

Radiation Effect on Mixed Convection Over a Vertical Wavy Surface in Darcy Porous Medium with Variable Properties

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Abstract

This paper presents an investigation of the influence of variable viscosity, variable thermal conductivity and thermal radiation on mixed convective flow due to a vertical wavy surface immersed in a fluid saturated Darcy porous medium. The viscosity and thermal conductivity of the fluid are assumed to be varying with respect to temperature. The Rosseland approximation is used to describe the radiative heat flux. The governing flow, momentum, energy and concentration equations are transformed into a set of ordinary differential equations using the similarity transformation and then solved numerically. The obtained results are compared with previously existing results admit an excellent accuracy. The effects of various physical parameters on velocity, temperature and concentration as well as Nusselt number and Sherwood number are presented graphically in two cases, i.e., aiding flow and opposing flow.

Key Words: Vertical Wavy Surface, Variable Properties, Thermal Radiation, Darcy Porous Medium

1. Introduction

The study of mixed convective heat and mass transfer problems is of considerable practical interest. Such a system is used in a wide range of industrial, geothermal engineering and engineering applications, for example atmospheric boundary layers, solar collectors, heat exchangers, electronic equipment, thermal insulation systems, oil separation from sand by steam, packed bed chemical reactors, underground disposal of nuclear waste materials and food storage. Mixed convection on heat and mass transfer over a vertical surface embedded in a porous medium has been examined extensively in the literature. Cheng and Lin [1] studied unsteady mass transfer in mixed convective heat flow from a vertical plate embedded in a liquid-saturated porous medium with melt-

ing effect. Their results showed that the rate of dynamic mass transfer at the solid–liquid interface is reduced with increasing the melting strength.

Bakier et al. [2] studied the melting effect on MHD mixed convection flow from radiate vertical plate embedded in a saturated porous medium. They observed that the stream function, velocity and the rate of heat transfer at the plate surface increases with increase of radiation parameter, mixed convection parameter and magnetic field. Makinde and Aziz [3] investigated MHD mixed convection from a vertical plate embedded in a porous medium with a convective boundary condition. Their results revealed that both the fluid velocity and temperature increase with an increase in the convective heat transfer parameter. Guedda et al. [4] considered the MHD mixed convection over a vertical flat plate embedded in a porous medium and found that the problem has multiple similarity solutions under VWT condition.

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Kaya [5] considered the influence of conjugate heat transfer on steady MHD mixed convective heat transfer flow over a thin vertical plate embedded in a porous medium with high porosity. Merkin et al. [6] studied mixed convection boundary layer flow on a vertical surface in a porous medium with convective boundary conditions.

The prediction of heat transfer from irregular surfaces is a topic of fundamental importance. Irregular surfaces are encountered in many practical applications for which convective heat transfer is of interest, for instance, condensation process, heat transfer devices such as flat plate solar collectors and flat plate condensers in refrigerators, grain storage container where walls are buckled. The presence of roughness elements disturb the flow and alter the heat transfer rate. A few studies are carried out to examine mixed convection over a wavy (irregular surface) surfaces. Jang and Yan [7] numerically analyzed mixed convection heat and mass transfer along a vertical wavy surface. Molla and Hossain [8] analyzed the radiation effect on mixed convection along a vertical wavy surface. They observed that the local Nusselt number and the average rate of heat transfer increase when the value of the radiation-conduction parameter and surface heating parameter increase. Mahdy [9] presented numerical results for mixed convection heat and mass transfer on a vertical wavy plate embedded in a saturated porous media. He noticed that, as the amplitude-wavelength ratio increases, the amplitude of the local Nusselt number and the local Sherwood number increase.

In recent years, progress has been considerably made in the study of radiation effect on mixed convective heat and mass transfer flow due to its importance in several engineering and industrial areas such as nuclear power plants, various propulsion devices for missiles, satellites, space vehicles and aircraft. Aydin et al. [10] investigated the radiation effect on MHD mixed convection flow about a permeable vertical plate. They noticed that an increase in the radiation parameter decreases the local skin friction parameter and increases the local heat transfer parameter. Hassanien and Al-arabi [11] studied the problem of non-Darcy unsteady mixed convection flow near the stagnation point on a heated vertical surface embedded in a porous medium with thermal radiation and variable viscosity. They concluded that the surface heat transfer decreases with radiation parameters. Hayat et al.

