Static Security Assessment of Large Power Systems Under Contingency Cases

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***Abstract*—The rapid expansion of the power system network, increasing power demand and integration of renewable energy resources have made the power system vulnerable to outages. A power system should be in a secure state under normal and contingency conditions. An uncontrollable and cascading outage may lead to a partial or complete blackout. To avoid these catastrophic events, security assessment of N-1 and N-2 contingencies is very essential. The static security assessment (SSA) is computationally burdensome and requires a fast calculation methodology or strategies to assess the critical contingencies and appropriate remedial actions. This paper proposes a conjugate gradient method (CGM) based strategy for efficient contingency screening and ranking. The proposed method has been tested on standard IEEE-9, 118 bus system and 246 bus Northern Regional Power Grid (NRPG) network of Power Grid Corporation of India Limited (PGCIL). Results are compared with the various linear equation (LE) solver-based methods to show the computational benefits in the N-1 and N-2 SSA of the large power system.**

***Index Terms*—Blackouts, Contingency Analysis, Linear Equation Solver, Conjugate Gradient Method, Security Assessment.**

I. Introduction

SECURITY assessment of a power system is highly desirable to ensure reliable and secure operation even in the event of one or more contingencies (grid component outages). Cascaded contingencies are some of the most catastrophic disasters in modern power systems that result in enormous economic damage and may lead to a severe blackout. A few of the worst blackouts recorded in history are India's 2012 blackout because of tripping of circuit breakers due to overloading of 400 kV Gwalior-Bina transmission line followed by a relay failure [1], USA 2003 world largest blackout [2] caused due to outage of FE's Eastlake unit 5 generator followed by Chamberlin–Harding 345-kV line tripping. To avoid such events North American Electric Reliability (NERC) [3] and other regulatory agencies enforced strict security standards that require operators to satisfy N-1 and N-2 security constraints. According to the NERC compliance program [3], analysis of static security is essential to ensure a reliable power supply.

Static security analysis (SSA) can be defined as the ability of the power network to maintain stability and operation within the acceptable limits following the contingency (disturbance) cases. N-1 static security analysis involves studying the impact of a single element outage, whereas N-k SSA is carried out for simultaneous outage of k components. For a system with N components (lines, transformers, generators), the number of possible N-1 contingencies is N and possible number of N-2 contingencies is $\frac{N!}{2!(N-2)!}$. In the case of N-1-1 contingencies, two contingencies are applied sequentially instead of simultaneously where remedial action is accounted after the first contingencies. There are total possible N-1-1 contingencies is $N(N-1)$. Therefore, for a large number of possible contingencies, a systematic and fast computational method is required to assess the security of the real-time power systems.

Earlier, contingency testing was restricted only to a set of critical cases which used to be selected by operator's experience. This kind of approach may result in omitting some real critical contingencies. In order to prevent this, various methods were proposed in the last few decades. One of those prominent methods consists post contingency performance indices (PIs) values, where PIs have been calculated by checking bus voltages and line flow violations during every contingency. Those which have a higher PI value indicate higher risk [4]. But this ranking algorithm will not give a perfect order and requires tuning for adaptive stopping criterion to balance the overall execution time. These drawbacks have been taken care in [4], [5], [6] using first order estimation of the PI equation. However, these methods can grossly underestimate the severity of a contingency due to the approximations involved and nonlinear impact of the contingencies on the system voltages and line flows. In [7], line outage sensitivity factors (LOSF) and generator outage sensitivity factors (GOSF) are used for faster computation. Contingency assessment techniques based on the network topology analysis is proposed in [8]. These papers have been focused on efficiently ranking of all possible contingencies during SSA without omitting any critical case contingency.

