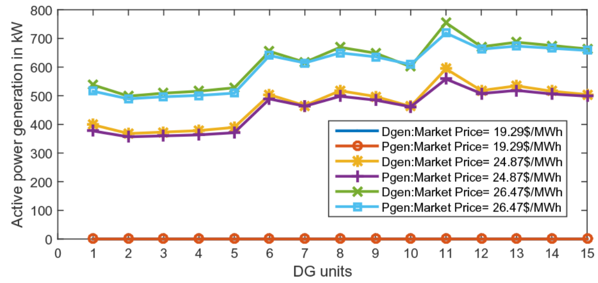

Fig. 9 *PGEN and DGEN of each DG unit at different market prices*

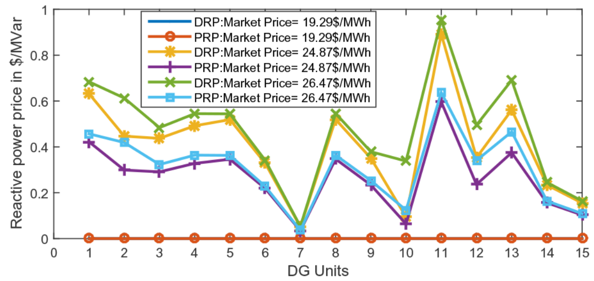

Fig. 10 *PRPP and DRPP of each DG unit at different market prices*

each market price, which is essential to maintain fair competition among DG owners in a deregulated environment. DISCO's extra benefit is defined as the amount of financial benefit DISCO received in the presence of DG units due to the active power loss reduction. This additional benefit is mainly due to the DG units' contribution to loss reduction. This is the additional benefit for the DISCO and it is not related to the profit due to the price gap between the revenue collected from consumers and the energy purchase from the grid and DG units by the DISCO. Fair competition among DG units can be maintained by providing financial incentives to private DG owners based on unit contribution in optimal operation of the distribution network in terms of APLs. So, to encourage the DG owners' participation in loss reduction, the DISCO will provide incentives to DG owners from DISCO's extra benefit until this reaches zero value. Fig. 7*b* presents variation of the APL of the network with respect to epochs. As epochs progress, the APL of the network decreases. This is due to providing incentives to the DG units which have more contribution in APLR.

3.2 Probabilistic approach

Fig. 8 shows probabilistic LMP (PLMP) of each DG unit based on its contribution to APLR. At a market price of 24.87 and 26.47 $/MWh, DG units received incentives as PLMP. As DG11 has more impact on APLR, DG11 receives more incentive over market price.

Fig. 9 shows the probabilistic active power generation (PGEN) of each DG unit. At market prices of 24.87 and 26.47 $/MWh, the values of PGEN depend on how much incentive DG units received in terms of PLMP. DG11 has more PGEN over remaining DG units as it receives more PLMP.

Similarly Fig. 10 shows probabilistic RPP (PRPP) of each DG unit. At market prices of 24.87 and 26.47 $/MWh, the PRPP value of each DG unit is based on DG's reactive power contribution on APLR. As reactive power of DG11 has more contribution to APLR it receives more PRPP over remaining DGs.

If the market price is 19.29 $/MWh and is less than '*b*' coefficients of all DG units, then no DG unit is able to generate the power as there is no incentive provided over market price. Owing to this, both deterministic LMP (DLMP) and PLMP values are the same and equal to the market price of 19.29 $/MWh. In the same way, both PGEN and active power generation with a deterministic approach (DGEN) are equal to zero at a market price of 19.29 $/MWh as shown in Fig. 9. Similarly, the PRPP value of each DG unit is equal to RPP at a substation bus, which is equal to zero.

