Arab J Sci Eng (2014) 39:1089–1106
DOI 10.1007/s13369-013-0699-6

RESEARCH ARTICLE - ELECTRICAL ENGINEERING

Estimation of Locational Marginal Price in a Restructured Electricity Market with Different Loss Cases using Seed Genetic Algorithm

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Received: 9 April 2012 / Accepted: 9 July 2013 / Published online: 8 September 2013
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Abstract In restructured electricity markets, an effective transmission pricing is required to address transmission issues and to generate correct economic signals. These prices depend on generator bids, load levels and transmission network constraints. A congestion charge is incurred when the system is constrained due to physical limitations. Locational marginal pricing (LMP) is a popular method in restructured power markets to address these issues. Seed genetic algorithms performs powerful global searches and is a well-proven optimization algorithm. This paper combines a seed Genetic Algorithm approach with DC optimal power flow (DCOPF) to estimate LMP at all buses while minimizing the net system generation costs or fuel cost in a constrained pool-based restructured electricity market. Various cases like LMP without loss, concentrated loss and distributed loss have been attempted. Both fixed bids and linear bids are considered for generators. Load is assumed to be inelastic. The developed models have been tested on IEEE 14 bus, New England 39 bus and 75 bus Indian Power systems. Comparison is made between linear programming-based DCOPF using Power World Simulator and the developed GA approach for all cases of fuel cost. In all the cases studied, GA approach is found to estimate better LMP and minimum fuel cost. ISO profits during congestion have also been evaluated in all cases. In this paper the proposed distributed loss model is stated to be the feasible operation compared with concentrated loss model.

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Keywords Concentrated loss · Distributed loss · Delivery factors · Fixed bid · Generation shift factors · Genetic algorithm · Linear bid · LMP · Seed · Shadow price

الخلاصة

إن تسعير الانتقال الفعّال - في أسواق الكهرباء المعاد هيكلتها - أمرٌ مطلوب لمعالجة قضايا النقل وتوليد الإشارات الاقتصادية الصحيحة. وتعتمد هذه الأسعار على مولد المناقصات ومستويات الحمل وقيود شبكة النقل. ويتم تكبد رسوم الازدحام عندما يتم تقييد النظام بسبب القيود المادية. إن التسعير الثانوي الموقعي (LMP) وسيلة معروفة في أسواق الطاقة المعاد هيكلتها لمعالجة هذه القضايا. وتنفذ خوارزميات البذور الجينية (GA) عمليات بحث عالمية قوية، وثبت أيضا أنها الخوارزمية الأمثل. وتجمع الورقة العلمية الحالية مقاربة خوارزمة البذور الجينية مع تدفق الطاقة المثلى (DCOPF) لتقدير التسعير الثانوي الموقعي في جميع الحافلات مع التقليل من صافي تكاليف توليد النظام أو تكلفة الوقود في سوق الكهرباء أعيدت هيكلتها ومستندة إلى بركة مقيدة. وقد تمت تطبيق محاولة مختلف الحالات مثل التسعير الثانوي الموقعي دون خسارة، والخسارة المركزة والفقدان الموزع، وأخذ في الاعتبار المزايدات الثابتة والمزايدات الخطية للمولدات، ومن المفترض أن يكون الحمل غير مرن. وقد تم اختبار النماذج المطورة على أنظمة حافلة IEEE 14، ونيو انغلاند 39 حافلة و 75 حافلة أنظمة الطاقة الهندي. وتم عمل مقارنة بين البرمجة الخطية (LP) استنادا DCOPF باستخدام محاكي الطاقة الدولي ونهج GA لجميع الحالات من تكلفة الوقود. وفي جميع الحالات التي تمت دراستها، وجد أن نهج GA يقدر LMPs بشكل أفضل وتكلفة وقود في الحد الأدنى. كما تم تقييم أرباح ISO في أثناء الازدحام في جميع الحالات. وفي هذه الورقة العلمية تم تأكيد أن نموذج الخسارة الموزع المقترح هو عملية مجدية مقارنة بنموذج الخسارة المركزة.

