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Magnetic Effect on Free Convection in a Non-Darcy Porous Medium Saturated with Doubly Stratified Power-Law Fluid

Natural convection heat and mass transfer along a vertical plate embedded in a doubly stratified power-law fluid saturated non-Darcy porous medium with uniform heat and mass flux is presented. The governing partial differential equations are transformed into ordinary differential equations using similarity transformations and then solved numerically. The effects of magnetic parameter, stratification parameter and power-law index on the velocity, temperature and concentration are illustrated graphically.

Keywords: hydromagnetic effects, free convection, non-Darcy porous medium, power-law fluid, double stratification

Introduction

Free and forced convection flows in a fluid saturated porous media are of great interest because of their various engineering, scientific and industrial applications in heat and mass transfer which occurs in the fields of design of chemical processing equipment, formation and dispersion of fog, distributions of temperature and moisture over agricultural fields and groves of fruit trees and damage of crops due to freezing and pollution of the environment, grain storage systems, heat pipes, packed microsphere insulation, distillation towers, ion exchange columns, subterranean chemical waste migration, solar power absorbers etc. A number of studies have been reported in the literature focusing on the problem of combined heat and mass transfer in porous media. The analysis of convective transport in a porous medium with the inclusion of non-Darcian effects has also been a matter of study in recent years. Due to its important applications in many fields, a full understanding for combined heat and mass transfer by non-Darcy natural convection from a heated flat surface embedded in fluid saturated porous medium is meaningful. The inertia effect is expected to be important at a higher flow rate and it can be accounted for through the addition of a velocity squared term in the momentum equation, which is known as the Forchheimer's extension of the Darcy law. A detailed review of convective heat transfer in Darcian and non-Darcian porous media including an exhaustive list of references can be found in the book by Nield and Bejan (2006).

The study of flow, heat and mass transfer in non-Newtonian fluids has gained much attention from the researchers because of its engineering and industrial applications such as the thermal design of industrial equipment dealing with molten plastics, polymeric liquids, foodstuffs, or slurries. Also, the non-linear behavior of non-Newtonian fluids in porous matrix is quite different from that of Newtonian fluids in porous media. The prediction of heat or mass transfer characteristics about natural convection of non-Newtonian fluids in porous media is very important due to its practical engineering applications, such as oil recovery and food processing. Several investigators have extended the convection of heat and mass transfer problems to fluids exhibiting non-Newtonian rheology. Different models have been proposed to explain the behavior of non-Newtonian fluids. Among these, the power-law model, which is merely an empirical relationship between the stress and velocity gradients, has been successfully applied to non-Newtonian fluids experimentally. Chen and Chen (1988) have studied the natural convection of a non-Newtonian fluid about a horizontal cylinder and sphere in a porous medium. Pascal and Pascal (1997) have considered the free convection in a non-Newtonian fluid saturated

porous medium with lateral mass flux. Free convection heat and mass transfer of non-Newtonian power law fluids with yield stress from a vertical flat plate in a saturated porous media was studied by Rami and Anna (2000). The flow of natural convection heat and mass transfer of non-Newtonian power law fluids with yield stress in porous media from a vertical plate with variable wall heat and mass fluxes was considered by Cheng (2006). Buoyant convection of power-law fluid in an enclosure filled with heat-generating porous media was considered by Kim and Hyun (2004). The study of free convection in boundary layer flows of power law fluids past a vertical flat plate with suction/injection was done by Sahu and Mathur (1996). Free convection from a horizontal line heat source in a power-law fluid-saturated porous medium was studied by Nakayama (1993).

There has been a renewed interest in MHD flow and heat transfer in porous and clear domains due to the important effect of magnetic field on the boundary layer flow control and on the performance of many systems using electrically conducting fluid such as MHD power generators, the cooling of nuclear reactors, plasma studies, purification of molten metals from non-metallic inclusion, geothermal energy extractions etc. Many problems of MHD Darcian and non-Darcian flow of Newtonian as well as non-Newtonian fluid in porous media have been analyzed and reported in the literature. Non-Darcy mixed convection in power-law fluids along a non-isothermal horizontal surface in a porous medium has been analyzed by Kumari and Nath (2004). The Effect of magnetic field on non-Darcy axisymmetric free convection in a power-law fluid saturated porous medium with variable permeability has been considered by Mansour and El-Shaer (2002).

