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TECHNICAL PAPER

Mixed convection in MHD doubly stratified micropolar fluid

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Abstract This paper analyzes the steady, mixed convection heat and mass transfer along a semi-infinite vertical plate with variable heat and mass fluxes embedded in a doubly stratified micropolar fluid. A uniform magnetic field of magnitude B_0 is applied normal to the plate. The governing nonlinear partial differential equations are transformed into a system of coupled nonlinear ordinary differential equations using similarity transformations and then solved numerically using the Keller-box method. The numerical results are compared and found to be in good agreement with previously published results as special cases of the present investigation. The study shows that increase in magnetic, thermal stratification and solutal stratification parameters increases the velocity and microrotation and decreases the temperature and concentration while the trend is reversed in case of coupling number. The micropolar fluids display more reduction in skin friction coefficient as well as wall couple stress than that exhibited by Newtonian fluids.

Keywords Micropolar fluid · Double stratification · MHD · Mixed convection · Heat and mass transfer

1 Introduction

Mixed convection flows are of great interest because of their various engineering, scientific and industrial applications in heat and mass transfer. Mixed convection of heat and mass transfer occurs simultaneously in the fields of design of chemical processing equipment, formation and dispersion of fog, distributions of temperature, moisture over agricultural fields, groves of fruit trees and damage of crops due to freezing and pollution of the environment ([1–3]). Extensive studies of mixed convection heat and mass transfer of a non-isothermal vertical surface under boundary layer approximation have been undertaken by several authors. The majority of these studies dealt with the traditional Newtonian fluids. It is well known that most fluids which are encountered in chemical and allied processing applications do not satisfy the classical Newton's law and are accordingly known as non-Newtonian fluids. Due to the important applications of non-Newtonian fluids in biology, physiology, technology and industry, considerable efforts have been directed toward the analysis and understanding of such fluids. A number of mathematical models have been proposed to explain the rheological behavior of non-Newtonian fluids. Among these, the fluid model introduced by Eringen [4] exhibits some microscopic effects arising from the local structure and micro motion of the fluid elements. Further, they can sustain couple stresses and include classical Newtonian fluid as a special case. The model of micropolar fluid represents fluids consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium where the deformation of the particles is ignored. Micropolar fluids have been shown to accurately simulate the flow characteristics of polymeric additives, geomorphologic sediments, colloidal suspensions, haematological suspensions, liquid crystals, lubricants etc. The heat and mass transfer in micropolar fluids is also important in the context of chemical engineering, aerospace engineering and also industrial manufacturing processes. The problem of mixed convection heat and mass

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transfer in the boundary layer flow along a vertical surface submerged in a micropolar fluid has been studied by a number of investigators.

The analysis of mixed convection in a doubly stratified (stratification of medium with respect to thermal and concentration fields) medium is a fundamentally interesting and important problem because of its broad range of engineering applications. These applications include heat rejection into the environment such as lakes, rivers, and seas; thermal energy storage systems such as solar ponds; and heat transfer from thermal sources such as the condensers of power plants. Although the effect of stratification of the medium on the heat removal process in a micropolar fluid is important, very little work has been reported in the literature. Cheng and Lee [5] analyzed the free convection on a vertical plate with uniform and constant heat flux in a thermally stratified micropolar fluid. Kumari and Nath [6] have solved the equations governing the unsteady mixed convection flow of an incompressible laminar electrically conducting fluid over an impulsively stretched permeable vertical surface in an unbounded quiescent fluid in the presence of a transverse magnetic field analytically using the homotopy analysis method as well as numerically by an implicit finite-difference scheme. Recently, Srinivasacharya and Ram Reddy [7] investigated the effect of doubly stratification on mixed convection in a micropolar fluid-saturated non-Darcy porous medium.

In recent years, several simple boundary layer flow problems have received new attention within the more general context of magnetohydrodynamics (MHD). The study of magneto-hydrodynamic flow for an electrically conducting fluid past a heated surface has important applications in many engineering problems such as plasma studies, petroleum industries, MHD power generators, cooling of nuclear reactors, the boundary layer control in aerodynamics and crystal growth. In addition, there has been a renewed interest in studying MHD flow and heat transfer in porous media due to the effect of magnetic fields on flow control and on the performance of many systems using electrically conducting fluids. The problem of MHD mixed convection heat and mass transfer in the boundary layer flow along a vertical surface submerged in a micropolar fluid has been studied by several investigators. Seddeek [8] investigated the analytical solution for the effect of radiation on flow of a magneto-micropolar fluid past a continuously moving plate with suction and blowing. Tzirtzilakis et.al [9] studied the action of a localized magnetic field on forced and free convective boundary layer flow of a magnetic fluid over a semi-infinite vertical plate. Mahmoud [10] analyzed the effects of slip and heat generation/absorption on MHD mixed convection flow of a micropolar fluid over a heated stretching surface. Hayat [11] studied the effects of heat and mass transfer on the mixed convection flow of a MHD micropolar fluid bounded by a stretching surface using homotopy analysis method. Das [12] considered the effects of partial slip on steady boundary layer stagnation point flow of an electrically conducting micropolar fluid impinging normally toward a shrinking sheet in the presence of a uniform transverse magnetic field.

From the literature survey, it seems that the similarity solutions for the effects of transverse magnetic field, thermal and solutal stratification on the laminar mixed convection heat and mass transfer along a vertical plate embedded in a micropolar fluid has not been reported so far. In view of this, the authors, in the present investigations, aim to study the mixed convection on a vertical plate with variable heat and mass fluxes embedded in a micropolar fluid in the presence of magnetic, thermal and solutal stratification effects. The novelty of this paper is the use of similarity transformations to find the solution of the problem. Most of the similar studies reported in the literature used local similarity transformations to solve the problems. It is established that similarity solutions are possible only when the variation in the temperature of the plate and the difference in the concentration are linear functions of the distance from the leading edge measured along the plate. Under these thermal and solutal boundary conditions, the governing system of partial differential equations is transformed into a system of nonlinear ordinary differential equations. The Keller-box method given in Cebeci and Bradshaw [13] is employed to solve the nonlinear system of this particular problem. The effects of various material parameters involved in the problem on the velocity, microrotation, temperature and concentration, heat and mass transfer rates are presented in the form of graphs.

