

# Mathematical Modelling and Analysis of High Voltage High Pulse Power Supply Performance on Various Loads

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**Abstract**— This paper presents the mathematical modeling and analysis of High voltage high pulse power system (HVHPPS) performance on various type of load like magnetron, klystron, particle accelerator etc.. The performance of high voltage high pulse power supply changes with the change in the load parameters i.e. dynamic impedance, cable length, temperature variation, cavity dimension change, applied cathode voltage, magnetic strength etc. The high voltage high pulse power supply for RF and Microwave types of load requires always a very good impedance matching, to achieve maximum power transmission and minimum reflection. In this paper the mathematical modeling of subsystem of High Voltage High Pulse Power System and load are generated. Accordingly the load behavior changes incorporated and analyzed the effect at the source side based on hardware results obtained years together. The mathematical modeling analysis result is validated through the experimental hardware model..

**Keywords**— *dynamic impedance, high voltage high pulse power supply, impedance mismatch*

## I. INTRODUCTION AND BACKGROUND

This high voltage high pulsed power is utilized for pulsed load applications i.e. magnetron, klystron accelerators etc. the performance will be better if pulse to pulse shaping and load matching observed. The utilization is divided in two categories a first category is of nuclear model and particle accelerators (pulsed power). The second categories are (high power modulators and pulser) discovered from pulsed radar[1][4]. High Voltage High Pulse Power System comprises of energy into nano to millisecond duration but intensive peak power as single pulse (with short rise and fall time, pulse width varies from Nano second to few micro second) at a particular repetitive rate. Pulse power system in a compact form is always attractive for directed energy weapon system defense, space

and commercial application like, particle accelerator, radar systems, plasma immersion ion implantation (PIII), food sterilization, package sealer, paint strippers, water treatment plants etc.[5] here such high voltage pulse power has specific use as futuristic weapon system against aerial targets i.e. mini, micro UAV's, main UAV and drones and space war counter attack. The five basic functional blocks content in any pulse power generator system i.e. main power supply system, charging and discharging circuit, pulse generation system, pulse boosting system and various nature of loads as illustrated in Fig.1.

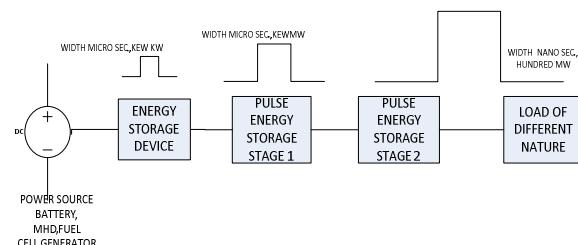


Fig.1.Functional Blocks of high voltage high pulse power functional block

The sub-system begins at the steady power source (batteries, solar cells, turbo-alternators, utility electric power) followed by energy storage device and ends at the interface with the dynamic impedance change load (laser, RF, X-ray, particle beam). There can be one or more stages of pulse formation. The circuit should be efficient for high powers.

## II. CIRCUIT DESCRIPTION AND DESIGN

**Line type pulser:** All of the stored energy is discharged during each pulse.

**Resonant charging:** When switch S is closed, current will begin to flow through the inductor L1 to charge the capacitor C1. Initially the reactance of L1 will limit the

current for resulting in voltage drop equal to the battery voltage appearing across L1.

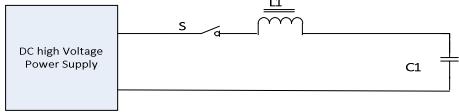


Fig.2. Resonant charging circuit

As the time passes, current begin to flow. This results in buildup of magnetic flux or field  $I$  in the core of L1. Eventually the charge across C1 approaches a value equal to  $Edc$ . The current through L1 at this point is maximum and both L1 and C1 have stored energy. Current begin to decrease because there no longer exists a voltage difference between C1 and Battery B1. This cause the magnetic field in L1 to collapse. This collapsing field produces a continuous of current flow in the L1 that create a voltage source that adds to the battery voltage. C1 now begin to charge higher than the battery voltage  $Edc$ , eventually approaching 2  $Edc$ , when all stored energy in L1 has been transferred to C1. there are now a voltage difference between the battery voltage  $Edc$  and capacitor voltage 2 $Edc$ . This causes the current to flow in the reverse direction in the circuit resulting in the extra stored energy in C1 flowing in L1. A magnetic field is again developed in L1 in opposite direction and L1 now has an excess stored energy. This cycle of event will continue until all original stored energy in dissipated in the  $I^2R$  loss. As a result of this cyclic reaction, a damped oscillation of current will flow between L1 and C1. This oscillation has a resonant time period determined by the values of L1 & C1. The resonant time is  $T_e = 2 \pi \sqrt{L_1 C_1}$ . The capacitor C1 will eventually have charge voltage equal to  $Edc$  after several cycles[2][3].

