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Flow Due to Slow Steady Rotation of a Porous Spherical Shell in a Visco-Elastic Fluid

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Abstract

In this paper we obtain the flow due to slow steady rotation of a porous spherical shell of negligible thickness about an arbitrary diameter in a visco-elastic fluid characterized by the constitutive relation given by Rivilin (1955). The non linear basic equations characterizing the flow are solved by a method of successive approximation suggested by Collins (1955). The stream line pattern of the secondary motion has been identified, also the expressions for the couple and the drag on the shell has been obtained. The result obtained coincides with that of Pattabhi Ramacharyulu (1965) in absence of porosity.

Keywords: Visco-elastic; Rotation Reynolds's Number; Porosity Coefficient; Secondary Motion.

1. Introduction

The problem of viscous incompressible fluid flow in the presence of porous spherical bodies has been a subject of extensive study during last three hand half decades not only for their theoretical interest but also on account of their possible industrial applications in Tribology like linear bearings (using spherical balls),Gas bearings, sintered, porous bearings, spherical air bearings for reduction of friction and application in physiological situations such as: flow in the vicinity of glands in various parts of living bodies. Lenov (1962) initiated analytical studies in this direction by investing the slow steady flow past a thin porous spherical shell. Gheorgitza (1963) investigated Lenov's problem for non-homogeneous permeability of the porous medium. This was followed by Rajvansi (1969)

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who investigated the problem for non-Newtonian fluids . Later Verma and Gaur(1972), Omprakash (1976), Iyengar (2000) examined the oscillating flows. A detailed   investigation of flow due to slow steady rotation of a porous spherical of   shell of negligible thickness in a viscous fluid about a diameter was carried out by Iyengar and Pattabhi Ramacharyulu (2001).Later this was extended by them to for two concentric porous spherical shells(2001) and also for a system of n- concentric porous spherical shells about a common diameter(2002). Patthabhi Ramacharyulu (1965) investigated the steady flow due to slow rotation of a sphere in a visco-elastic liquid characterised by the constitutive equations given by Revilin (1955).

   The aim of this work is to obtain the flow due to slow steady rotation of a homogeneous porous spherical shell of negligible thickness in a visco-elastic liquid with a small angular velocity about one of its diameters under the assumption of the continuity of the normal velocity on the surface. We employ a method of successive approximation suggested by Collins(1955) , in which it is assumed that the velocity components and the pressure can be expanded in ascending powers of a suitable parameter characteristic of the angular velocity (rotation Reynolds number) of the shell. Analytical expressions for stream function both outside and inside the rotating shell have been obtained from which the velocity field had been calculated in each stage of the perturbation procedure adopted. We notice a drag force due to the porous nature of the shell. This drag force will not arise when the shell is non-porous. Further, estimates have been made for the fictional couple and drag on the shell. The results obtained coincides with that of Pattabhi Ramacharyulu (1965) in absence of porosity.

   Rivilin (1955) considered a class of isotropic incompressible fluids, with rheological properties given by the equation of state, expressing $S_{i,j}$ the stress tensor as a polynomial in the kinematic symmetric tensors

$$E_{ij} = U_{i,j} + U_{j,i} \tag{1a}$$

   and

$$B_{ij} = A_{i,j} + A_{j,i} + 2U_{m,i}U_{m,j} \tag{1b}$$

where U_i and A_i denote the components of the velocity and acceleration in the I-th direction. In our present investigation, we consider a particular prototype of this class of fluids, characterized by the equation of state.

$$S = -PI + \phi_1 E + \phi_2 B + \phi_3 E^2 \tag{2}$$

where P is the hydrostatic mean pressure, I is the unit tensor of rank 2 and the coefficients ϕ_1, ϕ_2 and ϕ_3 are constants. These coefficients are in general functions of the invariants of the matrices E and B. Visco-inelastic liquids characterized by Reiner can formally be obtained from (2) when $\phi_2 = 0$. Further, if ϕ_3 also vanishes, the liquid is Newtonian.