1 Introduction

In April 2003 White Paper the US Federal Energy Regulatory Commission (FERC) proposed a market design for common adoption by US wholesale power markets. The electric power industry has undergone deregulation around the world, a core tenet of which is to build open-access, unambiguous and fair electricity markets. Proper and fair pricing of real power is an important issue in this competitive market. Core features of a market design include a two-settlement system consisting of a day-ahead market supported by a real-time market to ensure continual balancing of supply and demand for power, and grid congestion management by means of locational marginal pricing (LMP).

Under a deregulated electricity market environment, transmission networks hold a vital role in supporting the transaction between producers and consumers. One drawback of transmission constraint is congestion. Congestion occurs when transmission lines or transformers operate at or above its thermal limits and this prevents the system operators from dispatching additional power from a specific generator. Congestion can result in an overall increase in the cost of power delivery. Presently, there are two pricing structures [1] that are being used in a competitive energy market to account for congestion: the uniform pricing method market clearing price (MCP) and the nonuniform pricing method (LMP). In the first method, all generators are paid the same price (MCP) based on the bid of the marginal generator that would be dispatched in the absence of congestion. The second method (LMP) has been the basic approach in power markets to calculate nodal prices and to manage transmission congestion. The theory of spot price, which was first proposed by Schweppe et al. [2], is increasingly being employed in the form of (LMP) within an OPF framework. The LMP at a location is defined as the marginal cost to supply an additional increment of power to the location without violating any system security limits. Because of the effects of both transmission losses and transmission system congestion, LMP can vary significantly from one location to another. Mathematically, LMP at any node in the system is the dual variable (sometimes called a shadow price) for the equality constraint at that node (sum of injections and withdrawals is equal to zero). Or, LMP is the additional cost for providing one additional MW at certain node. Buyers pay ISO based on their LMP for dispatched energy. The ISO pays sellers based on their respective LMP. The LMP difference between two adjacent buses is the congestion cost which arises when the energy is transferred from one location (injection) to another (withdrawal). Marginal losses represent incremental changes in system losses due to incremental demand changes. Incremental losses yield additional costs which are referred to as the cost of marginal losses. Thus LMP is the summation of generation marginal cost, marginal loss cost and congestion cost. Therefore, LMP is defined as follows:

$$\text{LMP} = \text{generation marginal cost} + \text{congestion cost} + \text{loss cost} \quad (1)$$

In this decomposition model, LMP congestion component at Bus B, i.e. $\text{LMP}_\text{B}^\text{cong}$ remains invariant with reference to different reference buses, and the combination of the other two components, i.e. $\text{LMP}^\text{energy} + \text{LMP}_\text{B}^\text{loss}$, is also reference-independent. But each of LMP^energy or $\text{LMP}_\text{B}^\text{loss}$ is still reference-dependent. LMP can be derived using either an ACOPF model or a DCOPF model [3–9].

The objective function of OPF is meeting the load in the power system while maximizing social surplus and respecting operational constraints. In the absence of price elasticity load (which is mostly the case in the real-time market) maximising social welfare is equivalent to minimising the total production cost. In this paper we assumed load to be price inelastic. There are two approaches to calculate LMPs in RTM: ex post and ex ante. NY ISO uses ex ante prices as the real-time prices and penalises non-performing resources on the basis of reduced generation quantity [10], whereas ISO NE, PJM and MISO adopt the ex post pricing that provides dispatch incentives on the ground of rational prices [11,12]. Both ex ante and ex post approaches have their own merits and demerits. For example, ex ante pricing does not have a capability to penalise non-performing units, whereas ex post pricing has some difficulties in implementing co-optimisation of the energy and reserves [13]. In market planning and simulation, DC model is desired due to its robustness and speed. DCOPF is broadly employed by a number of industrial LMP simulators, such as ABB's Gid View™, GE's MAPS™, Siemens' Promod IV® and Power World [14,15]. Several papers have reported different models for LMP calculation. Reference [16] gave description about components of nodal prices for electric power systems. Reference [17] demonstrated the usefulness of dc power flow in calculating loss penalty factors, which has a significant impact on generation scheduling. It also pointed out that it is not advisable to apply predetermined loss penalty factors from a typical scenario to all cases. Reference [18] presented a slack-bus-independent approach to calculate LMPs and congestion components. Reference [19] presented LMP simulation algorithms to address marginal loss pricing based on the dc model. Literature shows that dc model can be acceptable in optimal power flow studies if the line flow is not very high, the voltage profile is sufficiently flat and the R/X ratio is less than 0.25 [20]. Reference [21] presented Nodal pricing with Genetic Algorithm for congestion management with DCOPF for lossless system. Reference [22] presented different methods and properties on LMP calculations based on DCOPF with and without loss. An LP-based approach for LMP without loss case with concentrated and distributed loss DCOPF model is presented in [23] with piecewise linear cost curves, but it does not give actual marginal cost of generation. Reference [24] presented Cumulant and Gram Charlier (CGC) method for calculating LMP and compared it with Monte Carlo and point estimation method. This method combines the concept of Cumulants and Gram-Charlier expansion theory to obtain probabilistic distribution functions (PDF) and cumulative distribution function (CDF) of LMP. This method is complex and time consuming. Reference [13] gives a systematic description on how the LMPs are produced; it also describes both the modelling and implementation challenges and solutions. Reference [25] described ACOPF-based LMP calculation considering distributed loss.