Stratification of fluid arises due to temperature variations, concentration differences or the presence of different fluids. In practical situations where the heat and mass transfer mechanisms run parallel, it is interesting to analyze the effect of double stratification (stratification of the medium with respect to the thermal and concentration fields) on the convective transport in a fluid. The analysis of free convection in a doubly stratified medium is a fundamentally interesting and important problem because of its broad range of engineering applications. The applications include heat rejection into the environment such as lakes, rivers and the seas; thermal energy storage systems such as solar ponds and heat transfer from thermal sources such as the condensers of power plants. Cheng (2009) considered the combined heat and mass transfer in natural convection flow from a vertical wavy surface in a power-law fluid saturated porous medium with thermal and mass stratification. Laxmi Narayana and Murthy (2007) have considered free convective heat and mass transfer in a doubly stratified porous medium saturated with a power law fluid.

From the literature survey, it seems that the problem of natural convection heat and mass transfer from vertical plate in non-Darcy

porous media saturated with power-law fluids with double stratification and magnetic effects has not been investigated so far. Thus this work aims to study the effects of double stratification on natural convection in a power-law fluid saturated non-Darcy porous medium with uniform heat and mass flux.

Nomenclature

A	= slope of ambient temperature
B	= slope of ambient concentration
B_0	= magnetic field strength
C	= concentration
$C_{\infty,0}$	= ambient concentration
c	= empirical constant
D	= solutal diffusivity
f	= reduced stream function
G	= modified Darcy parameter
g	= gravitational acceleration
K	= permeability constant
k	= thermal conductivity
Le	= lewis number
M	= magnetic parameter
N	= buoyancy ratio
n	= power-law index
q_w, q_m	= heat, Mass transfers from the plate
Ra_x	= the Darcy-Rayleigh number
T	= temperature
$T_{\infty,0}$	= ambient temperature
u, v	= Darcian Velocity components in x and y directions
x, y	= coordinates along and normal to the plate

Greek Symbols

α	= thermal diffusivity
β_T, β_C	= coefficients of thermal and solutal expansion
η	= similarity variable
θ	= dimensionless temperature
ϕ	= dimensionless concentration
ν	= kinematic viscosity
μ_e	= magnetic permeability
ρ	= density of the fluid
ψ	= stream function
σ	= electrical conductivity of the fluid
$\varepsilon_1, \varepsilon_2$	= thermal and solutal stratification parameters

Subscripts

∞	ambient condition
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Superscript

$'$	differentiation with respect to η
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Mathematical Formulation

Consider the two dimensional free convection flow of an electrically conducting fluid from the vertical flat plate in a doubly stratified non-Newtonian power-law fluid saturated non-Darcy porous medium. The x -axis is taken along the plate and y -axis normal to it. The physical model and coordinate systems are shown in Fig. 1. The porous medium is considered to be homogeneous and isotropic (i.e. uniform with a constant porosity and permeability) and is saturated with a fluid which is in local thermodynamic equilibrium with the solid matrix. The fluid has constant properties except the density in the buoyancy term of the balance of

momentum equation. The fluid flow is moderate and the permeability of the medium is low so that the Forchheimer flow model is applicable and the boundary effect is neglected. The plate is maintained at constant heat flux q_w and constant mass flux q_m . The ambient medium is assumed to be vertically non-linearly stratified with respect to both temperature and concentration in the form $T_{\infty}(x) = T_{\infty,0} + Ax^m$ and $C_{\infty}(x) = C_{\infty,0} + Bx^l$ respectively, where A and B are constants and varied to alter the intensity of stratification in the medium. A uniform magnetic field is applied normal to the plate. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected.

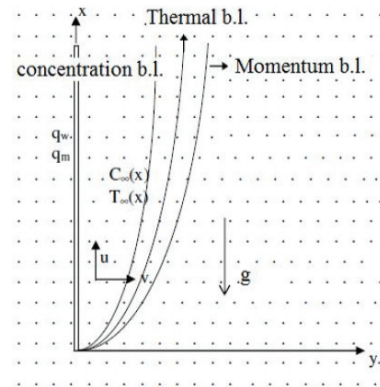


Figure 1. Physical model and coordinate system.

Using the Boussinesq and boundary layer approximations, the governing equations for the power-law fluid are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u^n + \frac{\sigma B_0^2 \mu_e^2 K}{\rho \nu} u + \frac{c \sqrt{K}}{\nu} u^2 = \frac{Kg}{\nu} (\beta_T (T - T_{\infty}) + \beta_C (C - C_{\infty})) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (4)$$

where u and v are Darcian velocity components along x and y directions respectively, T is the temperature, C is the concentration, n is the power-law index, K is the permeability, c is an empirical constant, g is the acceleration due to gravity, ν is the kinematic viscosity, ρ is the density, α is the thermal diffusivity and D is the solutal diffusivity of the medium, σ is the electrical conductivity of the fluid, μ_e is the magnetic permeability, B_0 is the strength of the magnetic field, β_T is the coefficient of thermal expansion, and β_C is the coefficient of solutal expansion.

