Optimization of Cooperative Spectrum Sensing under AWGN and Rayleigh Channels in cognitive Radio Network

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***Abstract*—This paper deals with cooperative spectrum sensing (CSS) under AWGN and Rayleigh channels with energy detection as sensing technique for cognitive radio network (CRN). The performance of m-out-of-N voting rule, OR fusion logic and AND fusion logic rules under varying threshold is studied, appropriate range of thresholds over which particular fusion rule can be applied has been investigated. An optimum voting rule has been used for AWGN and Rayleigh channels with an aim to improve the performance for real time implementation. A generalized optimal voting rule is derived to minimize the number of cognitive radios (CRs) required to meet target error bound. Simulations are performed for both the channels and the results are found to be comparable with the analytical results. It is observed that 2CRs for Rayleigh channel and 5 CRs for AWGN channel gives minimum error in a CRN of 10 CRs.**

keywords:*Cooperative spectrum sensing; Cognitive radio; Energy detection; AWGN channel; Rayleigh channel*.

I. INTRODUCTION

Over the last ten tears, wireless technologies have grown rapidly, hence there is a requirement of wide spectrum to cope up with technologies. Current spectrum licensing policy has resulted in very inefficient electromagnetic spectrum utilization. The underutilization of the spectrum leads to spectrum holes [1], hence contemporary CR technology has been advocated with a main objective to unravel spectrum inefficiency, which can get done with utilizing spectrum holes by unlicensed user (secondary user).

Cognitive radio (CR) has been extensively researched in recent years as a promising technology to improve spectrum utilization. CRs often called as spectrum or frequency agile radios senses the radio environment for spectrum holes and dynamically allocates different frequency channels. In order to utilize the unused spectrum of legacy wireless systems (primary users), we need to scan the radio environment to detect the spectrum holes, which is a critical component for cognitive radio.

One of the great challenges of implementing spectrum sensing is the hidden terminal and exposed terminal problems, which occurs when the cognitive radio is shadowed, receiver uncertainty problem, in severe multipath fading or inside buildings with high penetration loss, while a primary user (PU) is operating in the vicinity [2]. Due to these problems, a CR may fail to sense the presence of a primary user and access the legacy channel and possibility to cause interference to the licensed user. The main idea of cooperative sensing is to improve the sensing performance by exploiting the spatial diversity in the observations of spatially located cognitive radio users. By cooperation, CR users can share their sensing information for making a combined decision more accurate than the individual decisions [3]. The performance improvement due to spatial diversity is called cooperative gain, as the number of cooperative partners increases cooperative gain and spectrum sensing performance can be greatly enhanced [4], [3].

Energy detection mechanism for Spectrum Sensing does not require prior information of the primary user but cannot work in low SNR regime. The cooperation between CR terminals is one solution. Spectrum sensing via different mechanisms such as energy detection in low SNR region, Bayesian risk detection is studied in [5], [6] respectively. The optimal relaying schemes with amplify and forward(AF) is introduced in [7] which increases the system complexity and delay because of decentralized approach. In most recent work [8] multichannel threshold optimization techniques have been proposed in order to decrease the total error detection probability. In this paper, we considered efficiency of CSS with energy detection as sensing scheme at CRs to reduce the error probability and centralized decision fusion is applied for CRN. Accurate optimal value m in m-out-of-N rule is investigated in both AWGN and Rayleigh channels, extended this to OR and AND fusion logics.

The rest of this paper is organized as follows. In section II, spectrum sensing using energy detection for AWGN and Rayleigh channels with some realistic assumptions briefly introduced. In section III, basic principle of cooperative sensing with supporting equations. In section IV, the efficient CSS for optimization of number of CRs in a CRN is presented. Finally, we concluded the work done in section V.

II. SPECTRUM SENSING

We considered a cognitive radio network composed of N cognitive radios (secondary users), we deploy the fusion rules like AND Logic, OR Logic, and m-OUT-N Logic and a common receiver, We are performing centralized cooperative spectrum sensing in which each cognitive radio will sense the spectrum separately and then their distinct decisions are posted to the common receiver which can amalgamate all independent decisions and arrive at the absence or presence of primary user.

