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Multiple Constraints in Ecological Ammensalism- A Numerical Approach

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Abstract

The paper addresses the study of a mathematical model of "Multiple Conditions in Ecological Ammensalism". The mathematical model contains Ammensal-enemy species pair with cover for Ammensal, alternative resources for enemy, migrating for Ammensal species and immigrating for the enemy species. A couple of first order non- linear ordinary differential equations obtain the model. All possible solutions of the model are derived with the help of RK method of fourth order in a finite interval. The relation between the carrying capacity of the Ammensal species and dominance reversal time is identified by the numerical computational techniques.

Keywords: *Non-linear system, Ammensal species, Enemy species, Carrying capacity, Dominance reversal time.*

1 Introduction

A population means a group of individuals of identical or different species. The individuals survive together in a province. Population ecology is the study of populations in which how they live together with interactions among one another. A population may flourish or decline with so many factors like limited resources, migration, immigration, time delay and so on. In general, the nature sustains the carrying capacity of the species to maintain the population. Thus the Populations are limited by controlling their carrying capacities.Kapur J.N [15,16] examined various mathematical models in Biological and Medical sciences with innovative approach. Later N.C.Srinivas [19] investigated some competitive models, which are related to real life situations. Lakshmi Narayan with N.Ch.pattbahi Ramacharyulu [17] constructed the competitive ecological models and discussed their stability criteria. K.V.L.N.Acharyuly and N.Ch.Pattabhiramacharyulu [1-14] obtained few fruitful results in the mathematical models of ecological Ammensalism.

The authors concentrated on stability analysis of various mathematical models of ecological Ammensalism with analytical methods in the earlier work.But in this article,the model is investigated thoroughly by numerical study.A biological community, which exists for sufficiently long period in a more or less invariable state, should possess intrinsic abilities to resist perturbations coming in abundance from the environment. This ability of an ecosystem is usually termed as system-stability. A community is considered to be stable, if the number of member species remains constant over sufficiently long time intervals. On the other hand, an advanced theory of mathematical stability is available which deals with mathematical models of real objects. Therefore, if we have a good model of an ecosystem (in terms of differential or difference equations), the stability of real community can be deduced from our model by conventional methods of stability theory.For instance, the ecosystem may be considered to be stable, when the model trajectories of solutions of a system of equations in the phase space stay within a given bounded domain for a sufficiently wide range of perturbation.

Recently few mathematians like Rahnama Mohamad Bagher & Jahanshai Payman [18] and Wong Yee Leng & Siti Mariyam Shamsuddin [20] investigated and improved various real life mathematical models under complexity situations with the help of soft computing techniques like genetic algorithm.

In the present investigation, the authors derived the solutions numerically to this model with the aid of RK method of fourth order. From the numerical computations, the relation between the carrying capacity of the Ammensal species and dominance reversal time is identified.The paper examines a mathematical model of Ecological Ammensalism with multiple constraints. The mathematical model contains Ammensal-enemy species pair with cover for Ammensal, alternative resources for enemy, migrating for Ammensal species and immigrating for the enemy species. The model is obtained by a couple of first order non linear ordinary differential equations. The relations are investigated by changing the value of natural growth rate of Ammensal species while fixing all other parameters. The dominance reversal time provides the dominance criterion between the species in the exiting cases. The graphs are depicted with the aid of Mat lab wherever needed.

Notation Adopted:

N_1 (t) : The population of the Ammensal species (S_1) at time t

N_2 (t) : The population of the Enemy species (S_2) at time t

a_i : The natural growth rates of S_i, i = 1, 2.

a_{ii} : The rate of decrease of S_i; due to its own insufficient resources ,i=1,2.

a_{12} : The inhibition coefficient of S_1 due to S_2 i.e The Ammensal coefficient.

a_{21} : The inhibition coefficient of S_2 due to S_1

H_1(t) : The replenishment or renewal of S_1 per unit time

H_2(t) : The replenishment or renewal of S_2 per unit time

K_i : a_i/a_{ii} are the carrying capacity of N_i, i = 1, 2.