The boundary conditions are

$$v = 0, q_w = -k \frac{\partial T}{\partial y}, q_m = -D \frac{\partial C}{\partial y} \text{ at } y = 0 \quad (5a)$$

$$u = 0, T = T_{\infty}(x), C = C_{\infty}(x) \text{ as } y \rightarrow \infty \quad (5b)$$

where k is the thermal conductivity of the fluid.

In view of the continuity of Eq. (1), we introduce the stream function ψ by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (6)$$

Substituting Eq. (6) in Eqs. (2)-(4) and then using the following similarity transformations

$$\left. \begin{aligned} \eta &= \frac{y}{x} Ra_x^{n/(2n+1)}, \quad \psi = \alpha Ra_x^{n/(2n+1)} f(\eta), \\ \theta(\eta) &= \frac{k(T - T_\infty(x))}{q_w x} Ra_x^{n/(2n+1)}, \\ \phi(\eta) &= \frac{D(C - C_\infty(x))}{q_m x} Ra_x^{n/(2n+1)} \end{aligned} \right\} \quad (7)$$

we get the following system of equations

$$(f')^n + M f' + G(f')^2 = \theta + N\phi \quad (8)$$

$$\theta'' = \frac{1}{(2n+1)} \left(n f' \theta - (n+1) f \theta' + \varepsilon_1 f' \right) \quad (9)$$

$$\phi'' = \frac{Le}{(2n+1)} \left(n f' \phi - (n+1) f \phi' + \varepsilon_2 f' \right) \quad (10)$$

where the primes indicate partial differentiation with respect to η alone,

$$Ra_x = \frac{x}{\alpha} \left(\frac{K g \beta_T q_w x}{k \nu} \right)^{1/n}$$

is the Darcy-Reyleigh number, n is the

$$\text{power-law index parameter, } M = \frac{\sigma \mu_e^2 B^2 K}{\rho \nu} \left(\frac{x}{\alpha} \right)^{(n-1)} Ra_x^{-2n(n-1)/(2n+1)}$$

$$\text{is the magnetic parameter, } Le = \frac{\alpha}{D} \text{ is the Lewis number,}$$

$$G = \frac{c \sqrt{K}}{\nu} \left(\frac{x}{\alpha} \right)^{(n-2)} Ra_x^{-2n(n-1)/(2n+1)} \text{ is the modified Darcy}$$

$$\text{parameter and } N = \frac{\beta_C q_m k}{\beta_T q_w D} \text{ is the buoyancy ratio.}$$

$$\varepsilon_1 = \frac{(2n+1)k}{q_w} Ra_x^{n/(2n+1)} \frac{\partial T_\infty(x)}{\partial x} \text{ and } \varepsilon_2 = \frac{(2n+1)D}{q_m} Ra_x^{n/(2n+1)} \frac{\partial C_\infty(x)}{\partial x}$$

are the thermal and solutal stratification parameters. These parameters ε_1 and ε_2 will be independent of x only when $m=l=\frac{n}{(2n+1)}$, i.e., when $T_\infty(x) = T_{\infty,0} + A x^{n/(2n+1)}$ and $C_\infty(x) = C_{\infty,0} + B x^{n/(2n+1)}$, allowing similarity solution.

The boundary conditions (5) in terms of f , θ and ϕ become

$$f(0)=0, \theta'(0)=-1, \phi'(0)=-1 \quad (11a)$$

$$f'(\infty)=0, \theta(\infty)=0, \phi(\infty)=0 \quad (11b)$$

Numerical Procedure

The flow Eq. (8) coupled with the energy and concentration Eqs. (9) and (10) constitute a set of nonlinear non-homogeneous differential equation for which closed-form solution cannot be obtained. Hence, the problem has been solved numerically using shooting technique along with fourth order Runge-Kutta integration. The basic idea of shooting method for solving boundary value problem is to try to find appropriate initial condition for the computed solution "hit the target", so that the boundary conditions at other points are satisfied. Furthermore, the higher order non-linear differential equations are converted into simultaneous linear differential equations of first order and they are further transformed into initial valued problem applying the shooting method incorporating fourth order Runge-Kutta method. The iterative solution procedure was carried out until the error in the solution became less than a predefined tolerance level.