2. Basic equations

        Let a porous spherical shell of negligible thickness and radius *a* rotate steadily with a small angular velocity Ω about one of its diameters in a liquid characterized by (2), at rest at infinity, filling the shell both inside and outside. We employ a spherical polar coordinate system [R,θ, ϕ] : *R* is the distance measured from the center of the sphere, θ is the colatitude measured from the axis of rotation and ϕ is the azimuth. Let [U,V,W] denote the velocity components in the direction of [R,θ,ϕ] respectively.

        We shall also introduce the non-dimensional variables defined by the following equations:

$$(U,V,W) = \frac{\phi_1}{\rho a}(u,v,w); \qquad\qquad (A_R,A_\theta,A_\phi) = \frac{\phi_1^2}{\rho^2 a^3}(a_r,a_\theta,a_\phi);$$

$$P_{ij} = \frac{\phi_1^2}{\rho a^2} p_{ij}; \qquad\qquad S_{ij} = \frac{\phi_1^2}{\rho a^2} s_{ij};$$

$$E_{ij} = \frac{\phi_1^2}{\rho a^2} e_{ij}; \qquad\qquad B_{ij} = \frac{\phi_1^2}{\rho^2 a^4} b_{ij};$$

$$\phi_2 = \rho a^2 \beta; \qquad \phi_3 = \rho a^2 \gamma_c; \qquad\qquad G = \frac{a\phi_1^2}{\rho} g \;\; and \;\; D = \frac{\phi_1^2}{\rho} d \tag{3}$$

Where ρ is the density of the liquid, G is the couple due to the liquid friction on the sphere given by

$$G = -\int_0^\pi 2\pi [R^3 S_{R\phi}]_{R=a} \sin^2\theta \, d\theta \tag{4}$$

and D is the force (which will be along the axis of rotation) given by

$$D = \int_0^{2\pi} 2\pi [R^2(S_{RR}\cos\theta - S_{R\theta}\sin\theta)]_{R=a} \sin\theta \, d\theta \tag{5}$$

and $S_{RR}, S_{R\theta}, S_{R\phi}$ are the components of the stress tensor, relevant in the present analysis. The rotating shell (R = a) can now be represented by r = 1. The equations of steady motion and continuity for the liquid flow can now be written in the dimensionless form

$$\left[u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} - \frac{v^2+w^2}{r} \right] = \frac{1}{r^2}\frac{\partial(r^2 s_{rr})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta \, s_{r\theta})}{\partial \theta} - \frac{s_{\theta\theta}+s_{\phi\phi}}{r} \tag{6}$$

$$\left[u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + \frac{uv-w^2\cot\theta}{r} \right] = \frac{1}{r^2}\frac{\partial(r^2 s_{r\theta})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta \, s_{\theta\theta})}{\partial \theta} + \frac{s_{r\theta}-s_{\varphi\varphi}\cot\theta}{r} \tag{7}$$

$$\left[u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \theta} + \frac{uv+vw\cot\theta}{r} \right] = \frac{1}{r^2}\frac{\partial(r^2 s_{r\varphi})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta \, s_{\theta\varphi})}{\partial \theta} + \frac{s_{\varphi r}+s_{\theta\varphi}\cot\theta}{r} \tag{8}$$

and $\quad \dfrac{1}{r^2}\dfrac{\partial(r^2 u)}{\partial r} + \dfrac{1}{r\sin\theta}\dfrac{\partial(v\sin\theta)}{\partial \theta} = 0 \tag{9}$

The equation of continuity (9) is identically satisfied by introducing the (non-dimensional) Stoke's stream function defined by

$$u = \frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta}; \quad v = -\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r} \tag{10}$$