Another practical issue while performing contingency analysis is computation time. Computational burden is involved in the contingency ranking where a large number of power flow equations have to be solved, repeatedly. Computational time is considerably high even with the aid of linearized or DC load flow. To reduce the computational time, a certain sequence of instructions has to be executed repeatedly for small alterations in the network configurations. In this condition, contingency screening and ranking have to be performed as fast as possible with high accuracy [9]. Fast decoupled load flow (FDLF) with conventional linear equation(LE) solvers offers good performance but when the network size increases these solvers show inefficiency and more computation time. The conventional methods like LU decomposition (LUD) have a total number of flops(floating point operations) of order n^3 for n^{th} order matrix. So as size of the matrix increases this method exhibits slower performance. Another drawback of LUD is when applied on sparse matrices, factors contain more non-zeros than the original sparse matrix. This is implied as a fill-in. As the fill-in ratio increases, memory storage requirement and computation time also increase [10]. A similar problem can be observed in other direct methods such as Cholesky decomposition [11], QR factorization [12]. Alternative approaches to solving this system of linear equations are iterative methods [13] which can eliminate drawbacks observed in direct methods. Therefore, this paper proposes a novel conjugate gradient method (CGM) based strategy for faster contingency screening and ranking.

A. Major Contributions

For efficient and faster SSA of a large power system, this paper proposes

1) An efficient screening for N-2 and ranking strategy for faster N-1 and N-2 SSA.
2) CGM solver is used and its computational effectiveness is shown by comparing with other LE solvers.
3) The proposed techniques were used to perform N-1 and N-2 contingency analysis for standard IEEE test systems and North Region Power Grid system(NRPG).

The rest of the paper is organized as follows: Section II introduces the contingency screening and ranking methodology. The load flow modeling and CGM based linear equations solver is described in Section III. Section IV consists of simulation results and discussion of various test systems. Section V presents conclusion and future scope.

II. Contingency Screening and Ranking

Ranking contingencies according to their severity helps operator to determine critical components of system and in taking immediate action to make system stable. In order to rank contingencies after every single outage, post-contingency load flow analysis has to be performed to predict outage impact on system. To estimate system condition a composite performance index is proposed and expressed in (1)

$$PI_{ci} = PI_{pl} + PI_{vi} \tag{1}$$

Where PI_{pl} is branch power flow performance index when violation in i^{th} branch flow during an outage [4]. It can be calculated as

$$PI_{pl} = \sum_{i=1}^{l} \frac{W_l^P}{2n} [\frac{MVA_l}{MVA_{l,Base}}]^{2n} \tag{2}$$

Where,
MVA_l - MVA flow in l^{th} line during contingency.
$MVAB_{l,Base}$ - Base MVA flow value for the l^{th} line.
W_l^P - Weighting factor for l^{th} line.

In eq. (1), PI_{vi} is a voltage-based performance index when voltage limit violation at ith bus during the contingency. It can be calculated as

$$PI_v = \sum_{i=1}^{nb} \frac{W_i^V}{2n} [\frac{V_i - V_B^i}{\Delta V_i^{limit}}]^{2n} \tag{3}$$

Where,
V_i - Post contingency voltage at i_{th} bus.
V_B - Base case voltage obtained in pre-contingency load flow.
W_i^V - Weighting factor for i^{th} bus.

For line-flow based performance index, load flow has to be performed during every contingency to determine the line flow of every line. To avoid this burden, generally, line outage distribution factor (LODF) is employed [7] to directly calculate the line flows from base case load flow solution. It is based on DC approximations thereby it has less accuracy. AC power flow solutions capture the effect of an outage on system voltages and line flows better than dc power flows. Therefore, in this paper, an AC load flow based line distribution factor (ACLODF) is implemented to achieve the high accuracy in contingency screening and ranking. Details of the AC load flow model and its solution using the proposed conjugate gradient method are given in Section- III. The ACLODF [14] of an i^{th} line for outage of line j^{th} is computed as follows (4)

$$S_{LODF,ij} = [\frac{S_{ij} - S_{base,i}}{S_{base,j}}] * 100 \tag{4}$$

Where,
$S_{LODF,ij}$ - Percentage of MVA flow on line j that shows upon line i.
S_{ij} - MVA flow on line i^{th} for j^{th} line outage.
$S_{base,i}$ - Base case MVA flow on line i.
$S_{base,j}$ - Base case MVA flow on line j.

