Investigations on Distribution Load Flow Methods in a Radial Distribution Network

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***Abstract*—This paper presents a comparative Study of various Distribution Load Flow (DLF) methods for a Radial Distribution System. The DLF methods considered here are Current Injection Method based DLF (CIM), Vector Based DLF (VDLF), Matpower based DLF , Forward Backward sweep based DLF (FBLF) and Network Topology based DLF method (NTLF). Among all methods NTLF is found to be most efficient and uses graph theory and matrix algebra concepts based on simple matrix multiplication. On an IEEE 33 bus and PG&E 69 bus radial distribution test system, the convergence ability of all the approaches is evaluated by taking into account voltage profile, active and reactive power losses, iterations, and computational time.**

***Index Terms*—Current Injection Method (CIM), Forward Backward sweep Load Flow (FBLF), Network Topology based Distribution Load Flow (NTLF), Vector based Distribution Load Flow (VDLF).**

NOMENCLATURE

ϕ_i	angle between E_i and I_i
E_i	i^{th} bus voltage
I_i	i^{th} bus current
$I_b(k)$	Branch current at k^{th} line
$I_{eff}(i)$	Effective current supplied by i^{th} bus
I_p^{cal}	Calculated current at bus 'p'
I_p^{sp}	Specified current at bus 'p'
P_p^{sp}	Specified active power at bus 'p'
$P_{eff}(i)$	Active power supplied by i^{th} node
P_i^{inj}	Injected active power at bus 'i'
P_{load}	Active power load
P_{loss}	Active power loss
Q_p^{sp}	Specified reactive power at bus 'p'
$Q_{eff}(i)$	Reactive power supplied by i^{th} node
Q_i^{inj}	Injected reactive power at bus 'i'
Q_{load}	Reactive power load
Q_{loss}	Reactive power loss
R	Resistance
X	Reactance
Y_{bus}	Admittance matrix
Y_{pp}	Diagonal elements of Y_{bus}
Y_{pq}	Off diagonal elements of Y_{bus}
ADJL	Vector used for adjacent line number
ADJQ	Vector used for adjacent bus number
ITAGF	i^{th} bus tagging from
ITAGTO	i^{th} bus tagging to

I. INTRODUCTION

Distribution Load Flow is an emerging tool for analysing distribution networks and is used extensively in operations and planning studies. The majority of distribution systems are radial in nature, with high resistance to reactance ratio. Unbalanced operation and low voltage difficulties in distribution systems are an inevitable consequences of these characteristics, and conventional load flow algorithms fail to converge for such networks. Several distribution load flow methods have been proposed in the literature to take care of such problems.

In [1]- [2], the authors presented simple load flow techniques for radial distribution networks based on the formation of constant matrix using network topology and identification of branch and node. A directly solvable non-iterative DLF using bus indexing model and connectivity matrix to characterize the network is discussed in [3]. A direct approach of load flow solution by developing two matrices bus injection to branch current (BIBC) and branch current to bus voltage (BCBV) is explained in [4]. In [5], an efficient load flow solution works for radial and weakly meshed network based on the concept of partial network concept. In [6]- [7], current injection based load flow and forward backward sweeps load flow solutions were explained for various balanced and unbalanced systems. A simple and elegant algorithm for distribution system was presented in [8]- [10]. In [11], the authors have presented load flow study carried out with ETAP (14.0) software which simulates voltage profiles and active power flow as well as reactive power flows and losses. A distribution load flow for radial and meshed network based on the process of loop breaking by adding dummy nodes is explained in [12]. In [13], authors accounted for power flow solution for AC-DC distribution networks from employing graph theory and matrix algebra concepts.

In this paper, Comparative study of DLF algorithms have been proposed to investigate fast and efficient power flow solutions for radial distribution systems to mitigate active and reactive power losses and improve voltage profile. DLFs such as CIM, VDLF, DLF in Matpower, FBLF and NTLF have been applied on IEEE 33 bus RDS and PG&E 69 bus RDS for efficient distribution system planning studies. Among all DLFs, NTLF outperformed CIMLF, VDLF, DLF in Matpower and FBLF in terms of minimum real and reactive power losses.

