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Mixed Convection on a Vertical Plate in a Power-law Fluid Saturated Porous Medium with Cross Diffusion Effects

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Abstract

Mixed convection heat and mass transfer along a vertical plate embedded in a power-law fluid saturated Darcy porous medium with Soret and Dufour effects is studied. The governing partial differential equations are transformed into ordinary differential equations using similarity transformations and then solved numerically using shooting method. A parametric study of the physical parameters involved in the problem is conducted and a representative set of numerical results is illustrated graphically.

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Keywords: Mixed convection; Darcy porous medium; Power-law fluid; Soret and Dufour effects.

1. Introduction

The analysis of mixed convection boundary layer flow along a vertical surface embedded in porous media has received considerable theoretical and practical interest. The mixed convection flow occurs in several industrial and technical applications such as electronic devices cooled by fans, nuclear reactors cooled during an emergency shutdown, a heat exchanger placed in a low-velocity environment, solar collectors and so on. A number of studies have been reported in the literature focusing on the problem of mixed convection about different surface geometries in porous media. A review of convective heat transfer in porous medium is presented in the book by Nield and Bejan [1]. The majority of these studies dealt with the traditional Newtonian fluids. It is well known that most fluids which are encountered in chemical and allied processing applications do not satisfy the classical Newton's law and are accordingly known as non-Newtonian fluids. A number of mathematical models have been proposed to explain the rheological behavior of non-Newtonian fluids. Among these, a model which has been most widely used for non-Newtonian fluids, and is frequently encountered in chemical engineering processes, is the power-law model. Although this model is merely an empirical relationship between the stress and velocity gradients, it has been

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successfully applied to non-Newtonian fluids experimentally. Elgazery and AbdElazem [2] analyzed numerically a mathematical model to study the effects of a variable viscosity and thermal conductivity on unsteady heat and mass transfer in a non-Newtonian power-law fluid flow through a porous medium past a semi-infinite vertical plate with variable surface temperature in the presence of magnetic field and radiation. Chamkha et al. [3] studied the effects of melting, thermal radiation and heat generation or absorption on steady mixed convection from a vertical wall embedded in a non-Newtonian power-law fluid saturated non-Darcy porous medium for aiding and opposing external flows. Hayat et al. [4] investigated the Magneto hydrodynamic (MHD) mixed convection stagnation-point flow and heat transfer of power-law fluids towards a stretching surface.

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of a more intricate nature. It has been observed that an energy flux can be generated not only by temperature gradients but also by concentration gradients. The energy flux caused by a concentration gradient is termed the diffusion-thermo (Dufour) effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the thermal-diffusion (Soret) effect. In most of the studies related to heat and mass transfer process, Soret and Dufour effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. But these effects are considered as second order phenomena and may become significant in areas such as hydrology, petrology, geosciences, etc. The Soret effect, for instance, has been utilized for isotope separation and in mixture between gases with very light molecular weight and of medium molecular weight. The importance of these effects in convective transport in clear fluids has been reported in the book by Eckert and Drake [5]. Although the Soret and Dufour effects of the medium on the heat and mass transfer in a viscous fluid are important, very little work has been reported in the literature. Mahdy [6] presented a non-similar boundary layer analysis to study the flow, heat and mass transfer characteristics of non-Darcian mixed convection of a non-Newtonian power law fluid from a vertical isothermal plate embedded in a homogeneous porous medium with the effect of Soret and Dufour and in the presence of either surface injection or suction. Tai and Char [7] studied numerically on the combined laminar free convection flow with thermal radiation and mass transfer of non-Newtonian power-law fluids along a vertical plate within a porous medium in the presence of Soret and Dufour effects. Pal and Mondal [8] studied the influence of chemical reaction and thermal radiation on mixed convection heat and mass transfer over a stretching sheet in Darcy porous medium with Soret and Dufour effects.

