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SORET AND DUFOUR EFFECTS ON MIXED CONVECTION FLOW OF COUPLE STRESS FLUID IN A NON-DARCY POROUS MEDIUM WITH HEAT AND MASS FLUXES

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An analysis is presented to investigate the Soret and Dufour effects on mixed convection heat and mass transfer along a semi-infinite vertical plate embedded in a non-Darcy porous medium saturated with couple stress fluid with flux distributions. The governing nonlinear partial differential equations are transformed into a system of ordinary differential equations using similarity transformations and then solved numerically. Profiles of dimensionless velocity, temperature, and concentration are shown graphically for various values of Dufour number, Soret number, and couple stress parameter. Nusselt number and Sherwood numbers are calculated and presented in table form.

KEY WORDS: mixed convection, couple stress fluid, Soret and Dufour effects, non-Darcy porous medium, heat and mass fluxes

1. INTRODUCTION

A situation where both the forced and free convection effects are of comparable order is called mixed or combined convection. The analysis of a mixed convection boundary layer flow along a vertical plate embedded in fluid saturated porous media has received considerable theoretical and practical interest. The phenomenon of mixed convection occurs in many technical and industrial problems such as electronic devices cooled by fans, nuclear reactors cooled during an emergency shutdown, a heat exchanger placed in a low-velocity environment, solar collectors, and so on. Several authors have studied the problem of mixed convection about different surface geometries. The analysis of convective transport in a porous medium with the inclusion of non-Darcian effects has also been a matter of study in recent years. The inertia effect is expected to be important at a higher flow rate and it can be accounted for through the addition of a velocity squared term in the momentum equation, which is known as the

Forchheimer's extension of Darcy's law. A detailed review of convective heat transfer in Darcy and non-Darcy porous medium can be found in the book by Nield and Bejan (2006). Jayanthi and Kumari (2007) studied the effect of variable viscosity on non-Darcy free or mixed convection flow on a vertical surface in a non-Newtonian fluid saturated porous medium. Kairi and Murthy (2011) have investigated the viscous dissipation effect on natural convection heat and mass transfer from a vertical cone in a non-Newtonian fluid saturated non-Darcy porous medium. Recently Jaber (2012) analyzed the transient MHD mixed double diffusive convection along a vertical plate embedded in a non-Darcy porous medium with suction or injection. Most recently, non-Darcy mixed convection from a horizontal plate embedded in nanofluid saturated porous media have been considered by Rosca et al. (2012).

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of a more intricate nature. It has

been observed that an energy flux can be generated not only by temperature gradients but also by concentration gradients. The energy flux caused by a concentration gradient is termed the diffusion-thermo (Dufour) effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the thermal-diffusion (Soret) effect. In most of the studies related to heat and mass transfer process, Soret and Dufour effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. But these effects are considered as second-order phenomena and may become significant in areas such as hydrology, petrology, geosciences, etc. The Soret effect, for instance, has been utilized for isotope separation and in mixture between gases with very light molecular weight (H_2 , He) and of medium molecular weight (N_2 , air). The Dufour effect was found to be of an order of considerable magnitude such that it cannot be neglected as observed by Eckeret and Drake (1972). Dursunkaya and Worek (1992) studied diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface, whereas Kafoussias and Williams (1995) presented the same effects on mixed convective and mass transfer steady laminar boundary layer flow over a vertical flat plate with temperature-dependent viscosity. Postelnicu (2004) studied numerically the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Alam and Rahman (2006) have investigated the Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction. Both free and forced convection boundary layer flows with Soret and Dufour effects have been addressed by Abreu et al. (2006). The effect of Soret and Dufour parameters on free convection heat and mass transfers from a vertical surface in a doubly stratified Darcian porous medium has been reported by Narayana and Murthy (2007). Later Narayana and Sibanda (2010) extended their contribution to a wavy surface in Darcy porous media. Recently Cheng (2012a) presented the Soret and Dufour effects on free convection heat and mass transfer from an arbitrarily inclined plate in a porous medium with constant wall temperature and concentration. Later, Cheng (2012b) extended this study to mixed convection flow from a vertical wedge.

