Distribution of Loss Cost using Proportional Nucleolus Method in Competitive Power Markets

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***Abstract*—The present deregulated electricity industry has evolved into a distributed and competitive industry in which market forces drive the price of electricity and reduce the net cost of electricity. However the competitions in these markets require identification of the use of transmission networks, mainly the participation of utilities in losses caused in the transmission lines. This is because the consumers pay for their actual consumption where as generators are paid for their generation plus losses. Hence the loss cost allocation is of great importance as it should be allocated efficiently to both consumers and producers (both network users). Thus loss cost allocation in competitive electricity market requires a fair and efficient method with right economic signals. In this paper, loss cost allocation methodologies are presented in multilateral transaction frame work. Methodologies in cooperative game theory such as nucleolus, shapely and proportional nucleolus are presented. These are compared with conventional methods for loss allocation. Among all the methods presented, the proportional nucleolus is proved to be the efficient method with right economic signals. All the methods are implemented and results are compared for IEEE 14 bus, New England 39 bus and 75 bus Indian Power System.**

Keywords-cooperative game theory; deregulated electricity market; loss cost; multilateral transaction; nucleolus; proportional nucleolus; shapely

Nomenclature

y_i	**payoff allocated to player 'i'**
v(i)	**characteristic functional value**
e	**excess value**
φ_i	**Shapley value**
SC(i)	**Separable cost of player 'i'**
NSC	**Non Separable Cost**
x_i	**Cost allocated to player 'i'**
N	**Grand Coalition**
n	**Number of players**
φ	**Null Coalition**
n_S	**Number of players in Coalition S**

I. Introduction

The electricity industry throughout the world, which was long been dominated by vertically integrated utilities, is undergoing enormous changes. As a result, the present electricity market has been evolved into deregulated electricity industry in which market forces drive the price of electricity and reduce the net cost through increased competition. In this scenario, participants require and a fair and economic pricing structure that reflects both the share of power transacted as well as the cost of losses caused by users. This kind of allocation will have a influence on the decisions made by the market participants for their financial profits. Thus the loss cost allocation should reflect the participant's use of network. Though different cost allocation methods have been proposed till date, no method has gained economic significance/universal acceptance. Thus game theory concepts have been proposed for loss allocation in multiple transaction frame work [1].

Different loss allocation methods have been widely discussed in [2]-[6].Review of the methodologies is presented in [7].These methods have suggested loss cost allocations to generators, loads or to transactions. The cost allocation methodologies can be broadly classified into four as illustrated below:-

(a)Pro rata method: This method allocates the losses to generator or loads based on their power generated or consumed. As it allocates the cost based on the power transacted, this method possesses the main disadvantage of cross subsidization. That is it does not take into account the topology of the network.

(b)Incremental Transmission Loss method: These methods utilize the sensitivities with respect to nodal injections to allocate the losses to generator or loads. Selection of slack bus plays a major role in this method.

(c)Proportional sharing methods: These are tracing methods are used to compute the losses at each branch of the network.

(d)Transactional losses are computed according to the bilateral contracts/multi lateral contracts in competitive markets.

All these methodologies allocate the losses depending on a routine frame work established. However, they lack fairness and economic efficiency. Thus in order to overcome these drawbacks, game theory is implemented in loss cost allocation. However it remains a challenge to electricity market to choose the best allocation concept and distribute the losses among participants due to lack of fair allocation concepts.

II. Game Theory

Game Theory is the formal study of decision making where several players must make choices that potentially affects the choice of other players. Thus, game theory deals with any problem in which each player's strategy depends on what other players do. It looks at the rational behavior when each decision maker's well being depends on the decision of others as well as his own. It is assumed that the rationality of all players is of

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common knowledge. A player is said to be rational if he seeks to play in a manner which maximizes his own payoff. Payoff is the payment received at the end of game. It is mainly employed in power systems to prevent collusion due to market power i.e.discourage collusions that could minimize payoff [8]. The game theory methodologies can be used to identify non-competitive situations (from market co-coordinator point of view) and minimize the risks in price decisions (from participant's point). In this paper, Cooperative game theory is employed to allocate power system loss cost and mainly focuses on loss cost allocation in multilateral transaction framework. An extension of the core solution concept is presented for handling empty-core situations [13].