The purpose of the present work is to investigate the Soret and Dufour effects on natural convection heat and mass transfer from vertical plate in Darcy porous media saturated with power-law fluid with variable heat and mass flux conditions. Also, the effects of heat and mass transfer coefficient are illustrated in tabular form for various parameters.

2. Mathematical Formulation

Consider the mixed convection heat and mass transfer along a vertical plate in a non-Newtonian power-law fluid saturated Darcy porous medium. Choose the coordinate system such that *x*-axis is along the vertical plate and *y*-axis normal to the plate. The plate is maintained at variable surface heat flux $q_w(x)$ and mass flux $q_m(x)$. The temperature and concentration of the ambient medium are T_∞ and C_∞ respectively. Assume that the fluid and the porous medium have constant physical properties except for the density variation required by the Boussinesq approximation. The flow is steady, laminar, two dimensional. The porous medium is isotropic and homogeneous. The fluid and the porous medium are in local thermodynamical equilibrium. In addition the Soret and Dufour effects are taken in to consideration. Under these conditions, the governing equations describing the fluid flow can be written as follows.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u^n=u_\infty^n+\frac{gK}{\upsilon}\left(\beta_T\left(T-T_\infty\right)+\beta_C\left(C-C_\infty\right)\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\frac{D_m K_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

where u and v are the Darcian velocity components along x and y directions, T is the temperature, C is the concentration, k_T is the thermal diffusion ratio, υ is the kinematic viscosity, K is the permeability, g is the acceleration due to gravity, β_T is the coefficient of thermal expansion, β_C is the coefficient of concentration expansion, α_m is the thermal diffusivity, D_m is the mass diffusivity of the porous medium, C_p is the specific heat capacity, C_s is the concentration susceptibility, $T_m$is the mean fluid temperature and n is the power-law index. When $n = 1$, the Eq. (2) represents a Newtonian fluid. Therefore, deviation of n from a unity indicates the degree of deviation from Newtonian behavior. For $n< 1$, the fluid is shear thinning and for $n> 1$, the fluid is shear thickening.

The boundary conditions are

$$v=0,\ -k\frac{\partial T}{\partial y}=q_w(x),\ -D_m\frac{\partial c}{\partial y}=q_m(x)\ \text{ at } y=0 \text{ and } u\to u_\infty,\ T\to T_\infty,\ C\to C_\infty \text{ as } y\to\infty \tag{5}$$

3. Solution of the problem

In view of the continuity eq. (1), we introduce the stream function ψ by

$$u=\frac{\partial \psi}{\partial y},\quad v=-\frac{\partial \psi}{\partial x} \tag{6}$$

In order to explore the possibility for the existence of similarity, we assume

$$\left.\begin{aligned}&\eta=B\,y\,x^{b},\ \ \psi=A\ x^{a}\ f(\eta),\ T=T_\infty+\frac{q_w(x)}{k}\theta(\eta)\\ &\frac{q_w(x)}{k}=E\,x^{l}\ ,\ C=C_\infty+\frac{q_m(x)}{D}\phi(\eta),\frac{q_m(x)}{D}=F\ x^{m}\end{aligned}\right\} \tag{7}$$

where A, B, E, F, a, b, l and m are constants. Substituting (6) and (7) in (2),(3) and (4), it is found that similarity exists only if $a=\frac{2}{3}, b=\frac{-1}{3}, l=m=\frac{n}{3}$.Hence, appropriate similarity transformations are

$$\left.\begin{aligned}&\eta=B\,y\,x^{-1/3},\ \ \psi=A\ x^{2/3}\ f(\eta),T=T_\infty+\frac{q_w(x)}{k}\theta(\eta),\\ &\frac{q_w(x)}{k}=E\,x^{\frac{n}{3}},C=C_\infty+\frac{q_m(x)}{D}\phi(\eta),\frac{q_m(x)}{D}=F\,x^{\frac{n}{3}}\end{aligned}\right\} \tag{8}$$

Making use of the similarity transformations (8) in (2), (3) and (4) we get the following nonlinear system of differential equations.

