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RESEARCH PAPER

Steady rotation of a composite sphere in a concentric spherical cavity

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Abstract The problem of steady rotation of a composite sphere located at the centre of a spherical container has been investigated. A composite particle referred to in this paper is a spherical solid core covered with a permeable spherical shell. The Brinkman's model for the flow inside the composite sphere and the Stokes equation for the flow in the spherical container were used to study the motion. The torque experienced by the porous spherical particle in the presence of cavity is obtained. The wall correction factor is calculated. In the limiting cases, the analytical solution describing the torque for a porous sphere and for a solid sphere in an unbounded medium are obtained from the present analysis.

Keywords Rotation · Porous sphere · Solid core · Stokes flow · Brinkman equation · Stress jump coefficient · Torque · Wall correction factor

1 Introduction

The problems of the motion of a particle at the instant when it passes the center of the spherical container serves as a model of interaction in multi-particle systems. This class of problems is important because it provides some information on wall effects. Fluid flows past rotating axially symmetric bodies, which are of much fundamental importance, have received considerable attention due to their practical significance in the areas of chemical, biomedical and environmental engineering and science. One of the important physical quantities needed in different applications is the couple exerting on the rotating bodies by the fluid. The couple experienced by a rotating body is needed for designation and calibration of viscometers [1]. The rotational motion of axially symmetric bodies in steady incompressible Newtonian and non-Newtonian fluids has been studied. Slow rotation of spheroids in an infinite fluid was first solved by Jeffrey [2] using curvilinear coordinates. The Stokesian flow of a viscous liquid generated by the slow steady rotation of an axisymmetric body placed in an incompressible viscous liquid which is otherwise at rest was studied by Kanwal [1]. An expression for the couple experienced by the rotating body was also derived by Kanwal in terms of the toroidal velocity component. The slow steady rotation of an approximate sphere about its axis of symmetry in an incompressible micropolar fluid is studied by Iyengar and Srinivasacharya [3].

A survey of literature regarding the fluid flows past and within porous bodies indicates that while abundant information is available for flows in an infinite expanse of fluid, very little information is available for flows in enclosures. Cunnigham [4] and Williams [5], independently, considered the motion of a solid sphere in a spherical container. Haberman and Sayre [6] have made an analogous study for the motion of an inner Newtonian fluid sphere. Ramkissoon and Rahaman investigated the motion of inner non-Newtonian (Reiner-Revlin) fluid sphere in a spherical container [7] and a solid spherical particle in a spheroidal container [8]. The quasisteady translation and steady rotation of a spherically symmetric composite particle composed of a solid core and a surrounding porous shell located at the center of a spherical cavity filled with an incompressible Newtonian fluid was studied by Keh and Chou [9]. Motivated by an interest in the interaction of cells or microspheres with the glycocalix, Damiano et al. [10] have obtained exact solutions to the problem of translational and rotational motion of a sphere in a Stokes flow near a Brinkman medium. The quasisteady translation and steady rotation of a spherically symmetric porous shell located at the center of a spherical cavity filled with an incompressible Newtonian fluid was analytically investigated by Keh and Lu [11]. The flow problem of an incompressible axisymmetrical quasisteady translation and steady rotation of a porous spheroid in a concentric spheroidal container were analytically studied by Saad [12].

The flow problems of the motion of a porous particles in a container have been modelled by using the Stokes version of the Navier–Stokes equation for the flow inside the

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container and Darcy's law or Brinkman's equation for the flow within the porous particles. The boundary condition for the flow field across a porous-liquid interface has drawn the attention of many researchers. Several types of boundary conditions at the interface of the free fluid and porous region to link the different flow regimes were suggested in literature. Srinivasacharya [13] studied the motion of a porous sphere in a spherical container using the continuity of the velocity, pressure and tangential stresses at the porous-liquid interface. Deo and Gupta [14] investigated the problem of symmetrical creeping flow of an incompressible viscous fluid past a swarm of porous approximately spheroidal particles with Kuwabara boundary condition (i.e. the vanishing of vorticity on the boundary). By applying volume average techniques Ochoa-Tapia and Whitaker [15, 16] have investigated the boundary conditions at the porous-liquid interface and have shown that the equations require a discontinuity in the shearing stress but continuity in velocity components and normal stress.

