Design of Missile Roll Autopilot based on Quantitative Feedback Theory

2023 International Conference on Power, Instrumentation, Energy and Control (PIECON) | 979-8-3503-9976-9/23/$31.00 ©2023 IEEE | DOI: 10.1109/PIECON56912.2023.10085891

Joel Varghese Jacob
Electrical Engineering Department
National Institute of Technology, Warangal
Warangal, India

Murali Mohan Gade
Scientist F
Defense Research and Development Laboratory
Hyderabad, India

R. Jeyasenthil
Electrical Engineering Department
National Institute of Technology, Warangal
Warangal, India

Samran
Scientist B
Defense Research and Development Laboratory
Hyderabad, India

***Abstract*—This paper proposes the use of Quantitative Feedback Theory (QFT) as a controller design method for Roll attitude control of a missile. The main objective of this work is to demonstrate how QFT can be used to design a single controller at all Mach Numbers via an exceedingly simple example adapted from 'Guided Weapon Control Systems by P. Garnell and D.J. East'. As shown, the designed compensator has better Stability Margins and significant improvement in disturbance rejection. Relevant results arising out of simulations obtained in the time and frequency domain will also be discussed in this work.**

***Index Terms*—Non-linear control, QFT, Robust control, Disturbance rejection, Control in missiles**

I. Introduction

Missiles are subject to rolling moments for a variety of reasons throughout its flight. It can be due to accidental rigging errors, asymmetrical loading of lifting and control surfaces or atmospheric disturbances which results in the missile rolling about its axis by some angle. Moreover, Large roll rates cause the pitch and yaw channels to be coupled, hence making design of lateral autopilots very complicated. As such, Latax autopilots are usually designed with the assumption that the missile is roll stabilized, therefore simplifying the related 6-DOF equations, See [9], [13]. Roll attitude control of a missile is hence important to minimize the effect of large rolling disturbance torques, thus decoupling the pitch and yaw channels and simplifying the design process.

It is worth noting that existing literature proposes Roll autopilot design using various modern control strategies such as H_∞ ([3]) , μ analysis ([1], [2]), Higher Order Sliding mode techniques ([10]), LQR and Extended State Observer(ESO) based techniques ([11]). This paper suggests the use of QFT, to design an optimal roll attitude controller at different mach number for an air-to-air missile but it is applicable to air-to-surface and surface-to-air missiles as well. QFT is a technique, first proposed by Horowitz in 1963 for robust control of SISO systems and later extended to MIMO systems, See [6]–[8]. It takes into account various plant uncertainty and employs the nichols chart to propose a Robust controller. In recent times, the development of CAD tools have greatly simplified the bound calculation and loop shaping process ([4]).

In this paper, Quantitative feedback theory is used to systematically design an optimal controller by directly considering different operating points. A simple example of a Roll Autopilot is taken from [9] and a controller is designed for various mach number at an altitude of 1500m. It takes into consideration stability, bandwidth and disturbance rejection specification. The lead-lag compensator proposed is better, compared to that which is proposed by Garnell ([9], hereby referred to as the 'Garnell Compensator') both, in terms of disturbance rejection and also Stability Margins. This paper will be divided into 3 parts, the first part will define the problem and introduce the associated terminology. The second part applies QFT to the problem and will detail the design process. The third part will outline the results, advantages of the QFT controller (compared to the Garnell Compensator) and the scope for future work.