The variation in probabilistic values such as PLMP, PGEN, and PRPP over deterministic values such as DLMP, DGEN, and DRPP

Table 5 Comparison of deterministic and probabilistic approaches in terms of APL and DISCO's extra benefit

Load, MW	24.91		26.29		26.72	
Market price	19.29, $/MWh		24.87, $/MWh		26.47, $/MWh	
	Deterministic	Probabilistic	Deterministic	Probabilistic	Deterministic	Probabilistic
loss, kW	405.5	363.1	271.4	285.50	241.4	269.5
base loss, kW	405.5	407.2	453.70	455.8	469.3	471.5
extra benefit, $/h	0	0	0	0	0	0

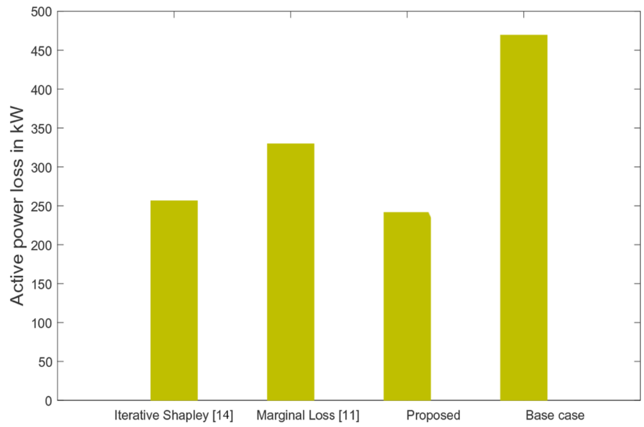

Fig. 11 *Comparison of deterministic approaches for LMP calculation based on APL*

Table 6 Comparison in terms of APL between the proposed method and existing methods in the probabilistic framework

	Active power losses in kW		
Load, MW	26.29	26.72	27.11
market price, $/MWh	24.87	26.47	29.28
proposed method	285.5	269.5	244.6
PSVIM	296.5	279.1	249.3
PMLM	306.4	294.7	277.5
base	453.7	469.3	598.9

is mainly due to the capability of the probabilistic approach in capturing the uncertainty present in the system.

3.3 Probabilistic approach versus deterministic approach

Table 5 presents a comparison between probabilistic and deterministic approaches in terms of APL. APL obtained in the probabilistic approach is different in comparison with the deterministic approach. This is due to the ability of the probabilistic approach to capture the uncertainties that exist in random input variables such as load and market price effectively. This variation shows the necessity of the probabilistic approach to handle the randomness in load and market price. However, both probabilistic and deterministic approaches provide zero DISCO's extra benefit at all market prices.

The base case means the distribution system without considering the DG units. As the load on the network is an uncertain variable, the $2m+1$ scheme of the PEM has been used to calculate base case losses. It is observed that the base case losses are less sensitive to randomness that exists in load on the distribution system

3.4 Comparative studies

Fig. 11 shows the comparison of the proposed deterministic method with existing methods such as marginal loss method (MLM) [8] and Shapley value-based iterative method (SVIM) [11] using load and price data at the eighth hour. The proposed method enables the DISCO to operate the distribution network with less APL in comparison with both MLM and SVIM. The proposed method operates the network with 241.4 kW, whereas SVIM [11] and marginal loss [8] methods operate the network with APLs of 256.4 and 329.6 kW, respectively. However the base case APL is 469.3 kW.

The proposed method has been compared with existing methods such as MLM [8] and SVIM [11] in terms of APL at different market prices in the probabilistic framework. Deterministic approaches MLM and SVIM have been implemented with $2m+1$: PEM to capture uncertainty in load and market price. Observed that the proposed probabilistic method operates the network at less APL in comparison with probabilistic MLM (PMLM) and probabilistic SVIM (PSVIM) at different market prices as shown in Table 6.

The proposed probabilistic approach has been compared with other probabilistic approaches such as MCS and 2 m schemes of the PEM. Table 7 presents comparisons among MCS, $2m$: PEM and $2m+1$: PEM in terms of mean and standard deviation of APL. The MCS method is more accurate and so considered the results of this method as a benchmark. However, MCS takes more computational time and required more data to process. Accuracy of MCS depends on the number of sample (data) points considered. Unlike MCS, the PEM takes very less computational time and less data samples to capture uncertainty in random input variables. $2m+1$: PEM achieves almost the same accuracy as MCS in terms of mean and standard deviation of the APL at less number of simulations. $2m+1$: PEM is more accurate than $2m$: PEM as the former one consider skewness of the input random variable. The main drawback of $2m$: PEM is that locations are directly varying with the number of random input variables.