2022 IEEE Students Conference on Engineering and Systems (SCES) | 978-1-6654-8072-7/22/$31.00 ©2022 IEEE | DOI: 10.1109/SCES55490.2022.9887642

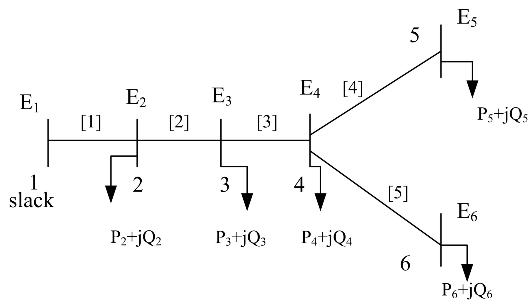

Fig. 1. Single line diagram of 6 bus RDS with representation of bus voltages

There are four sections in this paper: Mathematical approach for computing DLF employing various algorithms such as CIM, VDLF, DLF in Matpower, FBLF and NTLF are explained in section II. In section III, simulation results obtained by application of all DLF algorithms on two test cases viz.IEEE 33 bus RDS and PG&E 69 bus RDS are presented for identifying the most efficient DLF method reliable and Conclusions and scope for research are explained in Section IV.

II. Mathematical models to compute DLF

For system analyses, radial distribution networks are modelled as balanced single-phase equivalent networks. The shunt capacitance in a line, which is relatively modest, is ignored in distribution line modelling. The following is the mathematical way to computing DLF:

A. Vector based distribution Load flow (VDLF)

Consider a sample 6 bus radial network as shown in Fig 1.

Let us assume $E_1 = 1 + j0$; (taken as reference bus and constant for all iterations) $E_2 = E_3 = E_4 = E_5 = E_6 = 1 + j0$; From the given network as shown in Fig 1, $P_{eff}(i)$ including losses and loads beyond the i^{th} node and similarly $Q_{eff}(i)$ including losses and loads beyond the i^{th} node. The calculation of the P_{eff} and Q_{eff} starts from last bus to second bus. Initial losses are considered as zero but during the iterative process P_{loss} and Q_{loss} of each line are calculated and used to update the P_{eff} and Q_{eff} during the iterative process. The calculation of P_{eff} and Q_{eff} values are as shown in Table I.

TABLE I
CALCULATION OF P_{eff} AND Q_{eff}

$BusNo$	$P_{eff}(i)$	$Q_{eff}(i)$
6	$P_{load}(6)$	$Q_{load}(6)$
5	$P_{load}(5)$	$Q_{load}(5)$
4	$P_{load}(4)+P_{eff}(5)+P_{eff}(6)$	$Q_{load}(4)+Q_{eff}(5)+Q_{eff}(6)$
3	$P_{load}(3) + P_{eff}(4)$	$Q_{load}(3) + Q_{eff}(4)$
2	$P_{load}(2) + P_{eff}(3)$	$Q_{load}(2) + Q_{eff}(3)$

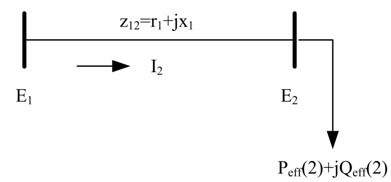

Fig. 2. A simple two bus system

$$E_1 = E_2 + I_2(r_1 + jx_1) \quad (1)$$

A simple 2 bus network taken into consideration as shown in Fig 2. The bus voltage is computed by using equation (1). It is observed from Fig 3 that OCD is a right angle triangle.

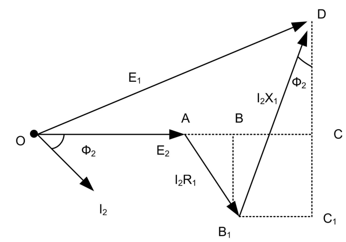

Fig. 3. Phasor diagram of sample network

$$OD^2 = OC^2 + DC^2 = (OA + AB + BC)^2 + DC^2 \quad (2)$$

where $OD = |E1|$; $OA = |E2|$; $AB = I_2R_1cos(\phi_2)$; $B_1B = C_1C = I_2R_1sin(\phi_2)$;$DC_1 = I_2X_1cos(\phi_2)$; $B_1C_1 = I_2X_1sin(\phi_2)$; $I_2cos(\phi_2) = P_{2eff}/E_2$; $I_2sin(\phi_2) = Q_{2eff}/E_2$; By substituting in equation (2) the expression as shown in equation (3) is obtained.

$$E_1^2E_2^2 = (E_2^2 + P_{eff}(2)R_1 + Q_{eff}(2)X_1)^2 + (P_{eff}(2)X_1 - Q_{eff}(2)R_1)^2 \quad (3)$$

The equation (3) is in fourth order which is highly non-linear and obtained without using any approximations. The solution of such highly non linear equation would help us quadratic fast convergence. It is observed that this works for radial type network and not for meshed type network. The flowchart of VDLF method is as shown in Fig 4.