$$\left(f'\right)^n=\left(1+\theta+\mathrm{N}\phi\right) \tag{9}$$

$$\theta''=\frac{1}{3}\left(n\,f'\theta-2\,f\theta'\right)-D_f\phi'' \tag{10}$$

$$\phi''=\frac{Le}{3}\left(n\,f'\phi-2\,f\phi'-3S_r\theta''\right) \tag{11}$$

where primes denote differentiation with respect to η alone, $S_r = \dfrac{D_m k_T q_w D}{\alpha_m T_m q_m k}$ is the Sore number $D_f = \dfrac{D_m k_T q_m k}{\alpha_m C_p C_s q_w D}$ is the Dufour number, $N = \dfrac{\beta_C F}{\beta_T E}$ is the buoyancy ratio, $Le = \dfrac{\alpha_m}{D_m}$ is the Lewis number.

The boundary conditions (5) in terms of f, θ, and ϕ become

$$f(0) = 0, \quad \theta'(0) = -1, \quad \phi'(0) = -1, \quad f'(\infty) = 1, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0 \tag{12}$$

The equations (9)-(11) have been obtained after properly choosing the dimensional constants A, B in the following form:

$$A = \left(\frac{EgK\beta_T \alpha_m^n}{\nu}\right)^{1/2n} \quad \text{and} \quad B = \left(\frac{EgK\,\beta_T}{\nu\alpha_m^n}\right)^{1/2n}$$

The parameters of engineering interest for the present problem are the Nusselt and Sherwood numbers, which are given by the expressions

$$\frac{Nu_x}{Bx^{2/3}} = \frac{1}{\theta\ (0)} \quad \text{and} \quad \frac{Sh_x}{Bx^{2/3}} = \frac{1}{\phi\ (0)} \tag{13}$$

4. Results and Discussions

The boundary value problem given by equations (9)-(11) along with the boundary conditions (12) are solved using the Shooting method by giving appropriate initial guess values for $f'(0)$, $\theta\ (0)$ and $\phi\ (0)$ to match the values with the corresponding boundary conditions at $f'(\infty)$, $\theta(\infty)$ and $\phi(\infty)$ respectively. In the present study, the boundary conditions for η at ∞ varies with parameter values and it has been suitably chosen at each time such that the velocity approach one, temperature and concentration profiles approach zero at the outer edge of the boundary layer.

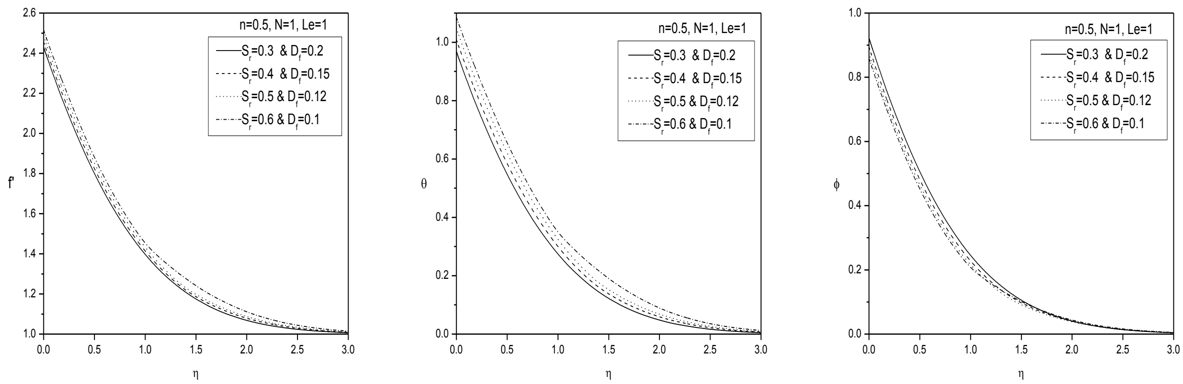

Figure 1: Velocity, Temperature and Concentration profiles for various values of S_r *and* D_f for pseudo-plastic fluids.