Ochoa-Tapia has derived the following stress jump boundary condition

$$\varepsilon^{-1}\frac{\partial u^{\mathrm{p}}}{\partial y}-\frac{\partial u^{\mathrm{l}}}{\partial y}=\frac{\sigma}{\sqrt{k}}u^{\mathrm{p}},$$

where u^{p}, u^{l} are tangential velocity components in porous region and liquid region respectively, ε is the porosity, k is the permeability of the homogeneous portion of the porous region, and σ is the stress jump coefficient. If $\sigma \neq 0$, there is a discontinuity in the shear stress at the porous-liquid interface. This jump condition is constructed to join Darcy's law with the Brinkman correction to Stokes equations. Experimentally it has been verified that the jump coefficient σ varies in the range from –1 to 1 [15–18]. Kuznetsov [17, 18] used this stress jump boundary condition at the interface between a porous medium and a clear fluid to discuss flow in channels partially filled with porous medium. Bhattacharyya and RajaSekhar [19] have used stress jump boundary condition while discussing the stokes flow of a viscous fluid inside a sphere with internal singularities, enclosed by a porous spherical shell. They concluded that the fluid velocity at a porous-liquid interface varies with the stress jump coefficient and it plays an important role in describing the flow field associated with porous medium. The flow of a viscous fluid in a spherical annulus formed by a solid sphere rotating with a constant angular velocity in a concentric spherical porous medium has been discussed by Srivastava et al. [20] using this stress jump boundary condition. They concluded that the torque on the rotating sphere also increases with the decrease of the permeability of the porous medium.

In this paper, we consider slow steady rotation of a porous sphere with an impermeable core in a spherical container. We have used the Brinkman's model for the flow inside the porous sphere and Stokes model for the flow within the spherical container. As boundary conditions, continuity of the velocity and the slip at the porous-liquid interface proposed by Ochoa-Tapia are employed [15, 16]. The hydrodynamic torque acting on the porous sphere in the presence of cavity and wall effects are studied numerically.

2 Formulation of the problem

Consider the steady rotational motion of a porous spherical particle of radius b located at the centre of a spherical vessel of radius c containing an incompressible Newtonian viscous fluid (see Fig. 1). Composite sphere consist of a spherical solid core of radius a and a permeable layer of thickness $b-a$ and permeability K''. The angular velocity of the porous particle is Ω about the axis of symmetry $\theta = 0$. We assume that the flow within the spherical container is Stokesian, and Brinkman's law [21] governs the flow inside the porous spherical particle.

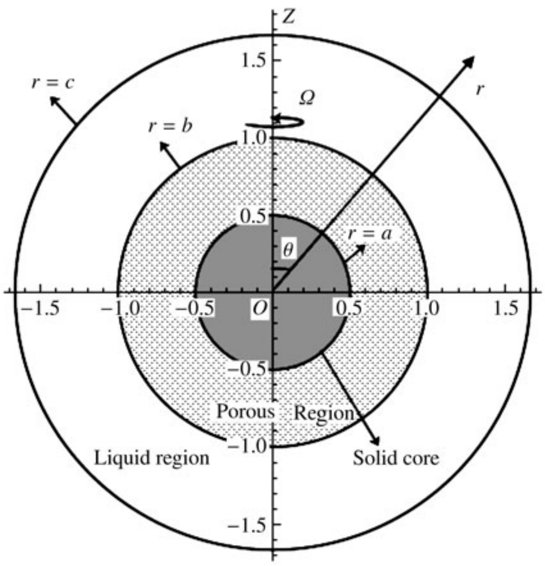

Fig. 1 The physical situation and the coordinate system

The equations of motion for the region within the spherical container are

$$\nabla \boldsymbol{q}^{(1)} = 0, \tag{1}$$

$$\nabla p^{(1)} + \mu\nabla\times\nabla\times\boldsymbol{q}^{(1)} = \mathbf{0}, \tag{2}$$

where $\boldsymbol{q}^{(1)}$ is volumetric average of the velocity, μ is the coefficient of viscosity and $p^{(1)}$ is the average of the pressure.