B. Current Injection Method based Load flow (CIM)

CIM load flow solution works for both radial type and meshed type of network. The mathematical expressions for understanding CIM are represented in equations (4)-(8). The current mismatch is given in equation (9). The equation (9) is non-linear and converges fast. It takes 3 to 4 iterations for any size of network. CIM takes huge memory and large CPU time. CIM's flow chart is depicted in Fig 5.

$$[I_{bus}] = [Y_{bus}][E_{bus}] \quad (4)$$

$$I_p^{cal} = Y_{pp}E_p + \sum_{q=1\&q\neq p}^{n} Y_{pq}E_q \quad (5)$$

$$I_p^{sp} = (P_p^{sp} - jQ_p^{sp})/E_p^* \quad (6)$$

$$F_p = \Delta I = I_p^{sp} - I_p^{cal} \quad (7)$$

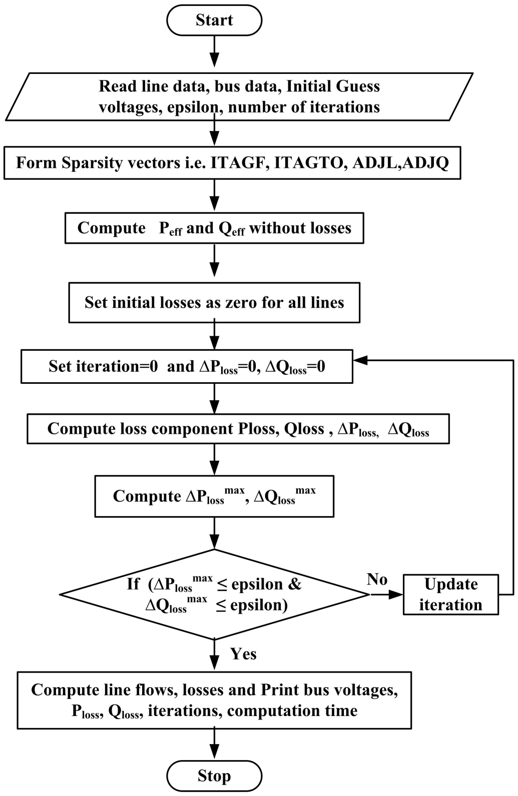

Fig. 4. Flow chart of VDLF method

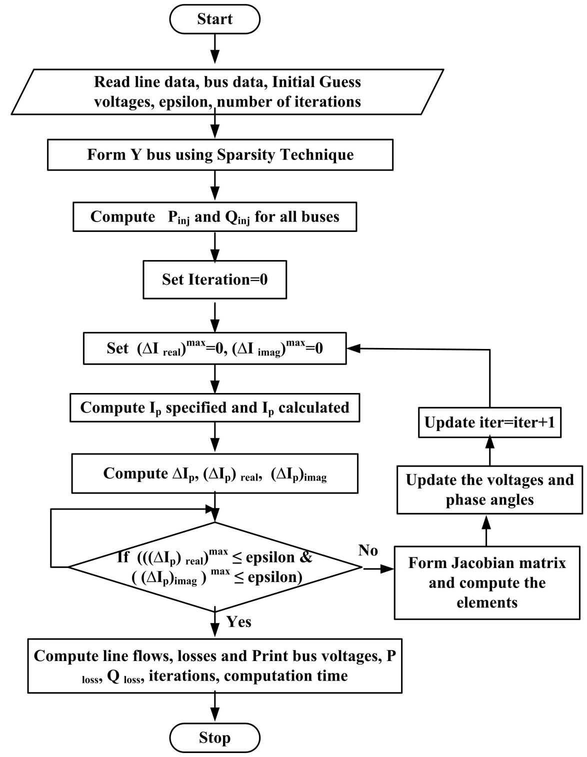

Fig. 5. Flow chart of CIM [6]

$$F_p = (P_p^{sp} - jQ_p^{sp})/E_p^* - Y_{pp}E_p - \sum_{q=1\&q\neq p}^{n} Y_{pq}E_q \quad (8)$$

$$F_p = \frac{(P_p^{sp} - jQ_p^{sp})(e_p^2 - f_p^2 + j2e_pf_p)(e_p - jf_p)}{|E_p^4|} - Y_{pp}E_p - \sum_{q=1\&q\neq p} Y_{pq}E_q \quad (9)$$