The non-dimensional velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$ are plotted for N= 1, Le = 1 in Fig. 1 with varying Soret and Dufour parameters for n = 0.5 (i.e., for pseudo-plastic fluids). It can be observed from Fig.1 that increasing the value of the Soret parameter(or simultaneously decreasing of Dufour parameter) the velocity of the fluid increases. Soret number is the ratio of temperature difference to the concentration. Hence, the bigger Soret number stands for a larger temperature difference and precipitous gradient. Thus the fluid velocity rises due to greater thermal diffusion factor. It is noticed from Fig.1 that the temperature of the fluid increases with the increase in the value of the Soret parameter. It is found from Fig.1 that the concentration of the fluid decreases with increase in the value of the Soret parameter.

The effects of Soret and Dufour parameters on the non-dimensional velocity, temperature and concentration is shown in Fig. 2 for$N=1$ and $Le=1$ by considering n = 1.0 (i.e., for Newtonian fluid). It isnoticed from Fig.2 that the velocity of the fluid is increased with increase in the value of the Soret parameter (or simultaneously decreasing of Dufour parameter). Fig.2 demonstrates that increase in the value of the Soret parameter increases the temperature of the fluid in the medium. It is seen from Fig.2 that the concentration of the fluid is decreased by increasing the value of the Soret parameter.

The variation of $f^{'}(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ with Soret and Dufour parameters is depicted in Fig.3 for N=1 and Le=1 for n =1.5 (i.e.,for dilatant fluids). Fig.3 illustratesthat the fluid velocity is increased with increase in the value of Soret parameter. It is seen from Fig.3 that the temperature of the fluid inthe medium is increased with increase in the value of the Soret parameter. It is found from Fig.3 that the concentration of the fluid is decreased by increasing the value of the Soret parameter(or simultaneously decreasing of Dufour parameter).

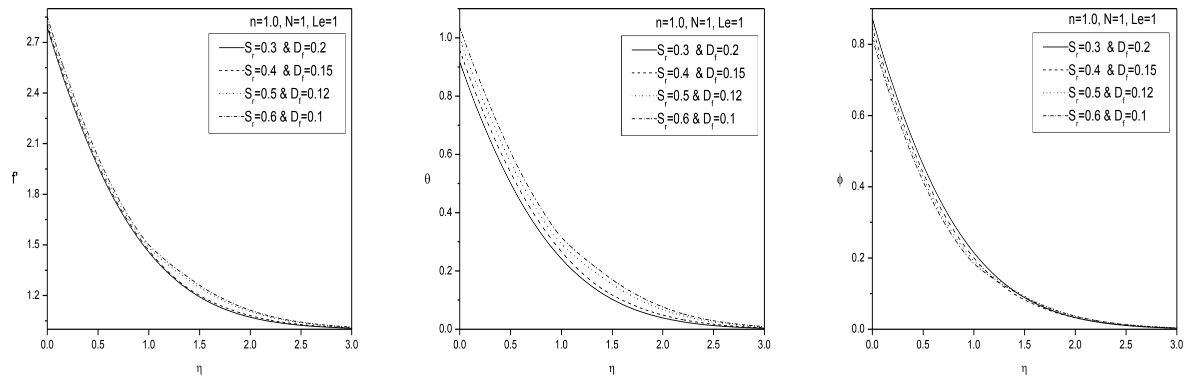

Figure 2: Velocity, Temperature and Concentration profiles for various values of S_r and D_f for Newtonian fluid.