The present paper proposed a seed GA-based DCOPF for without loss, concentrated and distributed loss models with linear incremental cost curve which gives true marginal cost of generation, and mismatch at the slack bus is removed. All these three models have been attempted for fixed generation bids and linear generation bids. The derivation of LMP and LMP components based on the above models is presented here. In concentrated loss DCOPF model total system loss is supplied by the slack bus which creates a burden on the slack bus. To eliminate a large mismatch at the slack bus, loss is distributed to all buses as an extra load.

Section 2 describes different types of bids used for generators. The problem formulation for LMP calculation using delivery factors has been discussed in Sect. 3 for all the models. Section 4 presented implementation of seed GA. Section 5 gives results and discussions for IEEE 14 bus system [26], New England 39 bus system [27] and 75 bus Indian power system [28]. Section 6 concludes the paper.

2 Types of Generator Bids

In general, generators bids depend on many factors, some of which (e.g. strategic behaviour) are difficult to model. To avoid excessive complexity, generator bids are assumed to be equal to their incremental costs for perfect competition. Two bidding models are generally used, namely fixed generator bids (corresponding to piecewise-linear heat rates) and linear bids (corresponding to quadratic heat rates).

2.1 Fixed Bids

The piecewise linear heat rate curve is converted to an approximate fixed incremental heat rate for each unit through linear regression method. The cost changes in steps with respect to generation. The main drawback of this bid is it does not give true marginal cost of generator. The fixed bid curve for piecewise linear cost characteristics is shown in Fig. 1.

2.2 Linear Bids

The non-smooth nature of the fixed bid in Fig. 1 may result in step changes in prices at certain load levels. One way to mitigate this is to use linear bids for the generating units. This will result in a much smoother supply curve as shown in Fig. 2 and also give the actual marginal cost of a generator. The generator cost curve is given by (2)