II. Problem Definition

Consider an air-to-air missile with Roll Moment of inertia, A = 0.96 $\mathrm{kg \cdot m^2}$ flying at a fixed altitude of 1500m. The variation of respective aerodynamic derivatives with respect to roll at various mach numbers are shown in table 1. We define also the transfer function between roll rate and aileron deflection in (1).

$$\frac{p}{\xi} = \frac{\frac{-L_\xi}{L_p}}{1 + s\tau_a} \tag{1}$$

The block diagram of the system is shown in Fig. 1. The Roll Autopilot is primarily designed as a disturbance rejection problem rather than a reference tracking one, and therefore the input command will be set to zero. This is not the case for bank-to-turn missiles where roll angle command will be provided to turn the missile. In the figure shown above, L is the disturbance torque in "$\mathrm{N \cdot m}$". L_ξ and L_p are roll aerodynamic

979-8-3503-9976-9/23/$31.00 ©2023 IEEE

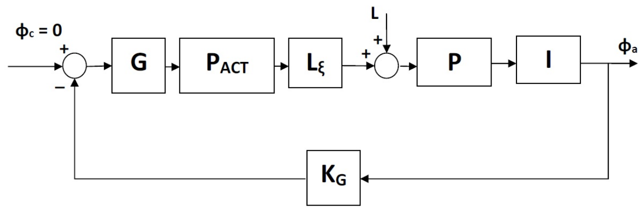

Fig. 1. Block diagram for Roll Autopilot

derivatives, See [9], [13]. The actuator transfer function is denoted by P_{ACT}, where

$$P_{ACT} = \frac{-K_s}{\frac{s^2}{\omega_{ns}^2} + \frac{2\mu_{ns}}{\omega_{ns}} + 1} \tag{2}$$

The natural frequency of the actuator, ω_{ns} will be taken as 180 rad/s. This value can be used to set the open loop gain crossover frequency (Bandwidth) of the autopilot, explained in section 2. μ_{ns}, the damping factor associated with the actuator is taken to be 0.7. K_G is the gyro, whose second-order dynamics we can consider as being approximately 1. I is an integrator that converts the roll rate to achieved roll angle. $K_s = 1.48$ See [9]. According to the transfer function,

$$P = \frac{\frac{-1}{L_p}}{T_a s + 1} \tag{3}$$

From practical experience, disturbance torque is modelled as a first order system response with time constant of 300ms which can readily be obtained if step signal is passed through a Low pass filter with cut-off frequency 10 rad/s. Note that disturbance torque is estimated to have a worst-case value of 1000 Nm at 2.8 Mach, and reduces with velocity. Finally, G is nothing but the controller that will be designed using QFT methodology.

A. The Garnell Compensator

The system, without a controller, is open loop unstable, as shown in bode plot. Fig. 2 shows negative gain and phase margins, which is undesirable. To solve this, the following controller (lead-lag compensator) is proposed in [9]:

$$G(s) = \frac{(1 + 0.05s)(1 + 0.0257s)}{(1 + 0.75s)(1 + 0.0018s)} \tag{4}$$

The bode plot of this controller is shown in Fig. 3. Frequency domain analysis of the open-loop system with this compensator is shown in Table I and Fig. 4.

The step response of the system with the Garnell compensator is also shown in Fig. 10. The disadvantages of this compensator, and the improvements made to it are discussed further.

III. QFT Design Procedure

A. Roll control system requirements

The autopilot should possess good stability margins, excellent disturbance rejection and should not have a bandwidth