[12] reported radiation effect and magnetic field on the mixed convection stagnation point flow over a vertical stretching sheet in a porous medium. Pal and Talukdar [13] presented an analytical model for combined effects of Joule heating and chemical reaction on unsteady MHD mixed convection of a viscous dissipating fluid over a vertical plate in porous media with thermal radiation. They observed that the value of the local Nusselt number decreases as the thermal radiation parameter increase. Makinde [14] analyzed heat and mass transfer by MHD mixed convection stagnation point flow toward a vertical plate embedded in a highly porous medium with radiation and internal heat generation. He noticed that the local skin-friction, local Nusselt number and local Sherwood number increase as radiation parameter increase. Narayana et al. [15] conducted a numerical study of viscous dissipation and thermal radiation effects on mixed convection from a vertical plate in a non-Darcy porous medium. Their analysis revealed that the effect of thermal radiation parameter was to increase the temperature gradient at the surface and thereby reduce the rate of heat transfer.

In all the above mentioned work, thermo-physical properties of the free stream region are assumed to be constant. These physical properties, particularly fluid viscosity and thermal conductivity may vary with temperature. For lubricating fluids heat generated by internal friction and the corresponding rise in the temperature affects the physical properties of the fluid. Therefore, to accurately predict the flow, heat and mass transfer rates, it is necessary to take into account the temperature dependent viscosity and thermal conductivity. There are many applications of this type, for instance, drawing of plastic films, study of spilling pollutant crude oil over the surface of seawater, in the process of hot cooling, wire drawing, paper production, and glass fiber production and gluing of labels on hot bodies. In view of this, the authors study the effects of variable properties on mixed convective heat and mass transfer flow past a vertical wavy surface in the presence of thermal radiation.

2. Formulation of the Problem

Consider the steady, incompressible, two-dimensional, laminar, mixed convection boundary layer flow over

a vertical wavy surface embedded in a fluid saturated porous medium. The fluid is assumed to be a gray, absorbing-emitting radiation but non-scattering medium. Figure 1 shows that the physical model of the wavy surface and two dimensional Cartesian coordinate system. We assume that surface profile is given by

$$\bar{y} = \bar{\sigma}(\bar{x}) = \bar{a} \sin\left(\frac{\pi \bar{x}}{l}\right) \quad (1)$$

where l is the characteristic length of wavy surface and \bar{a} is the amplitude of the wavy surface. The wavy surface is maintained at constant temperature T_w and constant concentration C_w which is higher than ambient fluid temperature T_∞ and concentration C_∞ . We consider the mixed convection-radiation flow to be governed by the following equations under Boussinesq approximations:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (2)$$

$$\frac{\partial}{\partial \bar{y}}\left(\frac{\mu}{K} \bar{u}\right) = \frac{\partial}{\partial \bar{x}}\left(\frac{\mu}{K} \bar{v}\right) \pm \rho g \left(\beta_r \frac{\partial T}{\partial \bar{y}} + \beta_c \frac{\partial C}{\partial \bar{y}} \right) \quad (3)$$

$$\begin{aligned} \bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} &= \frac{\partial}{\partial \bar{x}}\left(\alpha \frac{\partial T}{\partial \bar{x}}\right) + \frac{\partial}{\partial \bar{y}}\left(\alpha \frac{\partial T}{\partial \bar{y}}\right) \\ - \nabla \cdot \left(\frac{-4\sigma^*}{3k} \nabla T^4 \right) &= 0 \end{aligned} \quad (4)$$

$$\bar{u} \frac{\partial C}{\partial \bar{x}} + \bar{v} \frac{\partial C}{\partial \bar{y}} = D \left(\frac{\partial^2 C}{\partial \bar{x}^2} + \frac{\partial^2 C}{\partial \bar{y}^2} \right) \quad (5)$$

The associated boundary conditions are

$$\begin{aligned} \bar{u} = 0, \bar{v} = 0, T = T_w, C = C_w, \\ \text{at } \bar{y} = \bar{\sigma}(\bar{x}) = \bar{a} \sin\left(\frac{\pi \bar{x}}{l}\right) \\ \bar{u} \rightarrow U_\infty, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } \bar{y} \rightarrow \infty \end{aligned} \quad (6)$$

On the right hand side of eq. (3), (+) sign represents the case when the buoyancy force is “aiding” the uniform free stream, (-) sign represents the buoyancy force is “opposing” the uniform free stream. The last term on RHS of (4) is the radiative heat flux for an optically thick boundary layer according to the Rosseland ap-

proximation.

The fluid properties are assumed to be constant except fluid viscosity and thermal conductivity. We assume that the viscosity of the fluid is to be an inverse linear function of the temperature and it can be expressed as (Lai and Kulacki[16])

$$\frac{1}{\mu} = \frac{1}{\mu_\infty} (1 + \delta(T - T_\infty)) \text{ i.e. } \frac{1}{\mu} = b(T - T_r) \quad (7)$$

In the above relation, both $b = \delta/\mu_\infty$ and $T_r = T_\infty - 1/\delta$ are constants and their values depend on the reference state and the thermal property of the fluid (δ). μ_∞ is the coefficient of viscosity far away from the plate. Further, we assume that the fluid thermal conductivity α is to be varying as a linear function of temperature in the form (Slattery [17]):

$$\alpha = \alpha_0 (1 + E(T - T_\infty)) \quad (8)$$

where α_0 is the thermal conductivity at the wavy surface temperature and E is a constant depending on the nature of the fluid.