To verify the accuracy of the proposed method, PDF curves and cumulative distribution function (CDF) curves of the APL obtained from the PEMs are compared with the MCS using load and market price at the 17th hour as shown in Figs. 12 and 13. The PDF curve and CDF curve of the APL obtained from $2m+1$: PEM is following very closely with MCS than $2m$: PEM, which means that

Table 7 Comparison in terms of APL obtained from the proposed method with different probabilistic frameworks

	Active power losses, pu								
Market price	$2m$: PEM [27]			MCS [32]			$2m+1$: PEM [27]		
$/MWh	Mean	Std	Avg. time	Mean	Std	Avg. time	Mean	Std	Avg. time
32.24	0.0081	0.0028	68.03 s	0.0075	0.0017	66029.4 s	0.0074	0.0022	784.28 s
32.75	0.0088	0.0034		0.0080	0.0028		0.0079	0.0028	
32.79	0.0083	0.0031		0.0078	0.0020		0.0075	0.0025	
samples		4			1200			5	

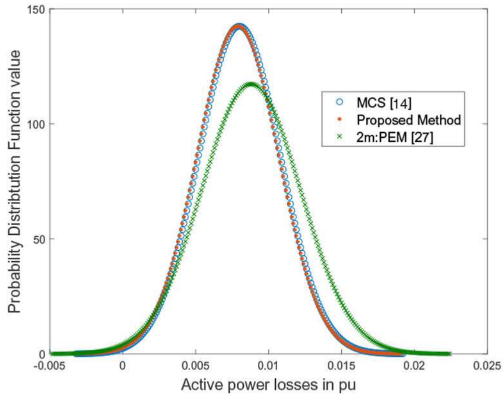

Fig. 12 *PDF curve of APL for different probabilistic approaches*

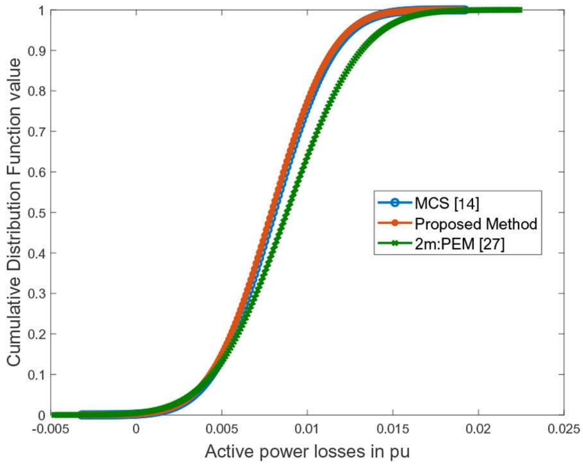

Fig. 13 *CDF curve of APL for different probabilistic approaches*

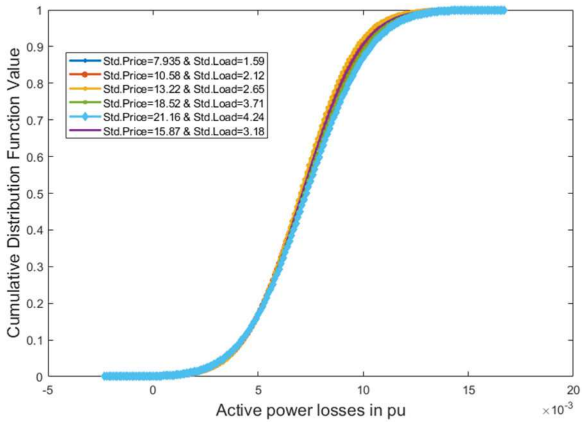

Fig. 14 *CDF curve of APL at different standard deviation values of load and market price*

$2m+1$: PEM is mostly fitting to MCS performance with less time and less sample data. The PDF curve of the APL obtained from $2m$: PEM is away from the MCS but not too far, but in case if the number of random inputs increases the deviation will increase.