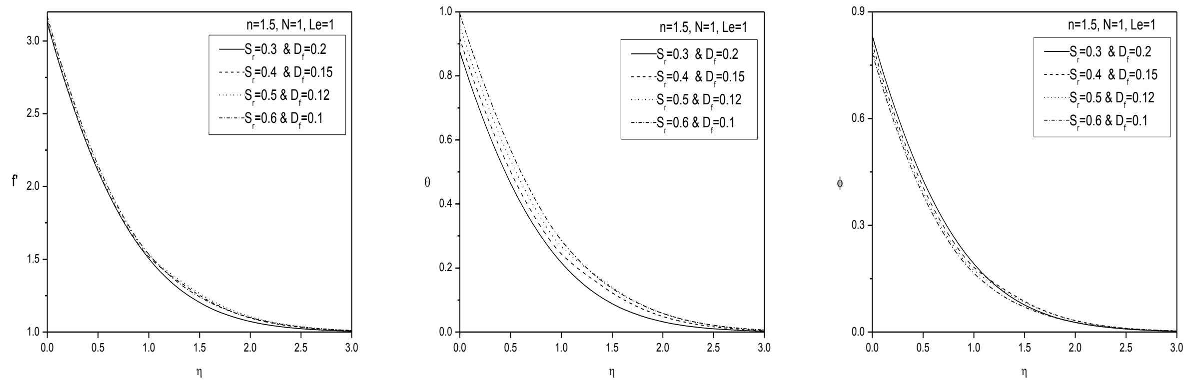

Figure 3: Velocity, Temperature and Concentration profiles for various values of S_r *and* D_f for dilatant fluids .

The non-dimensional velocity, temperatureand concentration with variation in power-law index are plotted in Fig.4. It is observed from Fig.4 that the fluid velocity is increased with increase in the value of the power-law index parameter. The effect of the increasing values of the power law index n is to increase the horizontal boundary layer thickness. That is, the thickness is much smaller for shear thinning (pseudo plastic; $n < 1$) fluids than that of shear thickening (dilatants; $n > 1$) fluids. In the case of a shear thinning fluid ($n < 1$), the shear rates near the walls are higher than those for a Newtonian fluid. It can be seen from Fig.4 that the temperature in the fluid is increased with increase in the value of the power law index parameter. Increasing the values of the power law index leads to increase the thermal boundary layer thickness. It can be found from Fig.4 that the concentration of the fluid increases with increase in the value of the power-law index parameter. Increasing the power-law index *(n)* tends to accelerate the flow and increases the solutal boundary-layer thickness. It is interesting to note that the velocity, temperature and concentration forNewtonian fluids is more than that plastic fluids and less than that of dilatants fluids.

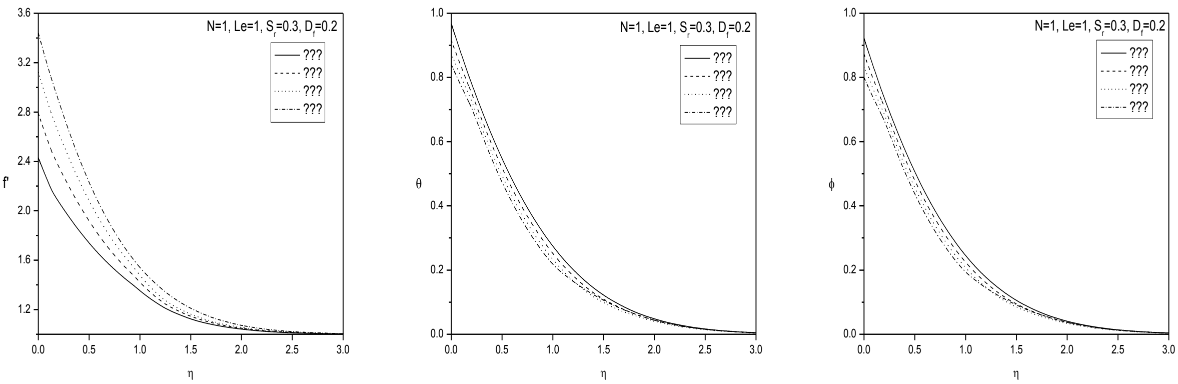

Figure 4: Velocity, Temperature and Concentration profiles for various values of Power-law index (n)