This resonant charging can be put to use to allow the capacitor C1 to maintain the stored energy in L1 and C1 by adding a diode CR1 in series with L1 and C1. When the capacitor C1 reaches a voltage equal to 2  $Edc$  the diode CR1 will block the reverse flow of current back through L1.

- The power source voltage required is only half that of capacitor stored energy value.
- The efficiency of transfer of energy is raised from approximately 50% to almost 100%.
- If is possible to regulate the voltage charge on the capacitor.

### III. MODELLING OF PULSE FORMING NETWORK

The pulse-forming network has dual purpose of storing the amount of energy required for a single pulse and discharging the energy into the load in the form of a pulse. Basically a PFN is a lumped constant transmission line of L & C. Now look at the basic circuit for generating pulses of arbitrary shape[6].

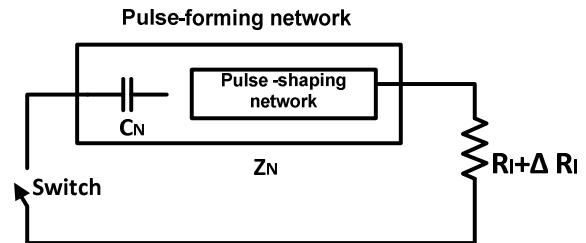


Fig.3. Basic circuit for generating pulses of arbitrary shape

From LT, the current is given by

$$i(s) = \int_0^\infty i(t)e^{-st} dt \quad (1)$$

Now apply the basic KVL equation in Laplace domain for the circuit, we get-

$$(R_l + \Delta R_l + Z_N + 1/C_{NS})i(s) = Q_N/C_{NS} = V_N/s \quad (2)$$

Where  $R_l + \Delta R_l$  = load resistance,  $Z_N$ =network impedance,  $C_N$ = network capacitance,  $Q_N$ = Charge on capacitor,  $V_N$ =Initial voltage on capacitor

Now rearranging equation (2),

$$Z_N = \frac{V_N}{s i(s)} - R_l + \Delta R_l - \frac{1}{C_{NS}} \quad (3)$$

If current pulse is a rectangular pulse with amplitude  $I_l$ , duration  $\tau$ , and  $R_l$  is a load resistance. Then  $i(s)$  is found from Eq.(1) to be-

$$i(s) = I_l/s(1 - e^{-s\tau}) \quad (4)$$

Substituting in Eq. (3),

$$Z_N = \frac{sV_N}{sI_l(1 - e^{-s\tau})} - R_l - 1/C_{NS} \quad (5)$$

also,

$$Z_N + 1/C_{NS} = R_l \left[ \frac{\left( \frac{V_N}{I_l R_l} - 1 + e^{-s\tau} \right)}{1 - e^{-s\tau}} \right] \quad (6)$$

Multiplying numerator and denominator by  $e^{s\tau/2}$ ,

$$Z_N + 1/C_{NS} = R_l \left[ \frac{\left( \frac{V_N}{I_l R_l} - 1 \right) e^{\frac{s\tau}{2}} + e^{-\frac{s\tau}{2}}}{e^{\frac{s\tau}{2}} - e^{-\frac{s\tau}{2}}} \right] \quad (7)$$

$$= R_l \left[ \coth \frac{s\tau}{2} \frac{\left( \frac{V_N}{I_l R_l} - 1 \right) e^{\frac{s\tau}{2}}}{e^{\frac{s\tau}{2}} - e^{-\frac{s\tau}{2}}} \right] \quad (8)$$

Choosing  $V_N = 2I_l V_l$ , there is obtained

$$Z_N + 1/C_{NS} = R_l \coth(s\tau/2) \quad (9)$$

The right-hand member of Eq. (9) is impedance function for an open-circuited lossless characteristic impedance  $Z_0 = R$ , and transmission time  $\delta = \tau/2$ . Hence, the pulse-shaping circuit plus the capacitance CN should be an electrical equivalent for line.

