Fast Identification of Ancillary Service Locations for Improving Hopf Bifurcation Margin Under Uncertainties

Bevin K C
Centre for Energy Studies
Indian Institute of Technology, Delhi
New Delhi, India
Email: bevinkc@gmail.com

Ram Krishan
Department of Electrical Engineering
National Institute of Technology, Warangal
Telangana, India
Email: rkrishan@nitw.ac.in

Ashu Verma*
Centre for Energy Studies
Indian Institute of Technology, Delhi
New Delhi, India
Email: averma@ces.iitd.ac.in

***Abstract*—Increasing penetration of renewable energy is posing challenges to stable operation of the power system. An important stability consideration is the small signal stability, which is characterized by Hopf bifurcation. The system operator can use ancillary services to maintain sufficient Hopf bifurcation margin to ensure stability. This work proposes a method to identify the optimum locations for ancillary services to improve Hopf bifurcation margin. The forecast uncertainties related to load and renewable based generation has been handled using boundary power flow based eigenvalue analysis. Simulation studies were performed on WSCC 9 bus system and New England 39 bus system. The accuracy of proposed method is proved by comparing the result with Monte-Carlo method.**

***Index Terms*—Hopf bifurcation, boundary power flow, ancillary services, eigenvalue sensitivity.**

I. Introduction

Power system operation has increased in complexity in recent times due to uncertainties in generation and load forecast [1]–[3]. Thus operating the system securely is a major challenge for the system operator. The system operator need to ensure sufficient margins for safe operation. Since power system operational conditions are continuously changing, dynamic stability of power system need to be considered. One method of finding this dynamic stability limit is by Hopf bifurcation method [4]–[6]. This method uses eigenvalues of system matrix. As power system network is progressively loaded, system may loose stability when the eigenvalues of state matrix crosses over to right half of s-plane. The additional loading which can be given until system looses stability is termed as Hopf bifurcation margin and this gives a measure of stability of power system. When the margins are stressed due to uncertainties, the system operator needs ancillary services to keep system secure.

The Indian Electricity Grid Code [7] defines ancillary services as "the services necessary to support the power system operation in maintaining power quality, reliability and security of the grid, eg. active power support for load following, reactive power support, black start, etc;". Authors in [8] and [9] have shown ancillary services will be inevitable for supporting large solar and wind power plants in future when renewable energy penetration further increases. The location of providing such ancillary services in the grid will have a large impact on the effectiveness of such services. In the previous works, location of ancillary services were found mainly based on loss reduction, economic operation and voltage support [10]–[12]. There is less focus on ancillary service placement for improving small signal stability. Previous works related to improving Hopf bifurcation margin focused on control actions like reactive power compensation, tap changing transformers, load shedding and adjusting set points of automatic voltage regulators and governors [13]–[16]. Improving Hopf bifurcation margin by finding optimal location of ancillary service is not yet attempted as per the knowledge of authors.

For the practical operation of power system, the uncertainties in loads and generation should also be taken into account. With higher penetration of weather dependent renewable energy resources in power system, uncertainties become predominant [1]–[3]. Uncertainties can be classified into two types- statistical and non-statistical. Stability analysis with statistical uncertainties are carried out in [17]–[19]. This require historical data which may not be available with required accuracy. Non statistical uncertainties have the advantage of not depending on any historical data to give accurate estimates. Robust control for stability analysis is used in [20]–[23], but it is computationally cumbersome and time consuming. An alternative approach is boundary power flow method, first proposed in [24] and later modified in [25]. This method gives the range of values of state variables for a particular range of uncertainty in input variables. This method uses sensitivity values which is calculated anyway during Newton Raphson load flow iterations. So this method is very fast and is used to handle the uncertainties in generation and load in the present work.

Previous studies have indicated that Hopf bifurcation margin is reducing when uncertainties are increased [26]. This increases the threat of instability. In order to mitigate this problem, our work focuses on how to improve the Hopf bifurcation margin in an uncertain scenario by finding the optimal location for providing ancillary service resources.

978-1-7281-6664-3/20/ $31.00 2020 IEEE

The main contribution of this paper is to propose a method to help system operator identify the best locations for ancillary service resources for improving Hopf bifurcation margin. The concept of boundary eigenvalue sensitivities is used to handle uncertainties in generation and load.