Substituting Eqns. (7) and (8) in Eqn. (3) and (4) and then introducing the stream function $\bar{\psi}$ ($\bar{u} = \partial \bar{\psi} / \partial \bar{y}$, $\bar{v} = -\partial \bar{\psi} / \partial \bar{x}$) and the following dimensionless variables

$$\begin{aligned} x = \frac{\bar{x}}{l}, y = \frac{\bar{y}}{l}, a = \frac{\bar{a}}{l}, \sigma = \frac{\bar{\sigma}}{l}, \\ \psi^* = \frac{\bar{\psi}}{\alpha_0}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \varphi = \frac{C - C_\infty}{C_w - C_\infty} \end{aligned} \quad (9)$$

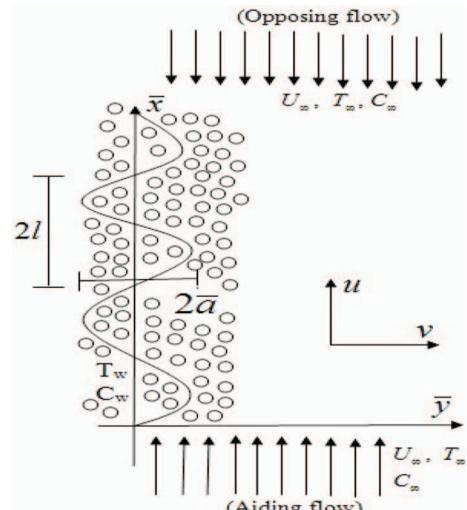


Figure 1. Physical model and coordinate system.

into the resulting equations and Eqn. (5), we get

$$\begin{aligned} \frac{1}{\theta - \theta_r} \left(\frac{\partial \theta}{\partial y} \frac{\partial \psi^*}{\partial y} + \frac{\partial \theta}{\partial x} \frac{\partial \psi^*}{\partial x} \right) + \frac{\partial^2 \psi^*}{\partial y^2} + \frac{\partial^2 \psi^*}{\partial x^2} \\ = \pm \Delta \left(1 - \frac{\theta}{\theta_r} \right) \left(\frac{\partial \theta}{\partial y} + N \frac{\partial \varphi}{\partial y} \right) \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{\partial \psi^*}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi^*}{\partial x} \frac{\partial \theta}{\partial y} = \beta \left(\left(\frac{\partial \theta}{\partial x} \right)^2 + \left(\frac{\partial \theta}{\partial y} \right)^2 \right) \\ + \left(1 + \beta \theta + \frac{4R}{3} \right) \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \end{aligned} \quad (11)$$

$$\frac{\partial \psi^*}{\partial y} \frac{\partial \varphi}{\partial x} - \frac{\partial \psi^*}{\partial x} \frac{\partial \varphi}{\partial y} = \frac{1}{Le} \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} \right) \quad (12)$$

where $Ra = g\beta_r(T_w - T_\infty)/(\alpha_0 v)$ is Darcy-Rayleigh number, $N = \beta_c(C_w - C_\infty)/(\beta_r(T_w - T_\infty))$ is the buoyancy ratio, $R = 4\sigma^* T_\infty^3/(\alpha_0 k)$ is radiation parameter $Pe = U_0 l/\alpha_0 k$ is Peclet number, $Le = \alpha_0/D$ is the Lewis number, and $\theta_r = (T_r - T_\infty)/(T_w - T_\infty) = -1/(\delta(T_w - T_\infty))$ is the variable viscosity parameter. The value of the parameter θ_r is determined by the operating temperature difference and viscosity of the fluid in consideration. For smaller values of θ_r , either the fluid viscosity changes considerably with temperature or the operating temperature difference is high. In either case, the variable viscosity effect is expected to become very important. On the other hand, for a larger values of θ_r , either $(T_w - T_\infty)$ or δ are small, and therefore the effects of variable viscosity can be neglected. $\beta = E(T_w - T_\infty)$ is the thermal conductivity parameter. $\Delta = Ra/Pe$ is the mixed convective parameter. It is clear from the definition of mixed convection parameter that the flow tends to natural convection when $\Delta \rightarrow \infty$ while it tends to forced convection when $\Delta = 0$. When the buoyancy flow is aiding the free stream flow, i.e. Δ is positive, the flow is called the aiding flow, otherwise the flow is called opposing flow (Δ is negative).