III. Cooperative Game Theory

A. Terminology

Consider game of N players with a characteristic function v. These players can form 2^N coalitions including the φ coalition. The characteristic function assigns to each coalition 'S' the minimum payoff under any adverse conditions. This can be found by applying max-min criteria to S and (N-S) players. An introduction to the cooperative game theory is presented in [12]. The set of all possible distribution of payoffs to the participants are called Imputations. A payoff vector $y = (y_1, y_2, \ldots\ldots y_n)$ is an imputation if it holds the following two conditions:

$$\sum_{i=1}^{n} y_i = v(N) \tag{1}$$

$$y_i \geq v(i),\ i = 1, 2, \ldots.\ n \tag{2}$$

There are numerous methods for the allocation of benefits among the participants of a cooperative game. Some of them are briefly described below:

B. The Core

One of the first solutions suggested for cooperative game is the core concept [10]. It is based on domination of imputations. That is, the core of a game is the set of all the imputations that are not dominated over any coalition.

For an imputation to belong to the core, it must satisfy

$$\sum_{i=1}^{n} y_i = v(N) \tag{3}$$

$$\sum_{i=1}^{n_s} y_i \geq v(S) \quad \forall S \subset N \tag{4}$$

It is clear that the core may include one or more than one imputation or may be even empty. Thus to choose a single solution whenever the core is non empty, Nucleolus concept was introduced in [9, 11].

C. The Nucleolus

It is based on the idea of minimizing the dissatisfaction of the most dissatisfied groups. For a coalition S, measure of its dissatisfaction is the excess e(S):

$$e(S) = v(S) - y(S) \tag{5}$$

where $y(S) = \sum_{i=1}^{n_s} y_i$

Thus the larger the excess, the more dissatisfied the coalition is with this Imputation. Thus it reduces to the following optimization problem.

$$\text{Min e} \tag{6}$$

$$y(S) + e \geq v(S) \tag{7}$$

$$y(N) = v(N) \tag{8}$$

One main drawback of nucleolus is that it is not monotonic that is even though the characteristic function v(s) of a coalition is increased, the payoff to the members of this coalition is not affected.

D. The Shapley Value

For the foundation of Shapley value [8], three axioms have been settled.

i. Symmetry: $\phi_i(v)$ is independent of the labeling of the players.

$$\phi_{\Pi(i)}(v) = \phi_i(v) \tag{9}$$

ii. Efficiency: The sum of the expectations must be equal to the characteristic functional value for the grand coalition N.

$$\sum_{i=1}^{n} \phi_i(v) = v(N) \tag{10}$$

iii. Additivity: The sum of expectations, for a player, by playing two games with characteristic values v_1 and v_2 must be equal to the value if he played both games together.

$$\phi_i(v_1 + v_2) = \phi_i(v_1) + \phi_i(v_2) \tag{11}$$

Thus the Shapley value which satisfies three axioms is given by

$$\phi_i(v) = \sum_{S, i \in S} \frac{(n_s - 1)!(n - n_s)!}{n!}[v(S) - v(S - \{i\})] \tag{12}$$

Its main advantage is it exhibits monotonicity. However its main disadvantage is that it may or may not lie inside the core.

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Interpretation [14]

- This is the mathematical expectation of the admission cost when all orders of formation of the grand coalition are equi-probable.
- Everything happens as if the players enter one by one, each of them receiving the entire saving he offers to the coalition formed just before him.
- All orders of formation of N are considered and intervene with same weight 1/n!

E. Proportional Nucleolus

In case of empty core, nucleolus cannot be implemented for loss cost allocation. Hence extended core concept is introduced in order to overcome this drawback. Thus non empty core is the main characteristic of proportional nucleolus. Just as the nucleolus chooses a unique solution from the core, the proportional nucleolus can be used to find the particular imputation from the extended core which is multi valued concept. It is important to find a unique solution from the multi valued core. The nucleolus formalizes the idea of a fair distribution of output in the sense of choosing the imputations that minimizes the biggest excess by any coalition as illustrated before. The proportional nucleolus differs from the original nucleolus in the definition of excess concerned with coalitions that suffer the biggest proportional excess of their worth as defined below

$$e(S)=\frac{v(S)-\sum_{j\in S}y_j}{v(S)} \quad \forall S\subset N \tag{13}$$

The proportional nucleolus can expand the core to obtain a unique solution in cases of both empty core and large core. Thus proving to be a better solution to both extended core and core selection problem. Thus it can be used to implement extended core as a solution concept.