2 Mathematical formulation

Consider a steady, laminar, incompressible, two-dimensional mixed convective heat and mass transfer along a semi-infinite vertical flat plate embedded in a free stream of doubly stratified, electrically conducting micropolar fluid with velocity $U(x)$. Choose the coordinate system such that x -axis is along the vertical plate and y -axis normal to the plate. The physical model and coordinate system are shown in Fig. 1. The plate is taken with variable surface heat flux $qw(x)$ and mass flux $qm(x)$. The temperature and the mass concentration of the ambient medium are assumed to be linearly stratified in the form $T_\infty(x) = T_{\infty,0} + A_1 x$ and $C_\infty(x) = C_{\infty,0} + B_1 x$, respectively, and increase linearly with respect to x, where A_1 and B_1 are constants and are given by A_1 and B_1 be the slopes of the ambient temperature profile and ambient concentration profile, respectively, with vertical distance and

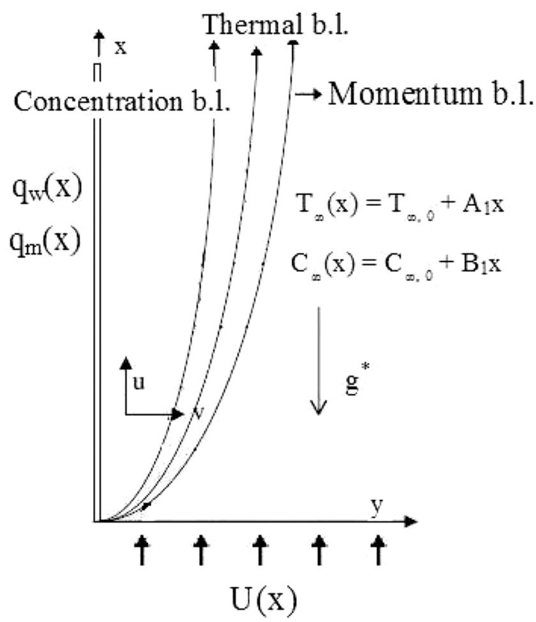

Fig. 1 Physical model and coordinate system

varied to alter the intensity of stratification in the medium. $T_{\infty,0}$ and $C_{\infty,0}$ are the ambient temperature and concentration at $x = 0$, respectively. A uniform magnetic field of magnitude B_0 is applied normal to the plate. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in comparison with the applied magnetic field. The Boussinesq approximation is invoked for the fluid properties to relate density changes, and to couple in this way the temperature and concentration fields, $\rho = \rho_\infty\left(1 - \beta_T\left(T - T_\infty\right) - \beta_C\left(C - C_\infty\right)\right)$ to the flow field.

For steady two-dimensional incompressible flow of a micropolar fluid with boundary layer and boussinesq approximations in the absence of the body couple, the governing equations for the micropolar fluid are given by [4, 14–16]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\left.\begin{aligned} &u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U(x)\frac{\mathrm{d}U(x)}{\mathrm{d}x} + \left(\frac{\mu+\kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \frac{\kappa\partial\omega}{\rho\,\partial y} + \\ &g^*(\beta_T(T - T_\infty) + \beta_c(C - C_\infty)) + \frac{\sigma B_0^2}{\rho}(U(x) - u) \end{aligned}\right\} \tag{2}$$

$$u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{\gamma}{\rho j}\frac{\partial^2 \omega}{\partial y^2} - \frac{\kappa}{\rho j}\left(2\omega + \frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{5}$$

where u and v are the components of velocity along x and y directions, respectively, ω is the component of microrotation whose direction of rotation normal to the xy-plane, g^* is the gravitational acceleration, T is the temperature, C is the concentration, β_T is the coefficient of thermal expansions, β_c is the coefficient of solutal expansions, B_0 is the coefficient of the magnetic field, μ is the dynamic coefficient of viscosity of the fluid, κ is the vortex viscosity, j is the micro-inertia density, γ is the spin-gradient viscosity, σ is the magnetic permeability of the fluid, v is the kinematic viscosity, α is the thermal diffusivity, D is the molecular diffusivity, σ is the magnetic permeability of the fluid, α is the thermal diffusivity and D is the molecular diffusivity.

Equations (1)–(3) represent the conservation of mass, conservation of momentum and conservation of angular momentum, respectively [4]. The last terms on the right-hand side of Eq. (2) stand for the Lorentz force term. The term $\left(\left(\sigma B_0^2/\rho\right)U(x)\right)$ represents the imposed pressure force in the inviscid region of the conducting fluid and $\left(\left(\sigma B_0^2/\rho\right)u\right)$ represents the Lorentz force imposed by a transverse magnetic field to an electrically conducting fluid. Equations (4) and (5) denote energy conservation law and conservation of species concentration, respectively.

The boundary conditions for the velocity, microrotation, temperature and concentration distributions for the present problem can be considered as:

$$\left.\begin{aligned} &u = 0, v = 0,\ \omega = 0,\ -k\frac{\partial T}{\partial y} = q_w(x),\ -D\frac{\partial C}{\partial y} = q_m(x) \ \text{ at } \ y = 0 \\ &u \to U(x), \omega \to 0, T \to T_\infty(x), C \to C_\infty(x) \text{ as } y \to \infty \end{aligned}\right\} \tag{6}$$

where the subscripts w and ∞ indicate the conditions at wall and at the outer edge of the boundary layer, respectively. The boundary condition $\omega = 0$ at wall i.e., at $y = 0$ in Eq. (6), represents the case of concentrated particle flows in which the microelements close to the wall are not able to rotate, due to the no-slip condition.

3 Method of solution

The continuity Eq. (1) is satisfied by introducing the stream function ψ such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{7}$$

To explore the possibility for the existence of similarity, we assume

$$\left.\begin{aligned} &\psi = A\,x^a f(\eta), \quad \eta = B\,yx^b, \quad \omega = E\,x^e\, g(\eta) \\ &T = T_{\infty,0} + \frac{q_w(x)}{Bk}\theta(\eta), \quad \frac{q_w(x)}{k} = M_1\,B\,x^m \\ &C = C_{\infty,0} + \frac{q_m(x)}{BD}\phi(\eta), \quad \frac{q_m(x)}{D} = N_1\,B\,x^n \end{aligned}\right\} \quad (8)$$

where $A, B, E, M_1, N_1, a, b, e, m$ and n are constants. We define $U(x) = A\,B\,x^d$. Substituting (7) and (8) in (2), (3), (4) and (5), it is found that similarity exists only if $a = 1, b = 0, c = d = m = n = 1$.