C. Matpower based DLF

Matpower has several number of power flow solutions for radial networks. Among them forward backward sweep method is popularly used. Additionally three types of ac power flow solutions are existing in Matpower i.e.current summation method, power summation methods and admittance summation methods. The DLF method under Matpower flow chart is given in Fig 6.

D. Forward backward sweep based Load flow (FBLF)

FBLF is easy to understand and simple to program. There are no approximations in deriving the basic mathematical equations and its convergence is usually in 3 to 4 iterations. It doest not require any big matrices and LU decomposition. Here, the concept of backward sweep and forward sweep approaches were utilized for DLF solution.

$$I_i = [-(P_i^{inj} + jQ_i^{inj})/E_i]^* \quad (10)$$

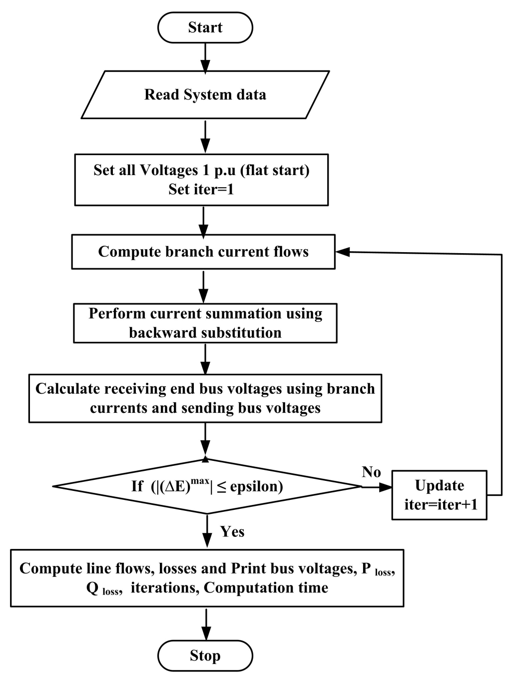

Fig. 6. Flow chart of DLF in Matpower [18]

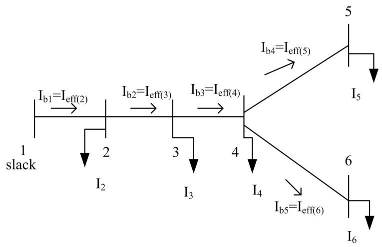

Fig. 7. Single line diagram of 6 bus radial network with representation of branch currents

1) Backward sweep: Backward sweep computes branch currents using equation (10) and I_{eff} from the following expressions whose significance is pictorially represented in Fig 7. $I_{eff(6)} = I_6$; $I_{eff(5)} = I_5$; $I_{eff(4)} = I_4 + I_{eff(5)} + I_{eff(6)}$; $I_{eff(3)} = I_3 + I_{eff(4)}$; $I_{eff(2)} = I_2 + I_{eff(3)}$; I_{eff} has interconnectivity including its own load current. As the iterations progress, the values of I_{load} and I_{eff} gets effected. The process of computing I_{eff} is referred as backward sweep approach.

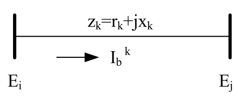

Fig. 8. A sample two bus system

2) Forward sweep: Forward sweep computes bus voltages from Fig 8 with the following expressions: $E_2 = E_1 - I_{b(1)}z_1$; $E_3 = E_2 - I_{b(2)}z_2$; $E_4 = E_3 - I_{b(3)}z_3$; $E_5 = E_4 - I_{b(4)}z_4$; $E_6 = E_4 - I_{b(5)}z_5$; The flow chart of FBLF method is as shown in Fig 9.