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The essence of spectrum sensing is a binary hypothesis-testing problem.

$$H_0 : \text{Primary user is not present}$$
$$H_1 : \text{Primary user is present}$$

By considering spectrum sensing only at jth cognitive radio , analytically, signal decision for hypothesis test is done for following cases [9]

$$y_j(t) = \begin{cases} w_j(t), & H_0, \\ h_j(t)x_j(t) + w_j(t), & H_1, \end{cases} \tag{1}$$

Where $y_j(t)$ is received signal at jth CR, $x_j(t)$ is the primary user signal, $w_j(t)$ is additive white Gaussian noise(AWGN) and $h_j(t)$ denotes the complex channel gain between the PU and the jth CR. We consider some realistic assumptions, as shown in Fig. 1 [8] that local sensing time smaller than total sensing period or coherence time of the channel we consider that channel is time invariant during local sensing period so $h_j(t)$ can be replaced by h_j without losing any information, under the assumption that PU is not changed in sensing period.

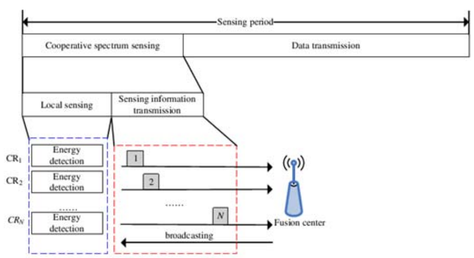

Fig. 1. Complete cooperative Spectrum sensing model

A. AWGN channel

It is well known that the energy detection is the optimal signal detector considering no prior information on the zero mean constellation signals. For the jth cognitive radio with energy detection, average probability of detection, average probability of false alarm, average probability of missed detection over AWGN channel us given by [10]

$$P_{f,i} = \frac{\Gamma(u, \lambda_i/2)}{\Gamma(u)} \tag{2}$$

$$P_{d,i} = Q_u(\sqrt{2\gamma_j}, \sqrt{\lambda_j}) \tag{3}$$

and

$$P_{m,j} = 1 - P_{d,j} \tag{4}$$

Where λ_j is threshold, γ_j is SNR , u is time-bandwidth product of energy detector for a jth CR, $\Gamma(a)$ is the gamma function, $\Gamma(a,x)$ is an incomplete gamma function given by $\Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}dt$, and $Q_u(a,x) = \frac{1}{a^{u-1}}\int_x^\infty t^u e^{-\frac{t^2+a^2}{2}} I_{u-1}(at)dt$, with $I_{u-1}(.)$ is the modified bessel function of the first kind and order $u-1$.

B. Rayleigh channel

Probability of false alarm will depend only on the channel, SNR and path distributions will not effect it so it remains same as AWGN. Rayleigh fading distribution occurs in non-line sight conditions which is most prominent in wireless communication environment and probability of detection of Rayleigh channel is given by

$$\begin{aligned} P_d &= e^{-\frac{\lambda}{2}} \sum_{n=0}^{u-1} \frac{1}{n!} \frac{\lambda}{2}^n + \frac{1+\bar{\gamma}}{\bar{\gamma}}^{u-1} \times \\ &[e^{-\frac{\lambda}{2(1+\bar{\gamma})}} - e^{-\frac{-\lambda}{2}} \sum_{n=0}^{u-1} \frac{1}{n!} \frac{\lambda\bar{\gamma}}{2(1+\bar{\gamma})}^n] \end{aligned} \tag{5}$$

using the relation $\Gamma(a,x) = (a-1)!e^{-x}\sum_{n=0}^{a-1}\frac{x^n}{n!}$ above equation can be written as

$$\begin{aligned} P_d &= \frac{\Gamma(u-1, \frac{\lambda}{2})}{(u-2)!} + (\frac{1+\bar{\gamma}}{\bar{\gamma}})^{u-1} \times \\ &e^{-\frac{\lambda}{2(1+\bar{\gamma})}}[1 - \frac{\Gamma(u-1, \frac{\lambda\bar{\gamma}}{2(1+\bar{\gamma})})}{(u-2)!}] \end{aligned} \tag{6}$$

III. COOPERATIVE SPECTRUM SENSING(CSS)

In cooperative spectrum sensing, binary hypothesis decision from each CR i.e. one bit decision (1 standing for the presence of licensed user, 0 stands for absence of licensed user) is forwarded to fusion centre. At the fusion centre, all 1-bit decisions are combined according to any one of the logic rule(AND Logic, OR Logic, and M-OUT-N Logic), a global decision will be taken, this final decision can be in favour of H_1 or H_0.

$$Y = \sum_{j=1}^{N} D_j \begin{cases} \geq m, H_1, \\ < m, H_0 \end{cases} \tag{7}$$

Where H_1 and H_0 are decisions taken by fusion centre that the primary user is transmitted or not transmitted respectively. Threshold m is an integer representing m-out of-N logic. From above the inference it can be seen that the AND logic corresponds to m=N and OR logic corresponds to m=1. The false alarm probability of cooperative spectrum sensing is given by

$$Q_f = Prob\{H_1|H_0\} = \sum_{l=m}^{N} \binom{N}{l} P_f^l (1-P_f)^{N-l} \tag{8}$$