α : a_{12}/a_{11} is the coefficient of Ammensalism.

h_1 : a_{11} H_1 is the rate of harvest of the Ammensal

h_2 : a_{22} H_2 is the rate of harvest of the enemy.

m_1 : Rate of decrease of the Ammensal due to harvesting.

m_2 : Rate of decrease of the enemy due to harvesting.

m : a constant characterized by the cover provided for the Ammensal species.

The state variables N_1 and N_2 as well as the model parameters a_1, a_2, a_{11}, a_{22}, K_1, K_2, α , h_1, h_2, m_1, m_2 and m are assumed to be non-negative constants.

2 Basic Equations

The model equations are formed by the non-linear ordinary different equations.

$$\frac{dN_1(t)}{dt} = (1-m_1)a_1N_1(t) - a_{11}N_1^2(t) - (1-m)a_{12}N_1(t)N_2(t) - h_1(t) \tag{1}$$

$$\frac{dN_2(t)}{dt} = (1-m_2)a_2N_2(t) - a_{22}N_2^2(t) + h_2(t) \tag{2}$$

with the conditions $N_i(0) = N_{i0} \geq 0, i = 1,2$;

3 Numerical Solutions of the Growth rate equations

The numerical illustartions of the mathematical model are computed by using the fourth order Runge-Kutta method .These are useful to know the interaction between the species .The derived solutions are given in Table-1.

Table-1

S.No	a_1	a_{11}	a_{12}	a_2	a_{22}	m	m_1	m_2	h_1	h_2	N_{10}	N_{20}	t*
1	1.593187	3.283874	4.228554	1.867148	4.419193	0.4	0.5	0.6	0.7	0.7	1.638284	0.768296	0.096
2	2.014261	2.34779	0.589065	4.436913	2.761909	0.4	0.5	0.6	0.7	0.7	1.938932	0.864447	0.100
3	1.830516	0.409059	2.510159	2.874402	3.568079	0.4	0.5	0.6	0.7	0.7	1.424785	0.821149	0.122
4	1.242465	2.687266	3.017212	2.081598	1.821775	0.4	0.5	0.6	0.7	0.7	0.509887	0.159696	0.243
5	1.242465	0.687956	0.314514	3.085399	1.679375	0.4	0.5	0.6	0.7	0.7	0.509887	0.159696	0.261
6	1.450382	3.644115	2.272611	1.73952	4.872066	0.4	0.5	0.6	0.7	0.7	1.937778	0.326983	0.293
7	1.460899	3.644115	2.227686	1.73952	4.872066	0.4	0.5	0.6	0.7	0.7	3.652344	0.326983	0.334
8	1.679163	2.276151	2.681616	1.817272	3.093791	0.4	0.5	0.6	0.7	0.7	2.030574	0.031174	1.003

The graphical depicted solutions are shown from Fig.1 to Fig.8

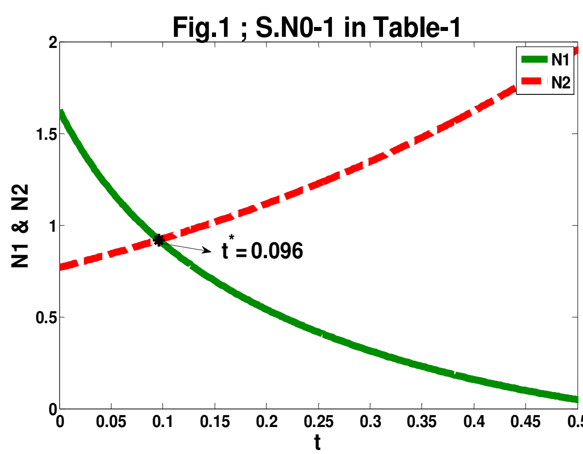


Fig.1: Dominance reversal time t*= 0.096 , when a_1=1.593187, a_{11}=3.283874, a_{12}=4.228554,a_2=1.867148, a_{22}= 4.419193, m = 0.4,m_1 = 0.5;m_2 = 0.6;h_1 = 0.7;h_2 = 0.7, N_{10}=1.638284 and N_{20}=0.768296

In Fig.1, the enemy dominates Ammensal species in natural growth but its initial strength is less than Ammensal. Ammensal out numbers enemy till the time instant t*=0.096 after which enemy is found to be going away from the equilibrium point while Ammensal species is asymptotic to the equilibrium point.