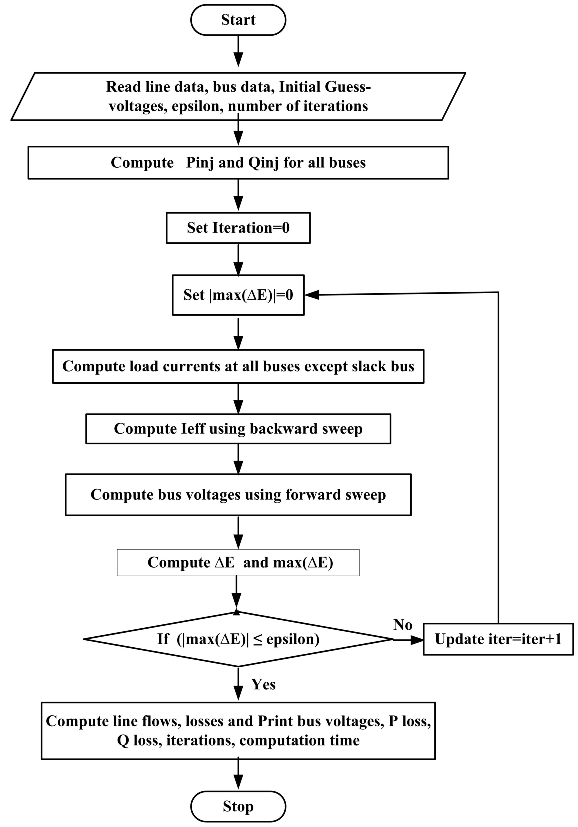

Fig. 9. Flow chart of FBLF [7]

E. Network Topology based Load flow (NTLF)

NTLF solution is an iterative process, utilizing graph theory and matrix algebra concepts. Four matrices have been generated and are utilised as input for the DLF solution. They are derived from Fig 10, which depicts the network structure.

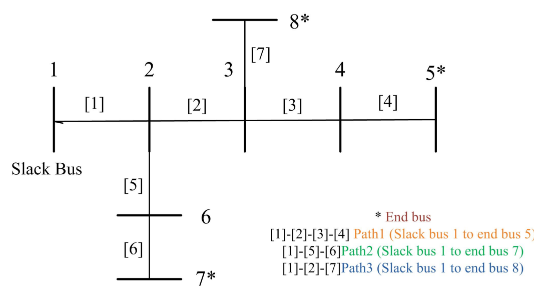

Fig. 10. Single line diagram of a 8-bus radial network

1) Load Beyond Matrix (LB): Load beyond matrix given in equation (11) finds all the loads beyond a line in the network with the size (l,n), where 'l' is line number and 'n' is bus number. If the $(n+1)^{th}$ bus is beyond the line $'l'$, $LB(l,n) = L(n+1)$ is used. If the $(n+1)^{th}$ bus does not cross line $'l'$, LB(l,n)=0. Bus $'n'$ has a load of L(n).

$$LB = \begin{bmatrix} L_2 & L_3 & L_4 & L_5 & L_6 & L_7 & L_8 \\ 0 & L_3 & L_4 & L_5 & 0 & 0 & 0 \\ 0 & 0 & L_4 & L_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_6 & L_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_8 \end{bmatrix} \tag{11}$$

2) Path Impedance Matrix (PI): The lines linking the slack bus and end bus in the network are found using the Path Impedance matrix (12). If the l^{th} branch in the p^{th} path connecting slack to the end bus is followed, $PI(p,l) = Z(l)$, and $PI(p,l) = 0$, if the p^{th} path number does not contain the l^{th} branch. The impedance of branch $'l'$ is $Z(l)$.

$$PI = \begin{bmatrix} Z_1 & Z_2 & Z_3 & Z_4 & 0 & 0 & 0 \\ Z_1 & 0 & 0 & 0 & Z_5 & Z_6 & 0 \\ Z_1 & Z_2 & 0 & 0 & 0 & 0 & Z_7 \end{bmatrix} \tag{12}$$

3) Path Drop Matrix (PD): The drop voltage of distribution network in each path is identified by the path drop matrix (13). $PD(p,l) = PI(p,l) * FC(l)$, If the l^{th} line in the p^{th} path connecting slack to the network's end bus is traced. $PD(p,l) = 0$, If the l^{th} line in the p^{th} path connecting slack to the network's end bus is not traced. Where FC is feeder current matrix.

$$PD = \begin{bmatrix} PD_1 & PD_2 & PD_3 & PD_4 & 0 & 0 & 0 \\ PD_1 & 0 & 0 & 0 & PD_5 & PD_6 & 0 \\ PD_1 & PD_2 & 0 & 0 & 0 & 0 & PD_7 \end{bmatrix} \tag{13}$$