The associated boundary conditions are given by

$$\left. \begin{aligned} \psi^* = 0, \theta = 1, \varphi = 1, \text{ on } y = a \sin(x), \\ \psi_y^* \rightarrow \frac{\alpha_0}{l} U_\infty, \theta \rightarrow 0 \text{ as } y \rightarrow \infty, \end{aligned} \right\} \quad (13)$$

Using the following transformations

$$x = \xi, \hat{\eta} = \frac{y - a \sin(x) Pe^{-1/2}}{\xi^{1/2} (1 + a^2 \cos^2 \xi)^{1/2}}, \psi^* = Pe^{1/2} \xi^{1/2} f(\hat{\eta}) \quad (14)$$

in Eqns. (10)–(13) and letting $Pe \rightarrow \infty$, we obtain the following boundary layer equations:

$$f'' + \frac{1}{\theta - \theta_r} \theta' f' = \pm \Delta \left(1 - \frac{\theta}{\theta_r} \right) (\theta' + N \varphi') \quad (15)$$

$$\beta(\theta')^2 + \left(1 + \beta \theta + \frac{4R}{3} \right) \theta'' + \frac{1}{2} f \theta' = 0 \quad (16)$$

$$\frac{1}{Le} \varphi'' + \frac{1}{2} f \varphi' = 0 \quad (17)$$

where prime denotes differentiation with respect to $\hat{\eta}$. The associated boundary conditions are

$$\begin{aligned} f = 0, \theta = 1, \text{ and } \varphi = 1 \text{ at } \hat{\eta} = 0 \\ f' \rightarrow 1, \theta \rightarrow 0 \text{ and } \varphi \rightarrow 0 \text{ as } \hat{\eta} \rightarrow \infty \end{aligned} \quad (18)$$

The engineering design quantities of physical interest include Nusselt number and Sherwood numbers which are defined as

$$Nu_\xi = \frac{-\theta'(0) Pe_\xi^{1/2}}{(1 + a^2 \cos^2(\xi))^{1/2}}, Sh_\xi = \frac{-\varphi'(0) Pe_\xi^{1/2}}{(1 + a^2 \cos^2(\xi))^{1/2}} \quad (19)$$

The set of nonlinear non-homogeneous differential equation (15)–(17) with corresponding boundary conditions (18) are solved numerically by employing Runge-Kutta method with shooting technique.

3. Results and Discussions

In the absence of variable viscosity, variable thermal conductivity and radiation parameter for vertical flat surface (i.e., $a = 0$) the results Local Nusselt number and the local Sherwood number have been compared with the values of Lai [18], and it was found that they are in good agreement, as shown in Table 1.

The velocity, temperature and concentration distribution for different values of variable viscosity parameter (θ_r) are shown in Figure 2. From Figure 2(a), it is clear that velocity decreases with increase in θ_r for both aiding and opposing flows. From Figures 2(b) and 2(c), it is noticed that the temperature and concentration pro-

Table 1. Comparison of Local Nusselt number and the local Sherwood number for $a = 0$, $\beta = 0$, $R = 0$ and $\theta_r \rightarrow \infty$ at $N = 0.5$, $Le = 1$

Δ	$Nu_\xi Pe_\xi^{-1/2}$		$Sh_\xi Pe_\xi^{-1/2}$	
	Lai [18]	Present	Lai [18]	Present
0.1	-0.5640	-0.5640	-1.6344	-1.6344
1.0	-1.1060	-1.1058	-1.9599	-1.9600
1.5	-1.3860	-1.3860	-2.3674	-2.3672
3.0	-1.5643	-1.5640	-2.6548	-2.6548
5.0	-2.2929	-2.2929	-3.6862	-3.6862
10	-2.2956	-2.2955	-5.0109	-5.0105

files for both aiding and opposing flow enhanced with increase in the values of θ_r . But this increment in temperature and concentration profiles is larger for the opposing flow. Hence, increasing θ_r tends to reduce velocity boundary layer thickness and enhance the thermal and solutal boundary layer thickness.

The variation of variable thermal conductivity parameter (β) on velocity, temperature and concentration distributions are shown in Figure 3. From Figure 3(a), it is observed that an increase in β leads to increase in velocity in case of aiding flow and increase in velocity profile near the surface and then decrease far away from the surface for opposing flow. From Figure 3(b), it is clear that increase in β tends to increase in the temperature for both aiding and opposing flows. Increase in β gives rise to decrease in concentration profile in both aiding and opposing flows as shown in Figure 3(c). It is clear that the values of temperature and concentration are larger for the opposing flow case.

The variation of thermal radiation parameter R on velocity, temperature and concentration distributions are shown in Figure 4. The radiation parameter R is the ratio of thermal radiation contribution relative to the thermal conduction. As $R \rightarrow \infty$, influence of thermal radiation is high in the boundary layer regime. For $R = 1$, thermal radiation and thermal conduction will give equal contribution. For aiding flow, with an increase in R there is an elevation in velocity as shown in Figure 4(a) in the boundary layer regime. For opposing flow, the effect of radiation parameter is to increase the velocity near the surface and then decrease far away from the surface. An increase in R causes a significant increase in velocity with distance in boundary layer, i.e. accelerates the flow.