The non-linear differential equations (8)-(10) are converted into the following system of linear differential equations of first order by the substitution

$$\{f, \theta, \phi, f', \theta', \phi'\} = \{z_1, z_2, z_3, z_4, z_5, z_6\}$$

$$\frac{dz_1}{d\eta} = z_4, \frac{dz_2}{d\eta} = z_5, \frac{dz_3}{d\eta} = z_6,$$

$$\frac{dz_4}{d\eta} = \frac{z_5 + N z_6}{n(z_4)^{n-1} + M + 2G z_4},$$

$$\frac{dz_5}{d\eta} = \frac{n z_3 z_4 - (n+1) z_1 z_5 + \varepsilon_1 z_4}{(2n+1)},$$

$$\frac{dz_6}{d\eta} = \frac{Le[n z_3 z_4 - (n+1) z_1 z_6 + \varepsilon_2 z_4]}{(2n+1)} \quad (12)$$

The boundary conditions in terms of $z_1, z_2, z_3, z_4, z_5, z_6$ are

$$z_1(0) = 0, z_2(\infty) = 0, z_3(\infty) = 0,$$

$$z_4(\infty) = 0, z_5(0) = -1, z_6(0) = -1 \quad (13)$$

Here, η at ∞ is taken as η_{\max} and chosen large enough so that the solution shows little further change for η larger than η_{\max} .

As the initial values for z_2, z_3 and z_4 are not specified in the boundary conditions (13), assume some values for $z_2(0), z_3(0)$ and $z_4(0)$. Then the equations (12) are integrated using the 4th order Runge-Kutta method from $\eta=0$ to $\eta=\eta_{\max}$ over successive steps $\Delta\eta$. The accuracy of the assumed initial values $z_2(0), z_3(0)$ and $z_4(0)$ is then checked by comparing the calculated values of $z_2(0), z_3(0)$ and $z_4(0)$ at $\eta=\eta_{\max}$ with their given value at $\eta=\eta_{\max}$ in (13). If a difference exists, another set of initial values for $z_2(0), z_3(0)$ and $z_4(0)$ must be assumed and the process is repeated. This

process is continued until the agreement between the calculated and the given condition at $\eta = \eta_{\max}$ is within the specified degree of accuracy.

In the present study, η_{\max} has been suitably chosen at each time such that the velocity, temperature and concentration profiles approach zero at the outer edge of the boundary layer. Extensive calculations have been performed to obtain the wall velocity, temperature and concentration fields for a wide range of parameters. The effect of thermal stratification parameter, solutal stratification parameter, magnetic parameter, non-Darcy parameter, and power law index parameter is studied on the velocity, temperature and concentration fields for uniform wall heat and mass flux condition is plotted for some selected combinations of parameter values.

Results and Discussion

The non-dimensional velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$ are plotted for $N = 0.5$, $M = 1$, $Le = 0.5$, $n = 0.5$, $G = 0.2$, $\varepsilon_2 = 0.4$ in Figs. 2-4 with varying thermal stratification parameter. It can be observed from Fig. 2 that the velocity of the fluid decreases with the increase of thermal stratification parameter. The thermal stratification reduces the effective convective potential between the heated plate and the ambient fluid in the porous medium. Hence, the thermal stratification effect reduces the velocity in the boundary layer. It can be noted from Fig. 3 that the temperature of the fluid is decreased with the increase in the value of the thermal stratification parameter. When the thermal stratification effect is considered, the effective temperature difference between the plate and the ambient fluid will decrease. Therefore, the thermal boundary layer is thickened and the temperature is reduced. It can be found from Fig. 4 that the concentration of the fluid is increased with increase in the value of the thermal stratification parameter.

The effect of solutal stratification parameter on the non-dimensional velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$ for $N = 0.5$, $M = 1$, $Le = 0.5$, $n = 0.5$, $G = 0.2$, $\varepsilon_1 = 0.8$ is depicted in Figs. 5-7. It is observed from Fig. 5 that the fluid velocity is decreased with increase in the value of solutal stratification parameter. It can be seen from Fig. 6 that the temperature of the fluid in the medium is increased with increase in the value of the solutal stratification parameter. It can be found from Fig. 7 that the concentration of the fluid is decreased by increasing the value of the solutal stratification parameter.