Hence the proportional nucleolus solution can be obtained by solving the following minimization problem

$$\min e \tag{14}$$

$$\frac{v(S)-\sum_{j\in S}y_j}{v(S)}\geq e \tag{15}$$

$$\sum_{i=1}^{n}y_i=v(N) \tag{16}$$

IV. Conventional Loss Allocation Methods

The conventional loss allocation methods are given in [14].

A. Terminology

The separable cost for player 'i' can be defined as

$$SC(i)=v(N)-v(N/(i)) \tag{17}$$

This represent marginal cost when i^{th} player participate in coalition N-{i}.Thus it is the minimum cost which must be allocated to player 'i' when he participates in the grand coalition at the last moment.

After separable costs for all players are calculated the Non Separable Cost (NSC) can be calculated by using

$$NSC=v(N)-\sum SC(i) \tag{18}$$

Based on the above definitions and standard nomenclature, the following models are formulated [14]

Let 'N' be the number of players

x_i be the cost allocated to player 'i'

i. *Equal Repartition of the Total Gain(ERTG)*

$$x_i=v(i)-\frac{1}{N}\left[\sum_j v(j)-v(N)\right] \tag{19}$$

ii. *Proportional Repartition of the Total Gain(PRTG)*

$$x_i=v(i)-\frac{v(i)}{\sum_j v(j)}\left[\sum_j v(j)-v(N)\right]=\frac{v(i)}{\sum_j v(j)}v(N) \tag{20}$$

iii. *Equal Repartition of the Non Marginal Costs(ERNMC)*

$$x_i=SC(i)+\frac{1}{N}\left[v(N)-\sum_j SC(j)\right] \tag{21}$$

iv. *Proportional Repartition of the Non Marginal Costs(PRNMC)*

$$x_i=SC(i)+\frac{SC(i)}{\sum_j SC(j)}\left[\sum_j v(j)-v(N)\right]=\frac{SC(i)}{\sum_j SC(j)}v(N) \tag{22}$$

v. *Separable Costs Remaining Benefits (SCRB)*

$$x_i=SC(i)+\frac{v(\{i\})-SC(i)}{\sum_{j\in N}v(\{j\})-SC(j)}[NSC] \tag{23}$$

vi. *Egalitarian-Non Separable Cost Allocation Methods(ENSC)*

$$x_i=SC(i)+\frac{1}{N}NSC \tag{24}$$

V. Loss Allocation in Multilateral Transaction Framework

In multilateral transaction framework, the first step is to form the transactions for the chosen bus. This is accomplished by grouping the loads present in the system based on locational marginal pricing (LMP).

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A. Formation of Transaction

Step 1: Group the loads based on locational marginal prices. LMPs can be obtained using power world simulator.

Step 2: After the grouping of loads run a DCOPF with one set of grouped loads (transactions) present in the system. This gives the optimal generations of all generators. Repeat this for all transactions.

This gives the multilateral transactions. This paper consider each multiple transaction as one player in cooperative game theory and all transactions happening simultaneously as a coalition

B. Algorithm for the loss cost allocation

Step 1: Read the power flow data of the system

Step 2: Read the transaction data of system and consider each transaction as a player. Let 'N' be the number of players.

Step 3: Read all the possible coalitions. Let 'K' be the total coalitions that can be formed by using 'N' players

Step 4: Start with first coalition S=1. Run the Newton Raphson power flow to compute the losses corresponding to coalition 'S' and then multiply the losses (MW) with loss cost per MW in order to obtain the total loss cost. This gives the characteristic functional value of coalition 'S' given by v(S). Repeat for all coalitions.