This paper is organized as follows. Section II gives the theoretical background of modeling the system, Hopf bifurcation and uncertainty handling using boundary load flow. In Section III, a method to find optimal location of ancillary services to improve Hopf bifurcation margin is proposed. Section IV shows the application of proposed method on 9 bus WSCC and 39 bus New England test systems. Finally, Section V gives the conclusion.

II. Background

This section gives details about modeling of power system, uncertainty handling by boundary power flow method and the concept of hopf bifurcation.

A. Modeling of power system

The non linear model of power system for dynamic analysis is given by the following differential algebraic equations [27], [28]:

$$\dot{x} = f(x, y, u) \tag{1}$$

$$0 = g(x, y) \tag{2}$$

For small signal analysis, (1) and (2) are linearized around the operating point and it can be written in matrix form as follows:

$$\begin{bmatrix} \Delta\dot{x} \\ 0 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} + E[\Delta u] \tag{3}$$

If D is non singular, the equation (3) can be reduced as:

$$\Delta\dot{x} = A_{sys}\Delta x + E\Delta u \tag{4}$$

where A_{sys} is the reduced system matrix given by:

$$A_{sys} = A - BD^{-1}C \tag{5}$$

The eigenvalues (σ) of the system can be calculated by solving:

$$|A_{sys} - \sigma I| = 0 \tag{6}$$

Loads are modeled as constant power loads. Since renewable energy sources are non dispatchable, they are assumed as negative loads. Therefore, uncertainties in prediction of renewable generation is also reflected in uncertainties in negative loads.

B. Boundary Power Flow for handling uncertainties

The boundary power flow method is essentially a variation of Newton Raphson method of load flow where uncertainties in input variables are also taken into account. The load flow problem is given by the following non-linear equation

$$L = f(W) \tag{7}$$

where,
W =vector of state variables like voltage magnitudes and angles at a bus
L = vector of input variables like real and reactive power injections to a bus

The boundary power flow method gives the extreme values of state variables for a given range of input variables.

$$W = f^{-1}(L) \tag{8}$$

Initial values for state variable W_0 is evaluated from the result of converged load flow solution with input variables L_0. The extreme values of W can be found by linearizing (8) around (W_0, L_0)

$$W = W_0 + M.(L - L_0) \tag{9}$$

M is the sensitivity of W with respect to L, evaluated at W_0. M will be same as the inverse of load flow Jacobian matrix. Each element of (L) can take either minimum value or maximum value among the range of input variables characterized by the uncertainties.

Each state variable (w_i) is given by

$$w_i = w_{0i} + \sum_{j=1}^{n} m_{ij}.(l_j - l_{0j}) \tag{10}$$

where, n is the number of state variables and m_{ij} are the elements of sensitivity matrix M. l_j can take either maximum value ($l_{j,max}$) or minimum value ($l_{j,min}$). The choice of value for l_j is made based on the sign of m_{ij} as per the criteria below.

If minimum value of w_i is desired,

$$l_j = \begin{cases} l_{j,min}, & \text{if } m_{ij} \geq 0 \\ l_{j,max} & \text{if } m_{ij} < 0 \end{cases} \tag{11}$$

If maximum value of w_i is desired,

$$l_j = \begin{cases} l_{j,max}, & \text{if } m_{ij} \geq 0 \\ l_{j,min} & \text{if } m_{ij} < 0 \end{cases} \tag{12}$$

The solution of (9) may not satisfy (7) readily. So these two equations are solved iteratively till W is converged.

C. Hopf bifurcation

It is observed that, for certain operating conditions, power system looses stability even before the saddle node point for voltage instability is reached [4], [29]. The reason of this phenomenon is Hopf bifurcation. Hopf bifurcation can be identified by observing the eigenvalues of A_{sys} matrix (from (6)).

For a system to be asymptotically stable, all the eigenvalues of system matrix should be in the left half of s-plane. When the operating conditions change, say increase in loading, the eigenvalues of the system also changes. For a specific condition, a pair of complex eigenvalues reaches the imaginary axis and start to crossover to the right half of s-plane. This condition where the system has a simple pair of purely imaginary eigenvalues is called Hopf bifurcation [5].

