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# Soret and Dofour effects on MHD free convection in a micropolar fluid

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**Abstract** This paper analyzes the flow and heat and mass transfer characteristics of the free convection on a vertical plate with variable wall temperature and concentration in a micropolar fluid in the presence of Soret and Dufour effects. A uniform magnetic field of magnitude $B_0$ is applied normal to the plate. The governing nonlinear partial differential equations are transformed into a system of coupled nonlinear ordinary differential equations using similarity transformations and then solved numerically using the Keller-box method. The numerical results are compared and found to be in good agreement with previously published results as special cases of the present investigation. The non-dimensional velocity, microrotation, temperature and concentration are presented graphically for various values of micropolar parameter, magnetic parameter, Dufour and Soret numbers. In addition, the Nusselt number, the Sherwood number, the skin-friction coefficient, the wall couple stress are shown in a tabular form.

**Keywords** Free convection · Micropolar fluid · Soret and Dufour effects · Heat and mass transfer

**Mathematics Subject Classification** 76A05 · 76E06 · 80A20 · 80E20

## 1 Introduction

Free convection of heat and mass transfer in non-Newtonian fluid have great importance in engineering applications; for instance, the thermal design of industrial equipment dealing with molten plastics, polymeric liquids, foodstuffs, or slurries. Several investigators have extended many of the available convection heat and mass transfer problems to include the non-Newtonian effects. Different models have been proposed to explain the behavior of non-

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Newtonian fluids. Among these, the fluid model introduced by Eringen [1] exhibits some microscopic effects arising from the local structure and micro motion of the fluid elements. Further, they can sustain couple stresses and include classical Newtonian fluid as a special case. The model of micropolar fluid represents fluids consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium where the deformation of the particles is ignored. Micropolar fluids have been shown to accurately simulate the flow characteristics of polymeric additives, geomorphological sediments, colloidal suspensions, haematological suspensions, liquid crystals, lubricants etc. The mathematical theory of equations of micropolar fluids and applications of these fluids in the theory of lubrication and in the theory of porous media are presented by Lukaszewicz [2]. The heat and mass transfer in micropolar fluids is also important in the context of chemical engineering, aerospace engineering and also industrial manufacturing processes. The problem of free convection heat and mass transfer in the boundary layer flow along a vertical surface submerged in a micropolar fluid has been studied by a number of investigators. Yurusoy and Pakdemirli [3] performed group classification of a non-Newtonian fluid flow problem using the classical Lie group approach and the equivalence transformations approach.

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of a more intricate nature. It has been observed that an energy flux can be generated not only by temperature gradients but also by concentration gradients. The energy flux caused by a concentration gradient is termed the diffusion-thermo (Dufour) effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the thermal-diffusion (Soret) effect. In most of the studies related to heat and mass transfer process, Soret and Dufour effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fouriers and Ficks laws. But these effects are considered as second order phenomena and may become significant in areas such as hydrology, petrology, geosciences, etc. The Soret effect, for instance, has been utilized for isotope separation and in mixture between gases with very light molecular weight and of medium molecular weight. The Dufour effect was found to be of order of considerable magnitude so that it cannot be neglected [4]. It is well known that the Soret coefficient has a considerable effect on convection process in liquids. The effects of the thermal-diffusion and the diffusion-thermo on the transport of heat and mass has been developed from the kinetic theory of gases by Chapman and Cowling [5]. Hirshfelder et al. [6] explained the phenomena and derived the necessary formulae to calculate the thermal-diffusion coefficient and thermal-diffusion factor for monatomic gases or for polyatomic gas mixtures.

In recent years, several simple boundary layer flow problems have received new attention within the more general context of magnetohydrodynamics (MHD). MHD flows have many applications in solar physics, cosmic fluid dynamics, geophysics and in the motion of earth’s core as well as in chemical engineering and electronics. Huges and Young [7] gave an excellent summary of applications. Several investigators have extended many of the available boundary layer solutions to include the effects of magnetic fields for those cases when the fluid is electrically conducting. Free convection in electrically conducting fluids through an external magnetic field has diverse applications in the fields such as nuclear reactors, geothermal engineering, liquid metals and plasma flows, petroleum industries, the boundary layer control in aerodynamics and crystal growth. Fluid flow control under magnetic forces is also applicable in magnetohydrodynamic generators and a host of magnetic devices used in industries. Postelnicu [8] has studied the Soret and Dufour effects and influence of magnetic field on heat and mass transfer by natural convection from a vertical surface in porous media. Soret and Dufour effects have been found to appreciably influence the flow field in steady MHD combined free-forced convective and mass transfer flow past a semi-infinite vertical plate [9]. Afify [10] carried out a numerical analysis to study the free convective heat and mass transfer of an incompressible electrically conducting fluid over a stretching sheet in the presence of suction and injection with the Soret and Dufour effects. Rahman [11] studied numerically the magnetic field, Soret and Dufour's effects on a stagnation point flowing over a flat stretching sheet. Makinde [12] considered the mixed convection flow of an incompressible Boussinesq fluid under the simultaneous action of buoyancy and transverse magnetic field with Soret and Dufour effects over a vertical porous plate with constant heat flux embedded in a porous medium. Makinde and Olanrewaju [13] studied unsteady mixed convection with Soret and Dufour effects past a porous plate moving through a binary mixture of chemically reacting fluid. Using the Lie group analysis, Yurusoy [14] obtained similarity solutions for creeping flow and heat transfer in second grade fluids. Pal and Talukdar [15] studied the influence of thermal radiation and first-order chemical reaction on unsteady MHD convective flow, heat and mass transfer of a viscous incompressible electrically conducting fluid past a semi-infinite vertical flat plate in the presence of transverse magnetic field under oscillatory suction and heat source in slip-flow regime. Possible new emerging engineering areas of these type of problems can be found in many industries such as in powder industry and in generating electric power in which electrical energy is extracted directly from moving electrically conducting fluid.