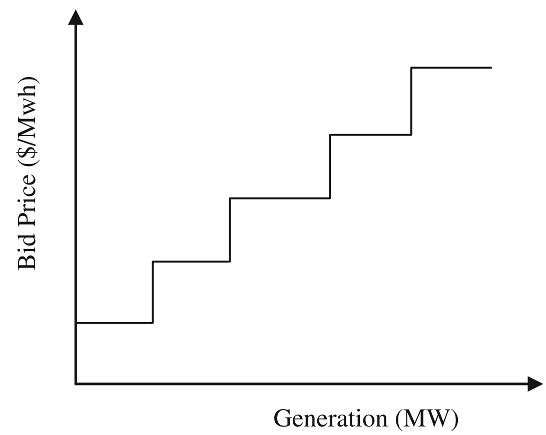


Fig. 1 Fixed bids

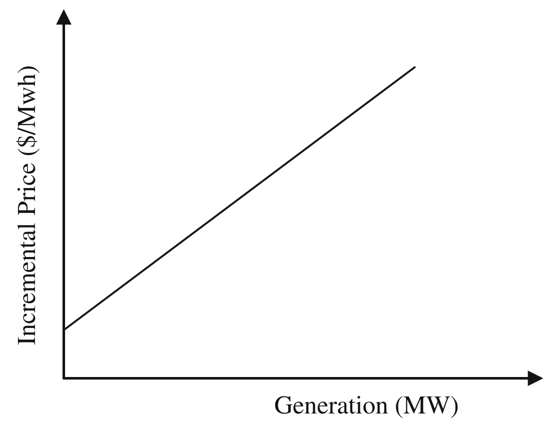


Fig. 2 Linear bids

$$C_i(P_{G_i}) = a_i + b_i P_{G_i} + c_i P_{G_i}^2 \ (\$/\text{h}), \tag{2}$$

where $C_i(P_{G_i})$ is the cost of unit i generating P_{G_i} MW, a_i is the no-load cost, b_i is the linear cost coefficient, and c_i is the quadratic cost coefficient of unit i. These a, b, c coefficients of generators are given by manufacturer of the generator. The incremental costs of the units can be modelled as fixed quantities (Fig. 1) or can be expressed as linear function with slope m of the unit outputs (Fig. 2) as in (3). The linear price curve introduces non-linearities in the problem; however, it is a more realistic representation of price than that of a fixed price for power.

$$dC_i(P_{G_i})\big/dP_{G_i} = b_i + 2c_i P_{G_i} \ (\$/\text{MWh}). \tag{3}$$

3 Mathematical Formulation for LMP Estimation

LMPs using DCOPF with and without considering line losses for fixed bids are solved with linear programming (LP) approach [23]. In the present paper active power generations of the generators except slack generation are considered

Table 1 Active power generations of generators for lossless, concentrated and distributed loss models of IEEE 14 bus system

Generator bus no.	Power generations in MW without loss			Power generations in MW with concentrated loss			Power generations in MW with distributed loss		
	LP approach	GA approach		LP approach	GA approach		LP approach	GA approach	
	Fixed bids	Fixed bids	Linear bids	Fixed bids	Fixed bids	Linear bids	Fixed bids	Fixed bids	Linear bids
1 (slack bus)	142	141.37	141.37	147.143	144.9	144.9	147.143	140.48	140.48
2	117	117.62	117.62	117	117.62	117.62	117	122.00	122.00
System loss (MW)				5.143	3.527	3.527	5.143	3.485	3.485
Fuel cost ($/h)	3,222.97	3,223.11	2,985.01	3,286.434	3,266.64	3,063.56	3,286.434	3,267.13	3,063.33
Generators paid ($/h)	3,178.51	3,178.17	5,681.85	3,238.464	3,232.49	5,919.08	3,238.464	3,231.06	5,744.59
Supplier surplus ($/h)	−44.46	−44.94	2,696.84	−47.97	−34.14	2,855.51	−47.97	−36.06	2,681.26
Load payment to ISO ($/h)	4,716.081	4,715.85	7,219.53	5,005.321	5,107.1	7,829.63	5,006.246	5,150.96	7,731.57
ISO profit ($/h)	1,537.571	1,537.67	1,537.67	1,766.861	1,874.61	1,910.55	1,767.782	1,919.89	1,986.98

Table 2 LMP at all buses for all models of IEEE 14 bus system

Bus no.	LMP at all buses ($/MWh)								
	Without loss model			Concentrated loss model			Distributed loss model		
	LP approach	GA approach		LP approach	GA approach		LP approach	GA approach	
	Fixed bids	Fixed bids	Linear bids	Fixed bids	Fixed bids	Linear bids	Fixed bids	Fixed bids	Linear bids
1	12.34	12.34	22.00	12.34	12.34	22.52	12.34	12.34	21.87
2	12.19	12.18	21.85	12.16	12.27	22.56	12.16	12.27	21.89
3	11.76	11.75	21.42	11.64	12.11	22.69	11.64	12.11	22.00
4	11.38	11.38	21.05	11.20	11.58	22.08	11.20	11.57	21.40
5	12.91	12.91	22.58	13.02	13.34	23.8	13.02	13.34	23.12
6	23.77	23.76	33.43	25.95	26.28	36.74	25.95	26.27	36.06
7	27.58	27.57	37.24	30.49	30.86	41.36	30.50	30.85	40.67
8	27.58	27.57	37.24	30.49	30.86	41.36	30.50	30.85	40.67
9	36.10	36.09	45.76	40.63	41.00	51.50	40.65	41.00	50.81
10	33.91	33.9	43.57	38.02	38.40	48.91	38.03	38.4	48.23
11	28.93	28.92	38.59	32.09	32.45	42.95	32.10	32.45	42.26
12	24.74	24.74	34.4	27.11	27.48	37.99	27.12	27.48	37.31
13	25.50	25.5	35.17	28.02	28.40	38.91	28.02	28.39	38.23
14	31.47	31.46	41.13	35.12	35.55	46.09	35.13	35.54	45.41

in chromosome using seed GA. The obtained PGs are used in calculation of LMP with and without loss for the congested transmission system. Generation shift factors (GSF) have been used for the calculation of transmission line flows. Delivery factors (DF) at all buses have been used to include the impact of marginal losses on LMP.