In a large power system, the outage of single component only affects a few line flows and generation outputs. Majority of line flows and generators experiences less or negligible impact. This impact can be calculated using equation 4. The large value of $S_{LODF,ij}$ indicates the substantially affected line for the disconnected component. In order to account for critical N-2 contingencies, candidate branch/generators can be screened by analysing pre contingent and post contingent. Methodolgy to rank for N-1 is given in section A. In section B screening for candidates for N-2 is explained and in section C ranking method :

A. N-1 Contingency Ranking

1) Input test case data.
2) Perform AC load flow for every outage and store MW flows for every line and voltage magnitude of every bus.
3) Now use eqs. (2), (3) to find the performance indices of every outage.

4) Using performance indices obtained in step 3, calculate the composite indices eq. (1) and rank them accordingly.

B. Screening for N-2 contingency

In interconnected power system, number of possible N-2 contingencies are considerably large. Performing ranking procedure to huge number of contingencies is time consuming process. So, screening lines which have large impact on system state is beneficial in terms of time. Procedure to screen N-2 contingencies:

1) Input line data, bus data of test systems.
2) Perform AC load flow analysis for all N-1 contingencies as described in fig. 1 and store base case line flows
3) Screening for line-line outages:
   a) First Initialize all N-1 line outage contingencies and perform AC load flow for every contingency and store the corresponding MVA flow of all lines.
   b) Calculate S_{LODF} using eq (4)
   c) Screen lines which have S_{LODF} value greater than specified threshold. These lines will be our candidate branches for second outage for that particular N-1 line outage contingency.
4) Screening for generator-line outage:
   a) Perform all N-1 generator outages and store MVA flow of every line.
   b) Calculate S_{LODF} using eq (4)
   c) Screen lines which have S_{LODF} value greater than specified threshold. These lines will be the candidate for second outage for that particular N-1 generator outage.
5) For generator-generator outage screening:
   a) Perform all N-1 generator outages and store MVA flow of every line.
   b) Calculate $MVAR_{LODF}$ as shown in equation 5

$$Q_{LODF,ij} = [\frac{Q_{ij} - Q_{base,j}}{Q_{base,j}}] * 100 \tag{5}$$

   Where,
   $Q_{LODF,ij}$ - Percentage of MVAR change by generator j when generator i is outaged.
   Q_{ij} - MVA flow on line i^{th} for j^{th} line outage.
   $Q_{base,i}$ - Base case MVAR generated by generator i.

   c) Screen generators that are producing higher difference of Q_{LODF} than given threshold value.

C. Ranking N-2 Contingencies

1) Update respective second outage for every N-1 contingency, second outage will be screened contingencies that are selected in subsection C.
2) Perform ranking procedure using proposed index (1) as mentioned in subsection B

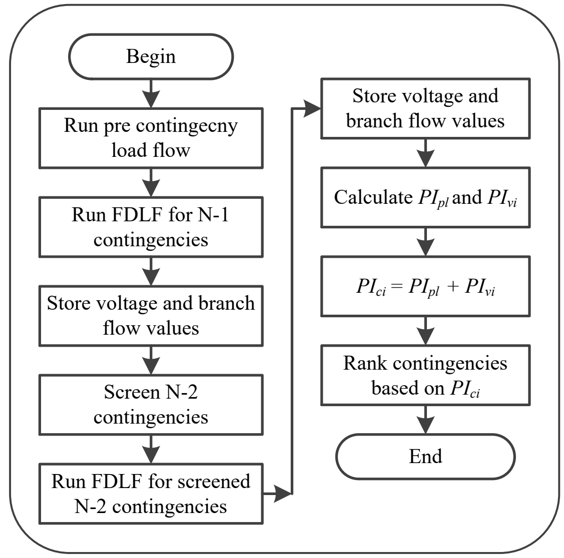

Fig. 1: Flow Chart for N-2 Contingency ranking

III. Load flow Modelling and Conjugate gradient Method

Execution time reduction of any program is not an easy task. It involves involves efficient mathematical modeling, efficient algorithms, examining computational bottlenecks, effective coding for utilization of parallel computing potentials of HPC (High Performance computing). In this study, modeling aspects and efficient algorithm are used to maximize the performance of FDLF solution.

