

## Optimization of Buffer Allocation In Tandem Closed Queuing Network Using Symbiotic Organisms Search

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### Abstract

In this paper, we present Symbiotic organisms search (SOS) approach to optimize buffer allocation in manufacturing systems with tandem topology. Manufacturing systems are specified as closed queuing networks. Maximization of throughput is the objective function of this problem. Expanded Mean Value Analysis (EMVA) is used to evaluate fitness function of this problem. SOS and EMVA are used in cyclic manner to optimize buffer allocation in closed queuing networks. Numerical experiments are shown to explain effectiveness of procedure.

**Key words:** Buffer Allocation Problem; Closed Queuing Network; Expanded Mean Value Analysis; Symbiotic organisms search.

### Introduction

Buffer Allocation Problem (BAP) is an important research issue in manufacturing system design. Buffer Allocation Problem (BAP) deals with allocation of optimal buffer sizes among intermediate buffer locations of a manufacturing system to achieve a specific objective. Blocking of machines can be minimized by providing buffer space in front of each machine. So throughput rate of manufacturing system increases. But buffer space needs additional capital investment and floor space. So more buffer space leads to high cost and less buffer space leads to less throughput because of blocking effect.

### Queuing Networks

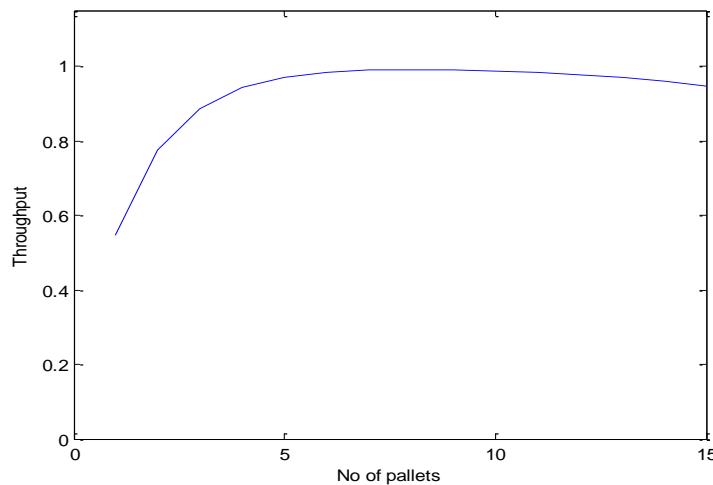
Queuing system is combination of server and queue. Queuing network is arrangement of queuing systems as a network. In manufacturing system each machine or service provider can be considered as a server and jobs waiting for operation in front of server

is treated as queue. Space provided for these jobs is buffer space. So manufacturing system can be considered as a queuing network. Queuing network with finite buffer capacities with external arrivals and departures is a Finite Open Queueing Network. Its work in process (WIP) is not constant. Network with finite buffers and without any arrival and departure is a Finite Closed Queueing Network. WIP of this network is constant.

Dead lock property and concavity property of finite closed queueing network are the important reasons for many research issues. Blocking of all servers is the dead lock situation in network. Chances for dead lock increases with number of jobs in closed queueing network. A necessary and sufficient condition for deadlock-free network under Blocking after service (BAS) is stated by Akyildiz [1] as follows:

*'The total number of customers (jobs) in closed queueing network must be less than the sum of buffer capacities in each cycle.'*

Throughput rate of network increases with number of jobs up to a maximum value and decreases onwards as shown in Fig.1. Throughput curve of finite closed queueing network is a quasi-concave curve in the size of population of network. This property is known as concavity property of closed queueing network.



**Figure 1:** Throughput graph for 3 node tandem network

### Buffer Allocation Problem In Finite Closed Queueing Network

Buffer sizes and number of jobs in network are the influencing parameters of BAP in finite closed queueing network. Based on space availability for buffers, BAP can be either constrained or unconstrained.

*Constrained space model:*

BAP with limited available space for buffer is known as constrained space model. Here objective function is optimization of fitness function, subject to sum of buffers equal to total available space for buffer allocation.

s.t. Sum of buffers = Total space available.

Deadlock free buffer allocation and optimum WIP are the research issues of this model.

*Unconstrained space model:*

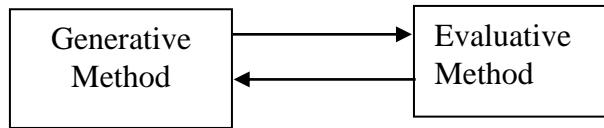
BAP with unlimited space for buffers is known as unconstrained space model. Objective function is either minimization of total cost or maximization of total profit for given WIP.