These are to be solved both outside and inside the shell subject to the following conditions

$$\lim_{r \to \infty}[u^+, v^+, w^+] = 0 \tag{11.1}$$

$$[u^+]_{r=1} = [u^-]_{r=1} ; [v^+]_{r=1} = [v^-] = 0; [w^+]_{r=1} = [w^-]_{r=1} = R_e \sin\theta \tag{11.2}$$

$$\lim_{r \to 0}[u^-, v^-, w^-] \to finite \tag{11.3}$$

where R_e stands for the rotation Reynold's numbers

$$R_e = \rho a^2 \Omega / \phi_1 \tag{12}$$

The superscripts + and – in the foregoing analysis and discussions represent, the values of the respective variables for the fluid flow outside and inside the shell respectively. To facilitate the investigation of the visco-elastic effects on the flow, we assume that the solution of the above equations can e expressed as a power series expansion in R_e :

$$X = R_e X^{(1)} + R_e^2 X^{(2)} + R_e X^{(3)} + ... \tag{13}$$

where X may stand for any one of the physical quantities. $u, v, w, \psi, e_{ij}, a_i, b_{ij}, s_{ij}, p, g, d, ...$

substituting such power series expansions for all the flow variables in the muster of the equations of motion (6) to (8) and in he boundary conditions (11.1) to (11.3) and also by equating like coefficients of , we obtain the basic equations and the boundary conditions both outside and inside the shell in various ordered approximations that characterize the fluid flow. Exact solutions for the equations that arise in the various ordered approximations together with the boundary conditions are obtained in the foregoing analysis.

3. First approximation

Substituting (13) in the Muster of (6) to (8) and equating the coefficients of R_e we get the equations for the stresses

$$s_{ij}^{(1)} = -p^{(1)}\delta_{ij} + e_{ij}^{(1)} \tag{14}$$

and hence the equations of motion

$$-\frac{\partial p^{(1)}}{\partial r} + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} E^2 \psi^{(1)} = 0 \tag{15}$$

$$-\frac{\partial p^{(1)}}{\partial \theta} - \frac{1}{\sin\theta} \frac{\partial}{\partial r} E^2 \psi^{(1)} = 0 \tag{16}$$

and $\quad E^2(w^{(1)} r \sin\theta) = 0 \tag{17}$

where $\quad E^2 = \dfrac{\partial^2}{\partial r^2} + \dfrac{\sin\theta}{r^2} \dfrac{\partial}{\partial \theta}\left(\dfrac{1}{\sin\theta}\dfrac{\partial}{\partial \theta}\right).$

However the boundary conditions in the present investigation are

$$\lim_{r \to \infty} [u^{(1)^+}, v^{(1)^+}, w^{(1)^+}] = [0, 0, 0] \tag{18.1}$$

$$[u^{(1)^+}]_{r=1} = [u^{(1)^-}]_{r=1} \quad ; \quad [v^{(1)^+}]_{r=1} = [v^{(1)^-}]_{r=1} = 0 \tag{18.2.1}$$

$$[w^{(1)^+}]_{r=1} = [w^{(1)^-}]_{r=1} = \sin\theta \tag{18.2.2}$$

$$\lim_{r \to 0} [u^{(1)^-}, v^{(1)-}, w^{(1)-}] \to finite \tag{18.2.3}$$

on eliminating $p^{(1)}$ from (15) and (16) we have the equation of $\psi^{(1)}$: $E^4 \psi^{(1)} = 0$

The equations (17) and (19) yields the following solutions, subject to the boundary conditions (18.1) to (18.3)