For the region inside the porous sphere, the equations of motion are

$$\nabla \boldsymbol{q}^{(2)} = 0, \tag{3}$$

$$\nabla p^{(2)} + \frac{\mu}{k}\boldsymbol{q}^{(2)} + \mu\nabla\times\nabla\times\boldsymbol{q}^{(2)} = \mathbf{0} \tag{4}$$

where $\boldsymbol{q}^{(2)}$ is volumetric average of the velocity, μ is the coefficient of viscosity, $p^{(2)}$ is the average of the pressure and k is the permeability of the porous medium.

Let (r, θ, ϕ) denote a spherical polar co-ordinate system with its origin at the center of the sphere of radius a with the diameter coinciding with the line of motion of the inner sphere taken as the initial line. Since the flow of the fluid is in the meridian plane and the flow is axially symmetric, all the quantities are independent of ϕ. Hence, we assume the velocity vectors $\boldsymbol{q}^{(1)}$ and $\boldsymbol{q}^{(2)}$ of the form of

$$\boldsymbol{q}^{(i)} = w^{(i)}(r,\theta)\boldsymbol{e}_\phi, \quad i = 1, 2. \tag{5}$$

where $(\boldsymbol{e}_r, \boldsymbol{e}_\theta, \boldsymbol{e}_\phi)$ are unit base vectors and $h_1 = 1$, $h_2 = r$ and $h_3 = r\sin\theta$ as the corresponding scale factors.

Since the fluid dynamic pressure is constant everywhere. From Eqs. (2) and (4), we get the following dimensionless equations

$$\mathrm{E}^2 r w^{(1)} \sin\theta = 0, \tag{6}$$

$$(\mathrm{E}^2 - \alpha^2) r w^{(2)} \sin\theta = 0. \tag{7}$$

Here $\alpha^2 = b^2/k$ and $\mathrm{E}^2 = \partial^2/\partial r^2 + [(1-\zeta^2)/r^2](\partial^2/\partial\zeta^2)$ is the Stokesian stream function operator.

3 Boundary conditions

The boundary conditions are

(1) Continuity of velocity component on the boundary of the porous sphere, i.e.,

$$w^{(1)}(r,\theta) = w^{(2)}(r,\theta), \text{ on } r = b, \tag{8}$$

(2) Ochoa-Tapia's stress jump boundary condition for tangential stress, i.e.,

$$\frac{\partial w^{(2)}}{\partial r} - \frac{\partial w^{(1)}}{\partial r} = \frac{\sigma}{\sqrt{k}} w^{(2)}, \text{ on } r = b, \tag{9}$$

where σ is the stress jump coefficient.

(3) on the impermeable core

$$w^{(2)} = 0, \text{ on } r = a, \tag{10}$$

(4) on the cell surface

$$w^{(1)} = -\Omega r \sin\theta, \text{ on } r = c. \tag{11}$$

4 Solution of the problem

The solution of Eq. (6) is

$$r w^{(1)} \sin\theta = (A r^2 + B r^{-1})\vartheta_2(\zeta), \tag{12}$$

and the solution of Eq. (7) is

$$r w^{(2)} \sin\theta = [C\sqrt{r}\mathrm{K}_{3/2}(\alpha r) + D\sqrt{r}\mathrm{I}_{3/2}(\alpha r)]\vartheta_2(\zeta), \tag{13}$$

where $\mathrm{I}_{3/2}(\alpha r)$ and $\mathrm{K}_{3/2}(\alpha r)$ denotes the modified Bessel functions of the first kind and second kind of order 3/2 and $\vartheta_2(\zeta) = (1-\zeta^2)/2$, $\zeta = \cos\theta$ is the Gegenbauer function of the first kind of order 2 and degree $-1/2$.