Step 5: Then allocate the loss cost to all players by using game theory namely Shapely, Nucleolus or proportional nucleolus.

Step 6: The cost is then allocated using the conventional methods given by (19) to (24).

Step 7: Tabulate the results and stop.

VI. CASE STUDIES

These conventional and game theoretic methods are implemented in case of IEEE 14 bus, New England 39 bus and Indian 75 bus systems. Here it is assumed that the total load at buying node is considered as it is in the transaction without partitioning.

Case Study 1:

The above algorithm is implemented in case of an IEEE 14 bus system [17]. The loads are grouped based on their locational marginal pricing and then we obtain the generator outputs by running a DC OPF and thus the obtained transactions are as shown in table I.

TABLE I. TRANSACTIONS DATA IN IEEE 14 BUS SYSTEM

No.	Power(MW)	S(j,k)	B(i)
1	29.3	(1,16.9419),(2,12.3803)	2,5
2	142	(1, 75.24),(2, 66.75293)	3,4
3	30.8	(1,17.277),(2,13.522)	6,12,13
4	56.9	(1,27.06),(2,29.83)	9,10,11,14

where

S(j,k)=Bus 'j' supplying load 'k' for transaction 'i'.

B(i)=Load Buses

TABLE II. CHARACTERISTIC FUNCTIONAL VALUES OF IEEE 14 BUS SYSTEM

Coalition (S)	Characteristic Functions v(S)
1	2.6997
2	45.5326
3	6.0747
4	11.2879
5	51.5066
6	7.7809
7	13.2551
8	59.2932
9	75.2844
10	19.5341
11	66.9071
12	83.3113
13	23.0674
14	94.8386
15	137.9590

Table II shows the characteristic functional values of 15 coalitions of IEEE 14 bus system with 4 players. Here first transaction is near to the generators and does not use much of the network. We can see the advantage of Proportional nucleus in allocating losses to first transaction.

Table III shows the allocation of 137.959002 € to four transactions using Conventional methods. The total loss cost is obtained by assuming that all the transactions are present in the system and then running a load flow. The total loss obtained is then multiplied with the loss cost per MW in order to obtain total loss cost.

TABLE III. LOSS COST ALLOCATION IN IEEE 14 BUS SYSTEM USING CONVENTIONAL METHODS

No.	ERTG (€)	PRTG (€)	ERNMC (€)	PRNMC (€)	SCRB (€)	ENSC (€)
1	20.8	5.68	6.68	20.97	16.11	6.68
2	63.6	95.76	78.45	55.86	68.54	78.45
3	24.2	12.77	18.21	26.57	22.18	18.21
4	29.3	23.74	34.61	34.55	31.11	34.61

Table IV shows the allocation of 137.959002 € to four transactions using Existing and Variant Nucleolus methods.

TABLE IV. LOSS COST ALLOCATION IN IEEE 14 BUS SYSTEM USING GAME THEORETIC APPROACHES

No.	Nucleolus(€)	Shapley(€)	Proportional Nucleolus(€)
1	20.67	13.86	3.82
2	63.50	70.98	98.05
3	24.04	21.17	20.12
4	29.75	31.95	15.97

Fig. 1 shows the comparison graph of loss cost allocation for IEEE 14 bus system using conventional and game theory methods. From Fig.1 it is observed that proportional nucleolus method gives the true loss cost allocation among the players.

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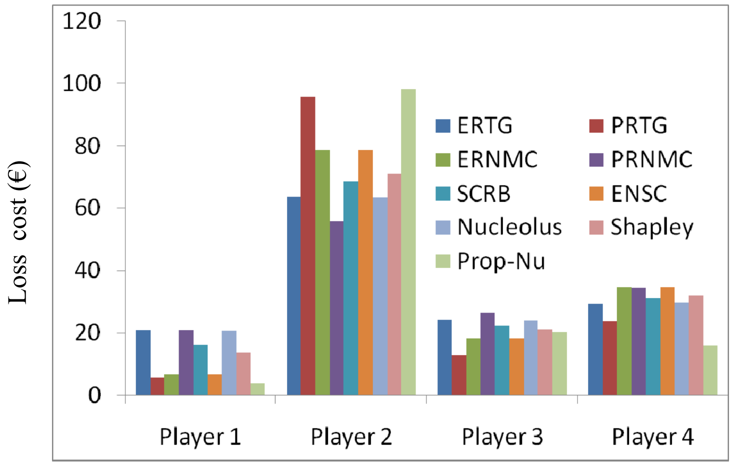

Fig. 1. Loss cost allocation using different methods for IEEE 14 bus system

Case Study 2:

The developed algorithms are tested on New England 39 bus system [18]. Loads are aggregated to form two transactions as shown in table V.