Objective function: Minimize (Total cost) or Maximize (Total profit)

s.t. Sum of buffers  $\geq$  WIP.

Deadlock free buffer allocation is the research issue of this model.

Evaluative and generative methods are required in cyclic manner to evaluate fitness function and to search for optimal solution respectively as shown in Fig. 2.



**Figure 2:** BAP solution process

## Literature Review

Gonzales [2] applied expansion and decomposition method to evaluate closed queueing network and applied Powell's method to optimize buffer allocation.

Daskalaki and Smith [3] attempted BAP, combined with routing in serial parallel queueing networks. An iterative 2-step method was used to solve BAP and routing problem. Expansion method was used as evaluative method and Powell's algorithm was used as generative method. Maximization of throughput and minimization of buffer size are the objective functions of this problem.

Smith and Cruz [4] solved BAP for general queueing networks. Minimization of total buffer size is objective function of this problem. The Generalized Expansion Method was used as evaluative method and Powell's algorithm was used as generative technique. Similar work was done by Smith et al. [5] for multi-server queueing networks.

Cruz et al. [6] solved the BAP in an arbitrary queueing networks. Aim was to find minimum buffer size in order to achieve desired throughput. Generalized Expansion Method was used as evaluative method and Lagrangian relaxation method was used as generative method.

Yuzukirmizi et al. [7] considered optimum buffer allocation for unconstrained space model of closed queueing networks. This is the first procedure to find optimum buffer allocation in closed queueing networks with general topologies and multiple servers. Expanded Mean Value Analysis was used to evaluate throughput of closed queueing network and Powell's algorithm was used to find optimum buffer allocation.

Cruz et al. [8] applied generalized expansion method and multi objective genetic algorithm to optimize throughput and buffer sizes for single server queuing network. Similar work was extended to optimize buffer size, throughput and server rate by Cruz et al. [9].

Soe and Lee [10] presented solutions for tandem queuing networks. Explicit expression was developed as an evaluative method to BAP.

K.L.Narasimhamu et al.[11] solved constrained space BAP for tandem closed queuing network with single server at each node. Objective function is to find optimum number of pallets and buffer allocation to maximize throughput.

## Model Description:

### Nomenclature

$M$	Number of nodes (Number of service stations)
$\mu$	Service rate of machine
$r_{ij}$	Routing probability that a job is transferred to $j^{\text{th}}$ node from $i^{\text{th}}$ node.
$C_p, C_g$	Coefficients (Constants)
$\lambda$	Throughput rate
$x_i$	Buffer space at $i^{\text{th}}$ node including server space
$c_i$	Number of servers at $i^{\text{th}}$ node
$K$	Buffer allocation vector ( $x_1, x_2, x_3, \dots, x_M$ )
$\xi$	Cycle in a network
$\pi_i(j   n)$	Conditional probability for $j$ pallets at node $i$ and $n$ pallets in the network.
$V_i$	Visit ratio at node $i$
$P_i(n)$	Blocking probability of a pallet at node $i$ with $n$ pallets in network.
rand	Random number

In the present work constrained space BAP model is considered. Manufacturing system is represented as a closed queuing network with single or multiple servers (machines) at each node. All these service centers are reliable and with exponential service rates. Buffer size includes the part being operated on machine. Blocking is considered after service. Topology of manufacturing system is tandem and there is a single class of customers (jobs).

Constrained space buffer allocation problem is formulated as follows.

*Find optimum buffer allocation  $X_1, X_2, X_3, \dots, X_M$  and optimum WIP*

*To Maximize ( $\lambda$ )*

*Subject to  $X_1 + X_2 + X_3 + \dots + X_M = \text{Total available space}$*

Expanded Mean Value Analysis, proposed by Yuzukirmizi et al. [7] is used as evaluative method to calculate throughput of manufacturing system. Symbiotic optimization search is used as search (generative) method to optimize buffer sizes.

### Symbiotic organisms search

Symbiotic organisms search (SOS) is a new meta-heuristic search algorithm proposed by Min yuan et al. [12]. It is a nature inspired technique. Mutualism, commensalism and parasitism are the symbiotic relationships in nature. Mutualism is a relationship between two different species in which both species will get benefit. Commensalism is a relationship between two different species in which one species will get benefit and other will not get benefit. Parasitism is a relationship between two different species in which one will get benefit and other will be effected negatively.