4) Slack bus to Other Bus Drop Matrix (SBOBD): The SBOBD matrix (14) is used to determine the overall voltage loss in each path i.e. between slack bus and existing other buses of the distribution network. $SBOBD(p,l) = 0$, if $PD(p,l) = 0$ otherwise $SBOBD(p,l) = \sum_{m=1}^{l} PD(n,m)$.

$$SBOBD = \begin{bmatrix} D_1 & D_2 & D_3 & D_4 & 0 & 0 & 0 \\ D_1 & 0 & 0 & 0 & D_5 & D_6 & 0 \\ D_1 & D_2 & 0 & 0 & 0 & 0 & D_7 \end{bmatrix} \tag{14}$$

The flow chart of NTLF method is as shown in Fig 11.

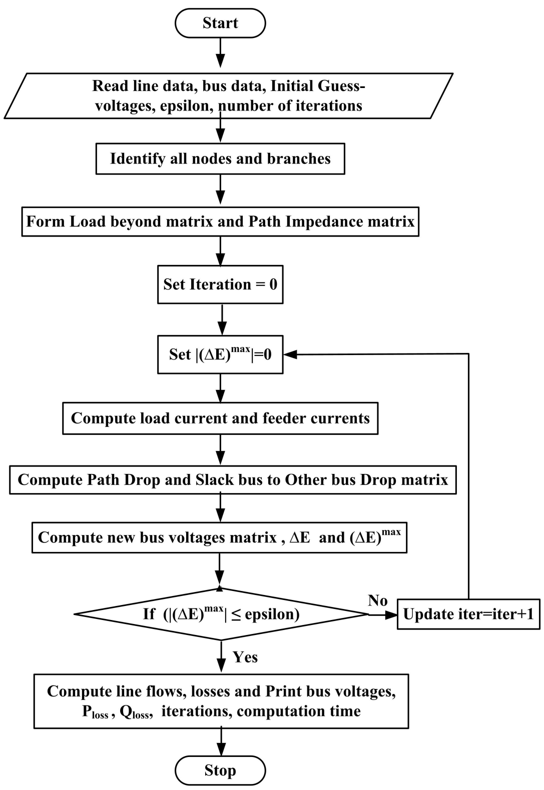

Fig. 11. Flow chart of NTLF

III. Results and Discussions

The DLF methods on IEEE 33 bus RDS and PG&E 69 bus RDS were implemented using MATLAB R2021b [17] software on a PC with an Intel (R) core i5 processor and 8GB RAM. The line data and bus data for IEEE 33 bus RDS and PG& E 69 bus RDS are available at [14] & [15], respectively. The single line diagram of IEEE 33 bus RDS and PG&E 69 bus RDS is shown in Fig 12 and Fig 14 respectively. The comparative study of DLF algorithms is done to study the fast and efficient power flow solutions for the radial distribution systems with minimum active and reactive power losses and better voltage profile. The simulation findings indicate that the NTLF algorithm is more efficient than the CIMLF, VDLF, MATPOWER, and FBLF algorithms.

TABLE II
Comparison of NTLF with other DLF methods for IEEE 33 bus RDS

Parameters	Distribution Load Flow Methods				
Type of Method	**CIM [6]**	**VDLF [16]**	**Matpower [18]**	**FBLF [7]**	**NTLF**
P Loss (kw)	211	210.98	203	188.72	182.18
Q Loss (kvar)	143.03	143.12	143	126.88	123.26
Iterations	4	4	3	3	1 (or) 2
Time (seconds)	0.1831	0.17003	0.17	0.10967	0.1526

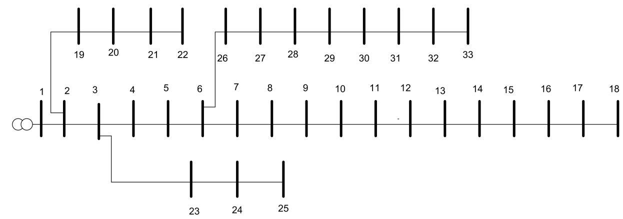

Fig. 12. IEEE 33 bus RDS single line diagram

A. Case Study I (IEEE 33 bus RDS [14])

A first case study is chosen to be IEEE 33 bus, 100 kVA, 12.66 kV, 32 branches of radial distribution system. The results shown in Table II gives comparative study of DLFs tested on IEEE 33 bus RDS for voltages, phase angles, active power losses, reactive power losses, iteration number and computational time. It is observed from the Table II that NTLF method is an efficient method in comparison with other methods. Fig 13 shows the voltage profiles for various DLFs on the IEEE 33 bus RDS.