#### A. Pulses Generated by a Lossless Transmission Line:

Since the microwave load for which pulsers designed require rectangular pulse.

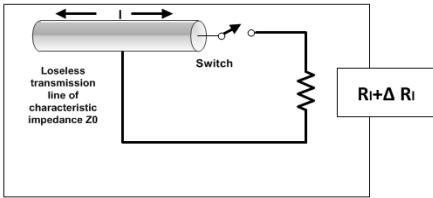


Fig.4. Schematic diagram for producing an ideal rectangular pulse

The a-c impedance of the transmission line, is  
 $Z = Z_0 \coth j\omega\delta$ ,

Applying Laplace Transform:  $Z(s) = Z_0 \coth s\delta$ .  
 The current transform is then:

$$i(s) = \frac{V_0}{s(R_l + Z_0 \coth s\delta)} \quad (10)$$

$$= \frac{V_0}{s(Z_0 + R_l)} \frac{1 - e^{-2s\delta}}{1 + \frac{Z_0 - R_l}{Z_0 + R_l} e^{-2s\delta}} \quad (11)$$

$$= \frac{V_0(1 - e^{-2s\delta})}{s(Z_0 + R_l)} \left[ 1 - \frac{Z_0 - R_l}{Z_0 + R_l} e^{-2s\delta} + \left( \frac{Z_0 - R_l}{Z_0 + R_l} \right)^2 e^{-4s\delta} - \dots \right] \quad (12)$$

where,  $V_0$  is the initial voltage.

The current is derived by applying the inverse transform,

$$i(t) = \frac{V_0}{Z_0 + R_l} (1 - U(t - 2\delta) - \frac{Z_0 - R_l}{Z_0 + R_l} [U(t - 2\delta) - U(t - 4\delta)]) + \dots \quad (13)$$

If  $R_l = Z_0$ , that is, if the line is matched to the load single rectangular pulse of amplitude  $I_l = V_0/2Z_0$  and duration  $\tau = 2\delta$ . Current and voltage pulses for  $R_l = Z_0$ ,  $R_l = 2Z_0$ , and  $R_l = 1/2Z_0$  are shown in Fig.5.

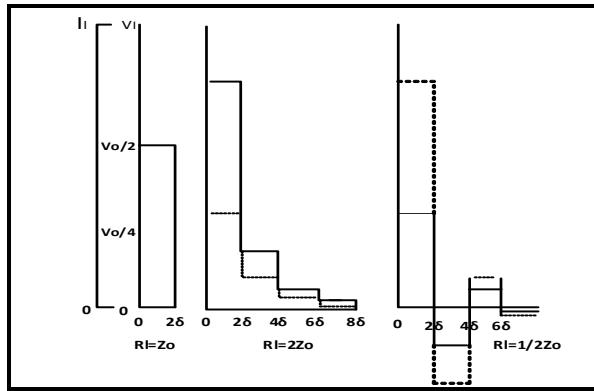


Fig.5. Current and voltage pulses for a lossless transmission line

The voltage and current pulses are represented in Fig 5. The effect of mismatching the load is to introduce a series of steps into the transient discharge. These steps are all of the same sign when  $R_l > Z_0$ , and alternate in sign when  $R_l < Z_0$ . A transmission line, having a one-way transmission time of  $\delta$ , generates a pulse of duration  $2\delta$ . Let's take an assumption of a pulse of 1 microsec duration and a signal velocity on the line of 500 ft/microsec, a line of length  $1 = 500 \times 1/2 = 250$  ft is required. High-voltage line or cable of this length would be impractical because of its large size and weight.

#### B. DeQuing

Pulse to pulse regulation provides constant input so as to get constant output from the RF. DEQ solid state switch in

series with resistance capacitor network is connected across the charging choke. As shown in Fig.8.