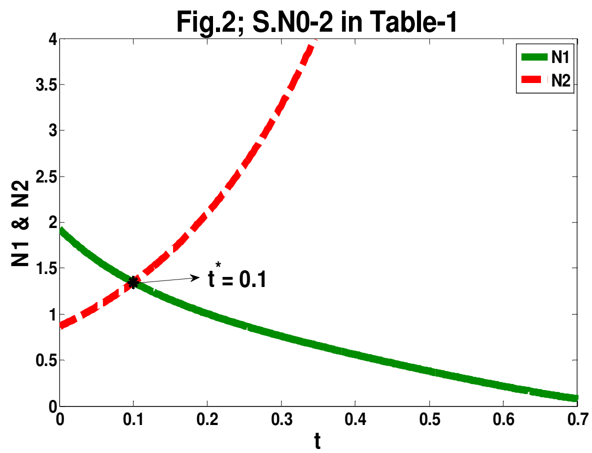


Fig.2: Dominance reversal time t*= 0.10, when a_1=2.014261, a_{11}=2.34779, a_{12}=0.589065,a_2=4.436913, a_{22}= 2.761909, m = 0.4,m_1 = 0.5;m_2 = 0.6;h_1 = 0.7;h_2 = 0.7, N_{10}=1.938932 and N_{20}=0.864447.

In Fig.2, it is noticed that Ammensal (S_2) while declining, dominates over the Enemy(S_2)species up to the time -instant t*=0.1 there after Enemy(S_2)species dominates over Ammensal (S_2) species and the Ammensal(S_1) species declines further.

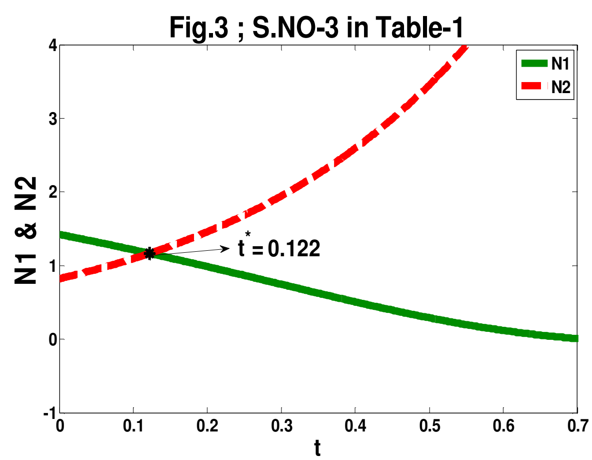


Fig.3: Dominance reversal time t*= 0.122, when a_1=1.830516, a_{11}=0.409059, a_{12}=2.510159,a_2=2.874402, a_{22}= 3.568079, m = 0.4,m_1 = 0.5;m_2 = 0.6;h_1 = 0.7;h_2 = 0.7, N_{10}=1.424785 and N_{20}=0.821149.

In Fig.3 Enemy has a constant natural growth rate through out the interval and is never effected by Ammensal species in any manner. Ammensal species persists to dominate enemy species up to the time -instant t*=0.122.