With the growing importance of non-Newtonian fluids in modern technology and industries, investigations on such fluids are desirable. In recent years, the study of convection heat and mass transfer in non-Newtonian fluids has received much attention and this is because

the traditional Newtonian fluid theory cannot precisely describe the characteristics of the real fluids. Considerable progress has been made in the study of heat and mass transfer in magneto-hydrodynamic flow of non-Newtonian fluids due to its application in many devices, such as the MHD power generator, aerodynamics heating, electrostatic precipitation, and Hall accelerator, etc. Different models have been proposed to explain the behavior of non-Newtonian fluids. Among these, couple stress fluids introduced by Stokes (1966) have distinct features, such as the presence of couple stresses, body couples, and nonsymmetric stress tensor. The main feature of couple stresses is to introduce a size-dependent effect. Classical continuum mechanics neglects the size effect of material particles within the continua. This is consistent with ignoring the rotational interaction among particles, which results in symmetry of the force-stress tensor. However, in some important cases such as fluid flow with suspended particles, this cannot be true and a size-dependent couple-stress theory is needed. The spin field due to microrotation of freely suspended particles sets up an antisymmetric stress, known as couple stress, and thus forming couple stress fluid. These fluids are capable of describing various types of lubricants, blood, suspension fluids, etc. The study of couple stress fluids has applications in a number of processes that occur in industry such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, and colloidal solutions, etc. Stokes (1966) discussed the hydromagnetic steady flow of a fluid with couple stress effects. An excellent treatise on couple stress (polar) fluid dynamics was reported by Stokes (1985). Lin and Hung (2007) studied the combined effects of couple stresses and fluid inertia on the squeeze-film characteristics between a long cylinder and an infinite plate. Jian et al. (2010) examined nonlinear dynamic analysis of a hybrid squeeze-film damper mounted rigid rotor lubricated with couple stress fluid and active control. Shivakumara (2010) presented the onset of convection in a couple stress fluid-saturated porous medium with the effects of nonuniform temperature gradients. Alyaqout and Elsharkawy (2011) investigated the optimal film shape for two-dimensional slider bearings lubricated with couple stress fluids. Hassan and Mohammed (2011) presented the effect of couple stresses on a pulsatile MHD biviscosity fluid flow with heat and mass transfer through a non-Darcy porous medium between two permeable parallel plates. Islam et al. (2011) discussed the steady incompressible flow of a couple stress fluid in a porous medium. Mahinder Singh and Pardeep Kumar (2011) studied the magneto and rotatory thermosolutal convection in cou-

ple stress fluid in porous medium. Gaikwad and Rama Prasad (2011) presented double diffusive convection in a porous medium saturated with couple stress fluid in the presence of Soret effect analytically. Recently, Srinivasacharya and Kaladhar (2012) studied the mixed convection flow of couple stress fluid between parallel vertical plates with Hall and ion-slip effects using the homotopy analysis method. However, study on mixed convection in couple stress fluids in non-Darcy porous medium along a vertical flat plate is not available, to the authors' knowledge. Therefore, in the present paper we investigate the effects of Soret and Dufour coefficients on the mixed convection flow in a non-Darcy porous medium using the Stokes (1966) couple stress model.

The objective of this paper is to investigate the Dufour and Soret effects on steady combined free-forced convective and mass transfer flow past a semi-infinite vertical flat plate in couple stress fluid. The Keller-box method (Cebeci and Bradshaw, 1988) is employed to solve the nonlinear problem. The effects of couple stress fluid parameter, and Soret and Dufour numbers are examined and are displayed through graphs.

2. MATHEMATICAL FORMULATION

Consider a two-dimensional mixed convective heat and mass transfer along a semi-infinite vertical plate embedded in a free stream of couple stress fluid with velocity u_∞ , temperature T_∞ , and concentrations C_∞ . Choose the coordinate system such that the x axis is along the vertical plate and the y axis normal to the plate. The plate is maintained at uniform and constant heat and mass fluxes q_w and m_w , respectively. In addition, the Soret and Dufour effects are considered (Kafoussias and Williams, 1995). The flow configuration and the coordinates system are shown in Fig. 1.

Using Boussinesq and boundary layer approximations, the governing equations for the couple stress fluid are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\begin{aligned} \frac{\rho}{\varepsilon^2} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) &= \frac{\mu}{\varepsilon} \frac{\partial^2 u}{\partial y^2} + \rho g [\beta_T (T - T_\infty) \\ &+ \beta_C (C - C_\infty)] - \eta_1 \frac{\partial^4 u}{\partial y^4} + \frac{\mu}{K_P} (u_\infty - u) \\ &+ \frac{\rho b}{K_P} (u_\infty^2 - u^2) \end{aligned} \quad (2)$$