The location of reference bus or slack bus will not impact LMP values, when ignoring system losses. But the individual components of LMP depend on the location of reference bus. If transmission losses are balanced at reference bus, i.e. in concentrated loss model, the bus LMPs depend on the location of reference bus. In distributed loss model the bus LMPs are independent of the choice of reference bus. It should also be noted that actual GSF values depend on the choice of slack bus, although the line flow based on GSF is the same with different references buses.

3.1 Generation Shift Factor

Generation shift factor is the ratio of change in power flow of line 'k' to change in power injection at bus 'i'. GSF can be computed using (3), where B^{-1} is inverse of B matrix (the imaginary part of Y bus matrix), X_k is reactance of line k, 'a' and 'b' are sending and receiving end bus of line k.

$$\mathrm{GSF}_{k-i} = \left(B^{-1}_{(a,i)} - B^{-1}_{(b,i)}\right) \Big/ X_k \tag{4}$$

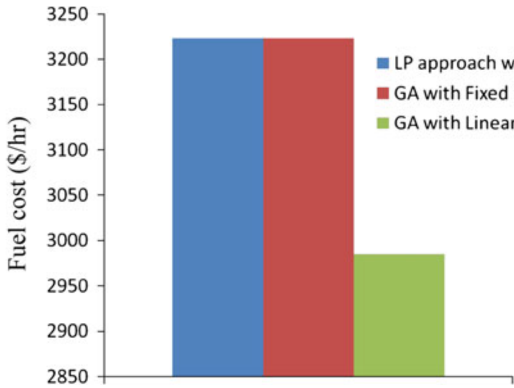


Fig. 3 Fuel cost comparison graph for without loss case of IEEE 14 bus system

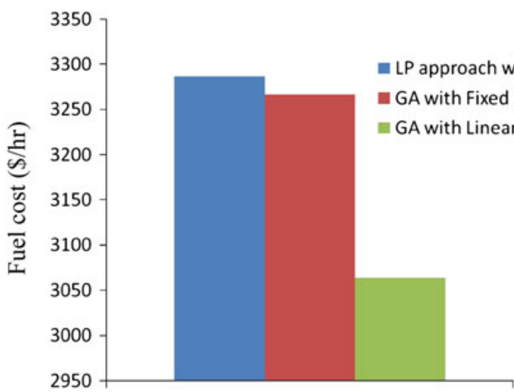


Fig. 4 Fuel cost comparison graph for concentrated loss case of IEEE 14 bus system

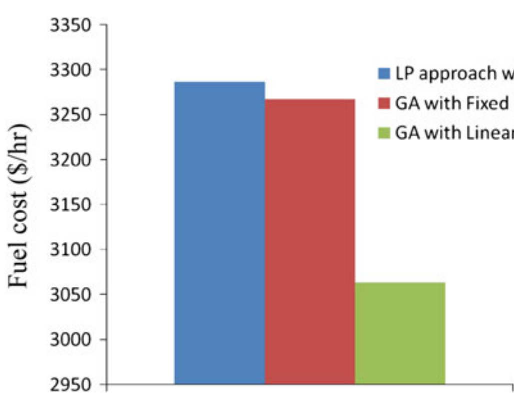


Fig. 5 Fuel cost comparison graph for distributed loss case of IEEE 14 bus system

3.2 Delivery Factor

The delivery factor (DF_i) at the ith bus represents the effective MW delivered to the customers to serve the load at that bus. It is defined as (4)