5 Determination of arbitrary constants

The boundary conditions (Eqs. (8) to (11)) in dimensionless form are

$$\begin{aligned} & w^{(1)} = w^{(2)}, \text{ on } r = 1, \\ & w_r^{(2)} - w_r^{(1)} = \alpha\sigma w^{(2)}, \text{ on } r = 1, \end{aligned} \tag{14}$$

$$w^{(2)}(r,\theta) = 0, \text{ on } r = \eta, \tag{15}$$

$$w^{(1)}(r,\theta) = -r\sin\theta, \text{ on } r = \frac{1}{\lambda}, \tag{16}$$

where $\eta = a/b$ and $\lambda = b/c$.

Using the boundary conditions Eqs. (14) to (16), we get the following system of equations

$$A + B - C\mathrm{K}_{3/2}(\alpha) - D\mathrm{I}_{3/2}(\alpha) = 0, \tag{17}$$

$$\begin{aligned} & -2A + B - C\left[(1+\alpha\sigma)\mathrm{K}_{3/2}(\alpha) + \alpha\mathrm{K}_{1/2}(\alpha)\right] \\ & -D\left[(1+\alpha\sigma)\mathrm{I}_{3/2}(\alpha) - \alpha\mathrm{I}_{1/2}(\alpha)\right] = 0, \end{aligned} \tag{18}$$

$$C\sqrt{\eta}\mathrm{K}_{3/2}(\alpha\eta) + D\sqrt{\eta}\mathrm{I}_{3/2}(\alpha\eta) = 0, \tag{19}$$

$$A\frac{1}{\lambda^2} + B\lambda + \frac{2}{\lambda^2} = 0. \tag{20}$$

The constants appearing in above system of equations are

$$\begin{aligned} A &= -\frac{2\alpha(z_1 - \sigma z_2)}{\Delta}, \\ B &= \frac{2[\alpha z_1 - (3+\alpha\sigma)z_2]}{\Delta}, \\ C &= \frac{6\mathrm{I}_{3/2}(\alpha\eta)}{\Delta}, \\ D &= -\frac{6\mathrm{K}_{3/2}(\alpha\eta)}{\Delta}, \end{aligned} \tag{21}$$

where

$$\begin{aligned} \Delta &= \alpha(1-\lambda^3)z_1 + [3\lambda^3 - \alpha\sigma(1-\lambda^3)]z_2, \\ z_1 &= \mathrm{K}_{3/2}(\alpha\eta)\mathrm{I}_{1/2}(\alpha) + \mathrm{K}_{1/2}(\alpha)\mathrm{I}_{3/2}(\alpha\eta), \\ z_2 &= \mathrm{K}_{3/2}(\alpha\eta)\mathrm{I}_{3/2}(\alpha) - \mathrm{K}_{3/2}(\alpha)\mathrm{I}_{3/2}(\alpha\eta). \end{aligned}$$

6 Torque on the body and wall effects

The hydrodynamic couple acting on the porous sphere with impermeable core in a spherical container is obtained as

$$T = 2\pi b^3 \int_0^\pi r^3 t_{r\phi}^{(1)}\Big|_{r=1} \sin^2\theta \mathrm{d}\theta \tag{22}$$

$$T = -4\pi b^3 \mu\Omega B. \tag{23}$$

The couple exerting on the rotating porous sphere in an unbounded medium is

$$T_\infty = -8\pi b^3\mu\Omega\left[\frac{\alpha z_1 - (3+\alpha\sigma)z_2}{\alpha(z_1 - \sigma z_2)}\right]. \tag{24}$$