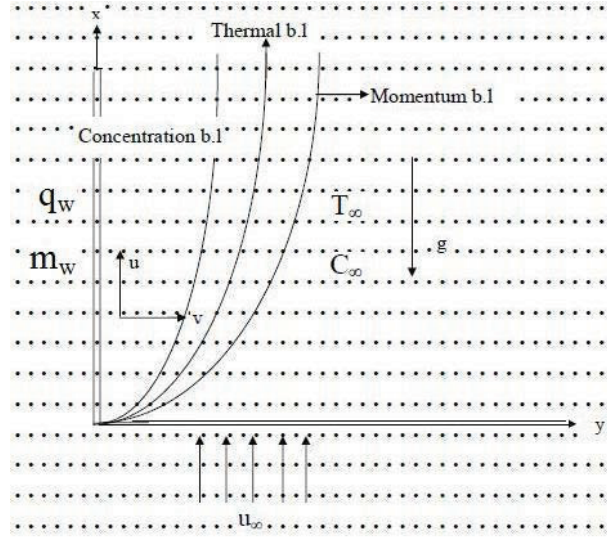


FIG. 1: Physical model and coordinate system

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{DK_T}{C_S C_P} \frac{\partial^2 C}{\partial y^2} \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{DK_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

where u , v are the velocity components in the x and y directions respectively, μ is the coefficient of viscosity, g is the acceleration due to gravity, ρ is the density, b is the Forchheimer constant, K_P is the permeability, ε is the porosity, β_T is the coefficient of thermal expansion, β_C is the coefficient of solutal expansion, α is the thermal diffusivity, D is the mass diffusivity, C_P is the specific heat capacity, C_S is the concentration susceptibility, T_m is the mean fluid temperature, K_T is the thermal diffusion ratio and η_1 is the couple stress fluid parameter. The third and fifth terms on the right-hand side of Eq. (2) stand for the first-order (Darcy) resistance and second-order porous inertia resistance, respectively, the second term indicates the couple stress effect, and the fourth term indicates the buoyancy force. The last terms on the right-hand side of the energy Eq. (3) and concentration Eq. (4) signify the Dufour (diffusion-thermo) effect and the Soret (thermal-diffusion) effect, respectively.

The boundary conditions are

$$\begin{aligned} u = 0, \quad v = 0, \quad \frac{\partial T}{\partial y} = -\frac{q_w}{k}, \quad \frac{\partial C}{\partial y} = -\frac{m_w}{D}, \\ \text{at } y = 0 \end{aligned} \quad (5a)$$

$$v_x = u_y \quad \text{at } y = 0 \quad \text{and } y \rightarrow \infty \quad (5b)$$

$$u = u_\infty, \quad T = T_\infty, \quad C = C_\infty \quad \text{as } y \rightarrow \infty \quad (5c)$$

where the boundary condition (5b) imply that angular velocity of the fluid particle on the boundary is equal to angular velocity of the boundary (i.e., $\text{Curl } \bar{q} = 0$).

Introducing the following similarity transformations,

$$\begin{aligned} \eta &= \frac{y}{x} \text{Re}_x^{1/2}, \quad u = u_\infty f'(\eta), \\ v &= -\frac{1}{2} \sqrt{\frac{u_\infty \nu}{x}} [f(\eta) - \eta f'(\eta)] \\ \theta(\eta) &= \frac{T - T_\infty}{(q_w/k)x} \text{Re}_x^{1/2}, \\ \phi(\eta) &= \frac{C - C_\infty}{(m_w/D)x} \text{Re}_x^{1/2} \end{aligned} \quad (6)$$

in Eqs. (1)–(4), we get the following nonlinear system of differential equations:

$$\begin{aligned} \frac{1}{\varepsilon} f''' + \frac{1}{2\varepsilon^2} f f'' + g_S \theta + g_C \phi + \frac{1}{\text{Da Re}_x} (1 - f') \\ + \frac{Fs}{\text{Da}} (1 - f'^2) - C_\alpha f^{(v)} = 0 \end{aligned} \quad (7)$$

$$\frac{1}{\text{Pr}} \theta'' + \frac{1}{2} [f\theta' - f'\theta] + D_f \phi'' = 0 \quad (8)$$

$$\frac{1}{\text{Sc}} \phi'' + \frac{1}{2} [f\phi' - f'\phi] + S_r \theta'' = 0 \quad (9)$$