The variation of the non-dimensional velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$ for $N = 0.5$, $Le = 0.5$, $n = 0.5$, $G = 1.0$, $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.8$ with magnetic parameter is shown in Figs. 8-10. It can be observed from Fig. 8 that the velocity of the fluid is decreased with increase in the value of the magnetic parameter. This is due to the fact that the introduction of a transverse magnetic field, normal to the flow direction, has a tendency to create the drag known as the Lorentz force which tends to resist the flow. Hence, the horizontal velocity profiles decrease as the magnetic parameter M increases. It can be found from Fig. 9 that increase in the value of the magnetic parameter increases the temperature of the fluid in the medium. It can be seen from Fig. 10 that the concentration of the fluid is increased by increasing the value of the magnetic parameter. As explained above, the transverse magnetic field gives rise to a resistive force known as the Lorentz force of an electrically conducting fluid. This force makes the fluid experience a resistance by increasing the friction between its layers and thus increases its temperature and concentration.

The effect of non-Darcy parameter on the non-dimensional velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$ for $N = 0.5$, $M = 1$, $Le = 0.5$, $n = 0.5$, $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.8$ is plotted in Figs. 11-13. It is observed from Fig. 11 that velocity of the fluid is decreased with increase in the value of the non-Darcy parameter. The increase in non-Darcy parameter implies that the porous medium is offering more resistance to the fluid flow. This results in reduction in the velocity profiles. It can be noted from Fig. 12 that by increasing the value of the non-Darcy parameter, there is an increase in the temperature in the fluid medium. It can be seen from Fig. 13 that the concentration of the fluid is increased if there is an increase in the value of the non-Darcy parameter. The increase in non-Darcy parameter reduces the intensity of the flow and increases the thermal and concentration boundary layer thicknesses.

The non-dimensional velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$ for $N = 0.5$, $M = 1$, $Le = 0.5$, $G = 0.2$, $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.8$ with a variation in power law index parameter is plotted in Figs. 14-16. It is observed from Fig. 14 that the fluid velocity is increased with increase in the value of the power law index parameter. The effect of the increasing values of the power law index n is to increase the horizontal boundary layer thickness. That is, the thickness is much smaller for shear thinning (pseudo plastic; $n < 1$) fluids than that of shear thickening (dilatants; $n > 1$) fluids. In the case of a shear thinning fluid ($n < 1$), the shear rates near the walls are higher than those for a Newtonian fluid. It can be seen from Fig. 15 that the temperature in the fluid is decreased with increase in the value of the power law index parameter. Increasing the values of the power law index leads to thinning of the thermal boundary layer thickness. It can be found from Fig. 16 that the concentration of the fluid increases with increase in the value of the power law index parameter. Increasing the power-law index (n) tends to retard the flow and increase the solutal boundary-layer thickness.

Conclusions

In this paper, a boundary layer analysis for free convection heat and mass transfer along a vertical plate in a non-Darcy porous media saturated with power-law fluid with uniform heat and mass flux conditions in the presence of magnetic field and double stratification is presented. Using the similarity variables, the governing equations are transformed into a set of ordinary differential equations, where numerical solution has been presented for a wide range of parameters. The higher values of the thermal stratification parameter result in lower velocity and temperature distributions, but higher concentration distribution. The higher values of the solutal stratification parameter result in lower velocity and concentration distributions, but higher temperature distribution. An increase in the values of the magnetic parameter results in lower velocity distribution, but higher temperature and concentration distributions. The same nature can be found in the case of non-Darcy parameter. Also, the higher values of the power-law index number result in lower velocity and temperature distributions, but higher concentration distribution within the boundary layer.

References

- Nield, D.A. and Bejan, A., 2006, "Convection in porous media", 3rd edition, New-York, Springer-Verlag.
- Chen, H.T. and Chen, C.K., 1988, "Natural convection of a non-Newtonian fluid about a horizontal cylinder and sphere in a porous medium", *Int. Commun. Heat Mass Transfer*, Vol. 15, pp. 605-614.
- Pascal, J.P. and Pascal, H., 1997, "Free convection in a non-Newtonian fluid saturated porous medium with lateral mass flux", *Int. J. Non-Linear Mechanics*, Vol. 32, pp. 471-482.

Rami, Y.J. and Anna, S.M., 2000, "Free convection heat and mass transfer of non-Newtonian power law fluids with yield stress from a vertical flat plate in a saturated porous media", *Int. Commun. Heat Mass Transfer*, Vol. 27, pp. 485-494.

Ching-Yang Cheng, 2006, "Natural convection heat and mass transfer of non-Newtonian power law fluids with yield stress in porous media from a vertical plate with variable wall heat and mass fluxes", *Int. Commun. Heat Mass Transfer*, Vol. 33, pp. 1156-1164.