A. Load Flow Method

In the SSA, evaluation of bus voltage and line flow is essential under all possible contingency cases. Solving massive load flow equations is a computationally exhaustive task. In the SSA, execution time can be improved either by parallel commutation or by implementing fast linear equation (LE) solving methods like Gaussian Elimination (GE), LU factorization, QR Factorization etc. In recent times, with the advancement in computational capability, performing a complete AC power flow for large systems has become computationally feasible and reliable than DC power flow. Fast Decoupled load flow (FDLF) method is the most preferred method to solve power flow equations because of its faster convergence rate. Unlike the Newton Raphson (NR) method where jacobian matrix need to be created and inverted in each iteration, fast decoupled method uses constant jacobian matrix. The basic FDLF model is presented in following eq (6) and (7).

$$\Delta P/V = B^{'} \Delta\delta \tag{6}$$

$$\Delta Q/V = B^{''} \delta V \tag{7}$$

In the FDLF model, the matrices $B^{'}$ and $B^{''}$ are obtained from the admittance matrix Y. $B^{'}$ and $B^{''}$ are real, sparse and have the structures of J_1 and J_2 in Jacobian matrix of NR method [15]. Here, the dependency between the active power flow and the bus voltage angle can be shown in the eq (6) and dependency between reactive power flow and the bus voltage in the equation (7). One iteration provides one solution for $\Delta\delta$ to update bus voltage angle δ and then one solution for ΔV to update to voltage magnitude. Separate convergence tests may be applied for the real and reactive power mismatches as follows:

$$\Delta P_{max} \leq \epsilon_p \tag{8}$$

$$\Delta Q_{max} \leq \epsilon_q \tag{9}$$

where ϵ_p and ϵ_q are the tolerances. In this paper, an efficient linear equation solver known as conjugate gradient method is proposed to solve the load flow model given in eq (6) and (7).

B. Linear equation solver

Linear equation solvers can be categorized into direct solvers, iterative solvers. Direct solvers like LU solver, contains traingular factorization and forward substitution and backward substitution. During power flow studies a numerous number of LEs, (6) and (7) which are in the form of $Ax = b$ has to be solved frequently. Direct inversion of matrix A is not recommended because of its inaccuracy and higher computation time. As system gets larger, size of matrix A also increases leading to less accurate values and more time to perform inversion of a matrix. A matrix is said to be well conditioned if a small change in A leads to a small change x and ill-conditioned otherwise. The LE solver used in this paper is conjugate gradient method (CGM). It is an iterative method employed to solve massive number of linear equations of the form $Ax = b$. This method is well known for its robustness and faster convergence rate [16], [17], [18]. To apply this method, A matrix should be symmetric, positive definite matrix (a symmetric matrix where every eigenvalue is positive). Since, B^1, B^{11} are symmetric positive definite matrices [19], [20], CGM is suitable to solve the load flow eqs. (6), (7). Algorithm and required equations to implement CGM are presented in figure 2. Table VII, VIII in section IV compares elapsed time while using LU decomposition, cholesky factorization, QR factorization and conjugate gradient method.

IV. Results and Discussions

In this section, the ability of the proposed method for screening and ranking power system contingencies according to their expected severity is tested on various standard test systems: IEEE 9 bus system, IEEE 118 Bus system and practical North Regional Power Grid (NRPG) 246 Bus system. The IEEE test system data are taken from the MATPOWER whereas NRPG data is taken from [21].

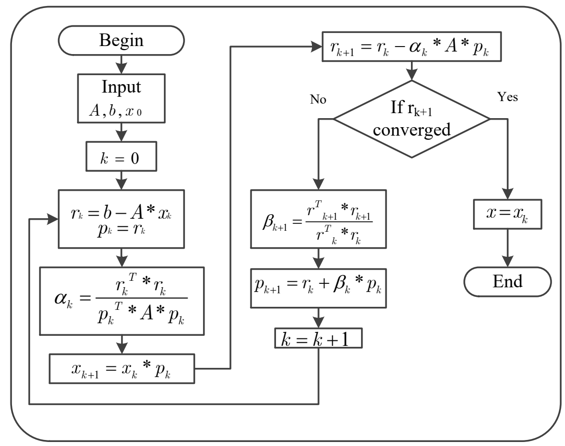

Fig. 2: Flow Chart for Conjugate Gradient Method

A. Test Case1: IEEE 9 Bus System

A standard IEEE 9 Bus system shown in Fig 3 consists of 3 generators, 9 buses, 9 lines and 3 loads. Bus 1 is considered as slack bus. The total number of possible N-1 and N-2 contingencies are 12 and 66 respectively. The number of screened N-2 contingencies is 26. All possible N-1 and screened N-2 contingencies are evaluated based on the performance indexes.