where primes denote differentiation with respect to η , $\text{Sc} = \nu/D$ is the Schmidt number; it relates the relative thickness of the hydrodynamic layer and mass transfer boundary layer. $\text{Pr} = \nu/\alpha$ is the Prandtl number, which relates the thermal and momentum boundary layer thickness. $\text{Re}_x = (u_\infty x)/\nu$ is the local Reynolds number. Reynolds number is a measure of the relative importance between the momentum flux by advection and by diffusion in the same direction. $S_r = [\text{DK}_T (T_w - T_\infty)]/[\nu T_m (C_w - C_\infty)]$ is the Soret number, $D_f = [\text{DK}_T (C_w - C_\infty)]/[\nu C_S C_P (T_w - T_\infty)]$ is the Dufour number, $\text{Gr}_x = [g\beta_T (T_w - T_\infty) x^3]/\nu^2$ is the local temperature Grashof number, $G_{c_x} = [g\beta_C (C_w - C_\infty) x^3]/\nu^2$ is the local mass Grashof number, $C_\alpha = \eta_1/\mu x^2 \text{Re}_x$ is the local couple stress parameter, $g_S = \text{Gr}_x \text{Re}_x^2$ is the temperature buoyancy parameter, $g_C = G_{c_x}/\text{Re}_x^2$ is the mass buoyancy parameter, $Fs = b/x$ is the Forchheimer number, which can be applied to all types of porous materials, as long as the permeability and non-Darcy coefficient can be determined experimentally, or empirically when no experimental data are available. $\text{Da} = K_P/x^2$ is the Darcy number and is a dimensionless

group used in the study of the flow of fluids in porous media. The term C_α in (7) gives the effect of couple stresses. Hence, as C_α increases, the effect of couple stresses increases.

Boundary conditions (5) in terms of f , θ , and ϕ become

$$\begin{aligned} f = 0, \quad f' = 0, \quad f'' = 0, \quad \theta' = -1, \\ \phi' = -1 \quad \text{at } \eta = 0 \end{aligned} \quad (10a)$$

$$f' = 0, \quad f'' = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{as } \eta \rightarrow \infty \quad (10b)$$

If $D_f = 0$ and $S_r = 0$, the problem reduces to mixed convection heat and mass transfer on a semi-infinite vertical plate with uniform heat and mass fluxes in a couple stress fluid without Soret and Dufour effects. In the limit, as $C_\alpha = 0$, the governing Eqs. (1)–(4) reduce to the corresponding equations for a mixed convection heat and mass transfer in a viscous fluid.

The heat and mass transfers from the plate are given by

$$q_w = -k \left[\frac{\partial T}{\partial y} \right]_{y=0}, \quad q_m = -D \left[\frac{\partial C}{\partial y} \right]_{y=0} \quad (11)$$

The nondimensional local Nusselt number $\text{Nu}_x = (xq_w)/[k(T_w - T_\infty)]$ and local Sherwood number $\text{Sh}_x = (xm_w)/[D(C_w - C_\infty)]$ are given by

$$\frac{\text{Nu}_x}{\text{Re}_x^{1/2}} = \frac{1}{\theta(0)}, \quad \frac{\text{Sh}_x}{\text{Re}_x^{1/2}} = \frac{1}{\phi(0)} \quad (12)$$

3. RESULTS AND DISCUSSION

The flow Eq. (7) together with the energy and concentration Eqs. (8) and (9) constitute a system of nonlinear nonhomogeneous differential equations for which closed-form solutions cannot be obtained. Hence the governing Eqs. (7)–(9) are solved numerically using the Keller-box implicit method (Cebeci and Bradshaw, 1988). The method has the following four main steps:

- (i) Reduce the system of Eqs. (7)–(9) to a first-order system;
- (ii) Write the difference equations using central differences;
- (iii) Linearize the resulting algebraic equations by Newton's method and write them in matrix-vector form;
- (iv) Use the block-tridiagonal-elimination technique to solve the linear system.

This method has been proven to be adequate and give accurate results for boundary layer equations. In the present study, the boundary conditions for η at ∞ are replaced by a sufficiently large value of η where the velocity approaches 1 and temperature and concentration approach zero. In order to see the effects of step size ($\Delta\eta$) we ran the code for our model with three different step sizes as ($\Delta\eta$) = 0.001, ($\Delta\eta$) = 0.01, and ($\Delta\eta$) = 0.05 and in each case we found very good agreement between them on different profiles. After some trials we imposed a maximal value of η at ∞ of 6 and a grid size of η as 0.01. In order to verify the accuracy of the present method, the results are compared with those cases reported by Lin and Lin (1987), as shown in Table 1 and the comparisons are found to be in a very good agreement. Therefore, prompted by this success, we used the developed code to study the problem considered in this paper.