7 Special cases

Case 1 If $a \to 0$ (or $\eta = 0$), the hydrodynamic couple acting on the porous sphere in an spherical container is reduced to

$$\begin{aligned}T = &-8\pi b^3\mu\Omega\{\alpha(3+\alpha\sigma)\cosh\alpha - [3+\alpha(\alpha+\sigma)]\sinh\alpha\}\\ &/\{\alpha[\alpha\sigma - \lambda^3(3+\alpha\sigma)]\cosh\alpha\\ &+[3\lambda^3 - \alpha(1-\lambda^3)(\alpha+\sigma)]\sinh\alpha\}.\end{aligned} \tag{25}$$

Case 2 If $a \to 0$ (or $\eta = 0$) and $c \to \infty$ (or $\lambda = 0$), the couple exerting on the rotating porous sphere in an unbounded fluid is

$$T_\infty = -8\pi b^3\mu\Omega\frac{\alpha(3+\alpha\sigma)\cosh\alpha - [3+\alpha(\alpha+\sigma)]\sinh\alpha}{\alpha[\alpha\sigma\cosh\alpha - (\alpha+\sigma)\sinh\alpha]}. \tag{26}$$

Case 3 For the steady rotational motion of a porous sphere in a spherical container with continuity of stress $\sigma = 0$, the expression for the hydrodynamic torque becomes

$$T = -8\pi b^3\mu\Omega\frac{3\alpha\cosh\alpha - (3+\alpha^2)\sinh\alpha}{-3\alpha\lambda^3\cosh\alpha + [3\lambda^3 - \alpha^2(1-\lambda^3)]\sinh\alpha}, \tag{27}$$

which agrees with the result obtained in Refs. [9, 12].

When $c \to \infty$ (or $\lambda = 0$), the couple exerting on the rotating porous sphere in an unbounded fluid is reduced to

$$T_\infty = -8\pi b^3\mu\Omega\frac{3+\alpha^2 - 3\alpha\coth\alpha}{\alpha^2}, \tag{28}$$

which agrees with the result obtained in Refs. [9, 12].

In the limiting case of $\alpha \to \infty$ (or $k = 0$),

$$T = -\frac{8\pi b^3\mu\Omega}{1-\lambda^3}. \tag{29}$$

This is the result for the rotation of an impermeable solid sphere in a cell model [12].

The wall correction factor W_c is defined as the ratio of the actual couple experienced by the particle in the enclosure and the couple on a particle in an infinite expanse of fluid. With the aid of Eqs. (23) and (24) this becomes

$$W_c = \frac{T}{T_\infty}. \tag{30}$$

Note that $W_c = 1$ as $\lambda = 0$ and $W_c \geqslant 1$ as $0 < \lambda \leqslant 1$.

The wall correction factor W_c in the case of porous sphere in spherical container is

$$W_c = \left\{1 - \lambda^3\frac{\alpha(3+\alpha\sigma)\cosh\alpha - [3+\alpha(\alpha+\sigma)]\sinh\alpha}{\alpha[\alpha\sigma\cosh\alpha - (\alpha+\sigma)\sinh\alpha]}\right\}^{-1} \tag{31}$$

If the stress jump coefficient is adopted as $\sigma = 0$, wall correction factor W_c is given by

$$W_c = [1 - \lambda^3(1 + 3\alpha^{-2} - 3\alpha^{-1}\coth\alpha)]^{-1}. \tag{32}$$

In the limiting case of $\alpha \to \infty$ (or $k = 0$), the torque acting in an unbounded medium is given by

$$T_\infty = -8\pi\mu\Omega b^3, \tag{33}$$

which is the well-known result for slow rotation of a rigid sphere of radius b in a viscous fluid around an axis passing through its centre [22, 23]. The wall correction factor W_c is given by

$$W_c = \frac{1}{1-\lambda^3}, \tag{34}$$

which is the well-known result obtained in Refs. [9, 12].