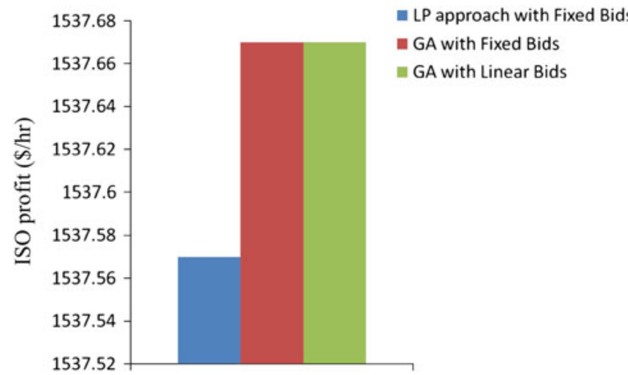


Fig. 6 ISO profit comparison graph for without loss case of IEEE 14 bus system

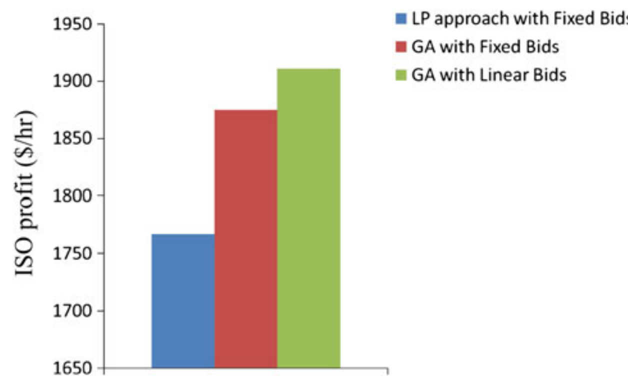


Fig. 7 ISO profit comparison graph for concentrated loss case of IEEE 14 bus system

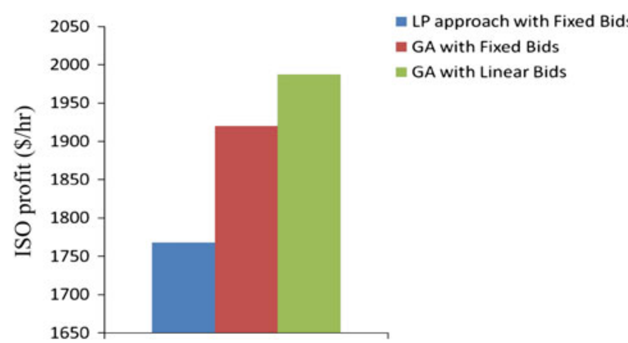


Fig. 8 ISO profit comparison graph for distributed loss case of IEEE 14 bus system

$$\mathrm{DF}_i = 1 - \mathrm{LF}_i = 1 - \partial P_{\mathrm{loss}} \big/ \partial P_i \tag{5}$$