In order to study the effects of couple stress fluid parameter C_α , non-Darcy parameter F_s , Soret number S_r , and Dufour number D_f explicitly, computations were carried out for the cases of $Pr = 0.71$ (air), $Sc = 0.22$ (hydrogen at 25°C and 1 atm pressure), $g_s = 1.0$, $g_c = 0.1$, $\varepsilon = 0.3$, and $Da = 1.0$. The values of Soret number S_r and Dufour number D_f are chosen in such a way that their product is constant according to their definition provided that the mean temperature T_m is kept constant (Kafousias and Williams, 1995). These values are used throughout the computations, unless otherwise indicated.

Figure 2 displays the nondimensional velocity for different values of Soret number S_r and Dufour number D_f with $C_\alpha = 1.0$ and $F_s = 0.5$. It can be observed from this figure that the velocity of the fluid increases with the increase of Dufour number (or decrease of Soret number). The dimensionless temperature for different values of Soret number S_r and Dufour number D_f with $C_\alpha = 1.0$ and $F_s = 0.5$ is shown in Fig. 3. It is clear that the temperature of the fluid decreases with the decrease of Dufour number (or increase of Soret number). Figure 4

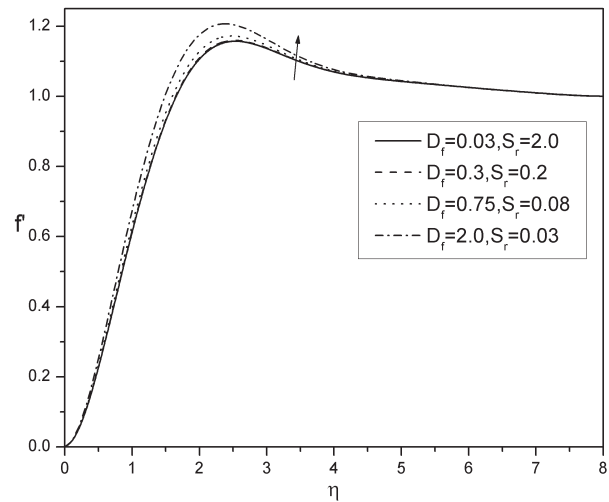


FIG. 2: Velocity profile for different values of D_f , S_r at $C_\alpha = 1.0$, $F_s = 0.5$

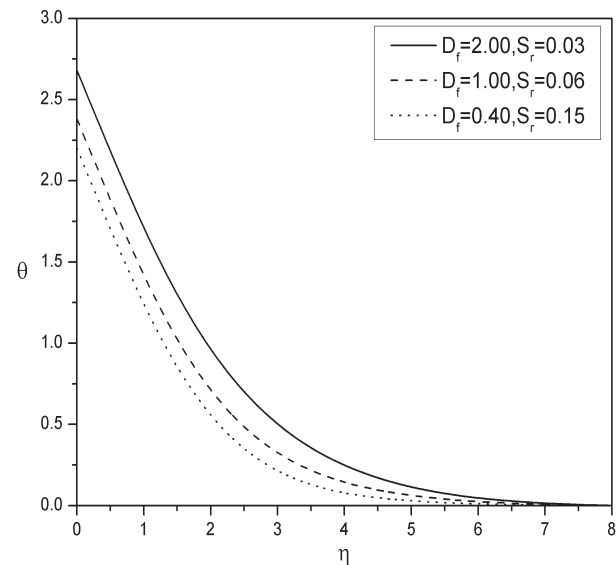


FIG. 3: Temperature profile for different values of D_f , S_r at $C_\alpha = 1.0$, $F_s = 0.5$

TABLE 1: Values of $Nu_x/Re_x^{1/2}$ for various values of Prandtl number with $C_\alpha = 0$, $S_r = 0$, $D_f = 0$, $g_s = 0$, $g_c = 0$ and $Sc \rightarrow 0$

Pr	Lin and Lin (1987)	Present (Nu_x)
0.01	0.0775587	0.0775588
0.1	0.200655	0.200657
1.0	0.458971	0.458973
10.0	0.997888	0.997910
100.0	2.15196	2.152223

demonstrates the dimensionless concentration for different values of Soret number S_r and Dufour number D_f with $C_\alpha = 1.0$ and $F_s = 0.5$. It is seen that the concentration of the fluid increases with decrease of Dufour number (or increase of Soret number).