The variation of non-dimensional torque $T_N = T/(4\pi b^3\mu\Omega)$ with permeability $k_1(=\sqrt{k}/b)$ is shown in Figs. 2 and 3 for various values of stress jump coefficient σ in the presence of cavity wall. Figure 2 shows the variation of torque with permeability in the presence of solid core. It is observed that the torque is decreasing as the stress jump coefficient σ is increasing. The torque is decreasing as the permeability is increasing when there is a jump in the stress at the boundary. Figure 3 shows the variation of torque with permeability in absence of solid core. Here similar behaviour is also observed, however, for some positive values of stress jump coefficient σ, there is a reversal in the behaviour of torque at a particular value of permeability (critical permeability K_c [19]). Beyond this value of K_c, the value of torque becomes negative which is not physically possible. Therefore, the positive values of σ can not be considered beyond that particular critical permeability. If negative σ is considered in the stress jump condition (9) the shear stress of external free flow region would become larger than that of the porous region which would generate a significant torque on the porous surface for any permeability. But, for positive values of σ, although the shear stress of the external region becomes low, for particular range of permeability a significant torque is generated on the surface. The variation of torque T_N with permeability k_1 for various values of separation parameter λ without the effect of stress jump coefficient is shown in Figs. 4 and 5. It is observed that the torque is increasing as the separation parameter λ is increasing and it is decreasing as the permeability is increasing.

The variation of wall correction factor W_c against the separation parameter λ with continuity of tangential stress ($\sigma = 0$) for various values of permeability k_1 is shown in Fig. 6. It can be observed that, as λ increases, the wall correction factor increases. For $k_1 > 1$, the particle mobility varies slowly with the separation parameter λ, compared with the case of lower permeability. The effect of the stress jump coefficient σ and separation parameter λ on wall correction factor W_c has been plotted in Fig. 7 for fixed value of k_1. It is observed that the wall corrector factor increases monotonically with increasing separation parameter λ. The correction factor decreases as the stress jump coefficient increases. Therefore, the wall correction factor depends not only on the separation parameter λ and permeability k_1 but also on the stress jump coefficient σ.

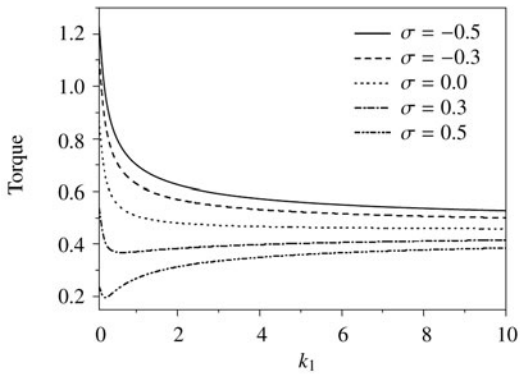

Fig. 2 Variation of torque T_N with permeability k_1 for various values of stress jump coefficient σ, with $\eta = 0.6$ and $\lambda = 0.6$

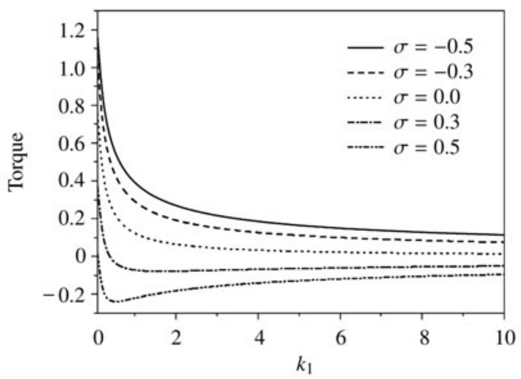

Fig. 3 Variation of torque T_N with permeability k_1 for various values of stress jump coefficient σ, with $\eta = 0$ and $\lambda = 0.6$