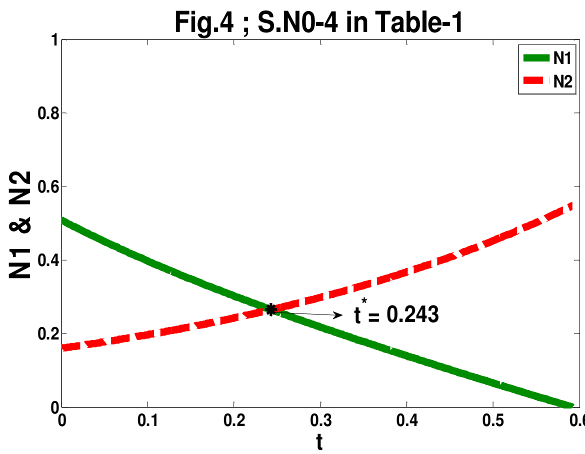


Fig.4: Dominance reversal time t*= 0.243, when a_1=1.242465, a_{11}=2.687266, a_{12}=3.017212,a_2=2.081598, a_{22}= 1.821775, m = 0.4,m_1 = 0.5;m_2 = 0.6;h_1 = 0.7;h_2 = 0.7, N_{10}=0.509887 and N_{20}=0.159696.

In Fig.4, Ammensal species reigns over enemy species till the time instant t*=0.096 after that the dominance is reversed.Enemy species flourishes through the interval, but its initial strength is less than the Ammensal.

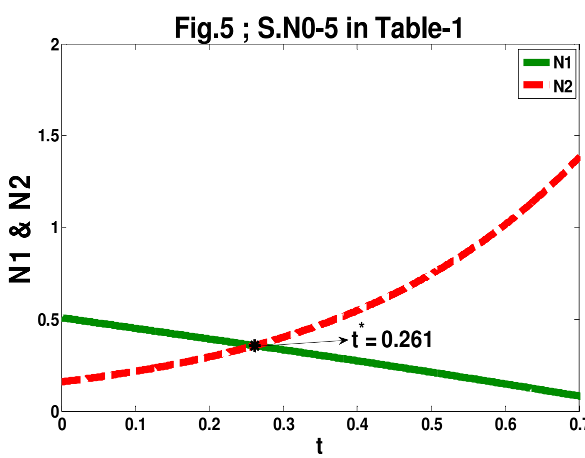


Fig.5: Dominance reversal time t*= 0.261, when a_1=1.242465, a_{11}=0.687956, a_{12}=0.314514,a_2=3.085399, a_{22}= 1.679375, m = 0.4,m_1 = 0.5;m_2 = 0.6;h_1 = 0.7;h_2 = 0.7, N_{10}=0.509887 and N_{20}=0.159696.

In Fig.5, Enemy dominates Ammensal in natural growth after the time instant t*=0.261. More over enemy species diverges from the equilibrium point and Ammensal gradually diminishes in the interval.

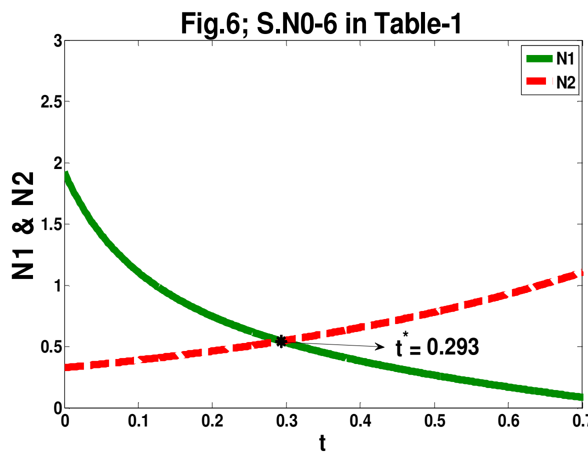


Fig.6: Dominance reversal time t*= 0.293, when a_1=1.450382, a_{11}=3.644115, a_{12}=2.272611,a_2=1.73952, a_{22}= 4.872066, m = 0.4,m_1 = 0.5;m_2 = 0.6;h_1 = 0.7;h_2 = 0.7, N_{10}=1.937778 and N_{20}=0.326983.

In Fig.6, Ammensal eclipses over Enemy species up to the time-instant t*=0.293 thereafter Enemy species eclipses over Ammensal species and the enemy species remains to exit with the exponential growth.