The dimensionless velocity component for different values of Forchheimer number F_s with $S_r = 2.0$, $D_f =$

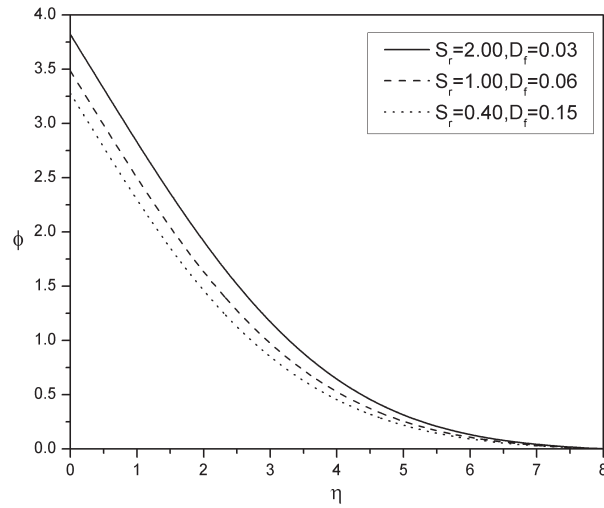


FIG. 4: Concentration profile for different values of D_f , S_r at $C_\alpha = 1.0$, $Fs = 0.5$

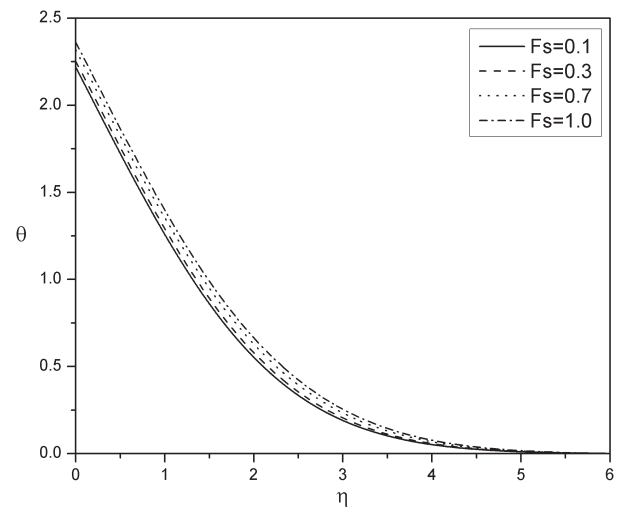


FIG. 6: Temperature profile for different values of Fs at $C_\alpha = 1.0$, $S_r = 2.0$, $D_f = 0.03$

0.03, and $C_\alpha = 1.0$, is depicted in Fig. 5. It shows the effects of Forchheimer (inertial porous) number on the velocity. In the absence of Forchheimer number (i.e., when $Fs = 0$), the present investigation reduces to a mixed convection heat and mass transfer in a couple stress fluid saturated porous medium with Soret and Dufour effects. It is observed from Fig. 6 that velocity of the fluid decreases with increase in the value of the non-Darcy parameter Fs . The increase in non-Darcy parameter implies

that the porous medium is offering more resistance to the fluid flow. This results in reduction of the velocity profile. The dimensionless temperature for different values of Forchheimer number Fs for $S_r = 2.0$, $D_f = 0.03$, and $C_\alpha = 1.0$, is displayed in Fig. 7. An increase in Forchheimer number Fs increases temperature values, since as the fluid is decelerated, energy is dissipated as heat and serves to increase temperatures. As such the temperature is minimized for the lowest value of Fs and maximized

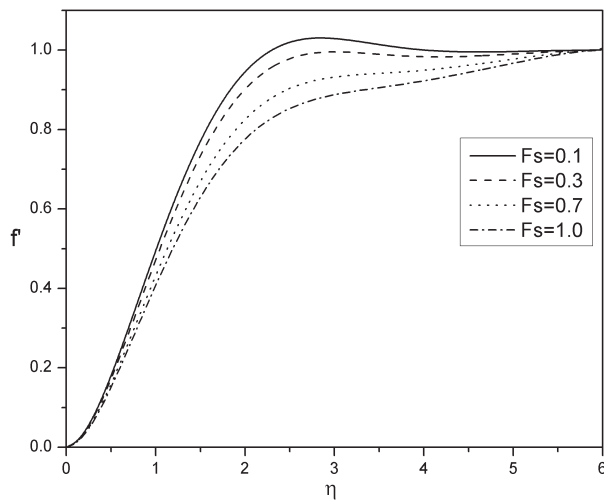


FIG. 5: Velocity profile for different values of Fs at $C_\alpha = 1.0$, $S_r = 2.0$, $D_f = 0.03$