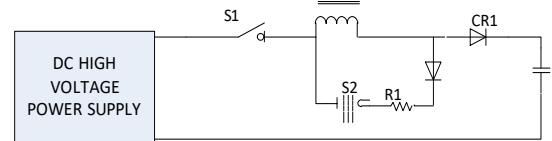


Fig.8. DeQuing switch circuit

As soon as pulse forming network changes to set level, the compared to a low level reference which in turn trigger DEQ switch and remain energy in the choke is dissipated in the resistor connected in the DEQ circuit.

#### IV. SIMULATION MODEL ON HIGH VOLTAGE HIGH PULSE POWER SUPPLY

Simulation studies have been done with following parameters: Single Phase AC Voltage Source with a peak voltage of 220V, 50Hz, Non-Linear Load as Diode Rectifier with R-Load with resistance, 1.5KΩ. The High Voltage High Power Pulse Generation Simulation Model is as shown in Fig.9.

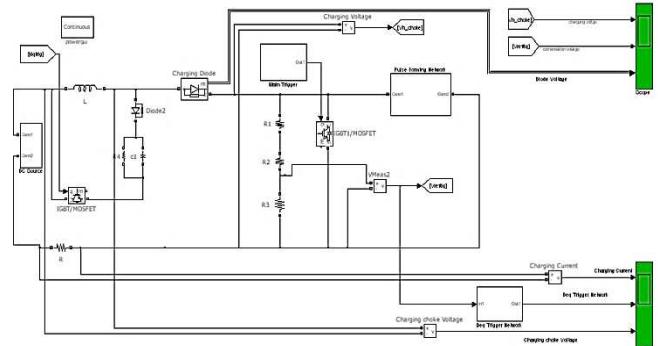


Fig.9. Simulation Model of High Voltage High Power Pulse Generation

#### V. SIMULATION RESULTS AND ANALYSIS

Developed high voltage high pulse power system tested with 1.5KΩ resistor connected through four RG8 (triaxial) cables to match the transmission line impedance of 12.5Ω. The system is fed from Single Phase AC Voltage Source with a peak voltage of 220V, 50Hz frequency.

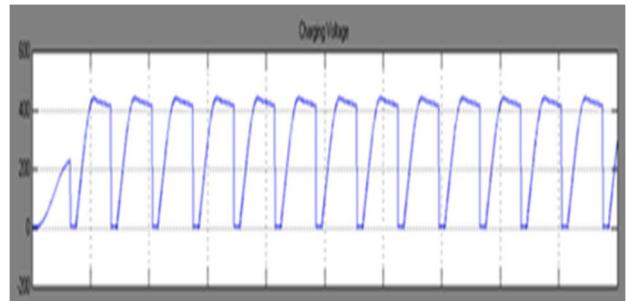


Fig.10. Charging Voltage waveform

Fig.10 shows charging voltage waveform shows the series resonance charging phenomenon between charging choke and the capacitor (lumped) of pulse forming

network in order to verify the proper charging, a fast RC compensation network (1:4) is developed and ensured low side end of charging choke voltage is  $V$  and due to resonance, the capacitor of Pulse forming network it charge up to  $2V$ .

DeQ trigger and Choke voltage waveforms are shown in Fig.11. Fast RC Compensation network sensed the extra charging voltage of choke. Then controller gives the trigger signal to DeQ switch to operate and dissipate the excess power across resistor network for pulse to pulse regulation to maintain constant charging voltage. Setting is done in such a way that maximum power dissipation should not exceed more than 25%.

During DeQ switching the charging voltage fall to zero, this can be seen in charging choke voltage waveform. If charging voltage is negative (between 5% - 10%) due to mismatch between pulse forming network and dynamic load, then reverse diode protection scheme reduce the pulse repetition rate by 20%. Due to action of shunt diode protection circuit the reverse overvoltage at the main switch stack is tolerant. The developed high voltage high pulse power system has high flexibility to reduce up to 50% of the output voltage with respect to the pulse width, the pulse repetition rate and voltage magnitude during reverse charging more the 25%.