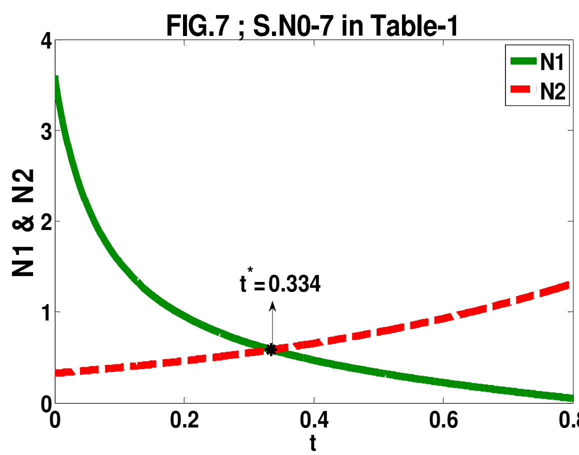


Fig.7: Dominance reversal time t*= 0.334, when a_1=1.460899, a_{11}=3.644115, a_{12}=2.227686,a_2=1.73952, a_{22}= 4.872066, m = 0.4,m_1 = 0.5;m_2 = 0.6;h_1 = 0.7;h_2 = 0.7, N_{10}=3.652344 and N_{20}=0.326983.

In Fig.7, Even though the initial growth rate of Ammensal (S_1) species is greater than the enemy (S_2) species, the enemy prevails over Ammensal in natural growth rate but its initial strength is less than Ammensal. Ammensal prevails over the enemy till the time instant t*=0.334.after which the enemy is observed to be moving away from the equilibrium point.

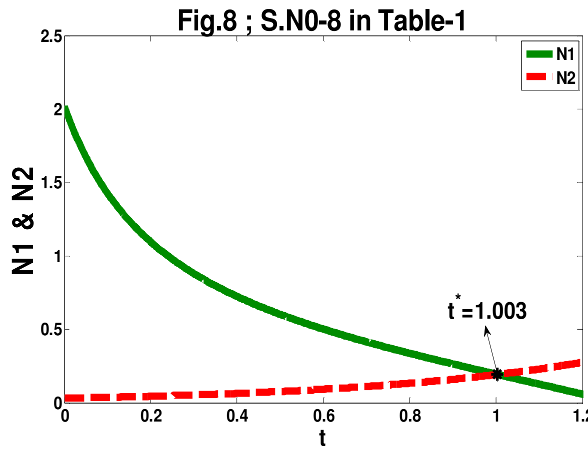


Fig.8: Dominance reversal time t*= 1.003, when a_1=1.679163, a_{11}=2.276151, a_{12}=2.681616,a_2=1.817272, a_{22}= 3.093791, m = 0.4,m_1 = 0.5;m_2 = 0.6;h_1 = 0.7;h_2 = 0.7, N_{10}=2.030574 and N_{20}=0.326983.

In Fig.8, Ammensal predominates enemy species till the time instant t*=1.003 and also in its initial strength. However Ammensal species turns to be extinct in the remaining interval and asymptotically converges to equilibrium point and the enemy species has a steady growth rate in the entire interval.

4 Results and Discussions

(i)The Ammensal species dominates the enemy species up to dominance reversal time even though the growth rate of Ammensal species declines.

(ii)The dominance is changed after dominance reversal time (t*).

(iii)Ammensal species goes down gradually and survives at a very low growth rate .

(iv)The enemy species has an unfaltering increase through out the interval.

(iv) In the course of time, it is observed that the enemy species has a steep rise as the Ammensal species becomes extinct after dominance reversl time(t*).

5 Relation between Carrrrying capacity(K_1) of Ammensal species and Dominance reversal time(t*)

The fixed parameters are regarded as **a_{11}=2.276151,a_{12}=2.681616, a_2=1.817272, a_{22}= 3.093791, N_{10}=2.030574,N_{20}= 0.0031174, m= 0.4,m_1 = 0.5,m_2 = 0.6,h_1 = 0.7,h_2 = 0.7.**

The varying variable is **a_1, i.e a_1= 0.679163, 1.679163, 2.679163, 3.679163, 4.679163, 5.679163, 6.679163, 7.679163, 8.679163, 9.679163, 10.679163**and then t* is deduced. The obtained solutions are tabulated in Table-2 and illustrated from Fig. (9) to Fig.(19).