Hence, appropriate similarity transformations are

$$\left.\begin{aligned} &\psi = A\,x f(\eta), \quad \eta = B\,y, \quad \omega = E\,x\,g(\eta) \\ &T = T_{\infty,0} + \frac{qw(x)}{B\,k}\theta(\eta), \quad \frac{qw(x)}{k} = M_1\,Bx \\ &C = C_{\infty,0} + \frac{qm(x)}{B\,D}\phi(\eta), \quad \frac{qm(x)}{D} = N_1\,Bx \end{aligned}\right\} \quad (9)$$

Making use of the dimensional analysis, the constants A, B, E, M_1 and N_1 have, respectively, the dimensions of velocity, reciprocal of length, the reciprocal of the product of length and time, the ratio of (temperature/length) and the ratio of (concentration/length).

Substituting (9) in view of (8) into Eqs.(2), (3), (4) and (5), we obtain

$$\left.\begin{aligned} \left(\frac{1}{1-N}\right)f''' + f f'' - (f')^2 + \left(\frac{N}{1-N}\right) g' + R_i(\theta + L\,\phi) + \\ (1-M)f' + 1 = 0 \end{aligned}\right\} \quad (10)$$

$$\lambda\, g'' + f\,g' - f'\,g - \left(\frac{N}{1-N}\right)\mathcal{J}\,(2g + f'') = 0 \quad (11)$$

$$\frac{1}{Pr}\,\theta'' + f\,\theta' - f'\,\theta - \varepsilon_1 f' = 0 \quad (12)$$

$$\frac{1}{Sc}\,\phi'' + f\,\phi' - f'\,\phi - \varepsilon_2 f' = 0 \quad (13)$$

where primes denote differentiation with respect to similarity variable η, $N = \frac{\kappa}{\mu+\kappa}$, $(0 \le N < 1)$ is the Coupling number, $R_i = \frac{Gr}{Re^2}$ is the mixed convection parameter, $Gr = \frac{g^*\,\beta_T\,M_1}{\nu^2\,B^4}$ is the thermal Grashof number, $Re = \frac{A}{\nu\,B}$ is the Reynolds number, $Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $Sc = \frac{\nu}{D}$ is the Schmidth number, $\mathcal{J} = 1/(jB^2)$ is the micro-inertia density, $\lambda = \frac{\gamma}{j\rho\nu}$ is the spin-gradient viscosity, $L = \frac{\beta_c\,M_1}{\beta_T\,N_1}$ is the buoyancy parameter, $M = \frac{\sigma B_0^2}{\mu B^2}$ is the magnetic field parameter, $\varepsilon_1 = \frac{1}{M_1}\frac{\mathrm{d}}{\mathrm{d}x}[T_\infty(x)]$ is the thermal stratification parameter and $\varepsilon_2 = \frac{1}{N_1}\frac{\mathrm{d}}{\mathrm{d}x}[C_\infty(x)]$ is the solutal stratification parameter.

The boundary conditions (6) in terms of f, g, θ and ϕ become

$$\left.\begin{aligned} &\eta = 0: \quad f = 0, f' = 0,\, g = 0,\, \theta' = -1,\, \phi' = -1 \\ &as \quad \eta \to \infty: \quad f' \to 1,\, g \to 0,\, \theta \to 0,\, \phi \to 0 \end{aligned}\right\} \quad (14)$$

The wall shear stress and the wall couple stress are

$$\tau w = \left[(\mu + \kappa)\,\frac{\partial u}{\partial y} + \kappa\,\omega\right]_{y=0} \quad \text{and} \quad mw = \gamma\left[\frac{\partial \omega}{\partial y}\right]_{y=0} \quad (15)$$

The non-dimensional skin friction $C_f = \frac{2\tau w}{\rho A^2}$ and wall couple stress $Mw = \frac{B}{\rho A^2}mw$, where A is the characteristic velocity, are given by

$$C_f = \left(\frac{2}{1-N}\right) f''(0)\bar{x}, \quad \text{and} \quad Mw = \frac{\lambda}{\mathcal{J}}\,g'(0)\bar{x} \quad (16)$$

where $\bar{x} = B\,x$.

4 Results and discussion

The nonlinear nonhomogeneous differential Eqs. (10) to (13) are solved numerically using an implicit finite-difference method known as the Keller-box scheme [13]. This method has four main steps. The first step is converting the Eqs. (10–13) into a system of first-order equations. The second step is replacing partial derivatives by central finite-difference approximation. The third step is linearizing the nonlinear algebraic equations by Newton's method and then casting as the matrix vector form. The last step is solving linearized system of equations using the block-tridiagonal-elimination technique. Here, the initial values for velocity temperature and concentration are arbitrarily chosen so that they satisfy the boundary conditions. The independence of the results at least up to 4th decimal place on the mesh density was examined. A convergence criterion based on the relative difference between the current and previous iterations was used. When this difference reached 10^{-5}, the solutions were assumed to have converged and the iterative process was terminated. This method has been proven to be adequate and give accurate results for boundary layer equations. In the present study, the boundary conditions for η at ∞ are replaced by sufficiently large value of η, where the velocity approaches to 1, microrotation temperature and concentration profiles approach to zero. After some trials, we have taken $\eta_\infty = 6$.

To see the effects of step size ($\Delta\eta$), we calculated $f'(0)$, $g(0)$, $\theta(0)$ and $\phi(0)$ for three different mesh sizes $\Delta\eta = 0.001$, $\Delta\eta = 0.01$ and $\Delta\eta = 0.05$ and the results are presented in Table 1. From the Table 1 it is found that there is a very good agreement between them on different

Table 1 The Convergence Analysis of $f'(0)$, $g(0)$, $\theta(0)$ and $\phi(0)$ for different mesh sizes ($\Delta\eta$)

$\Delta\eta$	$f''(0)$	$-g'(0)$	$\theta(0)$	$\phi(0)$
0.001	1.461034032	2.575746828	1.245967091	1.896104369
0.01	1.461034029	2.575746726	1.245967093	1.896104364
0.05	1.461034026	2.575746721	1.245967087	1.896104359

Table 2 Comparison of results for a vertical plate in viscous fluids without stratification case Ramachandran et al. [15] for $R_i = 1.0$

Pr	$f''(0)$		$1/\theta(0)$	
	[15]	Present	[15]	Present
0.7	1.8339	1.833886666	0.7776	0.777614531
7.0	1.4037	1.403649652	1.6912	1.691206973

profiles. To study the effects of the coupling number N, magnetic field parameter M, thermal stratification parameter ε_1 and solutal stratification parameter ε_2 on the physical quantities of the flow, the remaining parameters are fixed as $L = 1$, $\lambda = 1$ and $\mathcal{J} = 0.1$. The values of micropolar parameters λ and $\mathcal{J}$ are chosen so as to satisfy the thermodynamic restrictions on the material parameters given by [4].