In this study, the loading factor (λ) is varied to cause Hopf bifurcation. The loading factor which leads the system to Hopf

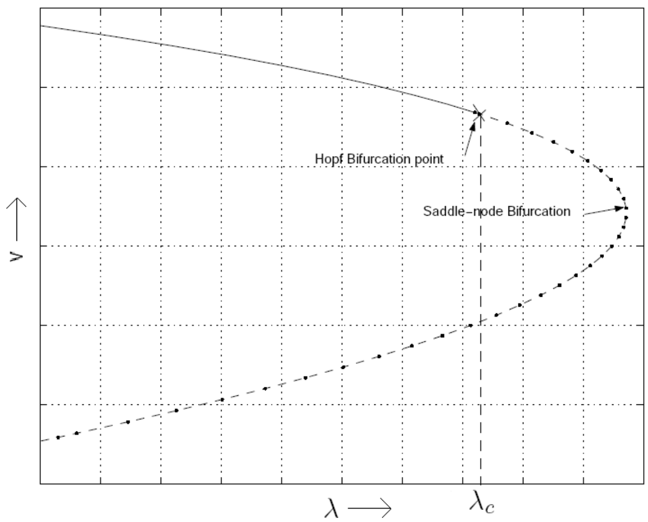

Fig. 1. Hopf bifurcation and saddle node bifurcation in power system

bifurcation is called critical loading(λ_c) and the corresponding eigenvalue which crosses over to right side of s-plane is called critical eigenvalue(σ_c). The amount by which loading can be increased till Hopf bifurcation is reached is termed as Hopf bifurcation margin (λ_m).

$$\lambda_m = \lambda_c - \lambda \tag{13}$$

This Hopf bifurcation margin serves as an important index for the stability of power system.

D. Effect of uncertainties on Hopf bifurcation margin

The critical loading for Hopf bifurcation, λ_c is evaluated at different levels of uncertainties through simulation studies on 9 bus WSCC system and 39 bus New England test system [30], [31]. The critical loading for Hopf bifurcation under various uncertainty scenarios are given in Table I.

TABLE I
CRITICAL LOADING FOR HOPF BIFURCATION

Condition	Critical loading for Hopf bifurcation (λ_c)	
	9 bus WSCC	39 bus New England
Crisp (No uncertainty)	0.87	0.256
3% uncertainty	0.817	0.178
5% uncertainty	0.78	0.156
10% uncertainty	0.7	0.103

From the Table I, it is clear that λ_c is reducing as uncertainties are increased. This in-turn reduces Hopf bifurcation margin, λ_m and makes the system more prone to small signal stability issues.

E. Boundary Eigenvalue Sensitivity

This section gives details of boundary eigenvalue analysis which was proposed in [26]. When a system is progressively loaded, the eigenvalues of the system calculated by (6) starts to cross over to right half of s-plane. The eigenvalue which cross over to right half first and lead to Hopf bifurcation is considered as the critical eigenvalue(σ_c). The sensitivity of real part of critical eigenvalue with loading is found by perturbation method.

$$s_j = \frac{\partial \; real(\sigma_c)}{\partial P_L} \tag{14}$$

For studying the effect of load uncertainties on eigenvalues, the choice of l_j in (10) is made based on sign of s_j instead of m_{ij} earlier. The maximum condition of boundary power flow is taken here since it gives the worst case scenario. The criteria for finding l_j is as follows.

$$l_j = \begin{cases} l_{j,max}, & \text{if } s_j \geq 0 \\ l_{j,min} & \text{if } s_j < 0 \end{cases} \tag{15}$$

Rest of the algorithm is similar to boundary power flow method given in Section II-B.

III. METHODOLOGY

This section provides the algorithms for identification of ancillary service locations to improve the Hopf bifurcation margin. The objective of algorithm is to provide enough ancillary service to improve the critical loading for Hopf bifurcation to a set target value of critical loading, λ_t. The algorithm should be able to handle uncertainties in load and generation forecast too.

The concept of boundary eigenvalue sensitivities is used to achieve this above objective. In this study λ_t is chosen as the crisp condition value given in first row of Table I. The algorithm shall work for any other λ_t as well according to the needs of the system operator. The target loading should be achieved with minimum usage of additional resources. The ancillary service can be provided from existing generators, battery banks, diesel generators, demand response or load curtailment. They are modeled as negative loads at the buses they are connected.