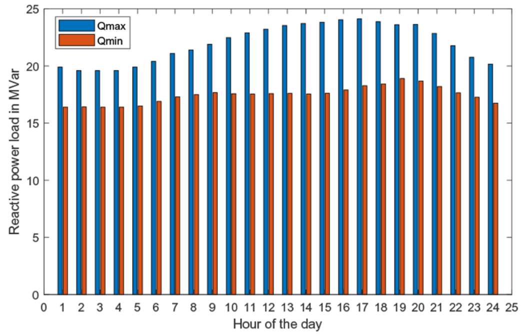

Fig. 15 *Reactive power load variation on test system*

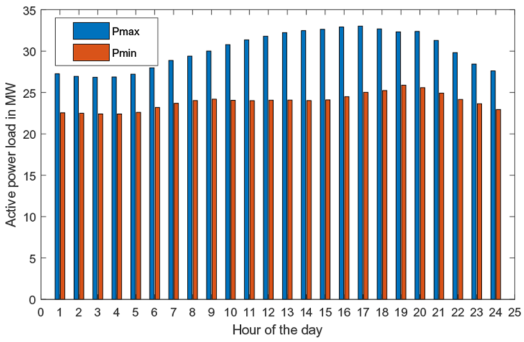

Fig. 16 *Active power load variation on test system*

The sensitivity of the proposed probabilistic approach is verified with respect to the standard deviation of input variables as presented in Fig. 14. A sensitivity study has been implemented based on load and price data at the 11th hour. The standard deviation of both load and market price is varying from 0.75 to 2 times of actual standard deviation in steps of 0.25. Simulated the proposed method at all different standard deviations and drawn CDF of APL. The output of the sensitivity analysis is the cumulative density function of APL. It has been observed from Fig. 14 that changing the standard deviation of the random parameters has very little effect on the estimated value of APL.

The TPC distribution test system having 15 DG units with each DG's capacity as 1 MW operating at 0.9 power factor lagging. Maximum generating active and reactive power by all DG units is equal to 15 MW and 7.26 MVar, respectively. However, the reactive power load on the test system is varying between 24.12 and 16.4 MVar as shown in Fig. 15. Similarly, the active power load on the test system is varying between 33.03 and 22.41 MW as shown in Fig. 16. From the above analysis, it has been observed that only DG units cannot supply the required active and reactive power demand. The deficiency in the active and reactive power can be supplied from the grid to maintain active and reactive power balance. Hence, DG units, which are operating at constant power factor, are not creating any power imbalance on the test system.

4 Conclusion

The proposed iterative algorithm with deterministic and probabilistic approaches was implemented on the TPC system. Both deterministic and probabilistic approaches are performing well over MLM and SVIM in terms of APL. The proposed method with a probabilistic framework $2m+1$: PEM performs with an almost similar accuracy of the MCS but with less computational time. The proposed method with the $2m+1$: PEM scheme performs well over the $2m$: PEM scheme by taking only one additional deterministic evaluation.

As the proposed method was developed based on uncertainties in load and market price, this method can be helpful to DISCO to provide financial incentives to DG owners, which are less sensitive to the uncertainty that exist in market price and load. The RPP of each DG unit was computed based on the actual contribution of that DG's reactive power on the APL of the network using sensitivity factors. Financial incentives to DGs have been allocated from the financial savings of the DISCO due to the APLR. PNT has been used for the calculation of the LMP at DG buses due to its superiority over other cooperative game theory concepts.

As the integration of DGs into the distribution system is increasing rapidly, this work can be helpful to the DISCO for efficient operation and maintenance of the distribution network. The proposed probabilistic approach for computing LMP at DG buses can be extended by considering emissions along with APL.

This work can be further extended with renewable energy sources such as wind power and photovoltaic power generation. As renewable energy sources are uncertain in nature, forecasting of wind power and photovoltaic power generation needs to be done before applying the proposed methodology. The forecasted electric power from renewable energy sources can be considered as the size of the DG unit at that particular instant.

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6 Appendices

6.1 Appendix 1: deterministic approach

In this study, the PNT-based iterative method was designed to compute LMP for each DG unit in the distribution system based on APLR. It was considered as the deterministic approach by assuming that load and market price does not change. The procedure for computing LMP, RPP, APL, and DISCO's additional benefit using the deterministic approach is presented in Fig. 17.