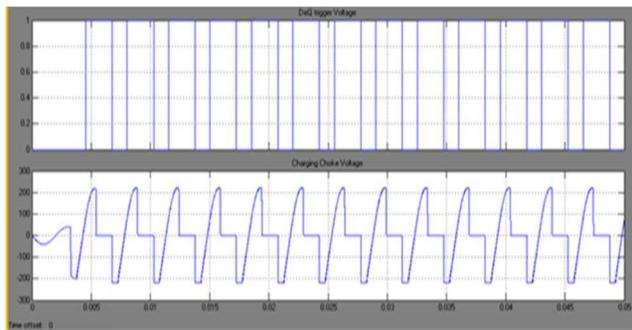


Fig. 11. DeQ trigger pulse and Choke voltage

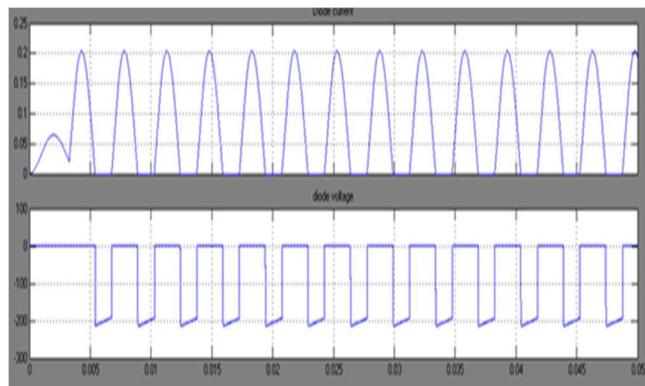


Fig. 12. Charging current and forward diode voltages

Fig.12. presents the charging current and forward diode voltage. Charging current and time is sensed through  $1\Omega$  resistor assembly. The overcurrent protection circuit turns off EHT supply when output current rapidly increases due to arc. The charging time depend upon the inductor value as the inductance is high the time will increase but the peak current will reduce and vice versa. The

diode voltage which is used to protect the reverse flow of current from PFN to the supply side.

Pulse Transformer is conventionally designed with bifilar winding. It is used to deliver a high voltage to the load and main switch stacks at low voltage. Six pieces of ferrite core are used in the pulse transformer. Figure 13 shows waveforms of output voltage at load. To sense 35 KV a fast compensation network is designed. For a load resistance of  $1.5K\Omega$  and a charging voltage of 3.2KV, the resulting output pulse voltage is 1.6KV. The rise time of load voltage is approx. 100ns (10%-90%).the initial peak of 5% is due to the first combination of L and C of pulse forming network. Second and third overshoot is due to mismatch of second and third combination of L and C of pulse forming network.

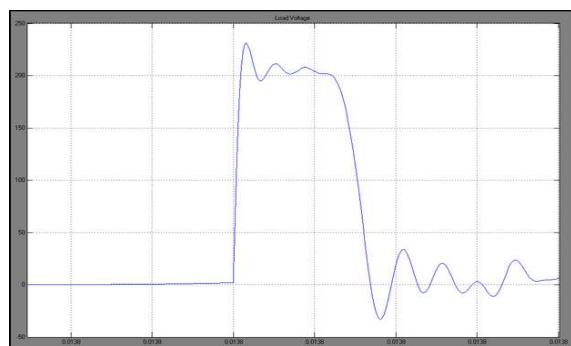


Fig.13.Simulated Output voltage at load

The initial few  $\mu$ second is required to settle down the initial mismatch of network and load. Rest of the pulse top is flat to make the total pulse width of 10micro second. The replica of initial pulse forming network, performance is seen during fall time of pulse. It has been experimented that, the output voltage waveform at load will deteriorate with increase of transmission line length. To compensate this deterioration on output wave form, a compensation circuit is designed and put across the primary side of pulse transformer.

Fig.13 waveform shows that resistive load will have no impedance mismatch but R-L load will have heavy negative mismatch. To minimize the negative mismatch on active impedance change of pulse forming network is implemented.