In the absence of coupling number N, magnetic parameter M , thermal stratification parameter ε_1, solutal stratification parameter ε_2 and buoyancy number L with $R_i = 1.0$, $\lambda \to 0$, $\mathcal{J} = 0$ and $Sc = 0.24$ the results have been compared with the Ramachandran et al. [15] for various values of Pr and found that they are in good agreement, as shown in Table 2.

The variation of the non-dimensional velocity, microrotation, temperature and concentration profiles with η for different values of magnetic parameter M is illustrated in

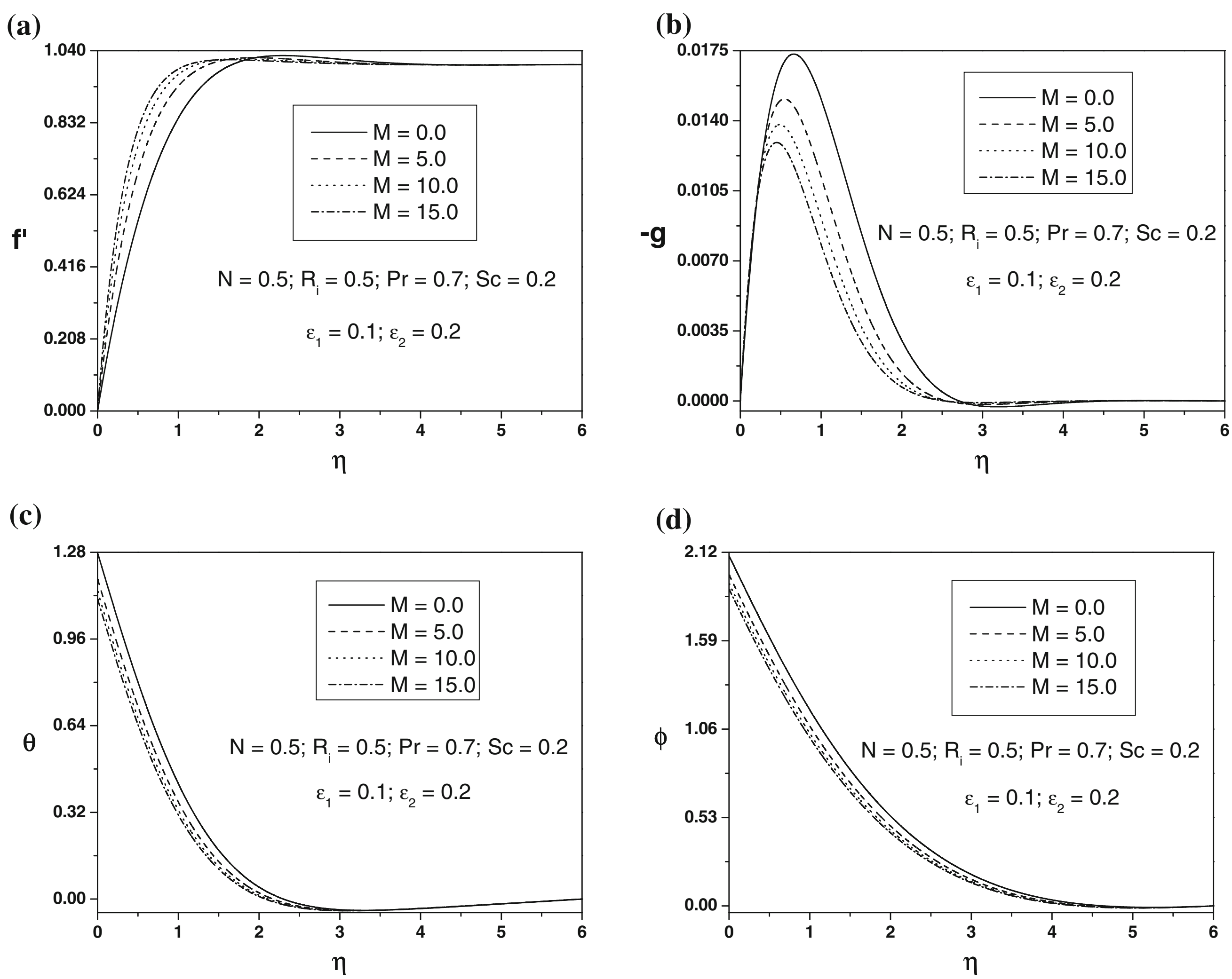

Fig. 2 Effect of Magnetic parameter M on **a** velocity **b** microrotation **c** temperature and **d** concentration