The problem of free convection heat and mass transfer in the boundary layer flow along a vertical surface submerged in a micropolar fluid has been studied by a number of investigators. El-Hakien et al. [16] studied the effects of the viscous and Joule heating on MHD-free convection flows with variable plate temperatures in a micropolar fluid. El-Amin [17] considered MHD free-convection and mass transfer flow in a micropolar fluid over a stationary vertical plate with a constant suction. The heat transfer process in a two-dimensional steady hydromagnetic natural convective flow of a micropolar fluid over an inclined permeable plate subjected to a constant heat flux condition have been analyzed numerically by Rahman [18]. Although the Soret and Dufour effects of the medium on the heat and mass transfer in a micropolar fluid is important, very little work has been reported in the literature. Beg [19] analyzed the two dimensional coupled heat and mass transfer of an incompressible micropolar fluid past a moving vertical surface embedded in a Darcy–Forchheimer porous medium in the presence of significant Soret and Dufour effects. A mathematical model for the steady thermal convection heat and mass transfer in a micropolar fluid saturated Darcian porous medium in the presence of significant Dufour and Soret effects and viscous heating is presented by Rawat and Bhargava [20]. Recently, Srinivasacharya and Ramreddy [21] considered the Soret and Dufour effects on mixed convection in a non-Darcy micropolar fluid. Yurusoy [22] obtained numerical solutions to the unsteady boundary layer equations of non-Newtonian fluids.

Thus motivated by the above investigations and applications mentioned, the aim of the present work is to investigate the effects of transverse magnetic field, Soret and Dufour on the free convection heat and mass transfer along a vertical plate with uniform wall temperature and concentration conditions embedded in a micropolar fluid. The present investigations can be utilized as a basis for studying more complex systems that arise in engineering and industrial application. The governing system of partial differential equations is transformed into a system of non-linear ordinary differential equations using similarity transformations. This system of nonlinear ordinary differential equations is solved numerically using Keller-box method given in Cebeci and Bradshaw [23]. The effects of various parameters on the Velocity, microrotation, temperature and concentration are presented graphically, and skin friction coefficient, wall couple stress, heat and mass transfer rates are tabulated.

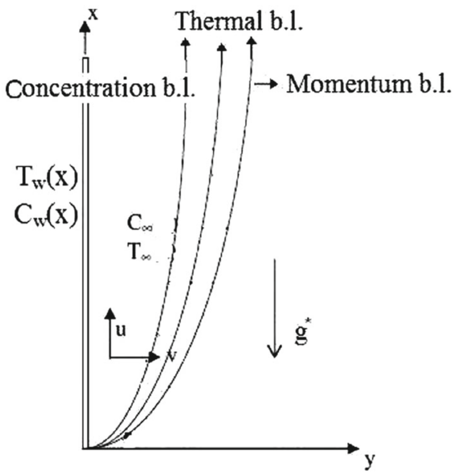


**Fig. 1** Physical model and coordinate systems

## 2 Mathematical formulation

Consider a steady, laminar, incompressible, two-dimensional free convective heat and mass transfer along a semi infinite vertical plate embedded in a free stream of electrically conducting micropolar fluid with temperature $T_\infty$ and concentration $C_\infty$. Choose the co-ordinate system such that $x$-axis is along the vertical plate and $y$-axis normal to the plate. The physical model and coordinate system are shown in Fig. 1. The plate is maintained at temperature $T_w(x)$ and concentration $C_w(x)$. These values are assumed to be greater than the ambient temperature $T_\infty$ and concentration $C_\infty$. A uniform magnetic field of magnitude $B_0$ is applied normal to the plate. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in comparison with the applied magnetic field. In addition, the Soret and Dufour effects are considered.

Using the boussinesq and boundary layer approximations, the governing equations for the micropolar fluid are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu+\kappa}{\rho}\frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho}\frac{\partial \omega}{\partial y} + g^*(\beta_T(T-T_\infty) + \beta_c(C-C_\infty)) - \frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{\gamma}{\rho j}\frac{\partial^2 \omega}{\partial y^2} - \frac{\kappa}{\rho j}\left(2\omega + \frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{D\,K_T}{C_s\,C_p}\frac{\partial^2 C}{\partial y^2} \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + \frac{D\,K_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{5}$$

where $u$ and $v$ are the components of velocity along $x$ and $y$ directions respectively, $\omega$ is the component of microrotation whose direction of rotation lies in the $xy-$plane, $g^*$