Pseudo code of general SOS algorithm is as follows:

```

Initialize ecosystem
Do {
    Mutualism phase
    Commensalism phase
    Parasitism phase
} while (termination)

```

### Proposed Optimization Procedure:

Following is the algorithm to solve constrained space buffer allocation problem using symbiotic organisms search.

Step 1: Generate initial eco system with constant sum of buffers

Step 2: Evaluate fitness value (Throughput). Find maximum throughput and corresponding number of pallets for each particle using EMVA algorithm.

Step 3: Iteration =0, i = 0

Step 4: iteration = iteration +1

Step 5: i = i+1, Find best organism ( $X_{best}$ )

Step 6: **Mutualism phase**

Step 6.1: Select one organism randomly  $X_j$ , where  $j \neq i$ .

Step 6.2: Determine Mutual relationship vector (Mutual\_vector) and Benefit factors ( $BF_1$ &  $BF_2$ )

$$\text{Mutual\_vector} = \frac{X_i + X_j}{2}$$

$BF_1$  = Random number either 1 or 2.

$BF_2$  = Random number either 1 or 2.

Step 6.3: Calculate change in  $X_i$  for all buffer spaces.

$$\text{Change in } X_i = \text{rand}(0,1) * (X_{best} - \text{Mutual\_vector} * BF_1)$$

Generate  $X_i$  new as follows

If change in  $X_i$  is positive then increase buffer size by one unit.

If change in  $X_i$  is negative then decrease buffer size by one unit.

Make adjustments to satisfy dead lock free buffer allocation.

Evaluate fitness values for  $X_i$  and  $X_i$  new . Update  $X_i$  with better fitness organism.

Step 6.4: Calculate change in  $X_j$  for all buffer spaces.

$$\text{Change in } X_j = \text{rand}(0,1) * (X_{best} - \text{Mutual\_vector} * BF_2)$$

Generate  $X_j$  new as follows

If change in  $X_j$  is positive then increase buffer size by one unit.

If change in  $X_j$  is negative then decrease buffer size by one unit.

Make adjustments to satisfy total buffer size.

Evaluate fitness values for  $X_j$  and  $X_{j\text{ new}}$ . Update  $X_j$  with better fitness organism.

**Step 7: Commensalism Phase**

Step 7.1: Select one organism randomly  $X_j$ , where  $j \neq i$ .

Step 7.2: calculate change in  $X_i$  for all buffer spaces.

$$\text{Change in } X_i = \text{rand}(-1,1) * (X_{\text{best}} - X_j)$$

Generate  $X_{i\text{ new}}$  as follows

If change in  $X_i$  is positive then increase buffer size by one unit.

If change in  $X_i$  is negative then decrease buffer size by one unit.

Make adjustments to satisfy total buffer size.

Step 7.3: Evaluate fitness function for  $X_{i\text{ new}}$ .

Compare  $X_{i\text{ new}}$  and  $X_i$ . Update  $X_i$  with organism having better fitness value.

**Step 8: Parasitism phase**

Step 8.1: Select one organism randomly  $X_j$ , where  $j \neq i$ .

Step 8.2: Generate  $X_{j\text{ new}}$  randomly

Step 8.3: Evaluate fitness function for  $X_{j\text{ new}}$ .

Compare  $X_j$  and  $X_{j\text{ new}}$ . Update  $X_j$  with organism having better fitness value.

Step 9: If  $i <$  size of eco system go to step 5. Otherwise go to step 10

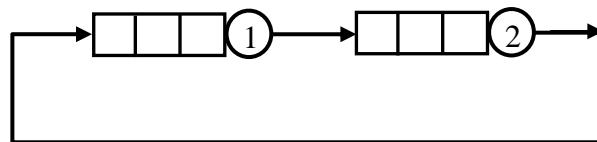
Step 10: If termination criteria is reached go to step 11. Otherwise go to step 4.

Step 11: Optimum buffer allocation =  $X_{\text{best}}$ .

## Numerical Experiments and Results

MATLAB code is written to find optimum buffer allocation in the closed queuing network using Symbiotic Organisms Search. Inputs to this program are network topology, number of nodes, service rates of machines and total available space. Tandem networks with different sizes are tested and shown below. Results obtained by above algorithm are verified with enumerative approach. For first four experiments, two tabular columns are shown under each experiment. First table shows results of enumeration approach and second table shows optimum result obtained by proposed algorithm. For remaining experiments optimum buffer allocation and optimum WIP are shown.