The missed detection probability of cooperative spectrum sensing is given by

$$Q_d = Prob\{H_1|H_1\} = \sum_{l=m}^{N} \binom{N}{l} P_d^l (1-P_d)^{N-l} \tag{9}$$

and

$$Q_m = 1 - Q_d$$

The OR-fusion logic condition can be derived by keeping m=1 in equation (9)

$$\begin{aligned} Q_{d,OR} &= Prob\{H_1|H_1\} = \sum_{l=1}^{N} \binom{N}{l} P_d^l (1-P_d)^{N-l} \\ &= 1 - (1-p_d)^N \end{aligned} \tag{10}$$

similarly for OR fusion logic by keeping m=N $Q_{d,AND} = p_b^N$

IV. Efficient cooperative spectrum sensing

The important aspect we are interested is : *Let total number of cognitive radios(N) is fixed then what is the m_{opt} that minimizes the total error rate. Here error in the received signal is because of Q_f and Q_m so total error rate F(m)=$Q_f + Q_m$?*

Without considering any analytical exact solutions intuitively Fig.2 and Fig.3 shows total error probability in terms of wide range of threshold for AWGN channel, Rayleigh fading channel respectively considering different voting rules from m=1 to m=10 in a CR network with N=10 nodes. It can be observed that m=5 is the optimum number to achieve minimum error for AWGN channel and m=2 for Rayleigh channel.

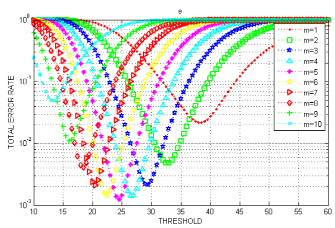

Fig. 2. Total error probability of cooperative spectrum sensing in 10dB AWGN channel; voting rules are m=1,2,. ..,10, N=10.

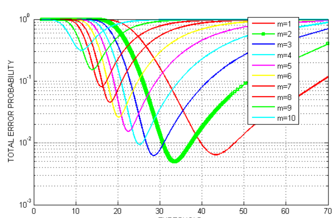

Fig. 3. Total error probability of cooperative spectrum sensing in 10dB Rayleigh channel; voting rules are m=1,2,. ..,10, N=10.

The specific true solution for optimum m is in following theorem [11].

Theorem : Given, N efficient m-out-of-N rule for CSS that minimizes F(m) is

$$m_{opt} = min(N, \lceil \frac{N}{1+\beta} \rceil), \tag{11}$$

where $\beta = \frac{ln\frac{p_f}{1-p_m}}{ln\frac{p_m}{1-p_f}}$ and $\lceil . \rceil$ represents the ceiling function.

Proof: Function F(m) is given by adding eqn(8) and eqn(9)

$$F(m) = 1 + \sum_{l=m}^{N} \binom{N}{l} [P_f^l (1-P_f)^{N-l} - P_m^l (1-P_m)^{N-l}]$$

Then we have

$$\begin{aligned} \frac{duF(m)}{dm} &\approx F(m+1) - F(m) \\ &= [P_m^{N-m}(1-P_m)^{N-m} - P_m^m (1-P_f)^{N-m}] \end{aligned} \tag{12}$$

once again differentiating eqn(12) it can show $\frac{du^2 F(m)}{dm^2} > 0 \Rightarrow$ minimum value. The optimum value for m is attained when $\frac{duF(m)}{dm}$ i.e.

$$P_m^{N-m}(1-P_m)^{N-m} = P_f^m (1-P_f)^{N-m}$$

which results in eqn(11)

Fig. 4 and Fig. 5 shows exact solution graphs of m in-terms of threshold evaluated from (eqn(11)). The results of certifies the intuitive remarks that we made above.

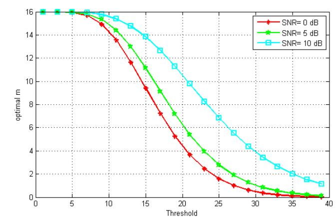

Fig. 4. Efficient voting rule versus detection threshold of CSS in AWGN channel with SNR(γ) =0,5,10dB,N=10

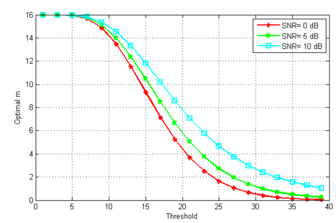

Fig. 5. Efficient voting rule versus detection threshold of CSS in Rayghleigh channel with SNR(γ) =0,5,10dB,N=10