is the gravitational acceleration, $T$ is the temperature, $C$ is the concentration, $\beta_T$ is the coefficient of thermal expansions, $\beta_c$ is the coefficient of solutal expansions, $C_p$ is the specific heat capacity, $B_0$ is the coefficient of the magnetic field, $\mu$ is the dynamic coefficient of viscosity of the fluid, $\kappa$ is the vortex viscosity, $\gamma$ is the spin-gradient viscosity, $\sigma$ is the magnetic permeability of the fluid, $\nu$ is the kinematic viscosity, $\alpha$ is the thermal diffusivity, $D$ is the molecular diffusivity, $K_T$ is the thermal diffusion ratio, $C_s$ is the concentration susceptibility, $T_m$ is the mean fluid temperature. We follow the work of many recent authors by assuming that $\gamma = (\mu + \kappa/2)\, j$ [24,25]. This assumption is invoked to allow the field of equations predicts the correct behavior in the limiting case when the microstructure effects become negligible and the total spin $\omega$ reduces to the angular velocity.

The boundary conditions are:

$$\left.\begin{aligned} &u = 0, v = 0, \omega = 0, T = T_w(x), C = C_w(x) \text{ at } y = 0 \\ &u \to 0, \omega \to 0, T \to T_\infty, C \to C_\infty \text{ as } y \to \infty \end{aligned}\right\} \tag{6}$$

where the subscripts $w$ and $\infty$ indicates the conditions at wall and at the outer edge of the boundary layer, respectively.

The continuity equation (1) is satisfied by introducing the stream function $\psi$ such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{7}$$

In order to explore the possibility for the existence of similarity, we assume

$$\left.\begin{aligned} &\psi = Ax^a f(\eta), \eta = Byx^b, \omega = E\,x^c\, g(\eta) \\ &\tfrac{T-T_\infty}{T_w(x)-T_\infty} = \theta(\eta), T_w(x) - T_\infty = M_1 x^l \\ &\tfrac{C-C_\infty}{C_w(x)-C_\infty} = \phi(\eta), C_w(x) - C_\infty = N_1 x^m \end{aligned}\right\} \tag{8}$$

where $A, B, E, M_1, N_1, a, b, c, l$, and $m$ are constants. Substituting (7) and (8) in (2)–(5), it is found that similarity exists only if $a = 1, b = 0, c = 1, l = m = 1$. Hence, appropriate similarity transformations are

$$\left.\begin{aligned} &\psi = A\,x\,f(\eta), \eta = B\,y, \quad \omega = E\,x\,g(\eta), \\ &T = T_\infty + M_1\,x\,\theta(\eta), T_w(x) - T_\infty = M_1\,x, \\ &C = C_\infty + N_1\,x\,\phi(\eta), C_w(x) - C_\infty = N_1\,x \end{aligned}\right\} \tag{9}$$

Making use of the dimensional analysis, the constants $A$, $B$, $E$, $M_1$ and $N_1$ have, respectively, the dimensions of velocity, reciprocal of length, the reciprocal of the product of length and time, the ratio of (temperature/length) and of the ratio (concentration/length).

Substituting (7) and (9) in (2)–(5), we obtain

$$(1 + K)f''' + Kg' + ff'' - (f')^2 + \theta + L\phi - Mf' = 0 \tag{10}$$

$$\left(1 + \frac{K}{2}\right)g'' + fg' - f'g - K\big(2g + f''\big) = 0 \tag{11}$$

$$\frac{1}{Pr}\theta'' + f\theta' - f'\theta + D_f\phi'' = 0 \tag{12}$$

$$\frac{1}{Sc}\phi'' + f\phi' - f'\phi + S_r\theta'' = 0 \tag{13}$$

where $Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $Sc = \frac{\nu}{D}$ is the Schmidth number, $K = \dfrac{\kappa}{\mu}$ is the micropolar parameter, $L = \frac{\beta_c}{\beta_T}\frac{N_1}{M_1}$ is the buoyancy parameter, $M = \frac{\sigma B_0^2}{\mu B^2}$ is the magnetic field parameter, $D_f = \frac{D\, K_T\, N_1}{C_s\, C_p\, \nu M_1}$ is the Dufor number and $S_r = \frac{D\, K_T\, M_1}{T_m\, \nu N_1}$ is the Soret number.

The primes denote differentiation with respect to similarity variable $\eta$.
The boundary conditions (6) in terms of $f, g, \theta$ and $\phi$ becomes

$$\left.\begin{array}{l} \eta = 0 : f(0) = 0, \quad f'(0) = 0, \quad g(0) = 0, \quad \theta(0) = 1, \quad \phi(0) = 1 \\ as \eta \to \infty : f'(\infty) \to 0, \quad g(\infty) \to 0, \quad \theta(\infty) \to 0, \quad \phi(\infty) \to 0 \end{array}\right\} \tag{14}$$

The wall shear stress and the wall couple stress are

$$\tau_w = \left[(\mu + \kappa)\frac{\partial u}{\partial y} + \kappa\,\omega\right]_{y=0} \quad \text{and} \quad m_w = \gamma\left[\frac{\partial \omega}{\partial y}\right]_{y=0} \tag{15}$$

The non-dimensional skin friction $C_f = \frac{2\tau_w}{\rho A^2}$ and wall couple stress $M_w = \frac{B}{\rho A^2} m_w$, where $A$ is the characteristic velocity, are given by

$$C_f = 2\,(1 + K)\, f''(0)\,\overline{x}, \quad \text{and} \quad M_w = \left[1 + \frac{K}{2}\right] g'(0)\overline{x} \tag{16}$$

where $\overline{x} = B\,x$.