A. Proposed method for identifying optimal locations for providing ancillary services

The method is based on the concept of boundary eigenvalue sensitivities explained in Section II-E. The buses are sorted in the decreasing order of boundary eigenvalue sensitivities. The most sensitive buses have a higher priority for providing ancillary services to improve Hopf bifurcation margin faster. The optimization problem formulation is given below.

Min $\sum_{i=1}^{n_b} P_{anc}(i)$

Subject to the following constraints:

1. $\lambda_c \geq \lambda_t$
2. $P_{anc}(i) \leq P_{anc\_max}$

where, $P_{anc}(i)$ is the real power supplied by ancillary services at i^{th} bus, n_b is total number of buses and P_{anc_max} is the maximum capacity of ancillary service at a bus.

The algorithm is given below.

The maximum amount of ancillary service to be allocated per bus is fixed so that transmission lines don't get overloaded. In this study, P_{anc_max} is taken as 0.063 p.u. for 9 bus system and 1.2 p.u. for the 39 bus system.

Algorithm 1: Proposed method to identify best locations for providing ancillary service

1 Initialize the variables, $\lambda = \lambda_c = P_{anc} = 0$.
Set the percentage uncertainty in loads and λ_t values.
2 Sort the buses in decreasing order of eigenvalue sensitivity wrt loading. Initialize i= bus number with highest sensitivity value.
3 If $\lambda_c \geq \lambda_t$, the algorithm **STOPS** and print results.
Else, continue.
4 If $P_{anc}(i) \geq P_{anc\_max}$, fix $P_{anc}(i) = P_{anc\_max}$ and update i with the next most sensitive bus number.
Else, $P_{anc}(i) = +0.01 p.u.$
5 Run boundary load flow with the current set of values for P_{anc} and λ. Find the critical eigenvalue, σ_c.
6 If $real(\sigma_c) > 0$, continue.
Else, $\lambda = +1\%$. Go to Step 5.
7 Set $\lambda_c = \lambda$. Go to Step 3.

B. Monte Carlo Method for verification of results

In Monte Carlo method, the values of $P_{anc}(i)$ are obtained by random sampling. The accuracy of this method increases with number of samples taken. Therefore, to obtain high accuracy, large number of samples need to be taken. Thus the method is time consuming, but it can be used to verify the results of eigenvalue sensitivity methods. This method is used to solve the same optimization as given in the previous section.

IV. Results

Simulation studies were done using MATLAB software on 9 bus WSCC system and 39 bus New England test system [30], [31].

A. Eigenvalue sensitivities

The eigenvalue sensitivities are calculated by applying a perturbation of 0.001 pu to the loads at various buses. For the 9 bus system, the most eigenvalue sensitive buses were found to be bus 5 and bus 6 with eigenvalue sensitivities 1.144 and 0.9173 respectively. The eigenvalue sensitivities at all buses for 9 bus system and 39 bus system are given are shown in Fig. 2 and 3 respectively.

B. Ancillary service location by proposed method

The algorithm explained in section III-A is implemented on the test systems by MATLAB code. The ancillary service required at each location to improve critical hopf bifurcation to λ_t are given in Fig. 4 and Fig. 5.

C. Verification of results by Monte Carlo method

Monte Carlo simulation was carried out with same conditions as the eigenvalue sensitivity methods in Section IV-B. The minimum generation addition to improve critical Hopf bifurcation to the target value is obtained for different uncertainty levels.