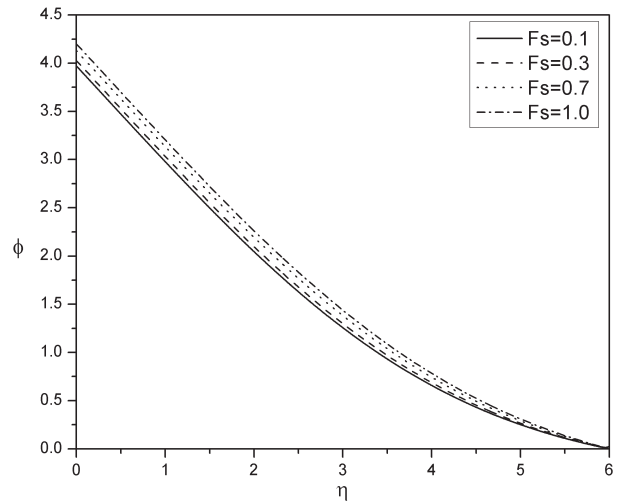


FIG. 7: Concentration profile for different values of Fs at $C_\alpha = 1.0$, $S_r = 2.0$, $D_f = 0.03$

for the highest value of F_s as shown in Fig. 6. Figure 7 demonstrates the dimensionless concentration for different values of Forchheimer number with $S_r = 2.0$, $D_f = 0.03$, and $C_\alpha = 1.0$. It is clear that the concentration of the fluid increases with the increase of Forchheimer number. The increase in non-Darcy parameter reduces the intensity of the flow and increases the thermal and concentration boundary layer thickness.

In Figs. 8–10, the effects of the couple stress parameter C_α at a given x location on the dimensionless velocity, temperature, and concentration profiles are presented for fixed values of $S_r = 2.0$, $D_f = 0.03$ and $F_s = 0.5$. As C_α increases, it can be observed from Fig. 8 that the maximum velocity decreases in amplitude and the location of the maximum velocity moves far away from the wall. This happens because of the rotational field of the velocity generated in couple stress fluid. It is clear from Fig. 9 that the temperature increases with the increase of couple stress fluid parameter C_α . It can be seen from Fig. 10 that the concentration of the fluid increases with the increase of couple stress fluid parameter C_α .

The variations of $1/\theta(0)$, $1/\phi(0)$ which are proportional to the rate of heat and mass transfers are shown in Table 2 for different values of the couple stress parameter with fixed Forchheimer, Soret, and Dufour numbers. From this table, it is observed that the value of the heat transfer rate and the mass transfer rate decreases with an increase in C_α . The heat and mass transfer is lower in the couple stress fluid compared to that of the Newtonian fluid. Hence, the presence of couple stresses in the fluids

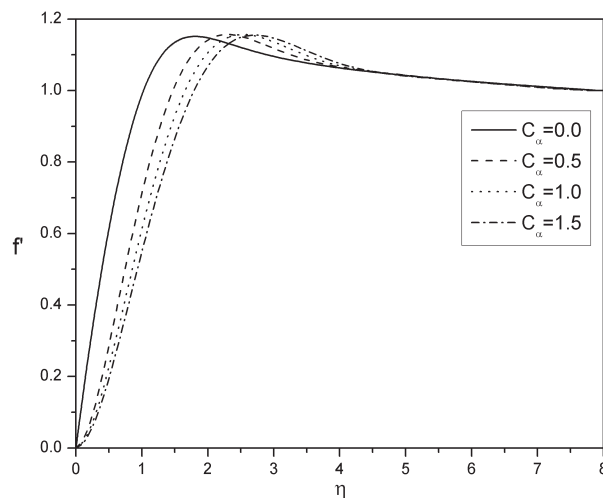


FIG. 8: Velocity profile for different values of C_α at $F_s = 0.5$, $D_f = 0.03$, $S_r = 2.0$