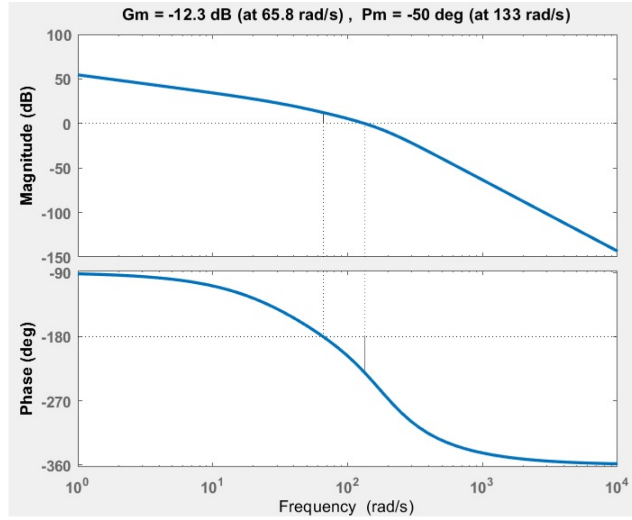

Fig. 2. Bode plot of the System without a controller at 2.8 Mach

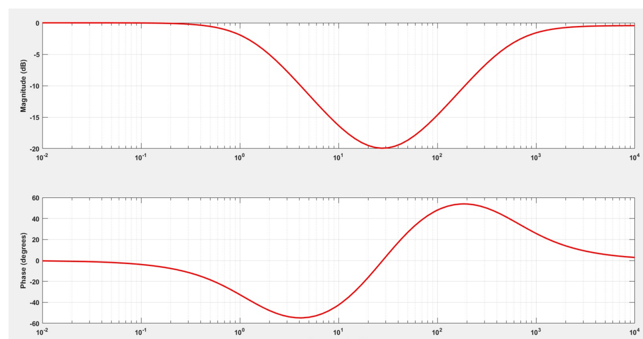

Fig. 3. Bode plot of the Garnell Compensator

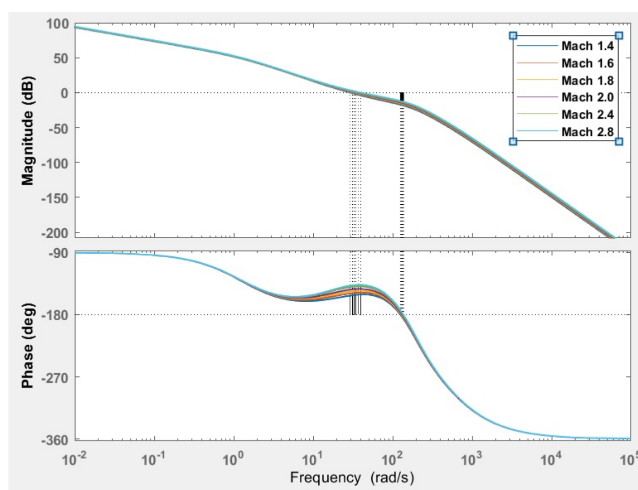

Fig. 4. Bode plot of Open-loop system with the Garnell Compensator at all Mach numbers

TABLE I
FREQUENCY DOMAIN PARAMETERS (GARNELL COMPENSATOR)

Mach	Gain Margin (dB)	Phase Margin (^{o})	Bandwidth (rad/s)
1.4	17.2	27.5	29.3
1.6	16.3	31	31.3
1.8	15.6	34.1	32.8
2.0	14.8	37	34.5
2.4	14	40.8	36.6
2.8	13	43.1	39.8

more than one-third of the actuator. These specifications are more concisely presented below:

1) Phase Margin $>= 30^o$, Gain Margin $>= 6$ dB
2) Steady state roll angle $<= 3^o$, when system is subjected to a disturbance Torque
3) Open-loop gain crossover frequency lies between 36 rad/s and 60 rad/s

B. Plant definition

As mentioned in Section 1, we will be defining the variation in plant parameters at different mach numbers, according to Table 1. The objective will be to design a single controller, G that fits all our specifications. Note that the choice of nominal plant, although unimportant, is the plant at Mach 1.4.

C. QFT Design specifications

To define the QFT specifications, the block diagram is modified as shown in Fig. 5. Note that:

$$P1 = \frac{-K_s L_\xi}{\frac{s^2}{\omega_{ns}^2} + \frac{2\mu_{ns}}{\omega_{ns}} + 1} \quad (5)$$

$$P2 = \frac{\frac{-1}{L_p}}{s(T_a s + 1)} \quad (6)$$

$$M = \frac{1}{0.3s + 1} \quad (7)$$

The QFT specifications are as follows:

1) Stability Margins:

$$\left|\frac{P1(j\omega)P2(j\omega)G(j\omega)}{1 + P1(j\omega)P2(j\omega)G(j\omega)}\right| <= 1.9 \quad (8)$$

for $\omega \; \epsilon$ [1,5,10,30,60,100,150,200] rad/s. This corresponds to a phase margin greater than 30^o

2) Disturbance Rejection:

$$\left|\frac{M(j\omega)P2(j\omega)}{1 + P1(j\omega)P2(j\omega)G(j\omega)}\right| <= \frac{10^{-4}(0.22s + 1)}{0.01s + 1} \quad (9)$$

for $\omega \; \epsilon$ [1,5,10,30] rad/s.