From above theorem we can make generalized remarks which can be applied for any CR network :

1. Usually for AWGN channel i.e. fading free channel P_f and P_m have same order i.e. $\beta = 1$, so the optimal choice of m is N/2 and the Fig. 2 resembles the same.
2. The OR rule is most advantages when ($P_f < P_m^{N-1}$) that means $\beta > N-1$. This mentions that $P_f < P_m$ which can be obtained when threshold (λ) is very large i.e. for a large threshold OR rule is optimal.
3. The AND rule is most advantages for very small (λ) which is obtained when $P_m < P_f$ i.e. ($\beta \to 0$)

Above remarks from the theorem is justified in Fig. 6.

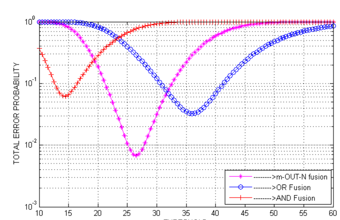

Fig. 6. Error detection probability comparison for fusion rules under different thresholds

Let us define the function $G(.,.)$ in terms of the variable N as

$$G(N, m_{opt}) = Q_f + Q_m - \varepsilon \qquad (13)$$

where ε is the desired error bound so that $Q_f + Q_m \leq \varepsilon$, N signifies the number of CRs in CSS and $m_{opt} = min(N, \lceil \frac{N}{1+\alpha} \rceil)$. The probabilities Q_f and Q_m are dependent on N and m_{opt} by eqn(8) and eqn(9), respectively. Here we are finding minimum number of cognitive radio networks to meet the desired error bond. Let us predict N_0 as the minimum required cognitive radios for CSS then then, from eq(13) we can write

$$G(N_0, m_{opt}) \leq 0, \qquad (14)$$
$$G(N_0 - 1, m_{opt,N_0-1}) < 0 \qquad (15)$$

From above equations N_0 is the least number of cognitive radios satisfying $Q_f + Q_m \leq \varepsilon$, we can quickly get the value of N_0 where N_0 is the first zero crossing point of the curve drawn form (13).

Fig. 7 and Fig. 8 depicts graphical approach of finding minimum CRs to get a required error bound for Rayleigh channel. Here we are considering $\varepsilon = 0.01$ for energy detection with an SNR of 10dB. in detail, if threshold is 40, it is appropriate to consider 12 CRs for AWGN and 16 CRs for Rayleigh channel. Fig. 9 shows receiver opting characteristics(ROC) under CSS considering different fusion rules and Non cooperative sensing(NCSS) ROC considering energy detection technique for spectrum sensing. From the graph we can say that AND fusion logic gives similar characteristics as with out cooperation, and overall performance of CSS is better than NCSS.

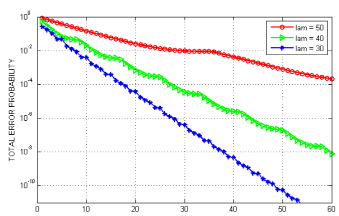

Fig. 7. Total error probability versus number of CRs in network for AWGN channel with N=60, $\gamma = 10dB$ and $\lambda = 50, 40, 30$

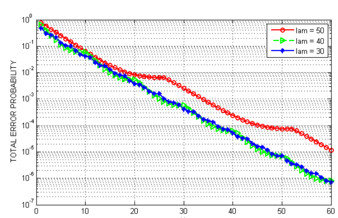

Fig. 8. Total error probability versus number of CRs in network for Rayleigh channel with N=60, $\gamma = 10dB$ and $\lambda = 50, 40, 30$

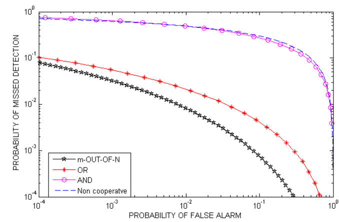

Fig. 9. Complementary ROC for Rayleigh channel under $\gamma = 10dB$

V. CONCLUSION

We examined the performance of cooperative spectrum sensing with energy detection technique under Rayleigh and AWGN channels in cognitive radio networks. It has been found that optimal decision voting rule for minimization of total error rate in AWGN channel is half-voting rule i.e. by considering distinct voting rules m=1,2,...10 it is noticed that m=5 gives minimum total error probability. Where as, for Rayleigh fading channel it has been shown that m=2 gives minimum probability error because Pf and Pm don't have same order. A method of selecting thresholds for distinct fusion rules is presented. Finally, an efficient spectrum sensing algorithm has been implemented which requires lesser than total number of CRs while matching a desired error bound.

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