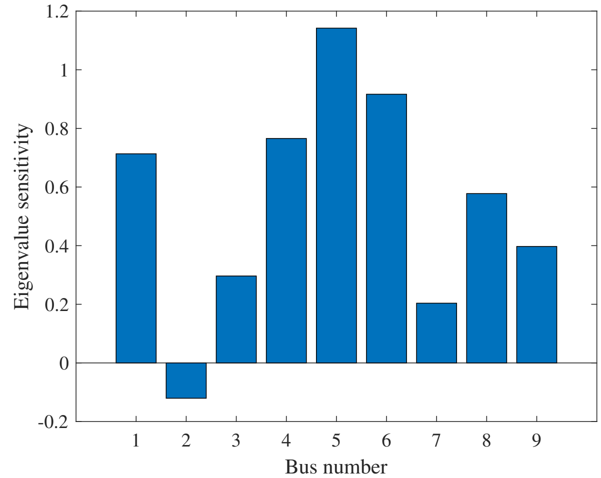

Fig. 2. Eigenvalue sensitivities for 9 bus system at critical loading

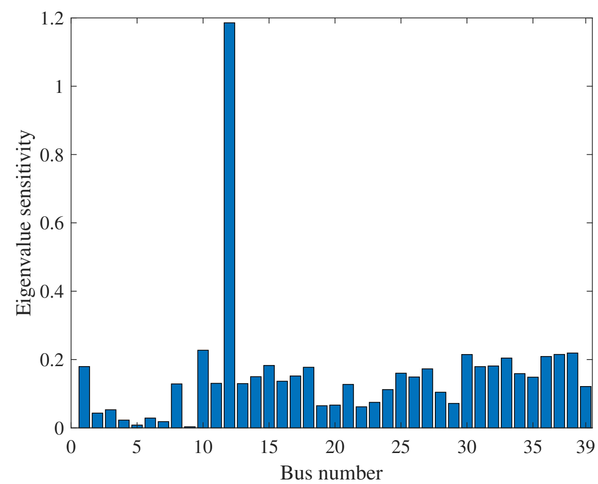

Fig. 3. Eigenvalue sensitivities for 39 bus system at critical loading

TABLE II
Total ancillary service power required to improve λ_c to λ_t by Eigenvalue Sensitivity method and Monte Carlo method

Uncertainty	WSCC 9 bus system (p.u.)		New England 39 bus (p.u.)	
(%)	Eigenvalue	Monte Carlo	Eigenvalue	Monte Carlo
3	0.071	0.078	2.2	2.4
5	0.129	0.142	3.2	3.6
10	0.356	0.378	5.7	6

A comparison of results obtained by eigenvalue sensitivity method and Monte Carlo method is given in Table II. It is clear that eigenvalue sensitivity method is able to give result close to Monte Carlo method. Also, in most cases the eigenvalue sensitivity method is able to achieve the target value of Hopf bifurcation margin with slightly lesser amount of total ancillary service power requirement. Thus eigenvalue sensitivity method is able to get solution more closer to the global optimum.

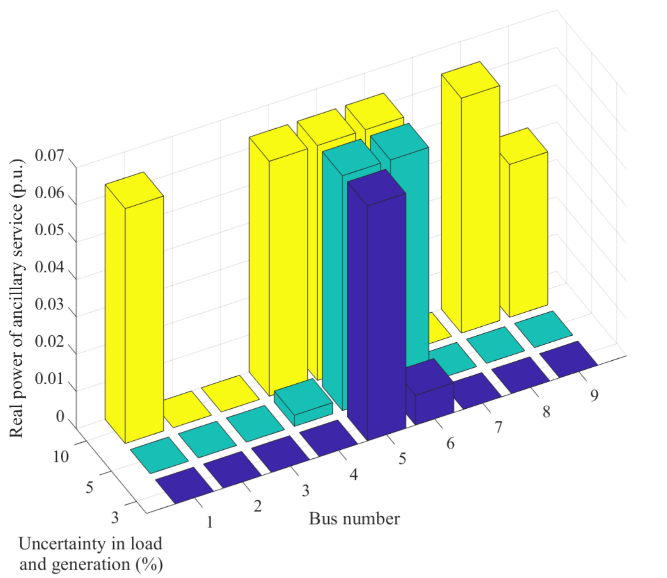

Fig. 4. Generation addition in p.u. at each bus after running Eigenvalue sensitivity algorithm on 9 bus system

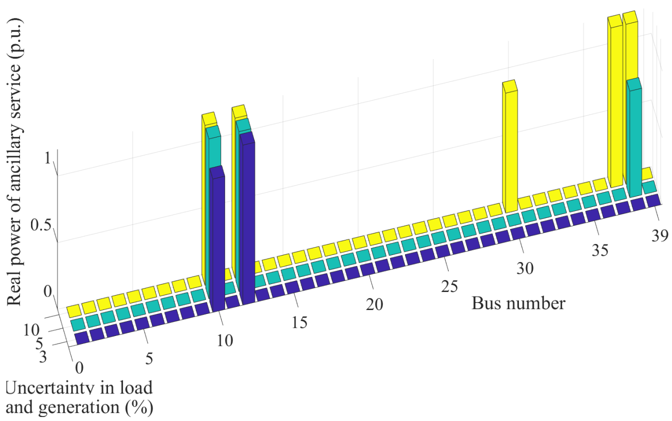

Fig. 5. Generation addition in p.u. at each bus after running Eigenvalue sensitivity algorithm on 39 bus system