The heat and mass transfers from the plate respectively are given by

$$q_w = -k\left[\frac{\partial T}{\partial y}\right]_{y=0} \quad \text{and} \quad q_m = -D\left[\frac{\partial C}{\partial y}\right]_{y=0} \tag{17}$$

The non dimensional rate of heat-transfer, called the Nusselt number $Nu = \frac{q_w}{B\,k\,(T_w - T_\infty)}$ and rate of mass transfer, called the Sherwood number $Sh = \frac{q_m}{D\,B\,[C_w - C_\infty]}$ are given by

$$Nu = -\theta'(0) \quad \text{and} \quad Sh = -\phi'(0). \tag{18}$$

## 3 Results and discussion

The flow Eqs. (10) and (11) which are coupled, together with the energy and concentration Eqs. (12) and (13), constitute non-linear nonhomogeneous differential equations for which closed-form solutions cannot be obtained. Hence the governing Eqs. (10)–(13) have been solved numerically using the Keller-box implicit method [23]. The method has the following four main steps:

(i) Reduce the system of Eqs. (10)–(13) to a first order system;
(ii) Write the difference equations using central differences;
(iii) Linearize the resulting algebraic equations by Newtons method and write them in matrix-vector form;
(iv) Use the block-tridiagonal-elimination technique to solve the linear system.

This method has a second order accuracy, unconditionally stable and is easy to be programmed, thus making it highly attractive for production use. A uniform grid was adopted, which is concentrated towards the wall. The calculations are repeated until some convergent criterion is satisfied and the calculations are stopped when $\delta f''(0) \leq 10^{-8}$, $\delta\theta'(0) \leq 10^{-8}$

**Table 1** Comparison between skin friction $f''(0)$ and $\theta'(0)$ calculated by the present method and that of Merkin [26] for $K = M = S_r = D_f = L = 0$ and $Pr = 1.0$

| $f''(0)$ | | $\theta'(0)$ | |
|---|---|---|---|
| Merkin [26] | Present | Merkin [26] | Present |
| 0.7395 | 0.7394 | −0.5951 | −0.5950 |

and $\delta\phi'(0) \leq 10^{-8}$. In the present study, the boundary conditions for $\eta$ at $\infty$ are replaced by a sufficiently large value of $\eta$ where the velocity, temperature and concentration approach zero. In order to see the effects of step size ($\Delta\eta$ ) we ran the code for our model with three different step sizes as $\Delta\eta = 0.001$, $\Delta\eta = 0.01$ and $\Delta\eta = 0.05$ and in each case we found very good agreement between them on different profiles. After some trials we imposed a maximal value of $\eta$ at $\infty$ of 6 and a grid size of $\Delta\eta$ as 0.01. In order to study the effects of the micropolar parameter $K$, magnetic field parameter $M$, Prandtl number $Pr$ and Schmidt number $Sc$, Dufour number $D_f$ and Soret number $S_r$ on the physical quantities of the flow, the remaining parameters are fixed as $L = 1$, $P_r = 1.0$ and $S_c = 0.24$.

In the absence of micropolar parameter $K$, Magnetic parameter $M$, Soret number $S_r$, Dufour number $D_f$ and buoyancy number $L$ with $Pr = 1.0$ and $Sc = 0.24$ the results have been compared with the exact values [26] and found that they are in good agreement, as shown in Table 1.

Majority of the papers that appears in the literature on Dufour and Soret effects on convective flows do not offer a physical basis to calculate Dufour and Soret coefficients ([27]). But, Benano-Melly et al. [28], while analyzing the problem of thermal diffusion in binary fluid mixtures, lying within a porous medium and subjected to a horizontal thermal gradient, presented list of references on the measurements and the Dufour coefficient. In the present anaysis the values of Soret number $S_r$ and Dufour number $D_f$ are chosen in such a way that their product is constant according to their definition provided that the mean temperature $T_m$ is kept constant. Kafoussias and Williams [29], Anghel et al. [30] and Postelnicu [27] have chosen the values of Soret number $S_r$ and Dufour number $D_f$ such that their product is 0.06. These authors have taken the values of the Dufour numbers as 0.03, 0.037, 0.05, 0.075, 0.6, 0.15 and the Soret numbers as 2.0, 1.6, 1.2, 0.8, 0.1, 0.4. Several authors have used the same combination of values to study the Soret and Dufour effects. In this study the same combination of values are used to investigate the effect on Dufour and Soret numbers on the skin friction, wall couple stress, heat and mass transfer rates. In order to clearly observe Dufour and Soret effects separately on the velocity, microrotation, temperature and concentration profiles of the flow, the analysis is carried out for various values of Soret number $S_r$ ranging from 0 to 3, Dufour number $D_f$ ranging from 0 to 1.2.