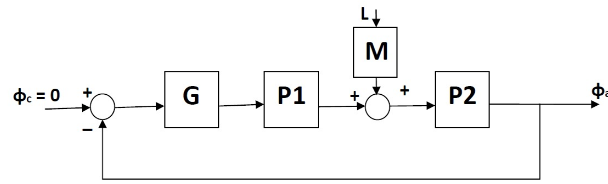

Fig. 5. Modified Block diagram for Roll Autopilot

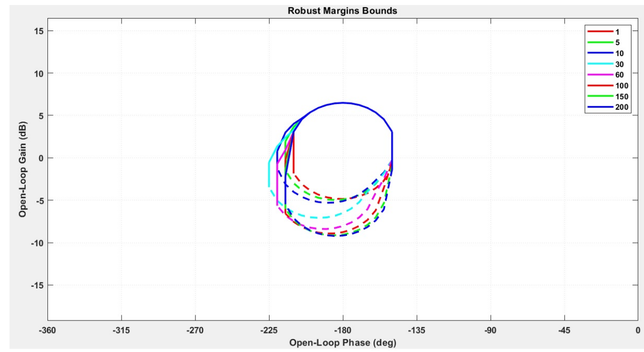

Fig. 6. Robust Stability Bounds

D. QFT Bounds

The equations (8) and (9) are converted into quadratic inequalities. Using an appropriate algorithm, these quadratic inequalities are solved and represented as bounds on the Nichols Plot, See [5], [12]. This process can be efficiently done using modern CAD Toolboxes. The stability bounds and disturbance rejection bounds are found and their intersection is taken at each frequency as shown in Figure 6, 7 and 8. These bounds express the plant model with uncertainty and the specifications at each frequency of interest.

E. Controller Design - Loop Shaping

The open loop nominal plant is used to perform loop shaping. Poles and zeroes are added until the nominal loop lies near the bounds. Note that the respective frequency points on the nominal loop should be below the bounds with dashed lines and above the solid-lined bounds to satisfy the criterion mentioned earlier. To make the design process easier, the Garnell compensator is taken as the initial controller from which further tuning is done to obtain the final QFT controller. A lead-lag compensator with a gain is finally obtained, See (10) and Fig. 9. Since we are not specifying any QFT criteria to meet our bandwidth constraints, we manually check whether our criteria is being met or not using bode plots at the lowest and highest mach numbers, i.e. 1.4 and 2.8.