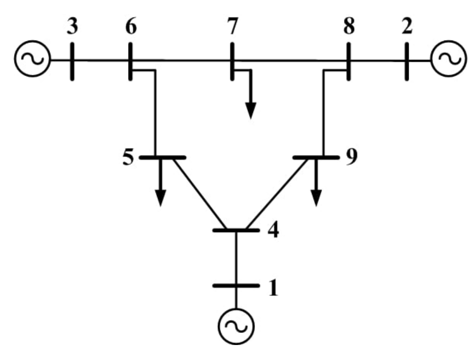

Fig. 3: Network Diagram of IEEE 9 bus system

The list of severe contingencies is given in Table-I and Table-II. In this system lines 4(3-6), and 7(8-2) they result in singular Jacobian matrices. In these cases, convergence of load flow solution is not possible and hence calculation of performance index is not feasible. Such types of contingencies can be considered the most severe contingencies and remaining contingencies ranking list is provided in Table I. From the results, it can be concluded that outage of generator 1, generator 2, line(7-8) are most severe cases for N-1 security assessment.

TABLE I: Set of N-1 Critical Contingencies on IEEE 9 Bus System

N-1 Outage	PI_{pl}	PI_{vi}	PI_{ci}	Rank
Generator 1	8.32	24.61	32.93	1
Generator 2	15.22	3.74	18.96	2
Line(7-8)	12.43	0.48	12.91	3
Line(8-9)	11.77	0.46	12.24	4
Line(9-4)	8.06	3.49	11.55	5
Generator 3	7.64	2.25	9.89	6
Line(5-6)	9.31	0.22	9.53	7
Line(4-5)	5.20	1.12	6.32	8
Line(6-7)	4.73	0.20	4.93	9

TABLE II: Top ten N-2 Critical Contingencies on IEEE 9 Bus System

N-2 Outage		PI_{pl}	PI_{vi}	PI_{ci}	Rank
Outage 1	Outage 2				
Generator 3	Line(7-8)	36.06	144.52	180.58	1
Generator 2	Line(6-7)	18.45	108.87	127.32	2
Generator 1	Line(7-8)	23.33	83.91	107.24	3
Generator 3	Line(4-5)	18.39	54.14	72.53	4
Generator 1	Line(4-5)	8.53	33.40	41.93	5
Generator 2	Line(4-5)	20.28	13.24	33.52	6
Generator 1	Line(9-4)	8.58	18.70	27.28	7
Generator 2	Line(7-8)	17.19	3.50	20.69	8
Generator 2	Line(8-9)	17.16	3.00	20.16	9
Generator 2	Line(5-6)	15.70	3.61	19.31	10

B. Test Case2: IEEE 118 Bus System

A Standard IEEE 118 Bus System consists of 19 Generators, 118 Buses, 186 lines/transformers and 91 loads. Total possible N-1 contingencies are 240 and N-2 contingencies are 28680. All possible N-1and screened N-2 contingencies are tested and top 10 critical contingencies are listed in Table III, IV. In this test case also the outages that result in singular matrix or close to singular matrices are not considered and ranking was done to remaining outages. was done to remaining outages

TABLE III: Top Ten N-1 Critical Contingencies on IEEE 118 Bus System

N-1 Outage	PI_{pl}	PI_{vi}	PI_{ci}	Rank
Line(25-27)	591.39	0.01	591.40	1
Line(8-5)	361.32	0.08	361.40	2
Line(27-115)	309.69	0.02	309.71	3
Line(23-25)	282.01	0.10	282.11	4
Line(26-30)	273.19	0.11	273.30	5
Line(82-83)	177.71	0.01	177.72	6
Line(23-32)	167.56	0.01	167.57	7
Line(38-65)	164.79	0.07	164.86	8
Line(30-17)	163.32	0.03	163.35	9
Line(38-37)	153.67	0.04	153.71	10