$$P_{\mathrm{loss}} = \sum_{k=1}^{M} F_k^2 \times R_k \tag{6}$$

$$F_k = \sum_{i=1}^{N} \mathrm{GSF}_{k-i} \times P_i \tag{7}$$

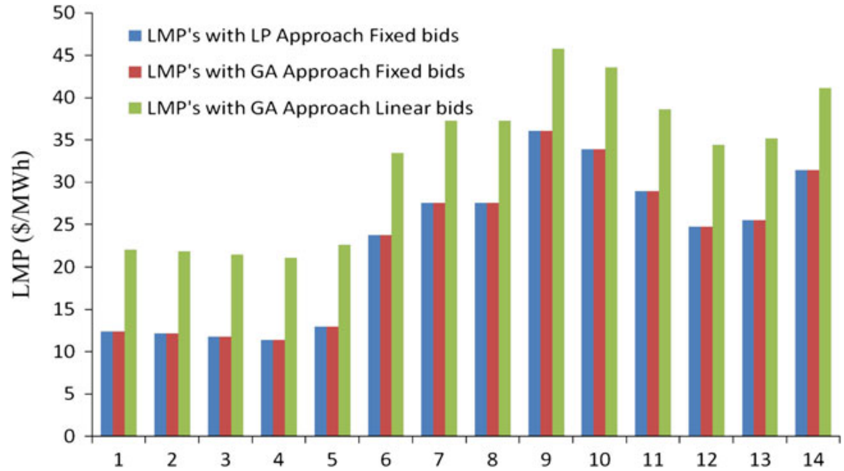


Fig. 9 LMP comparison graph for without loss case of IEEE 14 bus system

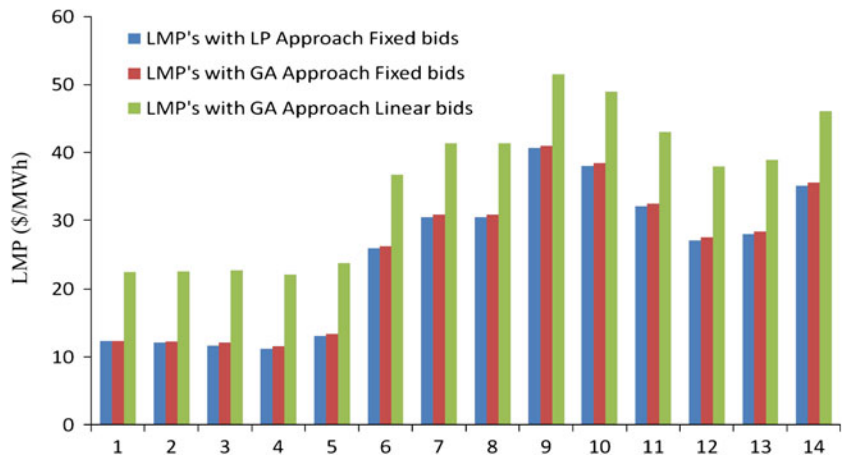


Fig. 10 LMP comparison graph for concentrated loss case of IEEE 14 bus system

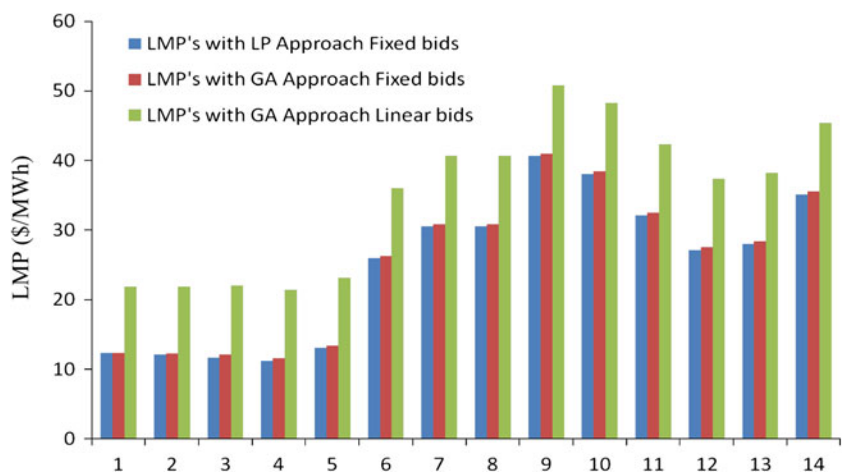
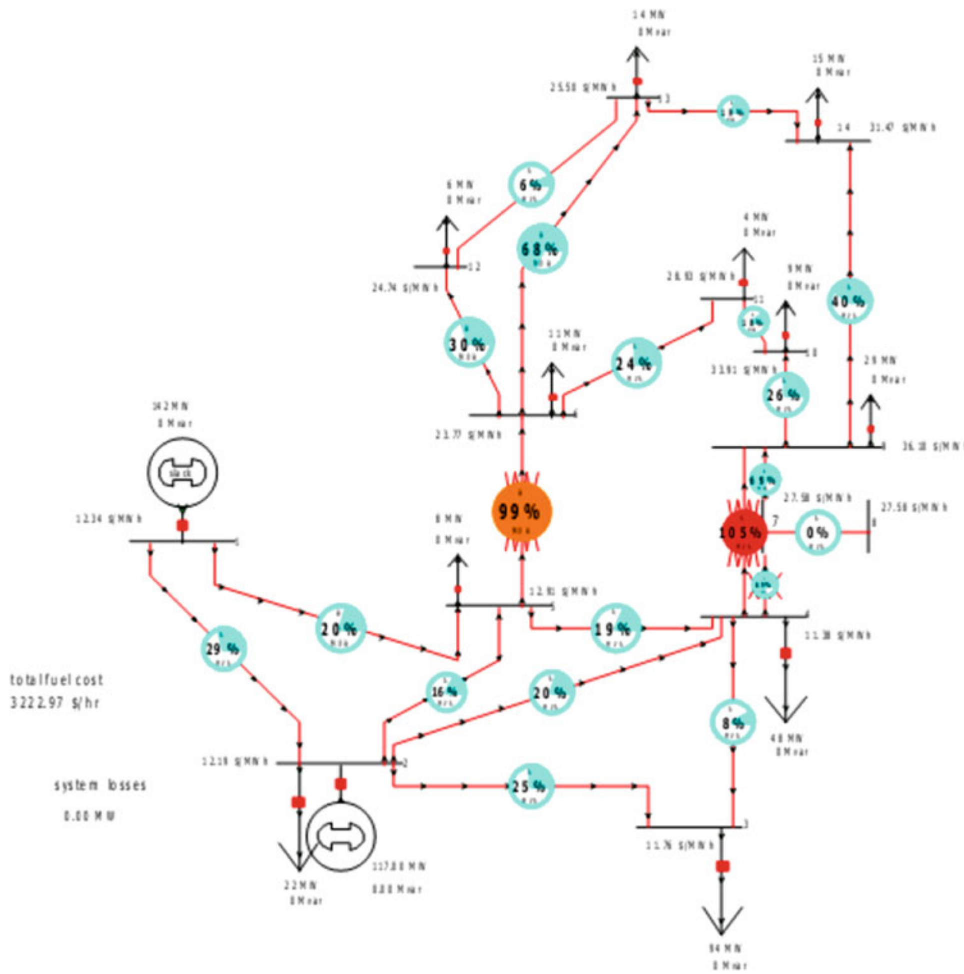