Figure 2 depicts the variation of micropolar parameter ($K$) on the non-dimensional velocity, microrotation, temperature and concentration. It is observed from Fig. 2a that the velocity decreases with the increase of $K$. The maximum of velocity decreases in amplitude and the location of the maximum velocity moves farther away from the wall with an increase of $K$. The velocity in case of micropolar fluid is less than that in the viscous fluid case (i.e.$K = 0$). It is seen from Fig. 2b that the microrotation component decreases near the vertical plate and increases far away from the plate with increasing coupling number $K$. For large value of $K$ microrotation is negative near the boundary and away from the boundary it becomes positive. Negative value of microrotation shows the reverse rotation. The reason is that the

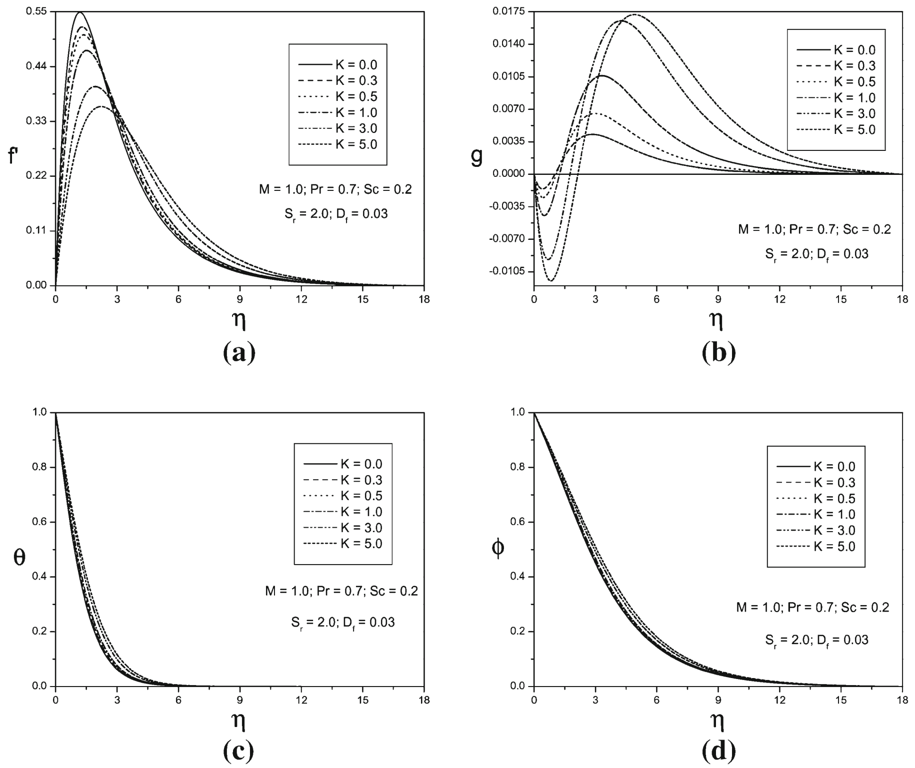


**Fig. 2** **a** Velocity, **b** microrotation, **c** temperature, **d** concentration profiles for various values of micropolar parameter $K$

microrotation field near the plate is dominated by a small number of particles spins that are generated by collisions with the boundary. The microrotation tends to zero as $K \rightarrow 0$. The second term in Eq. (10) shows that a negative microrotation gradient retards the fluid near the plate, while a positive microrotation gradient accelerates the fluid far away from the plate [31]. It is noticed from Fig. 2c that the non-dimensional fluid temperature increases with increasing values of micropolar parameter. It is clear from Fig. 2d that the non-dimensional fluid concentration increases with increasing values of $K$.

The variation of the non-dimensional velocity, microrotation, temperature and concentration profiles with $\eta$ for different values of magnetic parameter is illustrated in Fig. 3. It is observed from Fig. 3a that velocity decreases as the magnetic parameter ($M$) increases. From Fig. 3b, it is clear that the microrotation component increases near the plate and decreases far away from the plate for increasing values of $M$. It is noticed from Fig. 3c that the non-dimensional fluid temperature increases with increasing values of magnetic parameter. It is clear from Fig. 3d that the non-dimensional fluid concentration increases with increasing values of $M$. Application of a uniform magnetic field normal to the flow direction produces a force which acts in the negative direction of flow. This force is called the Lorentz force which tends to slow down the movement of the electrically conducting fluid in the vertical direction. This retardation effect is accompanied by an appreciable increase in the fluid temperature and concentration. These behaviors are clearly depicted in Fig. 3.