$$G(s) = \frac{1.1161(s + 45.8)(s + 13.72)}{(s + 395.1)(s + 1.303)} \quad (10)$$

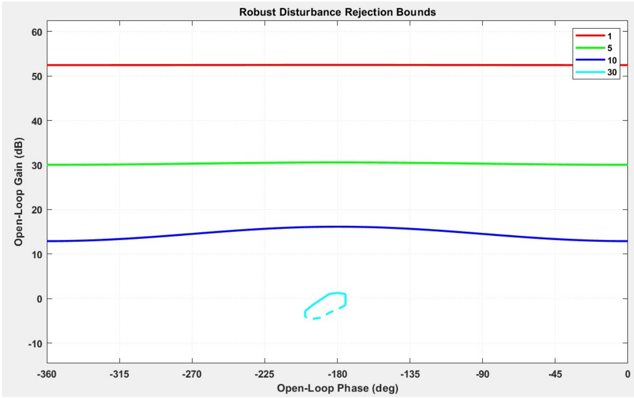

Fig. 7. Robust Disturbance Rejection bounds

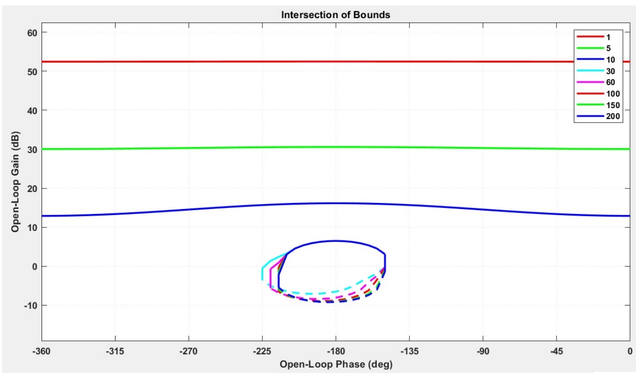

Fig. 8. Intersection of bounds

IV. RESULTS

A. Time Domain

If system is subject to a 1000 Nm disturbance Torque, the following are the steady state values of achieved angle (in degrees), see Table II. It can be seen that at low mach numbers, the roll angle is higher than the value suggested by our requirements but one should note that the disturbance Torque will practically be much lower than 1000 Nm at lower velocities, therefore such high roll angles will not be observed. Further analysis suggests that, for the given problem

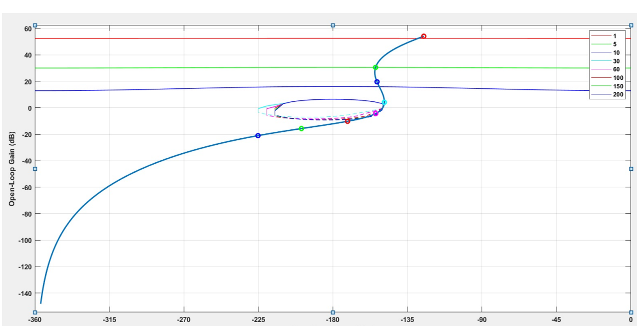

Fig. 9. Loop shaping of QFT controller

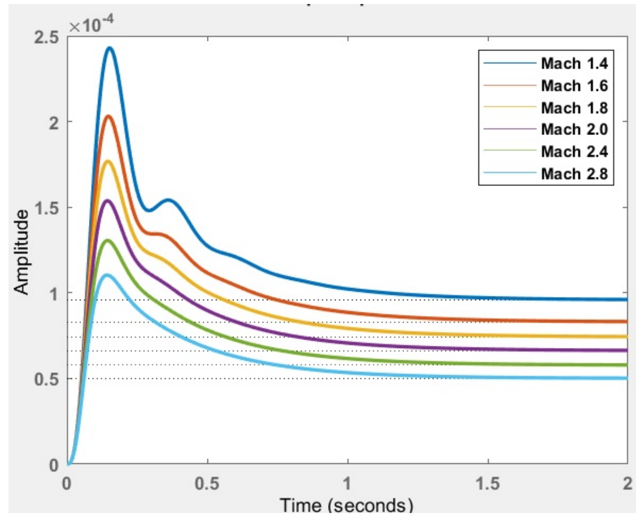

Fig. 10. Step response of Garnell compensator at all mach numbers

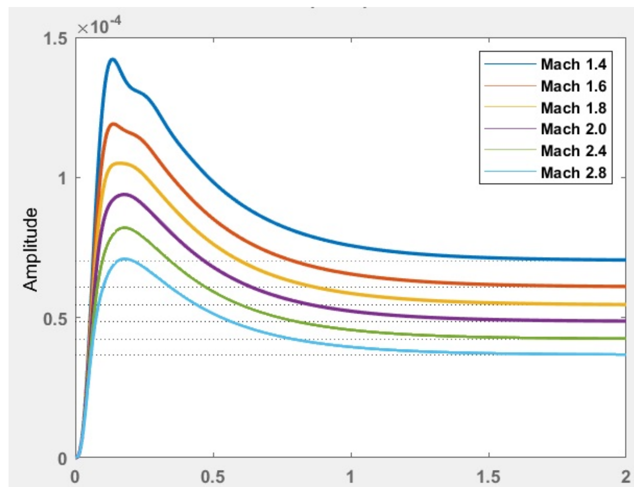

Fig. 11. Step response of QFT compensator at all mach numbers

conditions, our compensator can tolerate up to 750 Nm Rolling Torque at Mach 1.4.