6.2 Appendix 2: probabilistic approach

In the probabilistic approach, both inputs load and market price were considered as random variables. The PNT-based iterative method and $2m+1$: PEM were hybridised to compute LMP, RPP, APL, DISCO's extra benefit and also to capture uncertainty in load and market price simultaneously. The complete procedure for the probabilistic approach is presented in Fig. 18.

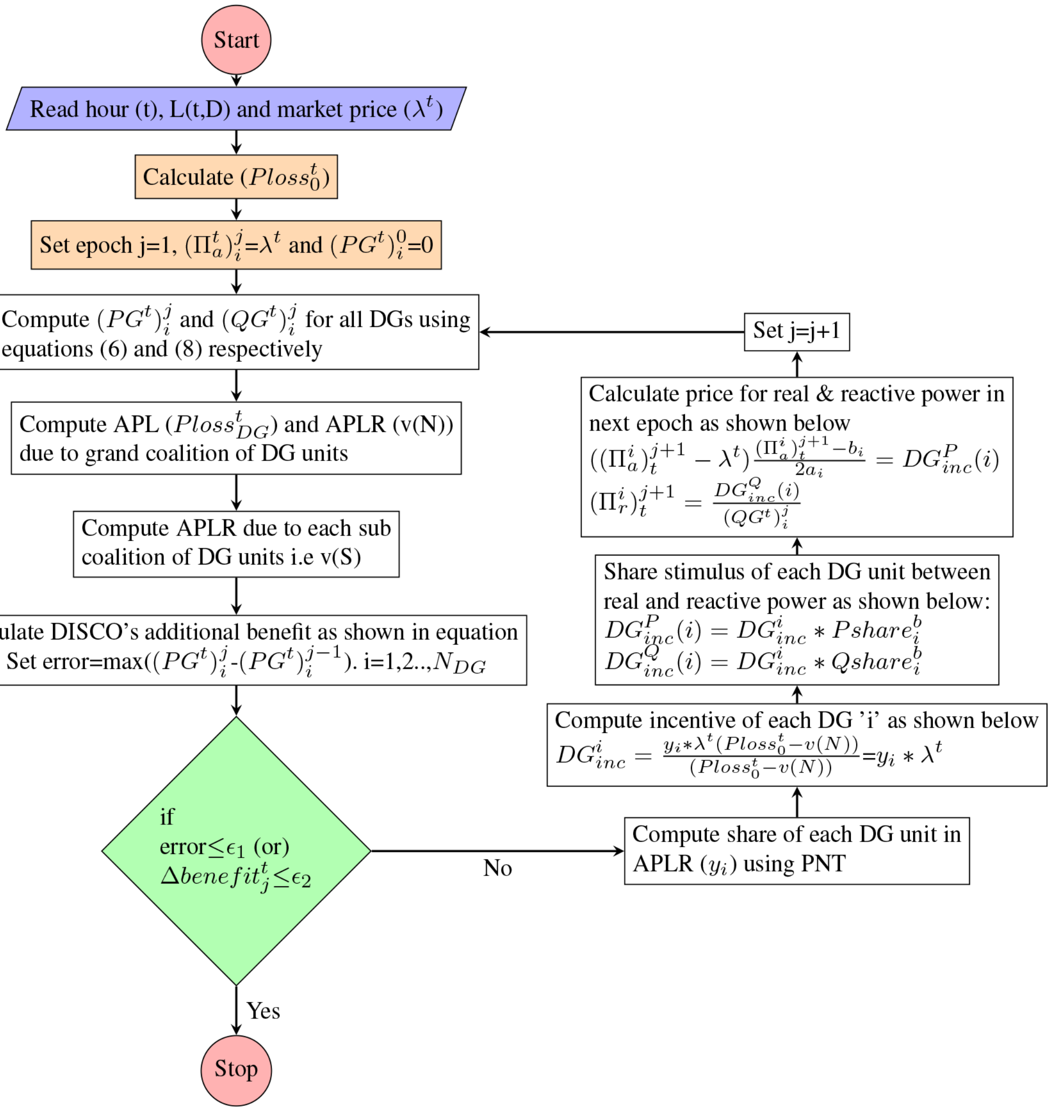

Fig. 17 *Deterministic approach for computation of LMP, RPP, APL, and DISCO's extra benefit*