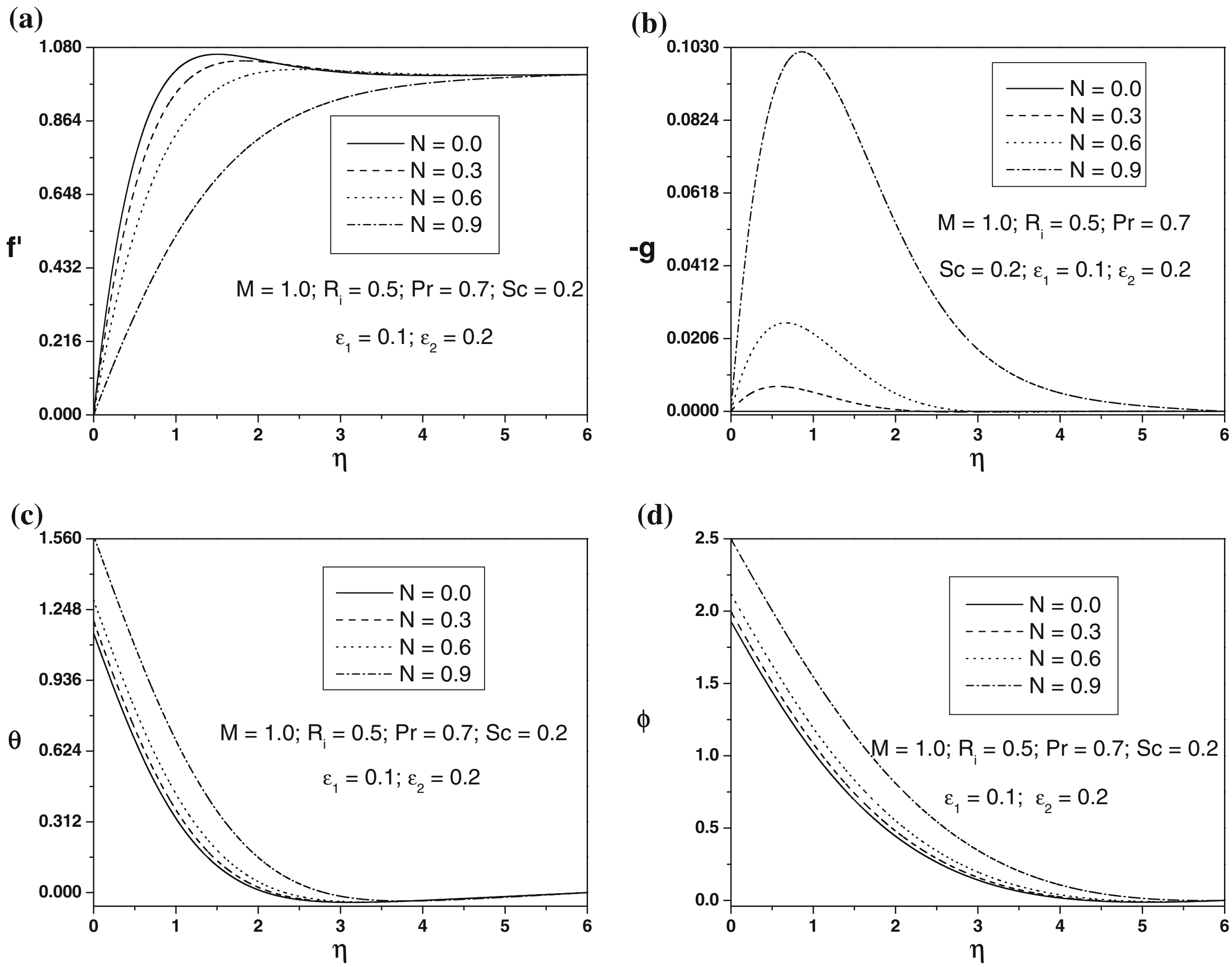

Fig. 3 Effect of Coupling number N on **a** velocity **b** microrotation **c** temperature and **d** concentration

Fig. 2. It is observed from Fig. 2a that momentum boundary layer thickness decreases i.e., velocity increase as the magnetic parameter (M) increases. From Eq. 2 when $U(x) > u$ (i.e., imposed pressure term dominates Lorentz force imposed by a transverse magnetic field normal to the flow direction), the effect of the magnetic interaction parameter will increase the velocity. Similarly, when $U(x) < u$,(when the Lorentz force dominates over the imposed pressure force), the effect of the magnetic interaction parameter will decrease the velocity. From Fig. 2b, it is observed that the microrotation component increases near the plate and decreases far away from the plate for increasing values of M. The reason is that the microrotation field in this region is dominated by a small number of particle spins that are generated by collisions with the boundary. It is noticed from Fig. 2c and d that the temperature and concentration decrease with increasing values of magnetic parameter. The magnetic field gives rise to a motive force to an electrically conducting fluid, this force makes the fluid experience an acceleration by decreasing the friction between its layers and thus decreases its temperature and concentration.

Figure 3 depicts the variation of velocity, microrotation, temperature and concentration with coupling number (N). The coupling number N characterizes the coupling of linear and rotational motion arising from the micromotion of the fluid molecules. Hence, N signifies the coupling between the Newtonian and rotational viscosities. As $N \rightarrow 1$, the effect of microstructure becomes significant, whereas with a small value of N the individuality of the substructure is much less pronounced. As $\kappa \rightarrow 0$ i.e., $N \rightarrow 0$, the micropolarity is lost and the fluid behaves as nonpolar fluid. Hence, $N \rightarrow 0$ corresponds to a viscous fluid. It is observed from Fig. 3a that the velocity decreases with the increase of N. The maximum of velocity decreases in amplitude and the location of the maximum velocity moves farther away from the wall with an