TABLE V. TRANSACTIONS DATA IN NEW ENGLAND 39 BUS SYSTEM

No.	Power(MW)	S(j,k)	B(i)
1	4794.8	(30 ,137.5),(31,584.43) (32,565.0),(33,647.91), (34, 607.85),(35,565.0) ,(36,538.12),(37,87.98) ,(38, 642.50), (39,418.48).	3,4 7,8 12,15,16,18,20,2 1,23,24,27
2	1965.7	(30 ,63.20)(31,180.48) (32, 194.81)(33,279.7) (34, 205.63) (35,195) (36, 172.5) (37,187.80) (38, 204.06) (39,282.5)	25,26,28,29,31,3 9

Table VI shows the allocation of 888.34€ to two transactions using Conventional loss allocation methods.

TABLE VI. LOSS COST ALLOCATION IN NEW ENGLAND 39 BUS SYSTEM USING CONVENTIONAL METHODS

No.	ERTG (€)	PRTG (€)	ERNMC (€)	PRNMC (€)	SCRB (€)	ENSC (€)
1	547.75	547.95	547.75	547.75	547.75	548.55
2	340.58	340.39	340.58	340.76	340.58	339.78

Table VII shows the allocation of 888.34€ to two transactions using Existing and Variant Nucleolus methods.

TABLE VII. LOSS COST ALLOCATION IN NEW ENGLAND 39 BUS SYSTEM USING GAME THEORETIC APPROACHES

No.	Nucleolus(€)	Shapley(€)	Prop-Nu(€)
1	547.76	547.76	547.94
2	340.58	340.58	340.40

Here we can see the effective allocation of Proportional nucleolus to second transaction which is smaller in magnitude and also does not use much of the network. Thus this in a way sends economic signals and also helps in checking market power.

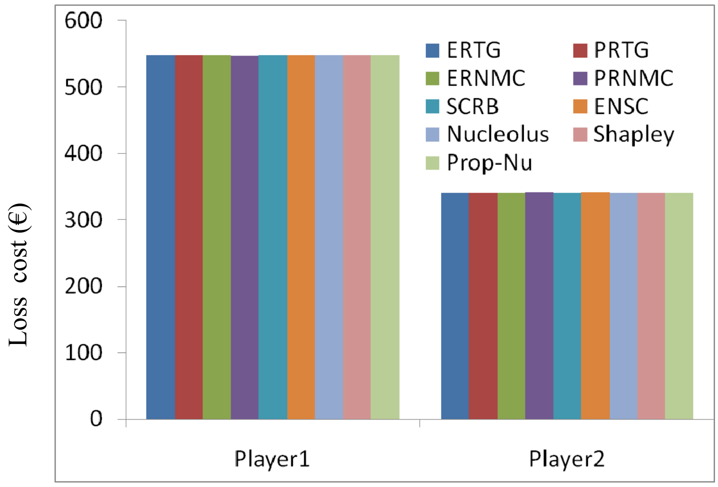

Fig. 2. Loss cost allocation using different methods for New England 39 bus system

Fig. 2 shows comparison graph of loss cost allocation for New England 39 bus system using conventional and game theory methods. Hence proportional nucleolus method shows the best one among all other methods.

Case Study 3:

In order to test the above techniques on a practical system, Indian 75 bus system [19] is chosen. The loads are aggregated based on locational marginal prices and an optimal power flow is run to find the optimal dispatch. The resulted transactions obtained are as shown in table VIII.