Table-2

Case	a_1	a_{11}	a_{12}	a_2	a_{22}	m	m_1	m_2	h_1	h_2	N_{10}	N_{20}	t*
1	0.679163	2.276151	2.681616	1.817272	3.093791	0.4	0.5	0.6	0.7	0.7	2.030574	0.0031174	0.880
2	1.679163	2.276151	2.681616	1.817272	3.093791	0.4	0.5	0.6	0.7	0.7	2.030574	0.0031174	1.003
3	2.679163	2.276151	2.681616	1.817272	3.093791	0.4	0.5	0.6	0.7	0.7	2.030574	0.0031174	1.158
4	3.679163	2.276151	2.681616	1.817272	3.093791	0.4	0.5	0.6	0.7	0.7	2.030574	0.0031174	1.340
5	4.679163	2.276151	2.681616	1.817272	3.093791	0.4	0.5	0.6	0.7	0.7	2.030574	0.0031174	1.527
6	5.679163	2.276151	2.681616	1.817272	3.093791	0.4	0.5	0.6	0.7	0.7	2.030574	0.0031174	1,689
7	6.679163	2.276151	2.681616	1.817272	3.093791	0.4	0.5	0.6	0.7	0.7	2.030574	0.0031174	1.815
8	7.679163	2.276151	2.681616	1.817272	3.093791	0.4	0.5	0.6	0.7	0.7	2.030574	0.0031174	1.914
9	8.679163	2.276151	2.681616	1.817272	3.093791	0.4	0.5	0.6	0.7	0.7	2.030574	0.0031174	1.990
10	9.679163	2.276151	2.681616	1.817272	3.093791	0.4	0.5	0.6	0.7	0.7	2.030574	0.0031174	2.061
11	10.679163	2.276151	2.681616	1.817272	3.093791	0.4	0.5	0.6	0.7	0.7	2.030574	0.0031174	2.106

The solution curves are illustrated from Fig.(9) to Fig.(19) as below.

While changing the natural growth rate of Ammensal species with small increments, the change in dominace reversal time and interaction between the both specis can be observed keenly from the following solution curves.

Fig.9; Case-1 in Table-2

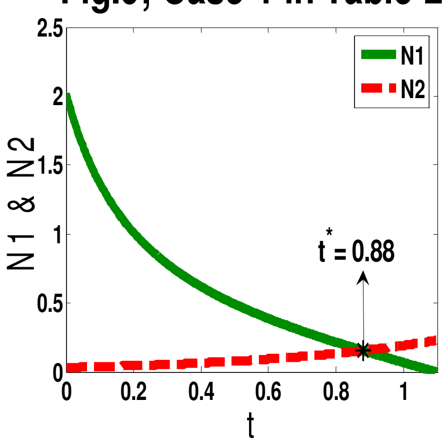


Fig.10; Case-2 in Table-2

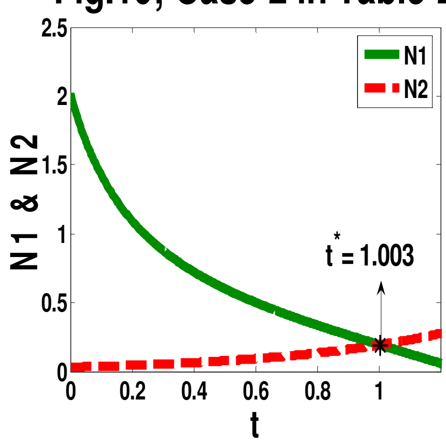
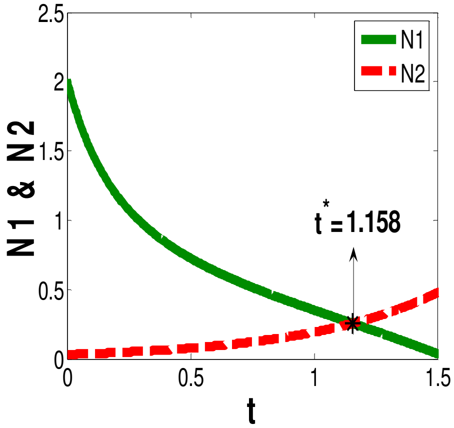