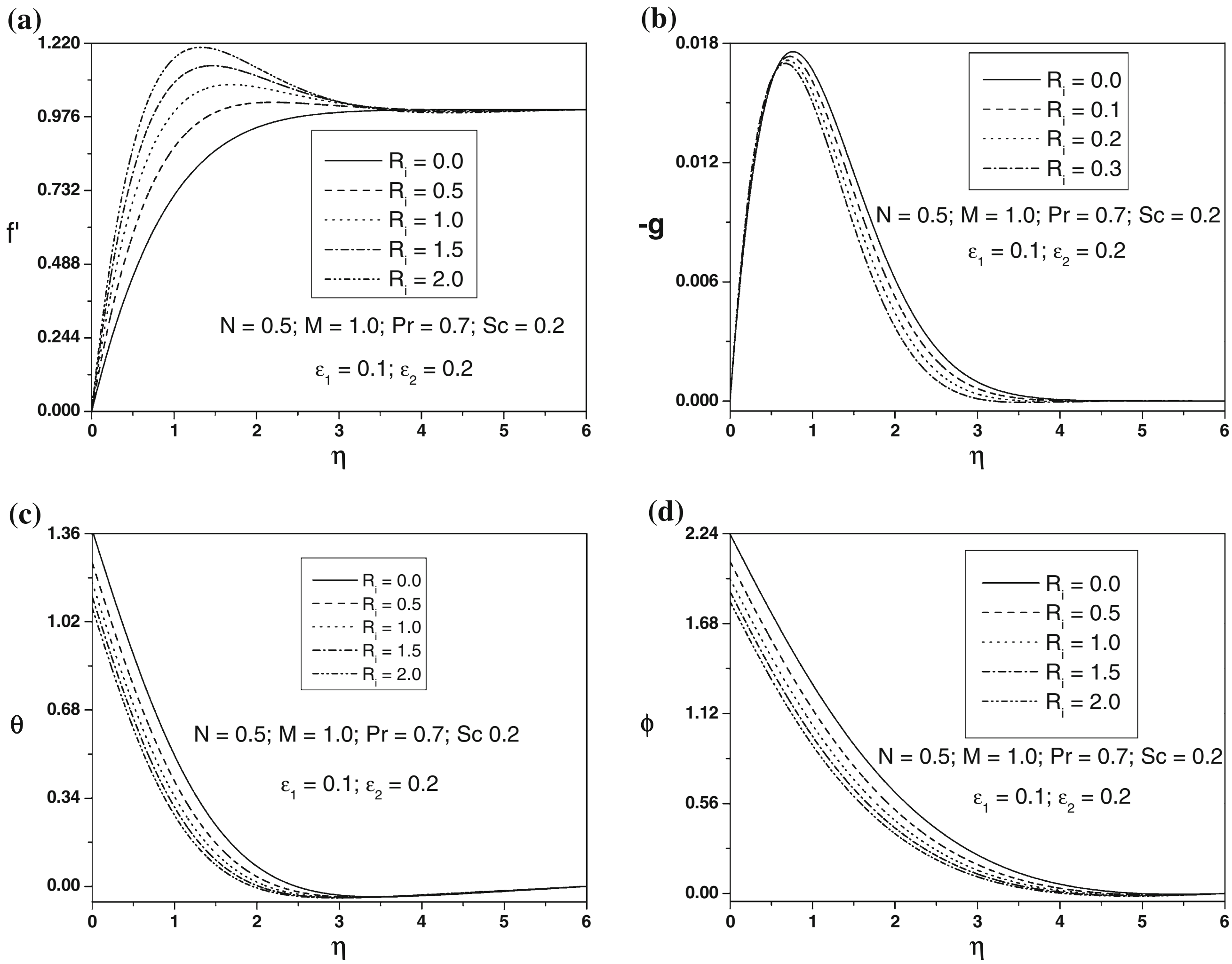

Fig. 4 Effect of mixed convection parameter R_i on **a** velocity **b** microrotation **c** temperature and **d** concentration

increase of N. The velocity in case of micropolar fluid is less than that in the viscous fluid case. It is seen from Fig. 2b that the microrotation component decreases near the vertical plate and increases far away from the plate with increasing coupling number N. The microrotation tends to zero as $N \to 0$ as is expected. It is noticed from Fig. 3c that the temperature increases with increasing values of coupling number. It is clear from Fig. 3d that the non-dimensional concentration increases with increasing values of N.

Figure 4 explains the effect of mixed convection parameter R_i on the non-dimensional velocity, microrotation, temperature and concentration profiles. Figure 4a shows that the dimensionless velocity rises as R_i increases. The higher value of R_i leads to the greater buoyancy effect in mixed convection flow, hence it accelerates the flow. It is seen from Fig. 4b, that within the boundary layer the microrotation is completely negative. Also, it is clear that the magnitude of the microrotation decreases with an increase in mixed convection parameter R_i. From Fig. 4c it is detected that the non-dimensional temperature of the fluid flow is decreasing from pure forced convection case ($R_i \to 0$) to the pure free convection case ($R_i > 1$). It is clear from Fig. 4d that the non-dimensional concentration decreases as R_i increases. As velocity is increasing with increase of R_i, as a consequence effect the concentration of the fluid decreases.

The effect of thermal stratification parameter ε_1 on the non-dimensional velocity, microrotation, temperature and concentration is shown in Fig. 5. It is observed from Fig. 5a that the velocity decreases with the increase of thermal stratification ε_1. This is because thermal stratification reduces the effective convective potential between the heated plate and the ambient fluid in the medium. Hence, the thermal stratification effect reduces the velocity in the boundary layer. From Fig. 5b, it is noticed that the values of microrotation change sign from negative to positive within the boundary layer. Also, it is clear that the magnitude of the microrotation increases with an increase in thermal