TABLE VIII. TRANSACTIONS DATA IN INDIAN 75 BUS SYSTEM

No.	Power (MW)	S(j,k)	B(i)
1	5199.26	(1,847.74),(2,331.63), (3,258.77),(4,25.91), (5,93.98),(6,205.75), (7,90.60),(8,68.56), (9,296.25),(10,62.00), (11,19.52),(12,1704.82), (13,806.88),(14,216.66), (15,170.1)	16,20,24,25, 27,28 ,30,32, 34 ,37,39,42, 46 ,47 ,48,49 50 ,51,52,53, 54 ,55,56,60, 61, 62, 63,64 65 ,66,67,68, 69, 70,71,72, 73 ,74 ,75
2	590.23	(1, 64.57) ,(2, 21.02), (3, 25.89) (4, 77.86) , (5,86.79), (6,64.86) (7,40.91) ,(8,16.60) , (9,36.72) (10,10) , (11, 53.09) , (12,56.14) (13,11.45) , (14, 12.93) (15,11.59)	57,58,59

Table IX shows the allocation of 2543.54€ to two transactions using Conventional methods.

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TABLE IX. LOSS COST ALLOCATION IN INDIAN 75 BUS SYSTEM USING CONVENTIONAL METHODS

No.	ERTG (€)	PRTG (€)	ERNMC (€)	PRNMC (€)	SCRB (€)	ENSC (€)
1	1870.5	1908.9	1870.58	1836.61	1870.5	1947.1
2	672.94	634.61	672.94	706.92	672.94	596.43

The following table X shows the allocation of 2543.54 € to two transactions using Existing and Variant Nucleolus methods.

TABLE X. LOSS COST ALLOCATION IN INDIAN 75 BUS SYSTEM USING GAME THEORETIC APPROACHES

No.	Nucleolus(€)	Shapley(€)	Prop-Nu(€)
1	1870.59	1870.59	1908.92
2	672.95	672.95	634.61

Here we can see the effective allocation of Proportional nucleolus to second transaction which is smaller in magnitude (590.23 MW) compared to first transaction.

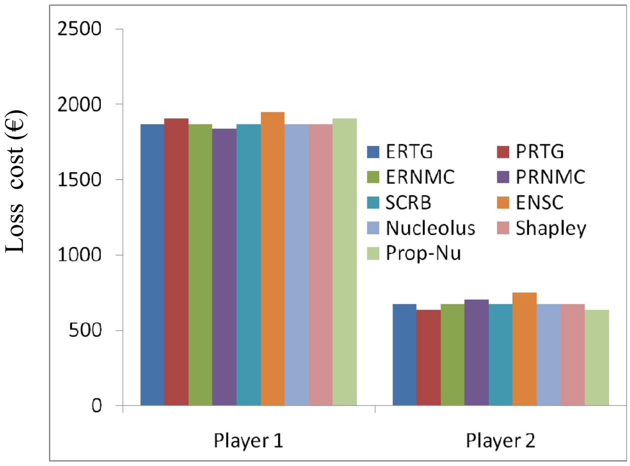

Fig. 3. Loss cost allocation using different methods for 75 bus system

Fig. 3 shows comparison graph of loss cost allocation for 75 bus Indian Power system using conventional and game theory methods. From Fig.3, it is proven that correct allocation of loss cost can be achieved with proportional nucleolus method.

VII. CONCLUSIONS

The conventional cost allocation methods fail to accomplish the economic signals and fair allocation. The ERNMC has an equivalent solution with GNSC. Though the practical cost allocation method (SCRB) is useful in terms of simplicity and fairness, it is not the best allocation method [16].

In case of Game Theoretic approaches, the Shapely value is monotonic. However it may or may not lie inside the core. In addition, as per the axioms on which Shapely value is formulated, it satisfies additive property. However loss allocation is non linear due to quadratic form of loss expression.

Though the nucleolus lies inside the core, its drawback is that it is not monotonic. Hence in order to overcome this drawback in case of nucleolus, proportional nucleolus is adopted. This method is monotonic unlike nucleolus. Similar to nucleolus, it lies inside the core as we are using the extended core concept. Thus it has great importance as it encourages empty core situations.

Hence Proportional Nucleolus is considered to be the most plausible concept for loss cost allocation.

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