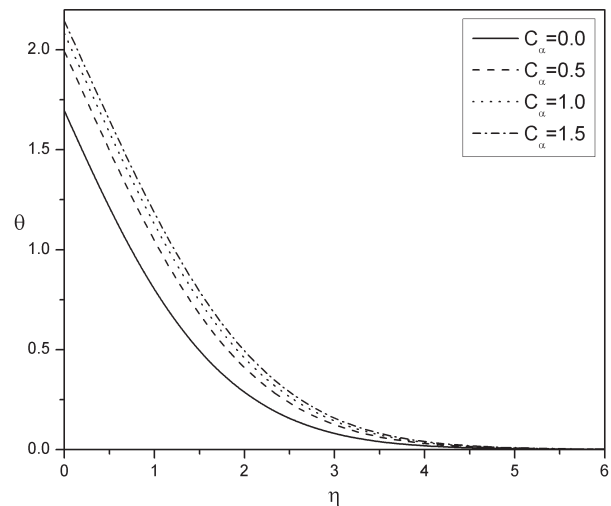


FIG. 9: Temperature profile for different values of C_α at $F_s = 0.5$, $D_f = 0.03$, $S_r = 2.0$

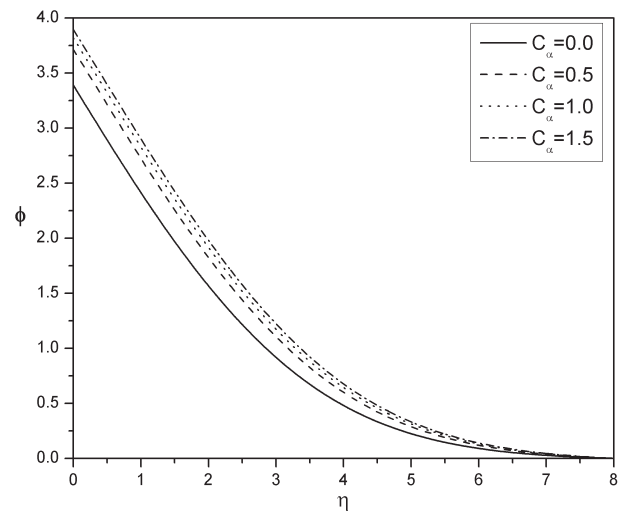


FIG. 10: Concentration profile for different values of C_α at $F_s = 0.5$, $D_f = 0.03$, $S_r = 2.0$

decreases the heat transfer coefficient and the mass transfer coefficient, which may be beneficial in flow, temperature, and concentration control of polymer processing. Also, it can be observed from this table that, for fixed values of C_α , S_r , and D_f , the heat and mass transfer coefficients are increasing with the increasing values of Forchheimer number F_s . Therefore, the inertial effects in couple stress fluid saturated non-Darcy porous medium increase the heat and mass transfer coefficients. Finally, the

TABLE 2: Effect of couple stress parameter, Forchheimer, Prandtl, Soret, and Dufour numbers on nondimensional heat and mass transfer coefficients

C_α	F_s	S_r	D_f	$1/\theta(0)$	$1/\phi(0)$
0.1	0.5	2	0.03	0.53857	0.283561
0.3	0.5	2	0.03	0.513432	0.275975
0.7	0.5	2	0.03	0.48891	0.268019
1	0.5	2	0.03	0.477371	0.264086
1	0.1	2	0.03	0.459678	0.256099
1	0.3	2	0.03	0.469166	0.260365
1	0.7	2	0.03	0.484557	0.267367
1	1	2	0.03	0.493833	0.271628
1	0.5	2	0.03	0.477371	0.264086
1	0.5	1.6	0.0375	0.476543	0.273542
1	0.5	1.2	0.05	0.475382	0.283725
1	0.5	1	0.06	0.474554	0.289123
1	0.5	0.8	0.075	0.473396	0.294753
1	0.5	0.5	0.12	0.470196	0.303719
1	0.5	0.2	0.3	0.458504	0.313847
1	0.5	0.1	0.6	0.440566	0.318434

effects of Dufour and Soret number on the rate of heat and mass transfer are shown in this table. The behavior of these parameters is self-evident from Table 2 and hence, for brevity, is not discussed.

4. CONCLUSIONS

In this paper, a boundary layer analysis for mixed convection heat and mass transfer in a non-Darcy couple stress fluid over a vertical plate with uniform and constant heat and mass flux conditions in the presence of Soret and Dufour effects is considered. Using the similarity variables, the governing equations are transformed into a set of nonsimilar parabolic equations where numerical solution has been presented for different values of parameters. The numerical results indicate that the rate of heat and mass transfers in the couple stress fluid are lower compared to that of the Newtonian fluid. The higher values of the Forchheimer number F_s indicate lower velocity, and the rate of heat and mass transfers, but higher wall temperature and wall concentration distributions. The present analysis has also shown that the flow field is appreciably influenced by the Dufour and Soret effects. The presence of couple stresses in the fluid decreases the velocity and increases temperature and concentration.

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