Fig.11; Case-3 in Table-2



Fig,12 ; Case-4 in Table-2

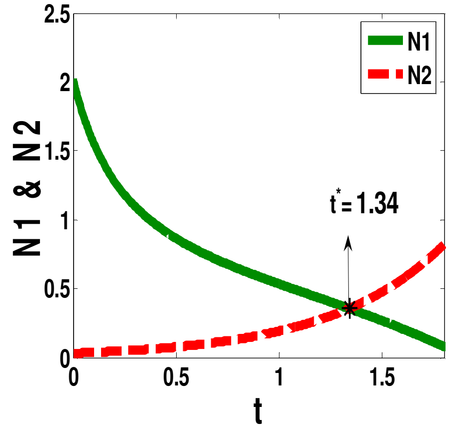


Fig.13; Case-5 in Table-2

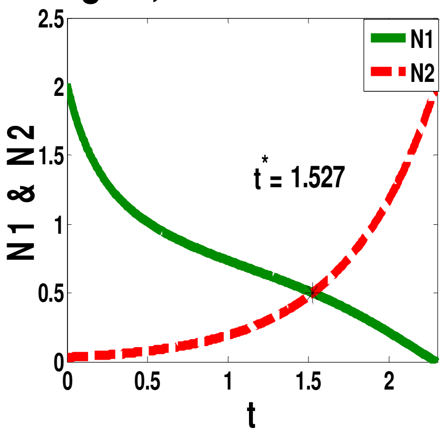


Fig.14 ;Case-6 in Table-2

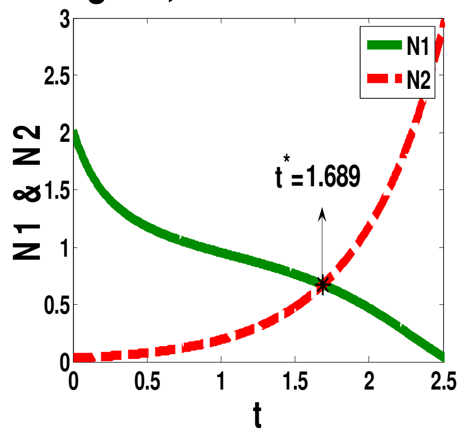


Fig.15; Case-7 in Table-2

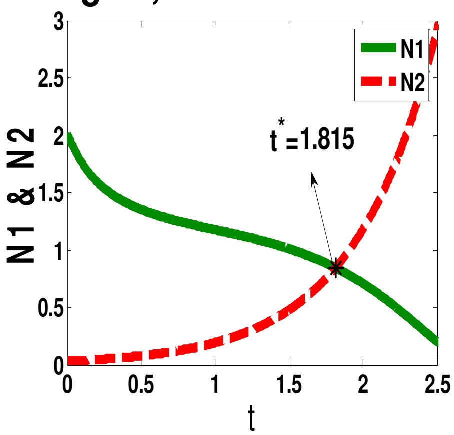


Fig.16 ;Case-8 in Table-2

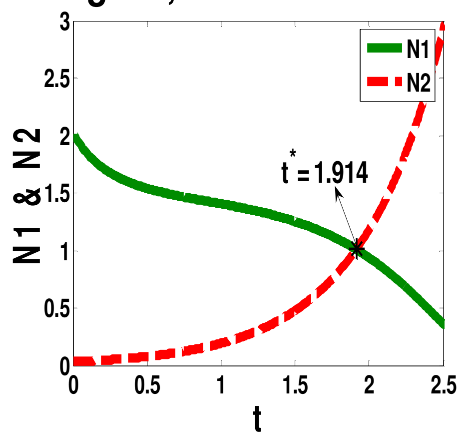


Fig.17 ;Case-9 in Table-2

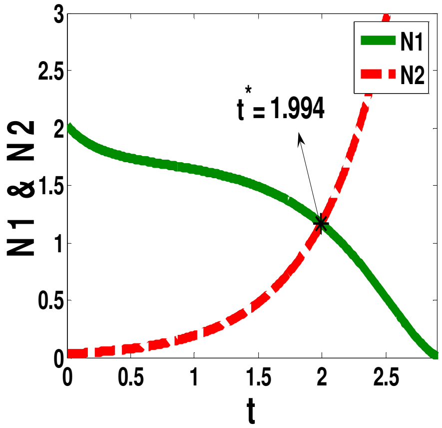


Fig.18 ;Case-10 in Table-2

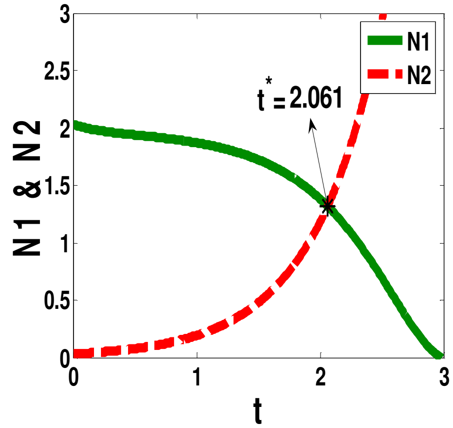
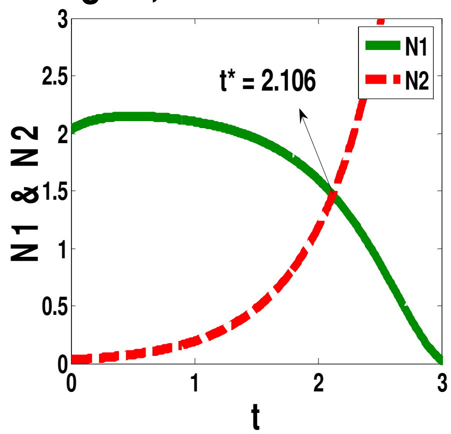


Fig.19;Case-11 in Table-2



The carrying capacity of Ammensal species is found by the ratio of the natural growth rate of Ammensal species and the rate of decrease of Ammensal species. This decrease happens due to its own insufficient resources. The values of Carrying capacity of Ammensal species according to the obtained numerical illustartions are tabulated in Table-3 along with the corresponding values of dominance reversal time(t*).

Table-3

S.NO	**K_1**	**t***
1	**0.29838**	**0.880**
2	**0.737720**	**1.003**
3	**1.177058**	**1.158**

4	**1.616396**	**1.340**
5	**2.055734**	**1.527**
6	**2.495073**	**1,689**
7	**2.934411**	**1.815**
8	**3.373749**	**1.914**
9	**3.813087**	**1.990**
10	**4.252425**	**2.061**
11	**4.691763**	**2.106**

6 Conclusions

The Ammensal species diminishes throughout the interval and eclipses the enemy species up to the dominance reversal time. It seems to be almost extinct with low growth rate. But the enemy species flourishes and exits with a steady growth rate forcing the Ammensal species to the extinction after dominance reversal time(t*). The distinguished relationships can be sorted as in Table-4,while keeping all the remaining parameters as constant.

Table-4

criterion	Conclusion
The carrying capacity and the growth rate of Ammensal species enhances	The dominance reversal time (t*) gradually raises

7 Future Work

One can investigate the following cases

I) A three species ecological Mathematical model consisting of a prey with unlimited resources, an immigrated predator with limited resources and a harvested Ammensal enemy to the prey, the effect of delay(s) of the interaction in the possible cases.

II) A Mathematical model of four species (S_1, S_2, S_3, and S_4); S_1 and S_2 are Ammensal to each other, S_2 is a predator living up on S_4, S_3 is a predator living up on S_1 and S_1 &S_4 are competitive struggling for existence.

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