Outside	**Inside**

$$\psi^{(1)^+} = \lambda\left(r + \frac{1}{r}\right)\sin^2\theta$$

$$u^{(1)^+} = \frac{2\lambda}{r}\left(1 + \frac{1}{r^2}\right)\cos\theta \qquad\qquad \psi^{(1)^-} = 2r^2\lambda(r - r^2)\sin^2\theta$$

$$\qquad\qquad\qquad\qquad\qquad\qquad u^{(1)^-} = 4\lambda(2 - r^2)\cos\theta$$

$$v^{(1)^+} = -\frac{\lambda}{r}\left(1 - \frac{1}{r^2}\right)\sin\theta \qquad\qquad v^{(1)^-} = 8\lambda(r^2 - 1)\sin\theta$$

$$\qquad\qquad\qquad\qquad\qquad\qquad w^{(1)^-} = r\sin\theta$$

$$w^{(1)^+} = \frac{\sin\theta}{r^2}$$

using $\psi^{(1)^+}, \psi(1)^-$ in (15),(16) we get pressure for both outside and inside as

$$p^{(2)^+} = \frac{2\lambda\cos\theta}{r^2} + constatnt \, ; \qquad\qquad p^{(2)^-} = -40\lambda r\cos\theta + constatnt. \tag{19}$$

From (4), (5) we calculate the couple and drag of outside and inside the shell as

$$g^{(1)^+} = 8\pi \qquad\qquad\qquad g^{(1)^-} = 0$$

$$d^{(1)^+} = -8\pi \qquad\qquad\qquad d^{(1)^-} = 0$$

In the above solutions λ stands for the parameter characteristic of shell porosity arising out of the continuity of the normal velocity at the Shell:

$$i.e., [u^{(1)^+}]_{r=1} = [u^{(1)^-}]_{r=1} = 4\lambda\cos\theta \tag{20}$$

The case λ =0 corresponds to the flow around a rotating non-porous Shell [9] the net frictional couple is: $g^{(1)^+} - g^{(1)^-} = 8\pi,$ which is the same as the one obtained by Stokes when the Shell is non-porous. The net drag on the Shell is given by $d^{(1)^+} - d^{(1)^-} = -8\pi\lambda,$ which reduces to zero when λ = 0.

4. Second approximation

Substituting (13) in (6) to (8) and collecting the co-efficients of $R_e^{\angle}$ we obtain the equations governing the flow for both outside and inside after dropping the superscript + and − are exactly the same as muster (6) to (8). However the boundary conditions in the present case are

$$\lim_{r \to \infty}[u^{(2)^+}, v^{(2)^+}, w^{(2)^+}] = [0, 0, 0] \tag{21.1}$$

$$[u^{(2)^+}]_{r=1} = [u^{(2)^-}]_{r=1} = 0; [v^{(2)^+}]_{r=1} = [v^{(2)^-}]_{r=1} = 0 \tag{21.2}$$

$$[w^{(2)^+}]_{r=1} = [w^{(2)^-}]_{r=1} = 0 \tag{21.3}$$

$$\lim_{r \to 0}[u^{(2)^-}, v^{(2)^-}, w^{(2)^-}] \to \text{finite} \tag{21.4}$$

Now the equations of motion outside the shell are

$$
\begin{aligned}
-\frac{\partial p^{(2)^+}}{\partial r} &+ \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} E^2 \psi^{(2)} \\
&= \left[-\frac{4\lambda^2}{r^3} - \frac{16}{r^5} - \frac{12\lambda^2}{r^7} \right] + \left[\frac{5\lambda^2}{r^3} + \frac{18\lambda^2}{r^5} + \frac{9\lambda^2}{r^7} - \frac{1}{r^5} \right] \sin^2\theta + \\
&\beta\left[\left(\frac{32\lambda^2}{r^5} + \frac{576\lambda^2}{r^7} + \frac{1296\lambda^2}{r^9} \right) - \left(\frac{24\lambda^2}{r^5} + \frac{576\lambda^2}{r^7} + \frac{864\lambda^2}{r^9} - \frac{72}{r^7} \right) \sin^2\theta \right] + \\
&\gamma_c\left[\left(\frac{40\lambda^2}{r^5} + \frac{408\lambda^2}{r^7} + \frac{864\lambda^2}{r^9} \right) - \left(\frac{40\lambda^2}{r^5} + \frac{396\lambda^2}{r^7} + \frac{576\lambda^2}{r^9} - \frac{45}{r^7} \right) \sin^2\theta \right]
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
-\frac{\partial p^{(2)^+}}{\partial \theta} &- \frac{1}{\sin\theta} \frac{\partial}{\partial r} E^2 \psi^{(2)+} = \left[-\frac{3\lambda^2}{r^6} - \frac{6\lambda^2}{r^4} + \frac{\lambda^2}{r^2} - \frac{1}{r^4} \right] \sin\theta\cos\theta - \\
&\beta\left[\frac{12\lambda^2}{r^4} - \frac{72\lambda^2}{r^6} - \frac{204\lambda^2}{r^4} \right] \sin\theta\cos\theta + \gamma_c\left[\frac{8\lambda^2}{r^4} + \frac{84\lambda^2}{r^6} + \frac{144\lambda^2}{r^8} + \frac{9}{r^6} \right] \sin\theta\cos\theta
\end{aligned}
\tag{23}
$$