Kim, G.B and Hyun, J.M., 2004, "Buoyant convection of power-law fluid in an enclosure filled with heat-generating porous media", *Numerical Heat Transfer Part A: Applications*, Vol. 45, pp. 569-582.

Sahu, A.K. and Mathur, M.N., 1996, "Free convection in boundary layer flows of power law fluids past a vertical flat plate with suction/injection", *Indian Journal of Pure and Applied Mathematics*, Vol. 27, 931 p.

Nakayama, A., 1993, "Free convection from a horizontal line heat source in a power-law fluid saturated porous medium", *Int. J. of Heat and Fluid Flow*, Vol. 14, pp. 279-283.

Kumari, M. and Nath, G., 2004, "Non-Darcy mixed convection in power-law fluids along a nonisothermal horizontal surface in a porous medium", *Int. J. of Engineering Science*, Vol. 42, pp. 353-369.

Mansour, M.A. and El-Shaar, N.A., 2002, "Effect of magnetic field on non-Darcy axisymmetric free convection in a power-law fluid saturated porous medium with variable permeability", *Journal of Magnetism and Magnetic Materials*, Vol. 250, pp. 57-64.

Ching-Yang Cheng, 2009, "Combined heat and mass transfer in natural convection flow from a vertical wavy surface in a power-law fluid saturated porous medium with thermal and mass stratification", *Int. Comm. Heat Mass Transfer*, Vol. 36, pp. 351-356.

Lakshmi Narayana, P.A and Murthy, P.V.S.N., 2007, "Free convective heat and mass transfer in a doubly stratified porous medium saturated with a power law fluid", *Int. J. of Fluid Mechanics Research*, Vol. 36, pp 524-537.

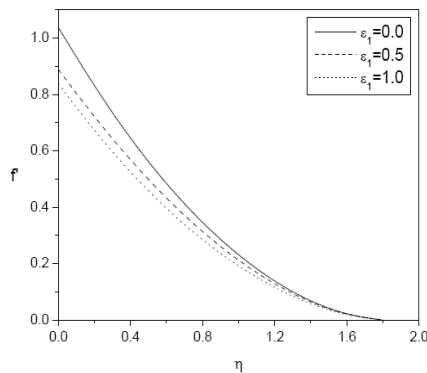


Figure 2. Velocity profiles for various values of ε_1 with $N = 0.5$, $M = 1$, $Le = 0.5$, $n = 0.5$, $G = 0.2$, $\varepsilon_2 = 0.4$.

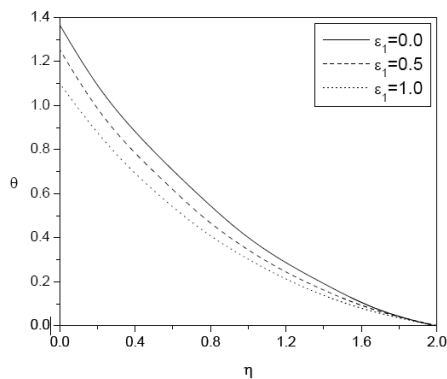


Figure 3. Temperature profiles for various values of ε_1 with $N = 0.5$, $M = 1$, $Le = 0.5$, $n = 0.5$, $G = 0.2$, $\varepsilon_2 = 0.4$.

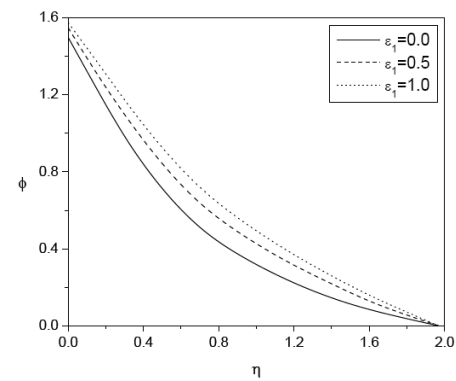


Figure 4. Concentration profiles for various values of ε_1 with $N = 0.5$, $M = 1$, $Le = 0.5$, $n = 0.5$, $G = 0.2$, $\varepsilon_2 = 0.4$.