TABLE III
COMPARISON OF AVERAGE COMPUTATION TIME FOR EIGENVALUE SENSITIVITY METHOD AND MONTE CARLO METHOD

Condition	Proposed method	Monte Carlo method
9 bus system	2 seconds	50 minutes
39 bus system	2 minutes	60 hours

The average computational time for both boundary eigenvalue sensitivity and Monte Carlo method is shown in Table III. It is clear that proposed method can provide results much faster than meta-heuristic methods like Monte Carlo.

V. CONCLUSION

This paper proposes a method to find optimal location for providing ancillary service for improving the Hopf bifurcation margin under uncertainties. The results show that providing enough reserves at the critical buses (identified by eigenvalue sensitivities) can greatly improve the Hopf bifurcation margin. The accuracy of the algorithm is verified by Monte-Carlo method. The algorithms presented in this paper can help system operator take preventive action to improve power system stability. This analysis can provide system operator vital information on the best locations for applying ancillary services to keep sufficient reserves for mitigating Hopf bifurcation. The method uses eigenvalue sensitivities, which can be easily calculated from boundary power flow. So no additional computation burden is incurred in finding these sensitivities. This method is fast and can be used for generator scheduling in real-time.

REFERENCES

[1] Y. Wang, N. Zhang, C. Kang, M. Miao, R. Shi, and Q. Xia, "An efficient approach to power system uncertainty analysis with high-dimensional dependencies," *IEEE Transactions on Power Systems*, vol. 33, no. 3, pp. 2984–2994, May 2018.

[2] A. Rabiee, S. Nikkhah, A. Soroudi, and E. Hooshmand, "Information gap decision theory for voltage stability constrained opf considering the uncertainty of multiple wind farms," *IET Renewable Power Generation*, vol. 11, no. 5, pp. 585–592, 2017.

[3] P. Andrianesis and G. Liberopoulos, "The "hidden cost" of renewable energy sources in electricity pool markets," in *2012 9th International Conference on the European Energy Market*, May 2012, pp. 1–8.

[4] V. Ajjarapu and B. Lee, "Bifurcation theory and its application to nonlinear dynamical phenomena in an electrical power system," *IEEE Transactions on Power Systems*, vol. 7, no. 1, pp. 424–431, Feb 1992.

[5] N. Mithulananthan, C. A. Canizares, and J. Reeve, "Indices to detect hopf bifurcations in power systems," in *North American Power Symposium*, 2000.

[6] M. Jazaeri and M. Khatibi, "A study on hopf bifurcations for power system stability analysis," in *2008 IEEE Canada Electric Power Conference*, Oct 2008, pp. 1–6.

[7] Central Electricity Regulatory Commission, "Indian electricity grid code," 2010.

[8] B. Crciun, T. Kerekes, D. Sra, and R. Teodorescu, "Frequency support functions in large pv power plants with active power reserves," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 2, no. 4, pp. 849–858, Dec 2014.

[9] G. Revel, A. E. Leon, D. M. Alonso, and J. L. Moiola, "Dynamics and stability analysis of a power system with a pmsg-based wind farm performing ancillary services," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 61, no. 7, July 2014.

[10] Y. K. SUN, Q. DING, H. XING, P. P. ZENG, and F. F. SUN, "Optimal placement and sizing of grid-scale energy storage in distribution networks with security constrains," in *2018 International Conference on Power System Technology (POWERCON)*, Nov 2018, pp. 1477–1482.

[11] D. Asija, K. M. Soni, S. K. Sinha, and V. K. Yadav, "Optimal placement of distributed generator for maximun load allowability and reduced losses mitigating congestion of transmission system," in *2016 7th India International Conference on Power Electronics (IICPE)*, Nov 2016, pp. 1–6.