$$\left(\nabla^2 - \frac{1}{r^2 \sin^2\theta} \right) w^{(2)+} = \left[-\frac{4\lambda}{r^4} + \beta\left(\frac{12\lambda}{r^6} \right) + \gamma_c\left(\frac{12\lambda}{r^6} \right) \right] \sin\theta\cos\theta \tag{24}$$

Eliminating $p^{(2)^+}$ from (22) and (23), we get

$$
\begin{aligned}
E^4 \psi^{(2)^+} = \Bigg[&\left(\frac{12\lambda^2}{r^3} + \frac{12\lambda^2}{r^5} - \frac{6}{r^5} \right) + \beta\left(-\frac{96\lambda^2}{r^5} - \frac{720\lambda^2}{r^7} - \frac{96\lambda^2}{r^9} + \frac{144}{r^7} \right) - \\
&\gamma_c\left(\frac{48\lambda^2}{r^5} + \frac{288\lambda^2}{r^7} - \frac{144}{r^7} \right) \Bigg] \sin^2\theta\cos\theta
\end{aligned}
\tag{25}
$$

Which yields the following solutions subject to the conditions (21.1) to (21.2.1)

$$\psi^{(2)^+} = \left(1 - \frac{1}{r}\right)^2 \left[\frac{1}{8}\left\{4\lambda^2(r+1)-1\right\} - \frac{\beta}{50r^3}\left\{\lambda^2(231r^3+262r^2+8r-4)-25r^2(r+2)\right\}\right.$$
$$\left. -\frac{\gamma_c}{2r}\left\{4\lambda^2(r+1)-(r+2)\right\}\right]\sin^2\theta\cos\theta$$

$$u^{(2)^+} = \frac{1}{r^2}\left(1-\frac{1}{r}\right)^2 \left[\frac{1}{8}\left\{4\lambda^2(r+1)-1\right\} - \frac{\beta}{50r^3}\left\{\lambda^2(231r^3+262r^2+8r-4)-25r^2(r+2)\right\}\right.$$
$$\left. -\frac{\gamma_c}{2r}\left\{4\lambda^2(r+1)-(r+2)\right\}\right](3\cos^2\theta-1)$$

$$v^{(2)^+} = \frac{1}{r^3}\left(1-\frac{1}{r}\right)\left[\left\{-\frac{1}{2}\lambda^2(2+2r+r^2)+1\right\} + \frac{\beta}{r^2}\left\{\frac{\lambda^2}{5r}(20r^3+77r^2+2r+2)\right\}\right.$$
$$\left. +\frac{\gamma_c}{8}\left\{2\lambda^2(r+3)-3\right\}\right]\sin\theta\cos\theta$$

Now from equation (24) subject to (21.1) and (21.2.2) we get

$$w^{(2)^+} = \frac{\lambda}{r^3}\left(1-\frac{1}{r}\right)[r-2(\beta+\gamma_c)]\sin\theta\cos\theta.$$