### Two node tandem network:



**Figure 3:** Two node tandem network

## Experiment I

**Table 1:** Complete enumeration for  $\mu_i = (8, 1)$ ,  $c_i = (1, 2)$ , available space = 6.

Number of pallets (WIP)	Buffer allocation				
	(1,5)	(2,4)	(3,3)	(4,2)	(5,1)
1	0.8889	0.8889	0.8889	0.8889	0.8889
2	1.7561	1.7561	1.7561	1.7561	1.7561
3	1.9319	1.9408	1.9408	1.9408	1.9056
4	1.9635	1.9831	1.9853	1.9765	1.9008
5	1.9690	1.9909	<b>1.9936</b>	1.9746	1.8895

**Table 2:** Solution using proposed algorithm for  $\mu_i = (8, 1)$ ,  $c_i = (1, 2)$ , available space = 6.

Optimum Buffer Allocation	Optimum size of WIP	Maximum Throughput
(3,3)	5	1.9936

## Experiment II

**Table 3:** Complete enumeration for  $\mu_i = (2, 3)$ ,  $c_i = (1, 2)$ , available space = 8.

Number of pallets (WIP)	Buffer allocation						
	(1,7)	(2,6)	(3,5)	(4,4)	(5,3)	(6,2)	(7,1)
1	1.2	1.2	1.2	1.2	1.2	1.2	1.2
2	1.7647	1.7647	1.7647	1.7647	1.7647	1.7647	1.7647
3	1.8157	1.9245	1.9245	1.9245	1.9245	1.9245	1.8868
4	1.8107	1.9348	1.9752	1.9752	1.9752	1.9627	1.9030
5	1.8029	1.9312	1.9794	<b>1.9918</b>	1.9876	1.9673	1.9036
6	1.7965	1.9185	1.9764	<b>1.9918</b>	1.9890	1.9664	1.9032
7	1.7916	1.9045	1.9668	1.9887	1.9873	1.9657	1.9031

**Table 4:** Solution using proposed algorithm for  $\mu_i = (2, 3)$ ,  $c_i = (1, 2)$ , available space = 8.

Optimum Buffer Allocation	Optimum size of WIP	Maximum Throughput
(4,4)	6	1.9918

### Experiment III

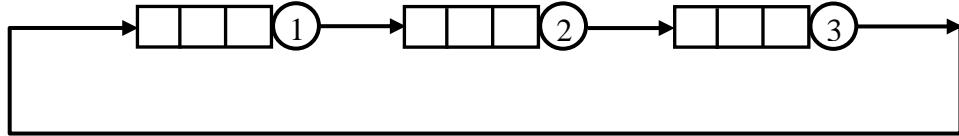
**Table 5:** Complete enumeration for  $\mu_i = (3, 2)$ ,  $c_i = (1, 3)$ , available space = 8.

Number of pallets (WIP)	Buffer allocation						
	(1,7)	(2,6)	(3,5)	(4,4)	(5,3)	(6,2)	(7,1)
1	1.2	1.2	1.2	1.2	1.2	1.2	1.2
2	2.069	2.069	2.069	2.069	2.069	2.069	2.069
3	2.4178	2.597	2.597	2.597	2.597	2.597	2.5042
4	2.4899	2.7292	2.8112	2.8112	2.8112	2.7696	2.5640
5	2.5026	2.7418	2.8693	2.9085	2.8888	2.7905	2.5405
6	2.5048	2.7150	2.8706	<b>2.9263</b>	2.8974	2.7660	2.5265
7	2.5052	2.6810	2.8355	2.9060	2.8733	2.7460	2.5231

**Table 6:** Solution using proposed algorithm for  $\mu_i = (3, 2)$ ,  $c_i = (1, 3)$ , available space = 8

Optimum Buffer Allocation	Optimum size of WIP	Maximum Throughput
(4,4)	6	2.9263

### Three Node Tandem Network



**Figure 4:** Three node tandem network

### Experiment IV

**Table 7:** Complete enumeration for  $\mu_i = (1, 0.1, 1)$ ,  $c_i = (1, 2, 1)$ , available space = 7.

Number of pallets (WIP)	Buffer allocation							
	(1,1,5)	(1,2,4)	(1,3,3)	(1,4,2)	(1,5,1)	(2,1,4)	(2,2,3)	(2,3,2)
1	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833
2	0.1644	0.1644	0.1644	0.1644	0.1644	0.1644	0.1644	0.1644
3	0.1884	0.1901	0.1901	0.1901	0.1898	0.1884	0.1901	0.1901
4	0.1931	0.1971	0.1974	0.1973	0.1965	0.1931	0.1971	0.1974
5	0.1936	0.1985	0.1993	0.1992	0.1981	0.1936	0.1985	0.1991
6	0.1931	0.1987	<b>0.1996</b>	<b>0.1996</b>	0.1984	0.1931	0.1986	0.194