TABLE IV: Top 10 N-2 critical contingencies in IEEE 118 Bus System

N-2 contingency		PI_{pl}	PI_{vi}	PI_{ci}	Rank
Outage 1	Outage 2				
Generator 69	Line(98-100)	998.36	0.34	998.7	1
Generator 69	Line(17-113)	997.63	0.36	997.99	2
Generator 69	Line(70-74)	996.88	0.33	997.22	3
Generator 69	Line(49-69)	995.99	0.35	996.35	4
Generator 69	Line(47-69)	994.94	0.35	995.3	5
Generator 69	Line(70-75)	992.46	0.35	992.81	6
Generator 69	Line(19-34)	991.95	0.35	992.3	7
Generator 69	Line(2-12)	989.34	0.78	990.12	8
Generator 69	Line(3-12)	988.61	0.45	989.07	9
Generator 69	Line(17-31)	988.32	0.33	988.65	10

C. Test Case3: NRPG Bus System

NRPG(North Regional Power Grid) Bus System consists of 5 Generators, 246 Buses, 376 lines, 42 generating units. Possible N-1 contingencies, N-2 contingencies are 418, 87153. All N-1 outages are tested and ranking is given in Table-V. Outages in which jacobian matrix is singular or not able to converge are considered most severe contingencies. In Table-V ranking is given to other outages.

TABLE V: Top 10 N-1 critical contingencies in NRPG

N-1 Outage	PI_{pl}	PI_{vi}	PI_{ci}	Rank
Line(140-144)	924.95	1.06E-05	924.95	1
Generator 37	846.58	0.07	846.65	2
Generator 20	632.02	0.06	632.08	3
Line(140-143)	608.18	3.14E-05	608.18	4
Line(139-145)	550.47	5.41E-05	550.47	5
Line(139-152)	404.08	0.0002	404.0802	6
Line(54-55)	394.48	0.001	394.481	7
Line(121-122)	381.12	0.62	381.74	8
Generator 21	281.55	0.12	281.67	9
Generator 19	273.45	0.02	273.47	10

TABLE VI: Top 10 N-2 critical contingencies in NRPG

N-2 contingency		PI_{pl}	PI_{vi}	PI_{ci}	Rank
Outage 1	Outage 2				
Generator 37	Line(165-170)	915.92	0.08	916.00	1
Generator 37	Line(165-37)	874.46	0.07	874.53	2
Generator 37	Line(165-171)	858.28	0.07	858.35	3
Generator 37	Line(175-177)	856.52	0.07	856.57	4
Generator 37	Line(169-170)	853.73	0.07	853.81	5
Generator 37	Line(181-230)	850.14	0.07	850.21	6
Generator 37	Line(238-230)	846.86	0.07	846.95	7
Generator 37	Line(166-175)	846.49	0.07	846.56	8
Generator 37	Line(166-167)	845.51	0.07	845.58	9
Generator 37	Line(181-37)	792.21	0.07	792.28	10

TABLE VII: Elapsed time of different test cases for N-1 ranking using various LE solvers

Test Case	Possible of N-1 Contingencies	LU	cholesky	QR	CGM
IEEE9	87	1.755	1.663	6.1033	1.13
IEEE118	240	18.413	12.727	51.5	12.37
NRPG	418	255.5	146.638	474.59	73.75

TABLE VIII: Elapsed time of different test cases for N-2 ranking using various LE solvers

Test case	Possible N-2 Contingencies	LU	cholesky	QR	CGM
IEEE9	3741	42.2	27.509	180.31	30.64
IEEE118	28680	1692	730	3456	861.45
NRPG	87153	-	-	-	-

V. Conclusion

This paper proposed a screening and ranking strategy of extensive contingencies in faster mode of execution for static security assessment of large power systems. For contingency ranking, a composite performance index based on line flow and bus voltage performance under the contingency conditions has been implemented. To achieve the acceptable result accuracy with faster calculation, FDLF with conjugate gradient linear equation solver has been implemented for the static security assessment of the large power system.

Depending on severity of contingency, remedial action may be selected before the consideration of second outage. In case is known as N-1-1 contingency. The violation in network can be avoided by using different remedial actions such as changing transformer tap position, varying the generator output power and use of shunt series compensation techniques. In the future works,proposed strategy can be implemented for the N-1-1 contingency.

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