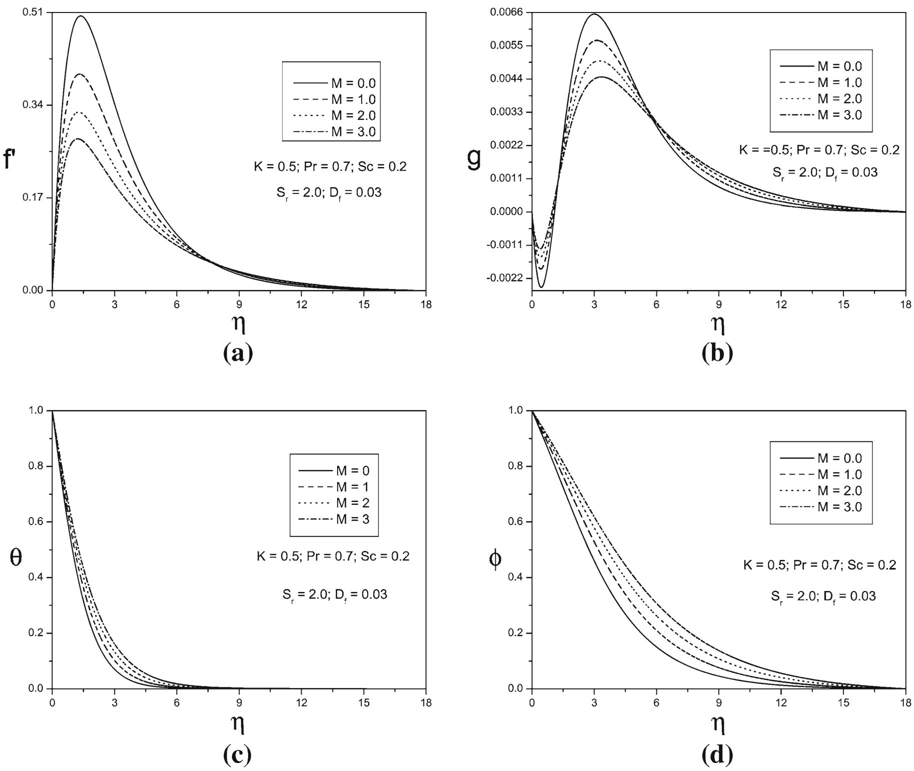


**Fig. 3** **a** Velocity, **b** microrotation, **c** temperature, **d** concentration profiles for various values of magnetic parameter $M$

The effect of Soret number $S_r$ on the non-dimensional velocity, microrotation, temperature and concentration is shown in Fig. 4. It is observed from Fig. 4a that the velocity increases with the increase of Soret number $S_r$. Soret number is the ratio of temperature difference to the concentration. Hence, the bigger Soret number stands for a larger temperature difference and precipitous gradient. Thus the fluid velocity rises due to greater thermal diffusion factor. From Fig. 4b, it is clear that the microrotation component decrease near the vertical plate and increase far away from the plate with increasing of Soret number, showing a reverse rotation near the two boundaries. The reason is that the microrotation field in this region is dominated by a small number of particles spins that are generated by collisions with the boundary. It is noticed from Fig. 4c that the temperature of the fluid decreases with the increase of Soret number. It is clear from Fig. 4d that the non-dimensional concentration of the fluid increasing with increase of Soret number $S_r$.

The effect of Dufour number $D_f$ on the non-dimensional velocity, microrotation, temperature and concentration is shown in Fig. 5. It is observed from Fig. 4a that the velocity increases with the increase of Dufour number $D_f$. From Fig. 4b, it is clear that the micro-rotation component decrease slightly near the vertical plate and increase slightly far away from the plate with increasing of Dufour number, showing a reverse rotation near the two boundaries. It is noticed from Fig. 4c that the temperature of the fluid increases with the increase of Dufour number $D_f$. It is clear from Fig. 4d that the non-dimensional concentration of the fluid decreasing with increase of Dufour number $D_f$. The Dufour number denotes

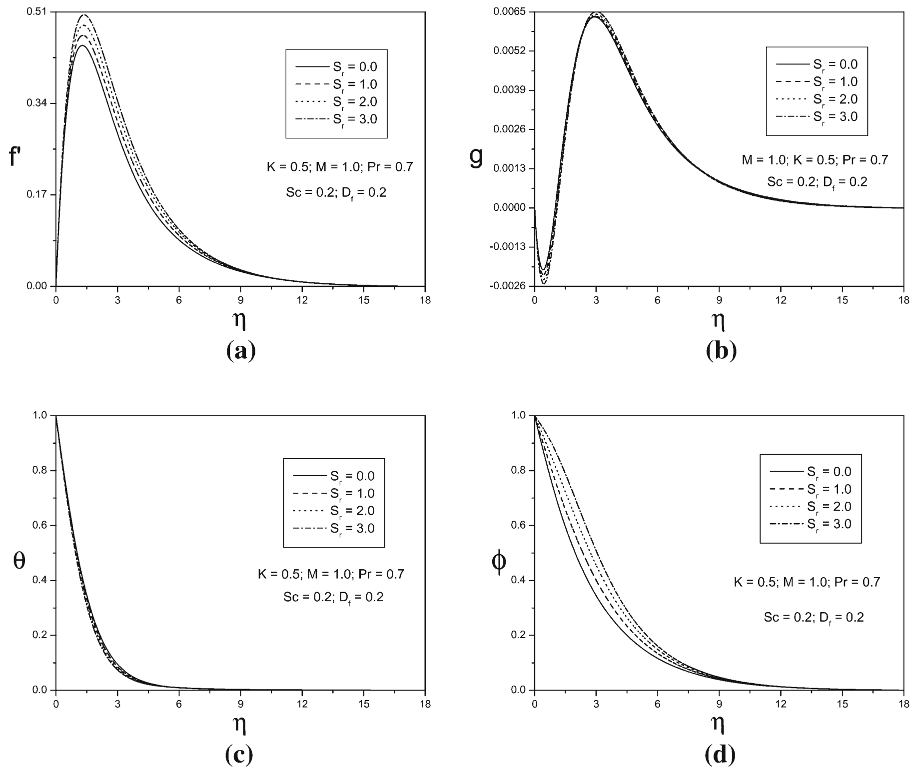


**Fig. 4** **a** Velocity, **b** microrotation, **c** temperature, **d** concentration profiles for various values of Soret number

the contribution of the concentration gradients to the thermal energy flux in the flow. It can be seen that an increase in the Dufour number causes a rise in the velocity and temperature and a drop in the concentration.