[12] M. Nick, M. Hohmann, R. Cherkaoui, and M. Paolone, "On the optimal placement of distributed storage systems for voltage control in active distribution networks," in *2012 3rd IEEE PES Innovative Smart Grid Technologies Europe (ISGT Europe)*, Oct 2012, pp. 1–6.

[13] A. A. P. Lerm and A. S. Silva, "Avoiding hopf bifurcations in power systems via set-points tuning," *IEEE Transactions on Power Systems*, vol. 19, no. 2, pp. 1076–1084, May 2004.

[14] T. Parreiras, S. Gomes, G. Taranto, and K. Uhlen, "Closest security boundary for improving oscillation damping through generation redispatch using eigenvalue sensitivities," *Electric Power Systems Research*, vol. 160, pp. 119 – 127, 2018.

[15] S. Wang, Y. Yi, Q. Jiang, X. Chen, and Y. Cao, "On-line control of hopf bifurcations in power systems," in *2008 IEEE Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century*, July 2008, pp. 1–7.

[16] A. I. Zecevic and D. M. Miljkovic, “The effects of generation redispatch on hopf bifurcations in electric power systems,” *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 49, no. 8, pp. 1180–1186, Aug 2002.

[17] F. Capitanescu, S. Fliscounakis, P. Panciatici, and L. Wehenkel, “Cautious operation planning under uncertainties,” *IEEE Transactions on Power Systems*, vol. 27, no. 4, pp. 1859–1869, Nov 2012.

[18] C. Peng, S. Lei, Y. Hou, and F. Wu, “Uncertainty management in power system operation,” *CSEE Journal of Power and Energy Systems*, vol. 1, no. 1, pp. 28–35, March 2015.

[19] L. Roald, S. Misra, T. Krause, and G. Andersson, “Corrective control to handle forecast uncertainty: A chance constrained optimal power flow,” *IEEE Transactions on Power Systems*, vol. 32, no. 2, pp. 1626–1637, March 2017.

[20] N. Yorino, M. Abdillah, Y. Sasaki, and Y. Zoka, “Robust power system security assessment under uncertainties using bi-level optimization,” *IEEE Transactions on Power Systems*, vol. 33, no. 1, pp. 352–362, Jan 2018.

[21] W. Grote, D. Kastsian, and M. Mnnigmann, “Guaranteed stability in optimal power generation dispatching under uncertainty,” *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 1103–1112, May 2013.

[22] Youyi Wang, D. J. Hill, and Guoxiao Guo, “Robust decentralized control for multimachine power systems,” *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 45, no. 3, pp. 271–279, March 1998.

[23] Hui Ni, G. T. Heydt, and L. Mili, “Power system stability agents using robust wide area control,” *IEEE Transactions on Power Systems*, vol. 17, no. 4, pp. 1123–1131, Nov 2002.

[24] R. N. Allan and A. M. L. da Silva, “Probabilistic load flow using multilinearisations,” *IEE Proceedings C - Generation, Transmission and Distribution*, vol. 128, no. 5, pp. 280–287, Sep. 1981.

[25] A. Dimitrovski and K. Tomsovic, “Boundary load flow solutions,” *IEEE Transactions on Power Systems*, vol. 19, no. 1, pp. 348–355, Feb 2004.

[26] R. Krishan, A. Verma, S. Mishra, and P. R. Bijwe, “Analysis of hopf bifurcation with forecast uncertainties in load/generation,” *IET Generation, Transmission Distribution*, vol. 11, no. 6, pp. 1531–1538, 2017.

[27] P. Sauer, M. Pai, and J. Chow, *Power System Dynamics and Stability: With Synchrophasor Measurement and Power System Toolbox*. Wiley, 2017.

[28] P. Kundur, N. Balu, and M. Lauby, *Power System Stability and Control*. McGraw-Hill Education, 1994.

[29] C. A. Canizares, “On bifurcations, voltage collapse and load modeling,” *IEEE Transactions on Power Systems*, vol. 10, no. 1, pp. 512–522, Feb 1995.

[30] University of washington power systems test case archive. [Online]. Available: http://labs.ece.uw.edu/pstca/pf14/pg_tca14bus.htm

[31] I. Hiskens, “IEEE PES task force on benchmark systems for stability controls,” Nov 2013.