From (22) and (23) using $\psi^{(2)^+}$, we obtain pressure distribution

$$p^{(2)^+} = \left[\left(\frac{\lambda^2}{r^2} - \frac{5\lambda^2}{r^4} - \frac{2\lambda^2}{r^6} + \frac{1}{2r^4} - \frac{2\lambda^2}{r^3} - \frac{1}{2r^3}\right) + \beta\left(\frac{16\lambda^2}{r^4} + \frac{106\lambda^2}{r^6} + \frac{4062\lambda^2}{25r^8} - \frac{2}{r^6} - \frac{462\lambda^2}{25r^3} + \frac{2}{r^3}\right)\right.$$
$$\left. +\gamma_c\left(\frac{14\lambda^2}{r^4} + \frac{72\lambda^2}{r^6} + \frac{108\lambda^2}{r^8} - \frac{2}{r^6} - \frac{8\lambda^2}{r^3} + \frac{2}{r^3}\right)\right] + \left[\left(-\frac{2\lambda^2}{r^2} + \frac{6\lambda^2}{r^4} + \frac{3\lambda^2}{2r^6} - \frac{1}{r^4} + \frac{3\lambda^2}{r^3} + \frac{3}{4r^3}\right)\right.$$
$$\left. +\beta\left(-\frac{18\lambda^2}{r^4} - \frac{111\lambda^2}{r^6} - \frac{2718\lambda^2}{r^8} + \frac{15}{r^6} + \frac{693\lambda^2}{25} - \frac{3}{r^3}\right) + \gamma_c\left(\begin{matrix}-\frac{16\lambda^2}{r^4} - \frac{72\lambda^2}{r^6} - \frac{72\lambda^2}{r^8} + \\ \frac{21}{2r^6} + \frac{12\lambda^2}{r^3} - \frac{3}{r^3}\end{matrix}\right)\right]\sin^2\theta\right]$$

The couple is $g^{(2)^+} = 0$ and the drag is $d^{(2)^+} = 0$. Now the equations of motion inside the shell are

$$-\frac{\partial p^{(2)^-}}{\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}E^2\psi^{(2)^-} = (-64\lambda^2r+32\lambda^2r^3) + (96\lambda^2r-64\lambda^2r^3-r)\sin^2\theta$$
$$-\beta(1216\lambda^2r+96\lambda^2r\sin^2\theta)-\gamma_c(704\lambda^2r-176\lambda^2r\sin^2\theta) \tag{26}$$

$$-\frac{\partial p^{(2)^-}}{\partial\theta} - \frac{1}{\sin\theta}\frac{\partial}{\partial r}E^2\psi^{(2)^-} = \left[(96\lambda^2r^2-32\lambda^2r^4-r^2)-\beta(96\lambda^2r^2)\right.$$
$$\left. +\gamma_c(1764\lambda^2r^2)\right]\sin\theta\cos\theta \tag{27}$$

$$(\nabla^2 - \frac{1}{r^2 \sin^2 \theta})w^{(2)^-} = 8\lambda r^2 \sin \theta \cos \theta \tag{28}$$

Eliminating $p^{(2)^-}$ from (26),(27) we ge $\quad E^4 \psi^{(2)^-} = 0$ \hfill (29)

which yields subject to the condition (21.2.1) and (21.3) : $\psi^{(2)^-} = 0, \; u^{(2)^-} = 0 \; and \; v^{(2)^-} = 0$

Now from (28) subject to (21.2.1) to (21.2) we get $\; w^{(2)^-} = \dfrac{4\lambda}{7} r^2 (r^2 - 1) \sin \theta \cos \theta$

Now substituting $\psi^{(2)^-}$ in (26) and (27) we get

$$p^{(2)^-} = [32\lambda^2 r^2 - 8\lambda^2 r^4] - [48\lambda^2 r^2 - 16\lambda^2 r^4 - \frac{r^2}{2}]\sin^2 \theta$$
$$+ \beta[608\lambda^2 r^2 + 48\lambda^2 r^2 \sin^2 \theta] + \gamma_c[352\lambda^2 r^2 - 88\lambda^2 r^2 \sin^2 \theta] \tag{30}$$