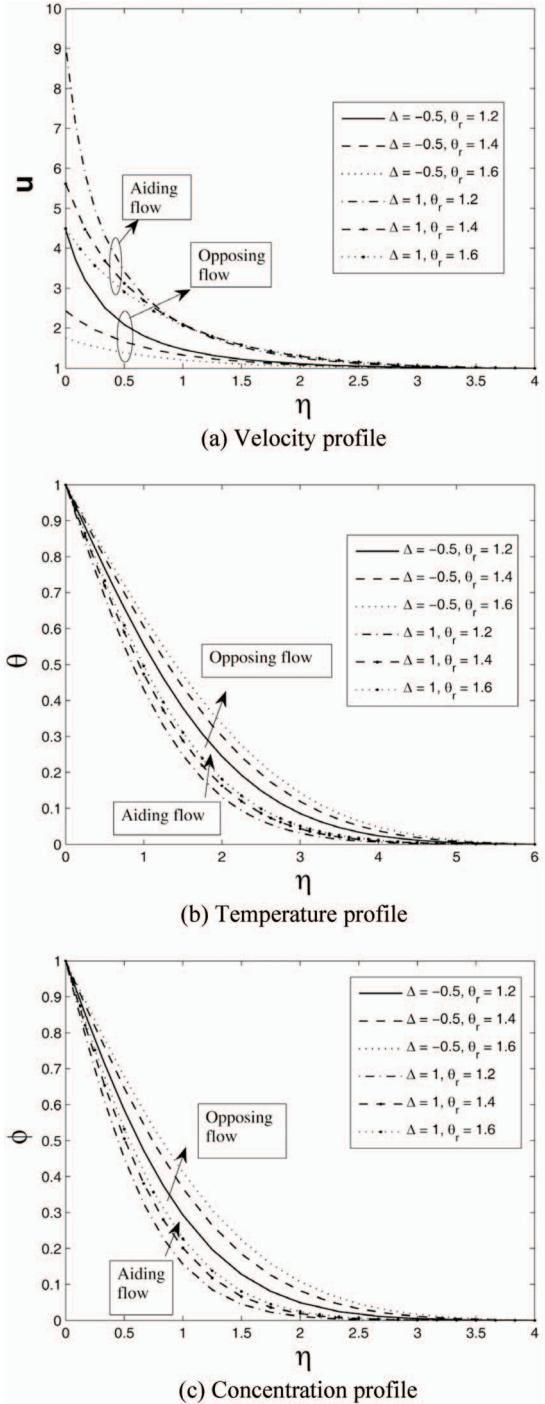
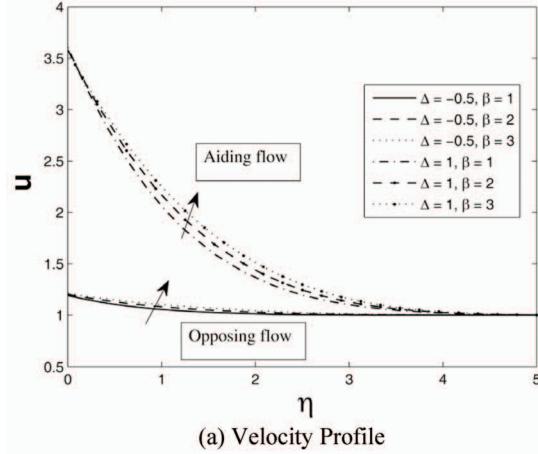
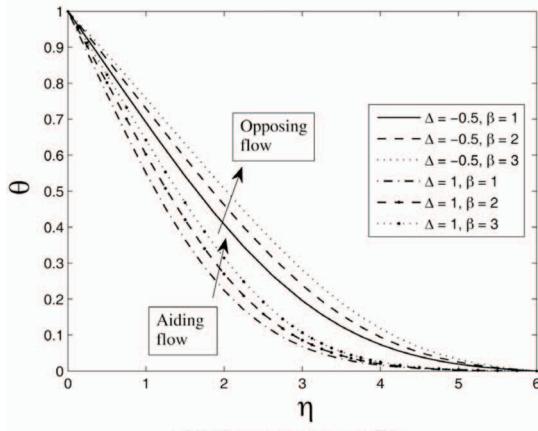


Figure 2. Velocity, temperature and concentration profiles for different values of θ_r for $N = 0.5$, $Le = 1$, $\beta = 0.5$ and $R = 1$.