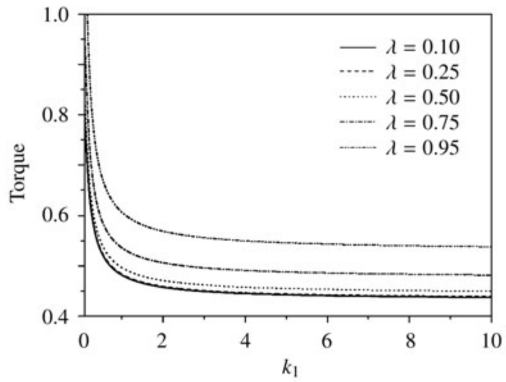

Fig. 4 Variation of torque T_N with permeability k_1 for various values of separation parameter λ, with $\sigma = 0$ and $\eta = 0.6$

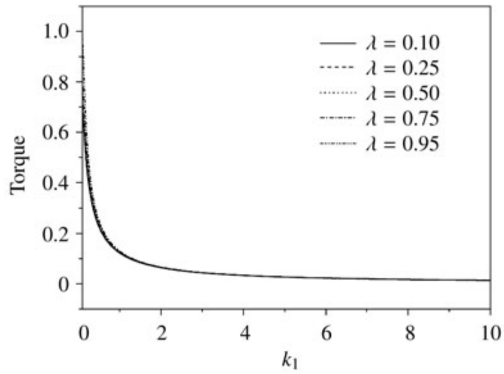

Fig. 5 Variation of torque T_N with permeability k_1 for various values of separation parameter λ, with $\sigma = 0$ and $\eta = 0$

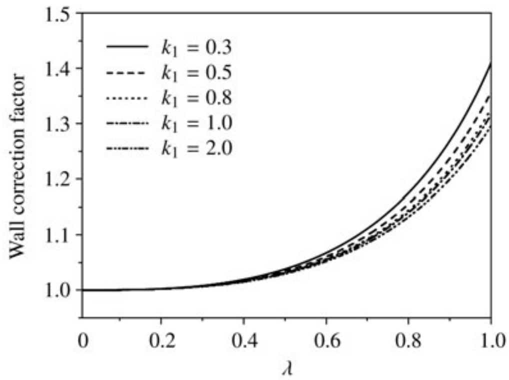

Fig. 6 Variation of wall correction factor W_c with separation parameter λ for various values of permeability k_1, with $\sigma = 0$ and $\eta = 0.6$

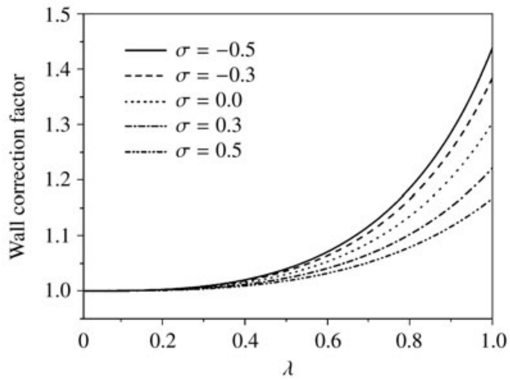

Fig. 7 Variation of wall correction factor W_c with separation parameter λ for various values of stress jump coefficient σ, with $k_1 = 1.5$ and $\eta = 0.6$

8 Conclusions

An exact solution for the problem of steady rotation of a composite sphere in a concentric spherical container is obtained by adopting Brinkman's law in the porous region and

Stokes' equation in the liquid region. At the porous-liquid interface Ochoa-Tapia and Whitaker's [15, 16] stress jump condition and continuity of the velocity have been used, together with no slip condition at the solid-porous interface. An expression is obtained for the hydrodynamic torque acting on the porous sphere with solid core in the presence of cavity and wall correction factor. The torque acting on the composite sphere is decreasing as the permeability k_1 is increasing, or as the stress jump coefficient σ is increasing. The wall correction factor W_c increases as the separation parameter λ increases and decreases as the stress jump coefficient σ increases. There is a considerable effect of the jump coefficient at the cavity wall. Therefore, the stress jump condition which is characterized by a stress jump coefficient cannot be ignored at the porous-liquid interface.

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