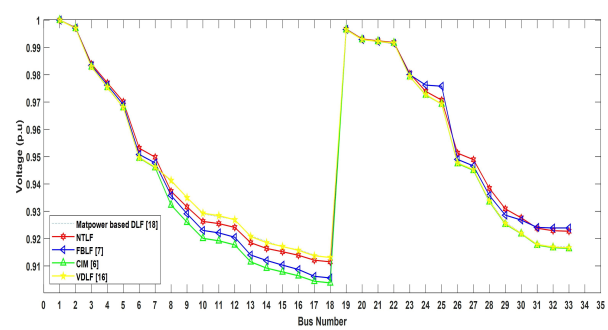

Fig. 13. Plot of Voltage Profiles of various DLF's on IEEE 33 bus RDS

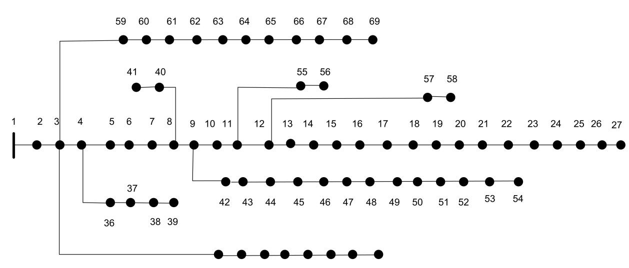

Fig. 14. Single line diagram of PG&E 69 bus RDS

B. Case Study II (PG&E 69 bus RDS [15])

The PG&E 69 bus, 100 kVA, 12.66 kV and 68 lines of radial distribution system are investigated in the second case study. The results shown in Table III gives comparative study of DLF's tested on PG&E 69 bus RDS for voltages, phase angles, active power losses, reactive power losses, iteration number and computational time. It is observed from the Table III that NTLF method is an efficient method in comparison with other methods. The plot of voltage profiles for various DLF's on PG&E 69 bus RDS is as shown in Fig 15.

TABLE III
COMPARISON OF NTLF WITH OTHER DLF METHODS FOR PG&E 69 BUS RDS

Parameters	Distribution Load Flow Methods				
DLF Method	CIM [6]	VDLF [16]	Matpower [18]	FBLF [7]	NTLF
P Loss (kw)	224.96	225.2	225	224.93	191.47
Q Loss (kvar)	102.15	100.21	100	102.41	87.783
Iterations	7	4	3	3	1 (or) 2
Time (seconds)	0.3312	0.17562	0.17	0.06296	0.1778

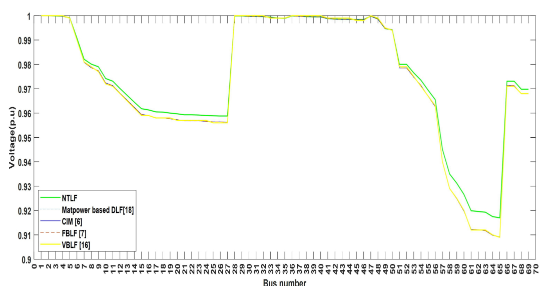

Fig. 15. Plot of Voltage Profiles of various DLF's on PG&E 69 bus RDS

IV. CONCLUSIONS

This paper has investigated the various DLF algorithms on standard test cases to find the most acceptable DLF method by comparing voltage profiles, active power losses, reactive power losses, iterations and time taken to compute the solution. Five DLF methods i.e. CIMLF, VDLF, MATPOWER, FBLF and NTLF algorithm were explored in this paper. The simulation findings show that NTLF outperforms the other DLFs in terms of losses, voltage profiles, and iterations. The efficient DLF technique can be used in a variety of ways, including load modelling, reactive power compensation studies, optimal integration of renewable DGs and network reconfiguration problems. There is a scope of achieving DLF solutions for the AC-DC systems as the existing AC distribution network is getting transformed into Hybrid network because of DC loads.

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