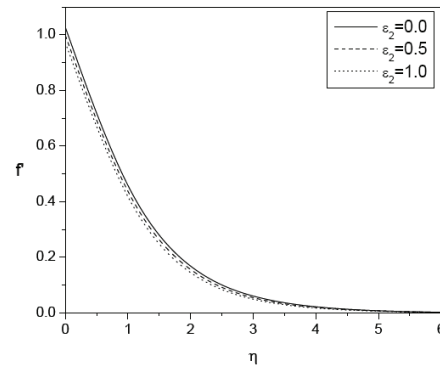


Figure 5. Velocity profiles for various values of ε_2 with $N = 0.5$, $M = 1$, $Le = 0.5$, $n = 0.5$, $G = 0.2$, $\varepsilon_1 = 0.8$.

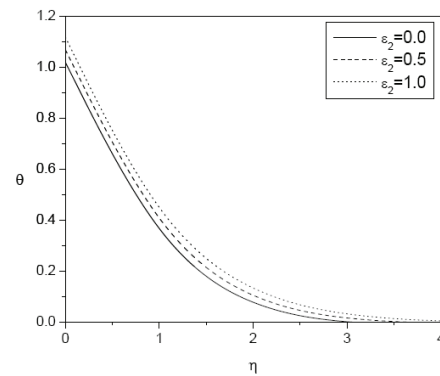


Figure 6. Temperature profiles for various values of ε_2 with $N = 0.5$, $M = 1$, $Le = 0.5$, $n = 0.5$, $G = 0.2$, $\varepsilon_1 = 0.8$.

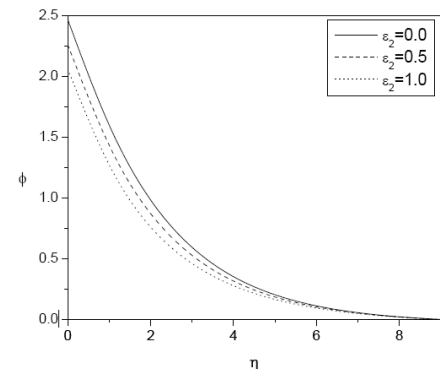


Figure 7. Concentration profiles for various values of ε_2 with $N = 0.5$, $M = 1$, $Le = 0.5$, $n = 0.5$, $G = 0.2$, $\varepsilon_1 = 0.8$.

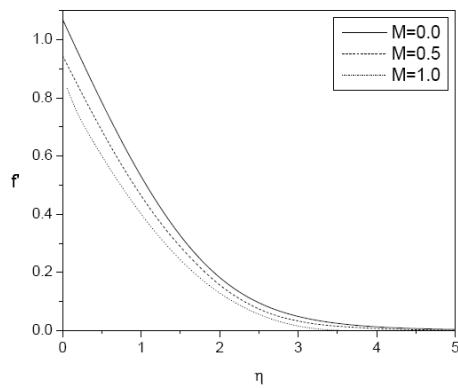


Figure 8. Velocity profiles for various values of M with $N = 0.5$, $Le = 0.5$, $n = 0.5$, $G = 1$, $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.8$.

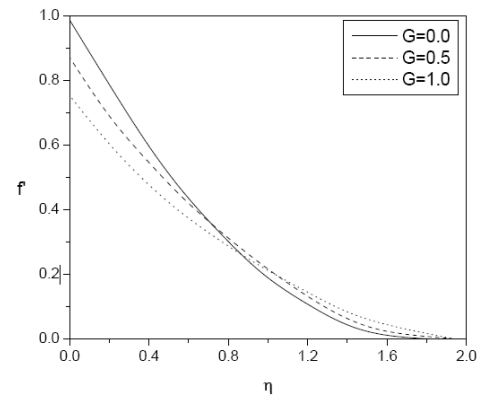


Figure 11. Velocity profiles for various values of G with $N = 0.5$, $Le = 0.5$, $n = 0.5$, $M = 1$, $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.8$.

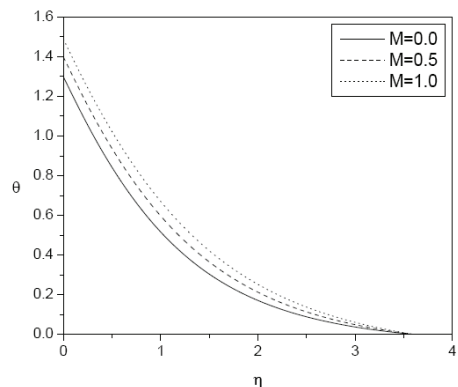


Figure 9. Temperature profiles for various values of M with $N = 0.5$, $Le = 0.5$, $n = 0.5$, $G = 1$, $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.8$.