Number of pallets (WIP)	Buffer allocation						
	(2,4,1)	(3,1,3)	(3,2,2)	(3,3,1)	(4,1,2)	(4,2,1)	(5,1,1)
1	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833
2	0.1644	0.1644	0.1644	0.1644	0.1644	0.1644	0.1644
3	0.1898	0.1884	0.1901	0.1898	0.1884	0.1898	0.1881
4	0.1965	0.1931	0.1970	0.1965	0.1931	0.1962	0.1923
5	0.1981	0.1936	0.1983	0.1980	0.1934	0.1972	0.1925
6	0.1984	0.1931	0.1984	0.1982	0.1929	0.1973	0.1920

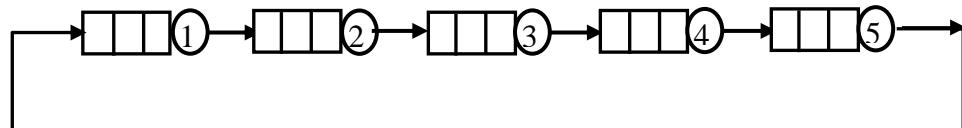
**Table 8:** Solution using proposed algorithm for  $\mu_i = (1, 0.1, 1)$ ,  $c_i = (1, 2, 1)$ , available space = 7.

Optimum Buffer Allocation	Optimum size of WIP	Maximum Throughput
(1, 3, 3)	6	0.1996

## Experiment V

Total available buffer space	Service rates	Number of servers	Optimum Buffer Allocation	Optimum size of WIP	Maximum Throughput
15	(1, 0.5, 1)	(1,2,1)	(5, 5, 5)	11	0.8039
25	(1, 0.5, 1)	(1,2,1)	(8, 9, 8)	15	0.7870
25	(1, 0.5, 2)	(1,2,1)	(4, 17, 4)	20	0.9398
50	(1, 0.5, 2)	(1,2,1)	(8, 36, 6)	39	0.9721

### Five node tandem network



**Figure 6:** Five node tandem network

## Experiment VI

Total available buffer space	Service rates	Number of servers	Optimum Buffer Allocation	Optimum size of WIP	Maximum Throughput
15	(0.3333, 1, 1, 0.3333, 0.3333)	(1, 2, 1, 1, 1)	(6,1,1,1,6)	14	0.2797
25	(0.3333, 1, 1, 0.3333, 0.3333)	(1, 2, 1, 1, 1)	(11,1,1,1,11)	21	0.2975
30	(0.3333, 1, 1, 0.3333, 0.3333)	(1, 2, 1, 1, 1)	(14,1,1,1,13)	24	0.3022
50	(0.3333, 1, 1, 0.3333, 0.3333)	(1, 2, 1, 1, 1)	(23,2,1,2,22)	36	0.3131
75	(0.3333, 1, 1, 0.3333, 0.3333)	(1, 2, 1, 1, 1)	(34,2,1,3,35)	51	0.3192
100	(0.3333, 1, 1, 0.3333, 0.3333)	(1, 2, 1, 1, 1)	(47,2,1,3,47)	66	0.3224

## Conclusion

This paper presents Symbiotic Organisms Search (SOS) approach to solve Buffer Allocation Problem (BAP) in tandem closed queueing networks. SOS is new metaheuristic algorithm inspired by biological interaction among various organisms. It follows three strategies of mutualism, commensalism and parasitism to find optimum result. Unlike other heuristic methods, SOS does not require any tuning parameter. Experimental results are shown for available space in closed queueing network up to 100 units. Optimum buffer allocation and optimum WIP obtained by proposed algorithm were tested with enumerative results. It is also observed that the throughput curve of above experiments for various buffer allocations follow quasi-concave shape in the size of population of closed queuing network. Hence it is proved that Symbiotic Organisms Search is suitable generative method to solve buffer allocation problem.

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