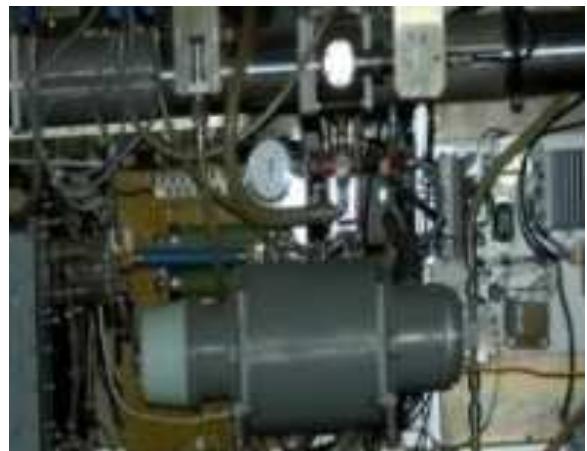


Fig.14. Photograph of Klystron based high voltage high pulse power supply (HVHPPS)

The simulated results were verified on klystron based high voltage high pulse power supply, output performance of power supply is being seen through high voltage compensation network of 135KV through klystron voltage as shown in Fig.14. The klystron which is used here is of 6 MWatt peak power and duty of .001, pulse width of 6 micro sec. The observed waveform of klystron current is shown in Fig 15. This waveform is almost perfectly matched and having no positive and negative mismatch.

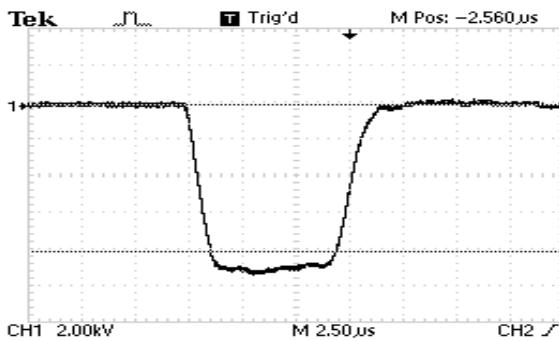


Fig15.Klystron voltage waveform

## VI. CONCLUSION

A mathematical model developed for subsystem module wise like, charging, discharging and load nature taken into consideration and impact were analyzed. The mathematical impact was represented in the output waveform. Based on mathematical model a Matlab simulation circuit model generated and sub system level output wave form analyzed, the result obtained from the model were compared with the hardware result on klystron performance analyzed finally after mathematical analysis, simulation result and actual hardware output result compared and found that load mismatch have significant impact on the output waveform. In the future work impedance matching algorithm may be developed to minimize the load mismatch should be evolved for wide band HVHPP system.

## REFERENCES

- [1] Hidenori Akiyama, Shunsuke Sakai, Takashi Sakugawa and Takao Namihira, "Environmental Applications of Repetitive Pulsed Power", IEEE Transactions on Dielectrics and Electrical Insulation Vol. 14, No. 4, Aug 2007.
- [2] Dongdong Wang, Jian Qiu and Kefu Liu, "All-Solid State Repetitive Pulsed-Power Generator using IGBT and Magnetic Compression Switches", IEEE Transactions on Plasma Science, Vol.38, No.10, October2010,pp-2633-2638.
- [3] Yifan Wu, Kefu Liu, Jian Qiu ,Xiao Xu Liu and Houxiu Xiao, "Repetitive and High Voltage Marx Generator Using Solid-state Devices", IEEE Transactions on Dielectrics and Electrical Insulation, Vol. 14, No. 4, August 2007,pp-937-940.
- [4] A.V. Akimov, P.V. Logachev, V.D. Bochkov, D.V. Bochkov, V.M. Dyagilev and V.G. Ushich, "Application of F TPI1-10k/50 Thyratron for building a modulator, intended for supply of inductive-resistive load in double pulse mode" IEEE International Pulsed Power Conference, 2007, Vol.2,pp-1339-13.
- [5] Sizhuang Liang, Youtong Fang and Xiaoyan Huang, "Simulation of an Electromagnetic Launcher With a Superconducting Inductive Pulsed Power Supply", IEEE Transactions on Applied Superconductivity, Vol.26, No.4, June 2016.
- [6] Zhenmei Li, Haitao Li, Xiaotong Zhang, Cunshan Zhang, Shucun Liu and Tong Lu,"Modelling and Simulation of Railgun System Driven by Multiple HTSPPT Modules",IEEE Transactions on Plasma Science, Vol.46, No.1, January 2018, pp-167-174.