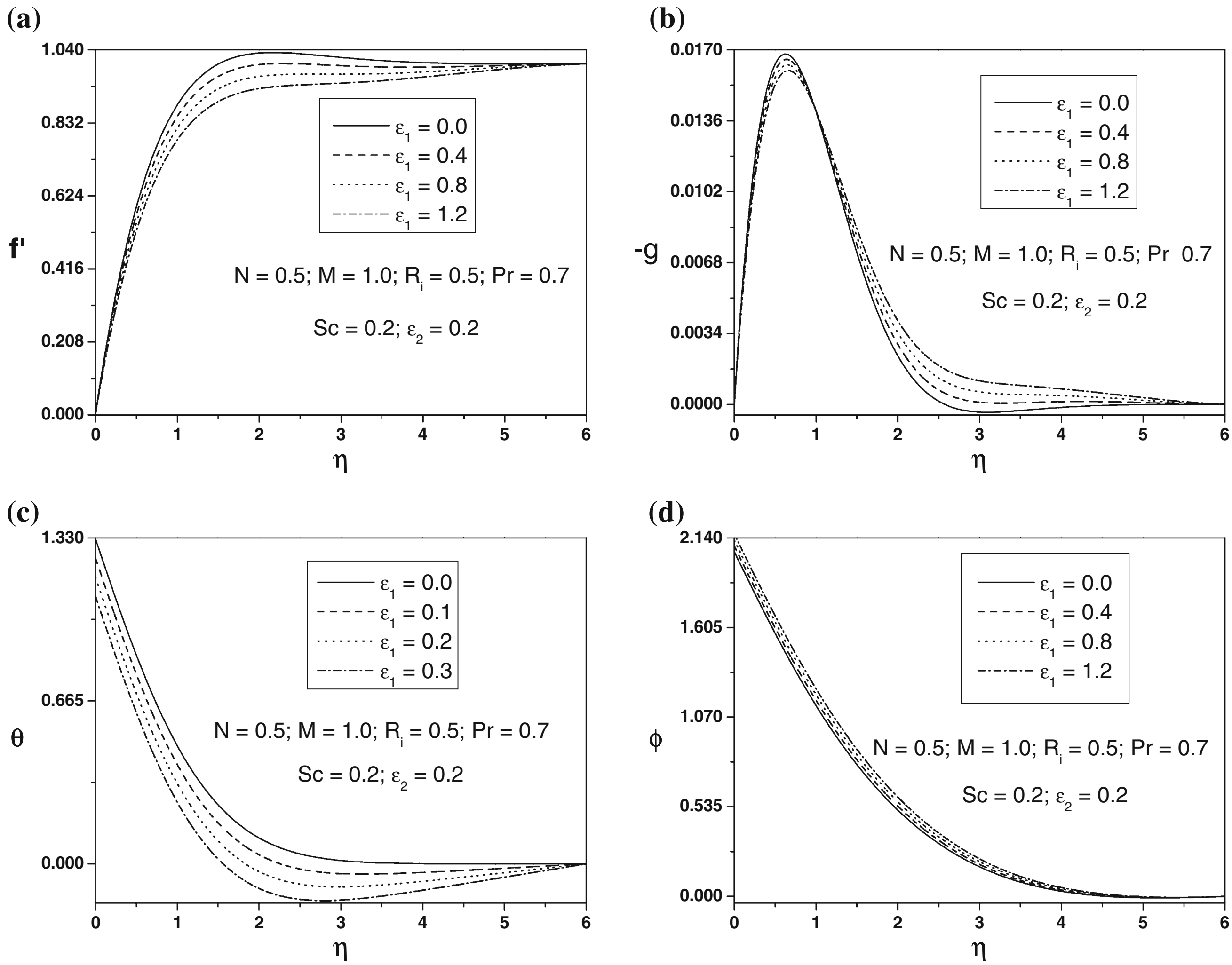

Fig. 5 Effect of thermal stratification parameterε_1 on **a** velocity **b** microrotation **c** temperature and **d** concentration

stratification parameter. It is depicted from Fig. 5c that the non-dimensional temperature of the fluid decreases with the increase of thermal stratification parameter. When the thermal stratification effect is taken into consideration, the effective temperature difference between the plate and the ambient fluid will decrease; therefore, the thermal boundary layer is thickened and the temperature is reduced. Figure 5d demonstrates that the concentration of the fluid increases with the increase of thermal stratification parameter. It is noticed that the effect of the stratification on temperature is the formation of a region with a temperature deficit (i.e., a negative dimensionless temperature). This is in tune with the observation made in references (Prandtl [17], Jaluria and Himasekhar [18], Gebhart et al. [19], Murthy et al. [20], Lakshmi Narayana and Murthy [21]).

The dimensionless velocity, microrotation, temperature and concentration for different values of solutal stratification parameter ε_2 are depicted in Fig. 6. From Fig. 6a it is observed that the velocity of the fluid decreases with the increase of solutal stratification parameter. From Fig. 6b it is depicted that the microrotation values change sign from negative to positive at the critical point $\eta = 1.16$ within the boundary layer. Also, it is clear that the magnitude of the microrotation increases with an increase in solutal stratification parameter. It is noticed from Fig. 6c the temperature of the fluid increases with the increase of solutal stratification parameter. It is clear from Fig. 6d that the non-dimensional concentration of the fluid decreases with the increase of thermal stratification parameter.

Table 3 shows the effects of the coupling number N, Prandtl number Pr, Schmidt number Sc, the magnetic parameter M, Mixed convection parameter R_i, thermal stratification parameter ε_1 and solutal stratification parameter ε_2 on the dimensionless skin friction C_f and wall couple stress M_w. It is seen from this table that the skin friction, wall couple stress decrease with increasing coupling