Table 2 shows the effects of the micropolar parameter $K$, Prandtl number $Pr$, Schmidt number $Sc$, the magnetic parameter $M$, Dufour number $D_f$ and Soret number $S_r$ on the skin friction $C_f$, dimensionless wall couple stress $M_w$, Nusselt number $Nu$ and Sherwood number $Sh$. It is seen from this table that the skin friction, wall couple stress and heat and mass transfer rates ($Nu$ and $Sh$) decrease with increasing micropolar parameter $K$. For increasing values of $K$, the effect of microstructure becomes significant, hence the wall couple stress decreases. The skin friction coefficient decreases and the wall couple stress increases with increasing Prandtl number and it is interesting to note that the reciprocal situation occurs in the case of heat and mass transfer coefficients. i.e., the Nusselt number increases whereas the Sherwood number decreases as Prandtl number increases. The skin friction coefficient and the Nusselt number decreases and the wall couple stress and Sherwood number increases with Schmidt number. Also, the effect of magnetic parameter is to decrease the skin friction coefficient, Nusselt number and Sherwood number whereas it increase the wall couple stress. Further, it is observed that the skin friction coefficient and the Nusselt number are decreasing and wall couple stress and Sherwood number are increasing with increasing values of Dufour number $D_f$ (or decreasing values of Soret number $S_r$).

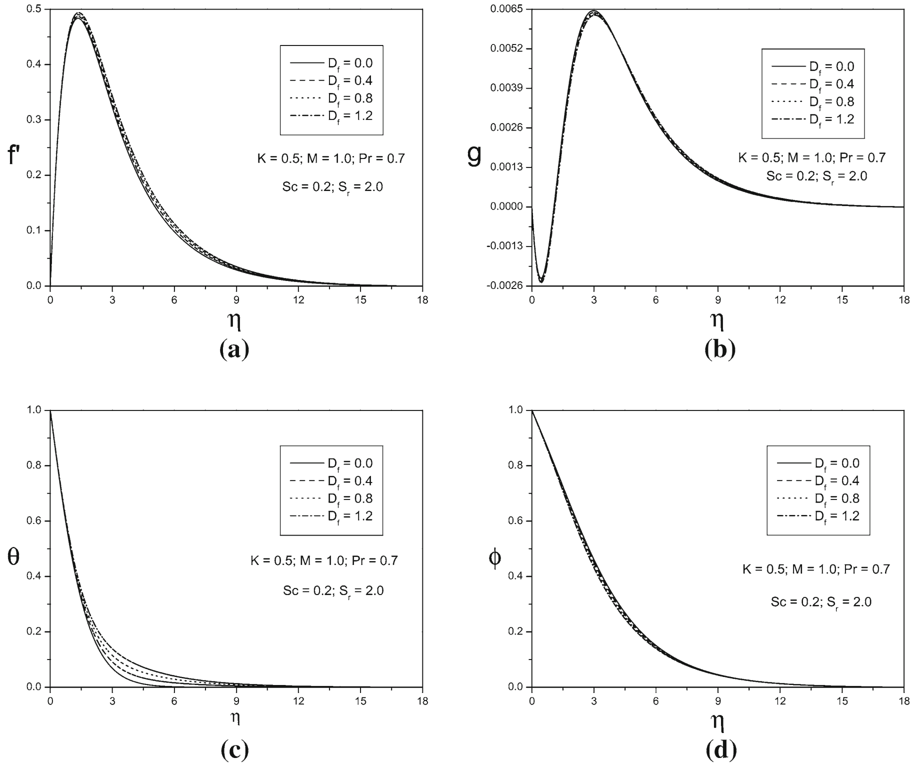


**Fig. 5** **a** Velocity, **b** microrotation, **c** temperature, **d** concentration profiles for various values of Dufour number

## 4 Conclusions

Free convection heat and mass transfer in an electrically conducting micropolar fluid over a vertical plate with magnetic, Soret and Dufour effects is considered. Using the similarity variables, the governing nonlinear partial differential equations are transformed into a system of coupled nonlinear ordinary differential equations and then solved numerically using the Keller-box method.

– The higher values of the micropolar parameter $K$ (i.e., for the case where the effect of microstructure becomes significant) resulting in lower velocity and microrotation distributions but higher wall temperature, wall concentration distributions in the boundary layer compared to the Newtonian fluid case ($K = 0$). The numerical results indicate that the skin friction and wall couple stresses in micropolar fluids are less than those obtained with Newtonian fluids. Also, non-dimensional heat and mass transfer coefficients decrease with the increase of the micropolar parameter.
– An increase in magnetic parameter, decrease the velocity, skin friction coefficient and heat and mass transfer rates accompanied by an increase in temperature and concentration distributions and the local wall couple stress. This is because of the Lorentz force, which tends to slow down the movement of the electrically conducting fluid in the vertical direction. This retardation effect is accompanied by an appreciable increase in the fluid temperature and concentration.