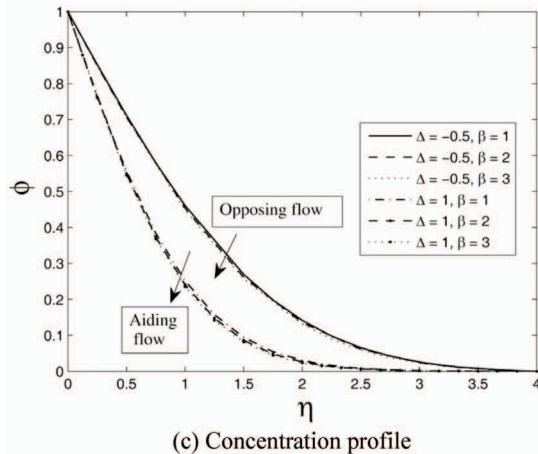
Therefore, it is an important in industrial flow process and polymeric process. The temperature is considerably increased with an increase in R for both aiding and opposing flows as shown in Figure 4(b). From Figure 4(c),



(a) Velocity Profile



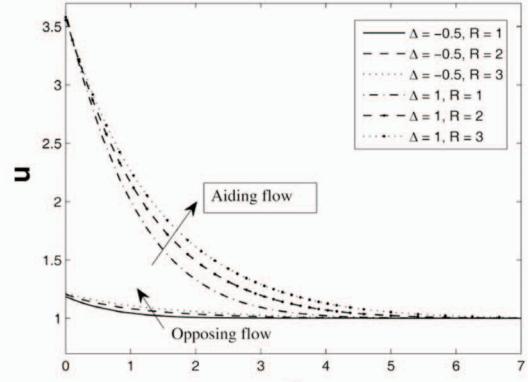
(b) Temperature profile



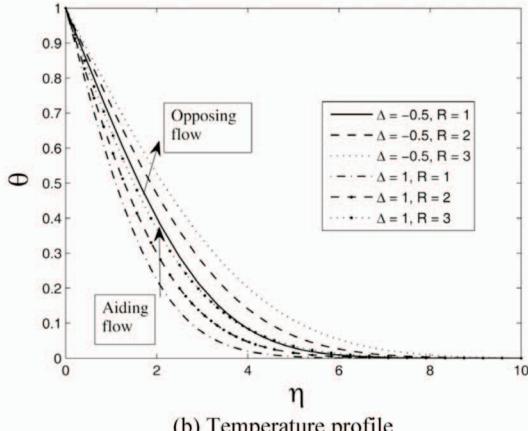
(c) Concentration profile

Figure 3. Velocity, temperature and concentration profile for different values of β for $N = 0.5$, $Le = 1$, $\theta_r = 2$ and $R = 1$.

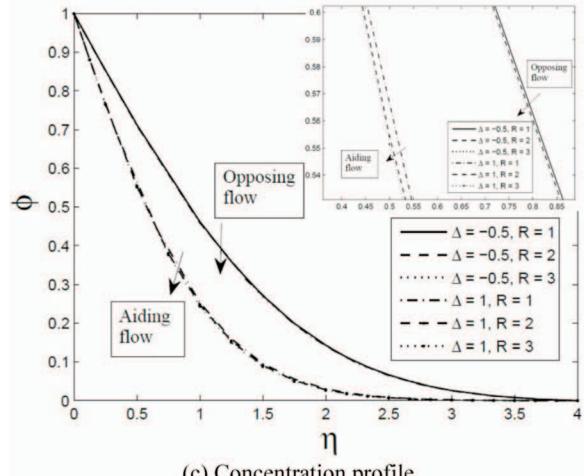
it is clear that concentration profile is slightly decreased with an increase in R in both cases of aiding and opposing flow. i.e. the thermal radiation contribution is less in species conservation equation.



(a) Velocity profile



(b) Temperature profile



(c) Concentration profile

Figure 4. Velocity, temperature and concentration profiles for different values of R for $N = 0.5$, $Le = 1$, $\theta_r = 2$ and $\beta = 0.5$.

Figure 5 represent the variable viscosity parameter θ_r on Nusselt number and Sherwood number with stream wise coordinate ξ . For both aiding and opposing flows, increase in θ_r results depreciation in the Nusselt num-

ber. It is an important to note that amplitude of the Nusselt number in aiding flow is more than the amplitude of the Nusselt number in opposing flow (Figure 5 (a)). From Figure 5(b), it is observed that for both aiding and opposing flows Sherwood number increases with increase in θ_r .

The variations of variable thermal conductivity (β) and radiation parameter (R) on Nusselt number and Sherwood number with stream wise coordinate ξ is displayed in Figures 6 and 7, respectively. From these figures it is observed that Nusselt number and Sherwood number are decreases in both cases of aiding and opposing flow by increase in β and R. But these results are larger in opposing flow and amplitude of the Nusselt number and Sherwood number is larger in aiding flow with increase in β and R.

The variation of mixed convective parameter Δ on Nusselt number and Sherwood number with stream wise

coordinate ξ is seen in Figure 8. It is noted that Nusselt number and Sherwood number decreases in case of aiding flow by increasing Δ but the opposite trend is noted in opposing flow.