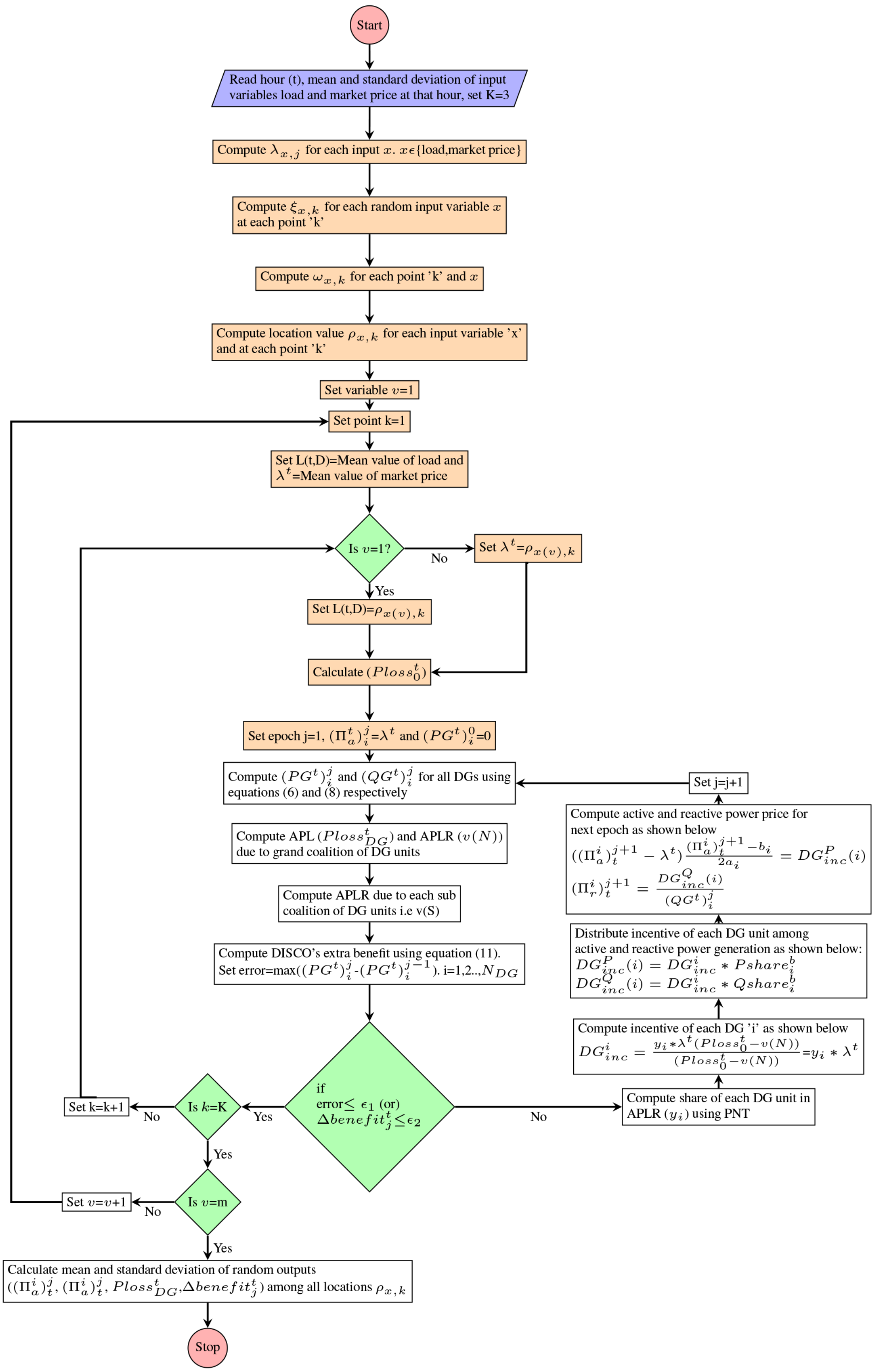

Fig. 18 *Probabilistic approach for calculation of LMP, RPP, APL, and DISCO's extra benefit*