**Table 2** Effect of $K$, $Pr$, $Sc$, $M$, $S_r$ and $D_f$ on skin friction, wall couple stress, heat and mass transfer rates

| $K$ | $Pr$ | $Sc$ | $M$ | $D_f$ | $S_r$ | $f''(0)$ | $-g'(0)$ | $Nu$ | $Sh$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.7 | 0.2 | 1.0 | 0.03 | 2.0 | 1.2236 | 0.0000 | 0.6155 | 0.1809 |
| 0.1 | 0.7 | 0.2 | 1.0 | 0.03 | 2.0 | 1.1554 | 0.0313 | 0.6073 | 0.1804 |
| 0.3 | 0.7 | 0.2 | 1.0 | 0.03 | 2.0 | 1.0431 | 0.0841 | 0.5933 | 0.1794 |
| 0.5 | 0.7 | 0.2 | 1.0 | 0.03 | 2.0 | 0.9538 | 0.1237 | 0.5813 | 0.1784 |
| 1.0 | 0.7 | 0.2 | 1.0 | 0.03 | 2.0 | 0.7927 | 0.1837 | 0.5573 | 0.1763 |
| 3.0 | 0.7 | 0.2 | 1.0 | 0.03 | 2.0 | 0.4974 | 0.2326 | 0.5004 | 0.1695 |
| 5.0 | 0.7 | 0.2 | 1.0 | 0.03 | 2.0 | 0.3758 | 0.2610 | 0.4678 | 0.1643 |
| 0.5 | 0.01 | 0.2 | 1.0 | 0.03 | 2.0 | 1.0595 | 0.1511 | 0.1296 | 0.3576 |
| 0.5 | 0.1 | 0.2 | 1.0 | 0.03 | 2.0 | 1.0264 | 0.1415 | 0.2510 | 0.3102 |
| 0.5 | 0.7 | 0.2 | 1.0 | 0.03 | 2.0 | 0.9538 | 0.1237 | 0.5813 | 0.1784 |
| 0.5 | 1.0 | 0.2 | 1.0 | 0.03 | 2.0 | 0.9386 | 0.1208 | 0.6709 | 0.1431 |
| 0.5 | 7.0 | 0.2 | 1.0 | 0.03 | 2.0 | 0.8605 | 0.1098 | 1.4165 | 0.1114 |
| 0.5 | 10 | 0.2 | 1.0 | 0.03 | 2.0 | 0.8476 | 0.1086 | 1.6291 | 0.0359 |
| 0.5 | 0.7 | 0.2 | 1.0 | 0.03 | 2.0 | 0.9572 | 0.0120 | 0.5803 | 0.1782 |
| 0.5 | 0.7 | 0.4 | 1.0 | 0.03 | 2.0 | 0.9441 | 0.0112 | 0.5714 | 0.1979 |
| 0.5 | 0.7 | 0.6 | 1.0 | 0.03 | 2.0 | 0.9371 | 0.0108 | 0.5663 | 0.2053 |
| 0.5 | 0.7 | 0.8 | 1.0 | 0.03 | 2.0 | 0.9328 | 0.0106 | 0.5630 | 0.2072 |
| 0.5 | 0.7 | 1.0 | 1.0 | 0.03 | 2.0 | 0.9300 | 0.0104 | 0.5607 | 0.2091 |
| 0.5 | 0.7 | 0.2 | 0.0 | 0.03 | 2.0 | 1.1650 | 0.0159 | 0.6505 | 0.2115 |
| 0.5 | 0.7 | 0.2 | 1.0 | 0.03 | 2.0 | 0.9572 | 0.0120 | 0.5803 | 0.1782 |
| 0.5 | 0.7 | 0.2 | 2.0 | 0.03 | 2.0 | 0.8270 | 0.0095 | 0.5280 | 0.1560 |
| 0.5 | 0.7 | 0.2 | 3.0 | 0.03 | 2.0 | 0.7375 | 0.0079 | 0.4876 | 0.1413 |
| 0.5 | 0.7 | 0.2 | 1.0 | 0.03 | 2.0 | 0.9572 | 0.0120 | 0.5803 | 0.1782 |
| 0.5 | 0.7 | 0.2 | 1.0 | 0.0375 | 1.6 | 0.9494 | 0.0118 | 0.5762 | 0.2050 |
| 0.5 | 0.7 | 0.2 | 1.0 | 0.06 | 1.0 | 0.9380 | 0.0115 | 0.5693 | 0.2448 |
| 0.5 | 0.7 | 0.2 | 1.0 | 0.12 | 0.5 | 0.9293 | 0.0112 | 0.5608 | 0.2777 |
| 0.5 | 0.7 | 0.2 | 1.0 | 0.3 | 0.2 | 0.9270 | 0.0111 | 0.5468 | 0.2984 |

– An increase in Dufour number $D_f$ (or decrease in Soret number $S_r$), decrease the velocity, concentration, skin friction coefficient and heat transfer rate but increase the temperature, mass transfer rate and the local wall couple stress.

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