The couple and drag are $\quad g^{(2)^-} = 0; \quad d^{(2)^-} = 0.$

Net couple and drag are $\quad g^{(2)^+} - g^{(2)^-} = 0; \quad d^{(2)^+} - d^{(2)^-} = 0.$

5. Discussion of the Results:

The stream line pattern of the secondary motion has been identified, also the expressions for the couple and the drag on the shell has been obtained. The result obtained coincides with that of Ramacharyulu (1965) in absence of porosity.

The secondary motion is given by ($\psi^{(1)^+}, \psi(1)^-$) noticed in the first approximation itself and is illustrated in Fig1 for $\lambda<0$.There will be a radial inflow in 2nd and 3rd quadrants and out flow in the 1st and 4th quadrants. The flow direction will be reversed for $\lambda>0$.
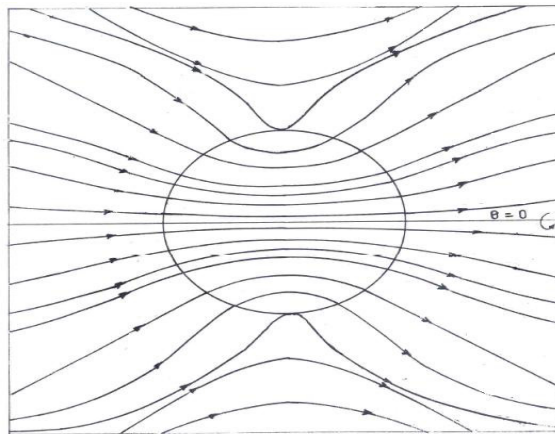

Fig.1: Stream line flow pattern in the first approximation for $\lambda= -1$

The secondary flow in the second approximation exists only outside of the shell and not inside.as shown in the

following figure 2.Here the stream pattern consists of 4 closed cells symmetrically placed in the axial plane and these are centered on the vortex points. Beyond the surface the dividing stream lines looks like as shown in the figure2.

6. Conclusions:

The secondary motion is characterized by $\psi^{(2)^+}$ .There will be a radial inflow with in the cone $C : \theta = \cos^{-1}(1/\sqrt{3})$ and out flow radially outside C. This suggests that the rotating porous shell functions like a centrifugal fan with suction near the pole with in C and this will be reversed (radially outwards) outside C. The vortex . The results coincide with Ramacharyulu (1965) in absence of porosity.

Problems of this type have applications in Tribology like linear bearings (uses spherical balls),Gas bearings, sintered and porous bearings, spherical air bearings for reduction of friction.
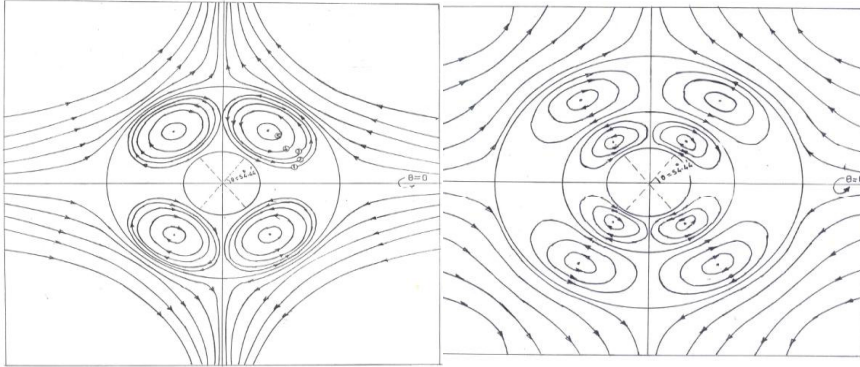

Fig2: Stream line flow pattern in the second approximation

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