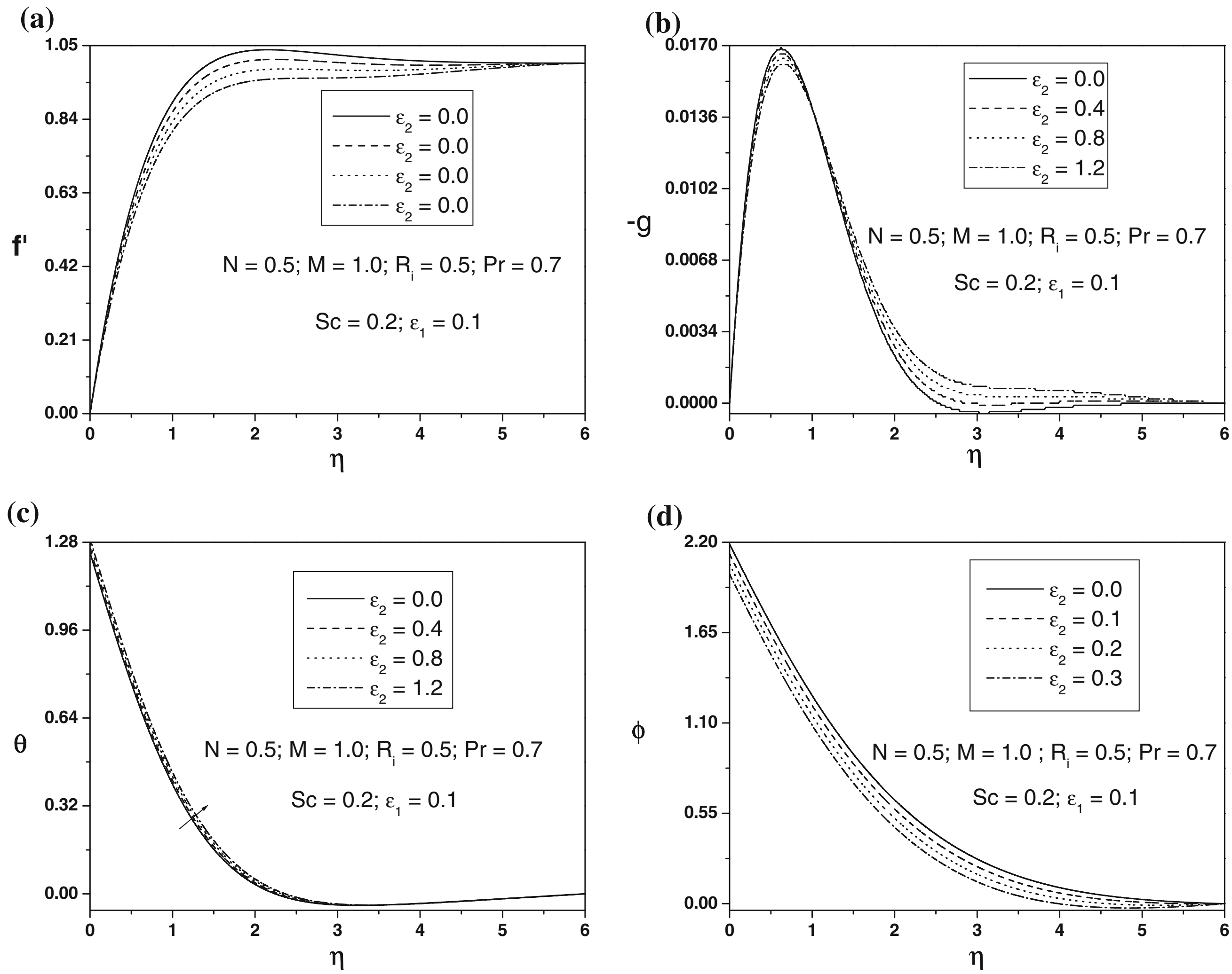

Fig. 6 Effect of solutal stratification parameterϵ_2 on **a** velocity **b** microrotation **c** temperature and **d** concentration

number N. For increasing values of N, the effect of microstructure becomes significant, hence the wall couple stress decreases. The skin friction coefficient decreases and the wall couple stress increases with increasing Prandtl number. The skin friction coefficient decreases and the wall couple stress increases with Schmidt number. The effect of magnetic parameter is to decrease the skin friction coefficient whereas increase the wall couple stress. From this table we observe that the increasing values of mixed convection parameter R_i increases the skin friction and decreases the wall couple stress. It demonstrates that the skin friction parameter, Nusselt number and Sherwood number decrease and the wall couple stress increases as ε_1 increases. It is clear that the skin friction parameter, Nusselt number and Sherwood number decrease and the wall couple stress increases as ε_2 increases. Furthermore, the skin friction parameter is higher, and wall couple stress parameter is lower for the unstratified fluid (i.e., $\varepsilon_1 = \varepsilon_2 = 0$) than for the stratified fluid (i.e., $\varepsilon_1 > 0$ and $\varepsilon_2 \neq 0$).

5 Conclusions

In this paper, a boundary layer analysis for mixed convection heat and mass transfer in an electrically conducting micropolar fluid over a vertical plate in the presence of a uniform magnetic field of magnitude with thermal and solutal stratification effects is considered;

- The fluid velocity decreases with increasing values of coupling number, thermal stratification parameter and solutal stratification parameters but increases with magnetic and mixed convection parameters.
- The temperature enhances for increasing values of coupling number and solutal stratification parameter but reduces for magnetic parameter, mixed convection and thermal stratification parameter.
- The concentration increases for increasing values of coupling number and thermal stratification parameter

Table 3 Effect of skin friction and wall couple stress for various values of N, R_i, Pr, Sc, M, ε_1 and ε_2

N	R_i	Pr	Sc	M	ε_1	ε_2	$f''(0)$	$-g'(0)$
0.0	0.5	0.7	0.2	1.0	0.1	0.2	2.2523	0.0000
0.3	0.5	0.7	0.2	1.0	0.1	0.2	1.8736	0.0297
0.6	0.5	0.7	0.2	1.0	0.1	0.2	1.3954	0.0906
0.9	0.5	0.7	0.2	1.0	0.1	0.2	0.6324	0.2945
0.5	0.0	0.7	0.2	1.0	0.1	0.2	1.1154	0.0555
0.5	0.5	0.7	0.2	1.0	0.1	0.2	1.5711	0.0641
0.5	1.0	0.7	0.2	1.0	0.1	0.2	1.9455	0.0706
0.5	1.5	0.7	0.2	1.0	0.1	0.2	2.2739	0.0760
0.5	2.0	0.7	0.2	1.0	0.1	0.2	2.5714	0.0807
0.5	0.5	0.01	0.2	1.0	0.1	0.2	2.2914	0.0841
0.5	0.5	0.1	0.2	1.0	0.1	0.2	1.8429	0.0713
0.5	0.5	0.7	0.2	1.0	0.1	0.2	1.5711	0.0641
0.5	0.5	1.0	0.2	1.0	0.1	0.2	1.5430	0.0635
0.5	0.5	7.0	0.2	1.0	0.1	0.2	1.4560	0.0620
0.5	0.5	10.0	0.2	1.0	0.1	0.2	1.4480	0.0619
0.5	0.5	100.0	0.2	1.0	0.1	0.2	1.4229	0.0616
0.5	0.5	0.7	0.2	1.0	0.1	0.2	1.5711	0.0641
0.5	0.5	0.7	0.4	1.0	0.1	0.2	1.4701	0.0614
0.5	0.5	0.7	0.6	1.0	0.1	0.2	1.4244	0.0602
0.5	0.5	0.7	0.8	1.0	0.1	0.2	1.3971	0.0596
0.5	0.5	0.7	1.0	1.0	0.1	0.2	1.3784	0.0591
0.5	0.5	0.7	0.2	0.0	0.1	0.2	1.4026	0.0623
0.5	0.5	0.7	0.2	1.0	0.1	0.2	1.5711	0.0641
0.5	0.5	0.7	0.2	2.0	0.1	0.2	1.7231	0.0656
0.5	0.5	0.7	0.2	3.0	0.1	0.2	1.8627	0.0669
0.5	0.5	0.7	0.2	1.0	0.0	0.2	1.5865	0.0646
0.5	0.5	0.7	0.2	1.0	0.4	0.2	1.5252	0.0627
0.5	0.5	0.7	0.2	1.0	0.8	0.2	1.4640	0.0608
0.5	0.5	0.7	0.2	1.0	1.2	0.2	1.4032	0.0589
0.5	0.5	0.7	0.2	1.0	0.1	0.0	1.5965	0.0649
0.5	0.5	0.7	0.2	1.0	0.1	0.4	1.5458	0.0633
0.5	0.5	0.7	0.2	1.0	0.1	0.8	1.4956	0.0618
0.5	0.5	0.7	0.2	1.0	0.1	1.2	1.4459	0.0602

but decreases for magnetic parameter, mixed convection and solutal stratification parameter.

- The skin friction coefficient as well as wall couple stress in the micropolar fluid are lower compared to that of the Newtonian fluid.
- The microrotation, skin friction increase and wall couple stress decreases with increase in the magnetic parameter.

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