On comparing with the compensator from Garnell, a 25% improvement in disturbance rejection is observed at low frequencies and 35% improvement at higher frequencies (10 rad/s), See Table III. Although Table III only summarizes the data for Mach 2.0, similar results can be observed at all mach numbers considered in this problem. See Fig. 10 and Fig. 11 for step response at different mach numbers. A 22% decrease in overshoot is observed at higher mach numbers while 33% decrease is observed at low mach number. This data is summarized in Table IV.

B. Frequency Domain

The gain margin and phase margin requirements are met for every plant condition. A key specification for us is a phase margin above 30^o. The Garnell compensator violates this requirement at Mach 1.4 whereas the QFT controller satisfies

TABLE II
STEADY STATE VALUE OF ACHIEVED ROLL ANGLE AT DIFFERENT MACH NUMBERS

Mach Number	Achieved roll (in degrees)
1.4	4.03
1.6	3.5
1.8	3.1
2.0	2.8
2.4	2.4
2.8	2.1

TABLE III
IMPROVEMENT IN DISTURBANCE REJECTION FOR DIFFERENT FREQUENCIES AT MACH 2.0

Frequency(rad/s)	Improvement (%)
0	26.58
1	25.66
5	28.00
10	34.82

this requirement at every point, See Fig. 13 and Table V. Gain Margins and bandwidth of both controllers are satisfactory and meet our requirements. As we have committed to designing a lead-lag compensator structure instead of a higher order controller, the open-loop gain crossover frequency is slightly higher than the design criteria, but this can be safely ignored. Results are summarized in Table 6 and The bode plot of the QFT controller is shown in Fig. 12.

V. CONCLUSION AND FUTURE WORK

As we have seen, QFT Design was able to propose a better controller, given some specific criterion. There was significant improvement in the lead-lag compensator that was proposed in terms of disturbance rejection and Stability Margins. A significant increase in disturbance rejection (about 25%-35%) is observed whilst also accurately attenuating high frequency disturbances. It is also clear that using QFT methodology will bring out the system limitations for which remedial action can be taken at the early stages of the design process. Albeit advantageous, Practical systems often have a variety of considerations that have to be taken into account. Conflicting specifications and requirements which are often asked of systems will require the engineer to use Hybrid methodology such as Gain Scheduling along with QFT. Combining such techniques can lead to powerful controllers and Robust Systems designed within a fairly low amount of time.

Future work will detail the use of QFT to build robust controllers at various flight conditions whilst also taking into account the uncertainty of aerodynamic derivatives and other plant parameters.

TABLE IV
IMPROVEMENT IN OVERSHOOT

Mach	Improvement in Overshoot (%)
1.4	33.33
1.6	34.20
1.8	32.82
2.0	29.46
2.4	24.59
2.8	22.50

TABLE V
FREQUENCY DOMAIN PARAMETERS (QFT CONTROLLER)

Mach	Gain Margin (dB)	Phase Margin (^{o})	Bandwidth (rad/s)
1.4	12.1	30.1	41.7
1.6	11.1	32.1	45.3
1.8	10.4	33.8	48.4
2.0	9.7	35.3	51.8
2.4	8.5	37.1	56.3
2.8	7.94	36.6	62.3