4. Conclusions

The effects of variable viscosity and variable thermal conductivity on mixed convective heat and mass transfer past a vertical wavy surface embedded in a fluid saturated porous medium with thermal radiation has been investigated theoretically. The main conclusions of the present investigations are as follows:

1. Increasing variable viscosity parameter results depreciation in velocity profile and Nusselt number for both aiding and opposing flow while temperature, concentration profile and Sherwood number results are increased.

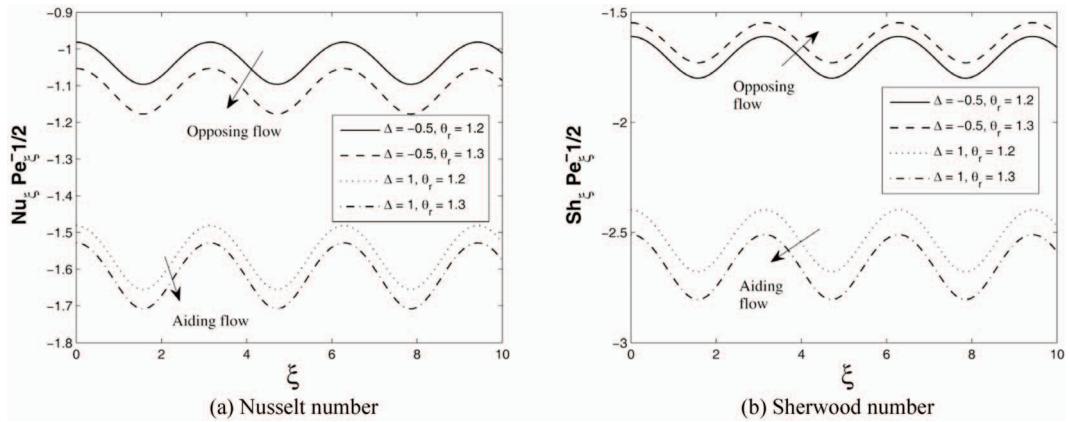


Figure 5. Axial distributions of Nusselt number and Sherwood number the for different values of θ_r , for $N = 0.5$, $Le = 1$, $a = 0.5$, $R = 1$ and $\beta = 0.5$.

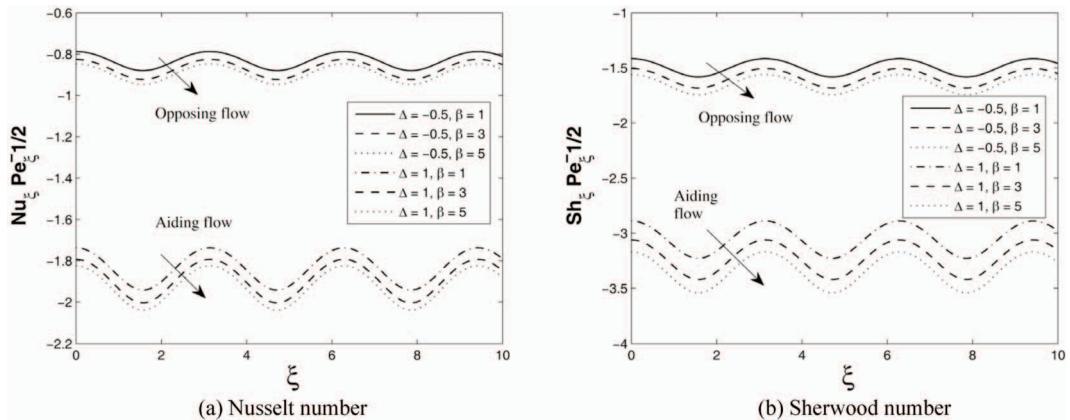


Figure 6. Axial distributions of the Nusselt number and Sherwood number for different values of β for $N = 0.5$, $Le = 1$, $a = 0.5$, $R = 1$ and $\theta_r = 1.5$.