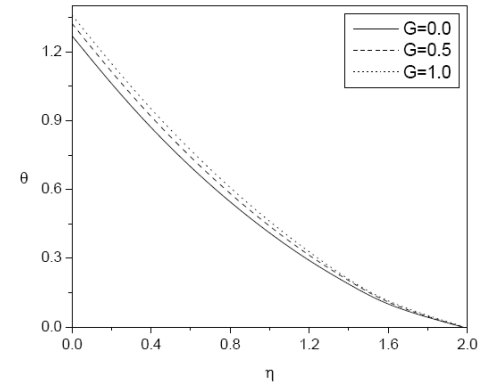


Figure 12. Temperature profiles for various values of G with $N = 0.5$, $Le = 0.5$, $n = 0.5$, $M = 1$, $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.8$.

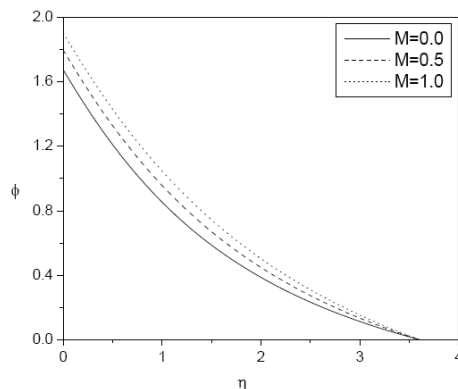


Figure 10. Concentration profiles for various values of M with $N = 0.5$, $Le = 0.5$, $n = 0.5$, $G = 1$, $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.8$.

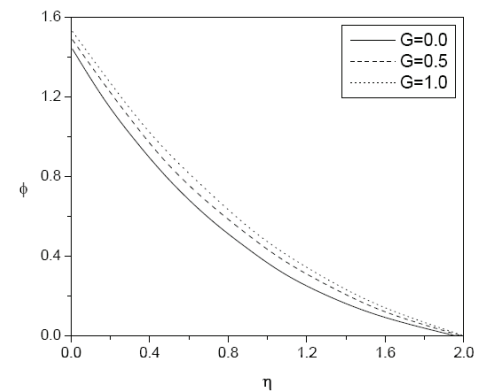


Figure 13. Concentration profiles for various values of G with $N = 0.5$, $Le = 0.5$, $n = 0.5$, $M = 1$, $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.8$.

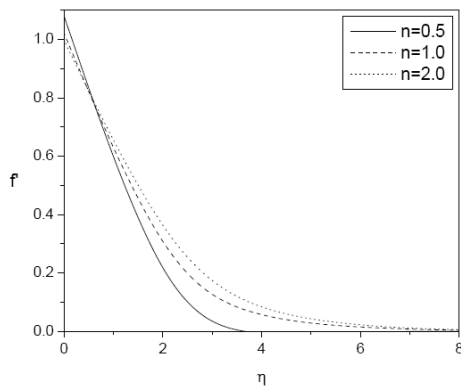


Figure 14. Velocity profiles for various values of n with $N = 0.5$, $Le = 0.5$, $G = 0.2$, $M = 1$, $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.8$.

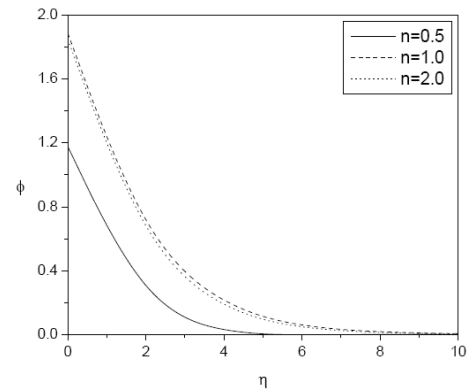


Figure 16. Concentration profiles for various values of n with $N = 0.5$, $Le = 0.5$, $G = 0.2$, $M = 1$, $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.8$.

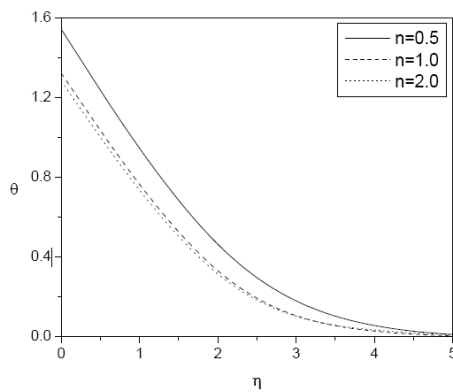


Figure 15. Temperature profiles for various values of n with $N = 0.5$, $Le = 0.5$, $G = 0.2$, $M = 1$, $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.8$.