Table 1: Variation of non-dimensional heat and mass transfer coefficients for various values of n, Le, N, S_r and D_f.

n	Le	S_r	D_f	N	$\frac{Nu_x}{Bx^{2/3}} = \frac{1}{\theta\ (0)}$	$\frac{Sh_x}{Bx^{2/3}} = \frac{1}{\phi\ (0)}$
0.0	1.0	0.3	0.2	1.0	0.963889	1.011445
0.5	1.0	0.3	0.2	1.0	1.032258	1.083950
1.0	1.0	0.3	0.2	1.0	1.090961	1.146183
1.5	1.0	0.3	0.2	1.0	1.142901	1.201230
2.0	1.0	0.3	0.2	1.0	1.189784	1.250903
2.5	1.0	0.3	0.2	1.0	1.232712	1.296381
1.5	0.0	0.3	0.2	1.0	2.255437	0.250000
1.5	0.5	0.3	0.2	1.0	1.248342	0.905918
1.5	1.0	0.3	0.2	1.0	1.142901	1.201230
1.5	1.5	0.3	0.2	1.0	1.094896	1.414549
1.5	2.0	0.3	0.2	1.0	1.066307	1.585874
1.5	2.5	0.3	0.2	1.0	1.046904	1.731101
1.5	1.0	0.3	0.20	1.0	1.142901	1.201230
1.5	1.0	0.4	0.15	1.0	1.091636	1.234548
1.5	1.0	0.5	0.12	1.0	1.046802	1.258393
1.5	1.0	0.6	0.10	1.0	1.006652	1.277202
1.5	1.0	0.8	0.075	1.0	0.936907	1.306808
1.5	1.0	1.0	0.06	1.0	0.877870	1.330700
1.5	1.0	0.3	0.2	0.5	1.405854	1.483001
1.5	1.0	0.3	0.2	0.6	1.309065	1.379424
1.5	1.0	0.3	0.2	0.7	1.243781	1.309524
1.5	1.0	0.3	0.2	0.8	1.198283	1.260755
1.5	1.0	0.3	0.2	0.9	1.165984	1.226074
1.5	1.0	0.3	0.2	1.0	1.142901	1.201230

Table.1 shows the effects of n, Le, S_r, D_f and N on the non-dimensional heat and mass transfer coefficients. It is seen from this table that both the heat and mass transfer rates increase with increasing power-law index n. For increasing value of Le, the heat transfer rate is decreasing whereas the mass transfer rate is increasing. The Lewis number (diffusion ratio) is the ratio of Schmidt number and Prandtl number The Schmidt number quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. Hence the rate of mass transfer is increased with the increase in Schmidt number or Lewis number. Similarly, decrease in Prandtl number i.e. increase in Lewis number is equivalent to increasing the thermal conductivities, and therefore heat diffuses away from the heated plate more rapidly. Hence the rate of heat transfer is reduced. The heat transfer rate is decreasing for increasing values of Soret parameter (or simultaneously decrease of Dufour parameter) but the mass transfer rate is increasing. There is decrease in both the heat and mass transfer rates with increase in the buoyancy ratio N.

5. Conclusions

In this paper, a boundary layer analysis for mixed convection heat and mass transfer along a vertical plate in a Darcy porous media saturated with power-law fluid with variable heat and mass flux conditions in presence of the Soret and Dufour effects has been considered. It can be concluded from the present analysis that the higher values of the Soret parameter (or lower values of Dufour parameter) result in higher velocity and temperature distributions but lower concentration distribution for the pseudo-plastic fluids with $n = 0.5$. The same behavior is seen for the dilatant fluids with $n= 1.5$. Also, the effects of Soret and Dufour parameters are studied for Newtonian case $n=1.0$ and found to be the same nature as above fluids within the boundary layer. Also, the higher values of the power-law index parameter result in higher velocity, but lower temperature and concentration distributions within the boundary layer.

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