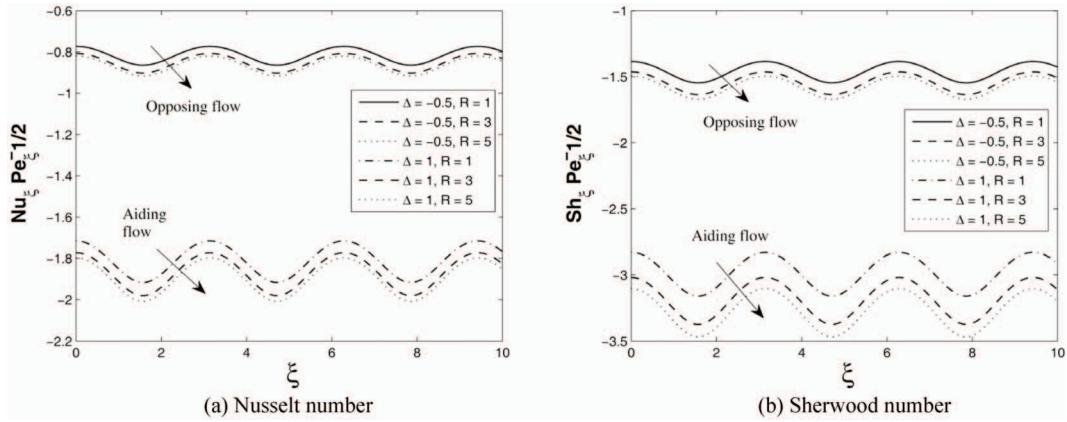


Figure 7. Axial distributions of the Nusselt number and Sherwood number for different values of R for $N = 0.5$, $Le = 1$, $\alpha = 0.5$, $\beta = 0.5$ and $\theta_r = 1.5$.

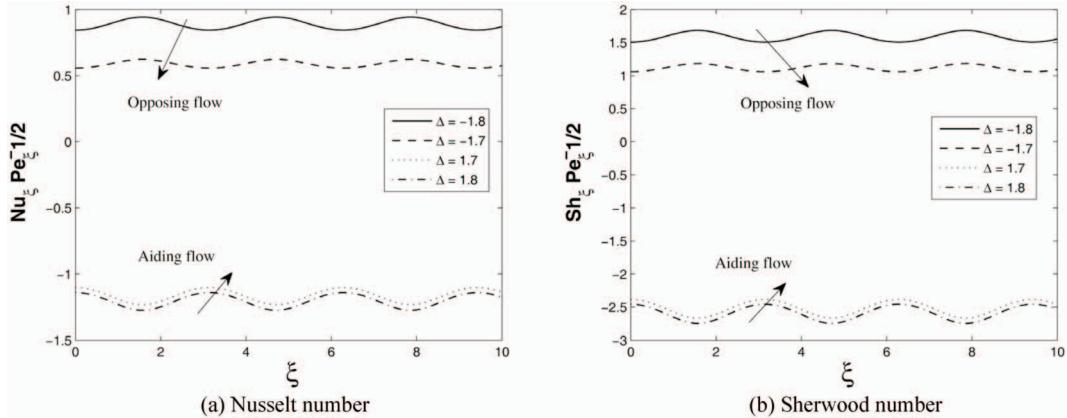


Figure 8. Axial distributions of the Nusselt number and Sherwood number for different values of Δ for $N = 0.5$, $Le = 1$, $\alpha = 0.5$, $R = 1$, $\beta = 0.5$ and $\theta_r = 1.5$.

2. Increase in variable thermal conductivity or radiation parameter tends to increase velocity for the aiding flow but opposite trends is observed in opposing flow. The temperature is increased whereas concentration, Nusselt number and Sherwood number results are decreased markedly with increase in variable thermal conductivity or radiation parameter for both cases of aiding and opposing flow.
3. Nusselt number and Sherwood number are decreased for aiding flow but the opposite trend is observed for opposing flow by increasing mixed convective parameter.

Acknowledgement

One of the authors Mr. B. Mallikarjuna wishes to thank to the Department of Science and Technology, New

Delhi, India for providing financial support to enable conducting this research work under Inspire Fellowship Program.

Nomenclature

\bar{a}	Amplitude of the wavy surface
l	Characteristic length of the wavy surface
\bar{u}	Velocity component in the x direction
\bar{v}	Velocity component in the y direction
T_w	Wall temperature
T_∞	Ambient fluid temperature
C_w	Wall concentration
C_∞	Ambient fluid concentration
K	Permeability of the porous medium
g	Acceleration due to gravity
D	Mass diffusivity

k Mean absorption coefficient

Ra Darcy - Rayleigh number

Pe Peclet number

N Buoyancy ratio

Le Lewis number

R Radiation parameter

U_∞ Uniform free stream velocity

Nu Nusselt number

Sh Sherwood number

\bar{x}, \bar{y} Coordinate system

Greek Symbols

$\bar{\sigma}$ Surface geometry function

μ Dynamic viscosity of the fluid

ν Kinematic viscosity of the fluid

ρ Fluid density

β_t Coefficient of thermal expansion

β_c Coefficient of mass expansion

α Thermal conductivity

α_o Thermal diffusivity

σ^* Stefan-Boltzmann constant

δ Thermal property of the fluid

$\bar{\psi}$ Stream function

θ Non-dimensional temperature

ϕ Non-dimensional concentration

θ_r Variable viscosity parameter

β Variable thermal conductivity parameter

Δ Mixed convective parameter

ξ Stream wise coordinate

η Similarity variable

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Manuscript Received: Mar. 20, 2015

Accepted: Jul. 9, 2015