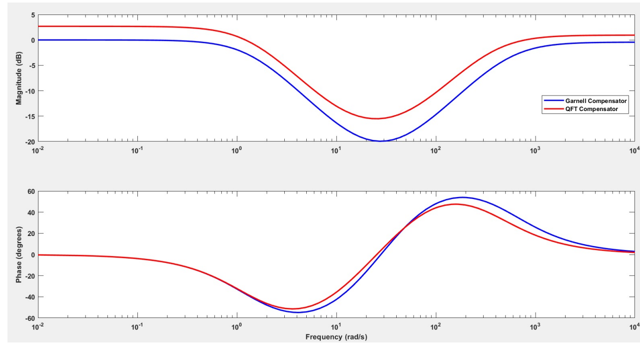

Fig. 12. Comparison of QFT and Garnell controllers

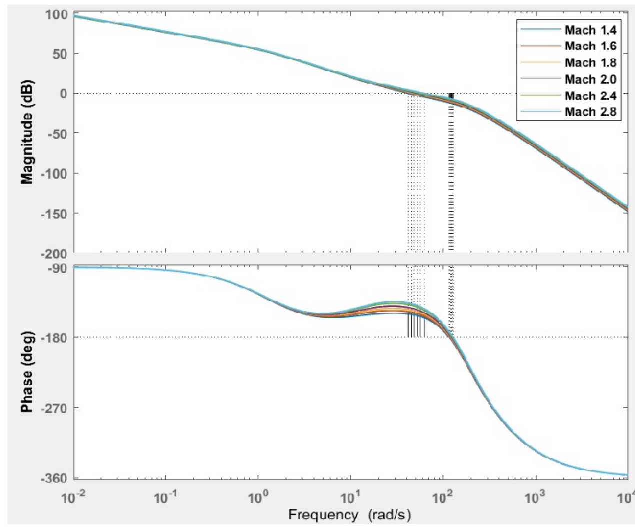

Fig. 13. Bode plot of all plants with QFT controller

ACKNOWLEDGMENT

This Research work was carried out at the Directorate of Systems, Defence Research and Development Laboratory (DRDL), Hyderabad, India.

References

[1] Buschek, H. (1999). Full envelope missile autopilot design using gain scheduled robust control. Journal of Guid- ance Control and Dynamics, 22, 115–122.

[2] Buschek, H. (2003). Design and flight test of a robust autopilot for the iris-t air-to-air missile. Control Engi- neering Practice, 11, 551–558.

[3] Colgren, R. (1984). Nonlinear H control of a missile's roll axis. Proceedings of the American Control Conference, IEEE Publications, 2109–2113.

[4] Craig Borghesani, Yossi Chait, O.Y. (1994). Quantitative feedback theory toolbox users guide.

[5] Garcia-Sanz, M. (2017). Robust Control Engineering: Practical QFT solutions. CRC Press, Taylor and Francis Group, Boca Raton, FL.

[6] Horowitz, I.M. (1963). Synthesis of feedback systems. Academic Press, New York.

[7] Horowitz, I.M. (1991). Survey of quantitative feedback theory (qft). International Journal of Control, 53, 255–291.

[8] I. M. Horowitz, S. (1972). Synthesis of feedback systems with large plant ignorance for prescribed time-domain tolerances. International Journal of Control, 16, 287–309.

[9] P. Garnell, D.E. (1977). Guided Weapon Control Systems. Pergamon Press, Oxford.

[10] Ram B. Sankar, Bijnan Bandyopadhyay, H.A. (2016). Roll autopilot design of a tactical missile using higher order sliding mode technique. Indian Control Conference (ICC), 298–303.

[11] S.E. Talole, A. A. Godbole, J.K. (2011). Design and flight test of a robust autopilot for the iris-t air-to-air missile. Journal of Guidance, Control, and Dynamics, 34, 107–117.

[12] Yossi Chait, O.Y. (1993). Multi-input/single-output computer-aided control design using the quantitative feedback theory. International Journal of Robust and Non-Linear Control, 47–54.

[13] Zarchan, P. (2012). Tactical and strategic missile guidance. In T.C. Lieuwen (ed.), Progress in Astronautics and Aeronautics, volume 239. American Institute of Astronautics and Aeronautics, Reston, VA, 6th edition.