Fig. 11 LMP comparison graph for distributed loss case of IEEE 14 bus system

$$
\begin{aligned}
\frac{\partial P_{\text{loss}}}{\partial P_i} &= \sum_{k=1}^{M} \frac{\partial}{\partial P_i}(F_k^2 \times R_k) \\
&= \sum_{k=1}^{M} R_k \times 2F_k \times \frac{\partial F_k}{\partial P_i} \\
&= \sum_{k=1}^{M} 2 \times R_k \times GSF_{\text{k-i}} \times \left(\sum_{j=1}^{N} \text{GSF}_{\text{k-j}} \times P_j \right)
\end{aligned} \tag{8}
$$

In (5)–(7), LF_i represents the loss factor (LF_i) at bus 'i' which is calculated using (8), F_k is the power flow in line k, R_k is the resistance of line k, P_i is the injected power at bus i, and GSF_{k-i} is the generation shift factor to line 'k' from bus 'i'. LF_i may be viewed as the change of total system loss with respect to 1 MW increase in injection at that bus. Interestingly, the loss factor at a bus may be positive or negative. When it is positive, it implies that an increase of

Fig. 12 Power world simulation diagram of IEEE 14 bus system for without loss model with LP approach fixed bids



injection at the bus may increase the total system loss. If it is negative, it implies that an increase of injection at the bus may reduce the total loss.

3.3 LMP Estimation

Case 1. Without Losses Using Seed GA-Based DCOPF

In this method the objective function is minimization of total production cost subjected to demand balance and line flow constraints. This problem is solved with seed GA; then LMPs are calculated from the obtained generator power outputs and then generators paid by ISO, supplier surplus, load payment to ISO and total ISO profit are calculated and compared with LP approach.

The objective function is

$$\text{Minimize } J = \sum_{i=1}^{N} MC_i \times P_{G_i} \tag{9}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} P_{G_i} = \sum_{i=1}^{N} P_{D_i} \tag{10}$$

$$F_k \leq \text{limit}_{k,} \quad k = 1, 2 \ldots, M \tag{11}$$

$$P_{G_i}^{\min} \leq P_{G_i} \leq P_{G_i}^{\max} \quad i = 1, 2, \ldots, N, \tag{12}$$

where N is number of buses, M is number of lines, MC_i marginal cost at bus i, i.e. $(b_i + 2c_i P_{G_i})$ in \$/MWh, P_{G_i} is output power of generator at bus i (MWh), P_{D_i} is the demand at bus i and limit_k is thermal limit of line k.

Case 2. With Concentrated Loss Using Seed GA-Based DCOPF

In this method the objective function is minimization of total production cost subjected to energy balance and line flow constraints. However, in LMP-based electricity markets, system marginal losses have significant impact on the economics of power system operation. So, system marginal losses have to be taken into account for obtaining more accurate LMPs. In this model it is assumed that total system loss is supplied by slack bus generator. This problem is solved with seed GA and the total fuel cost is compared with LP approach. In LP approach loss is calculated using ac load flow and is added to the slack bus as extra load by modifying the line data with

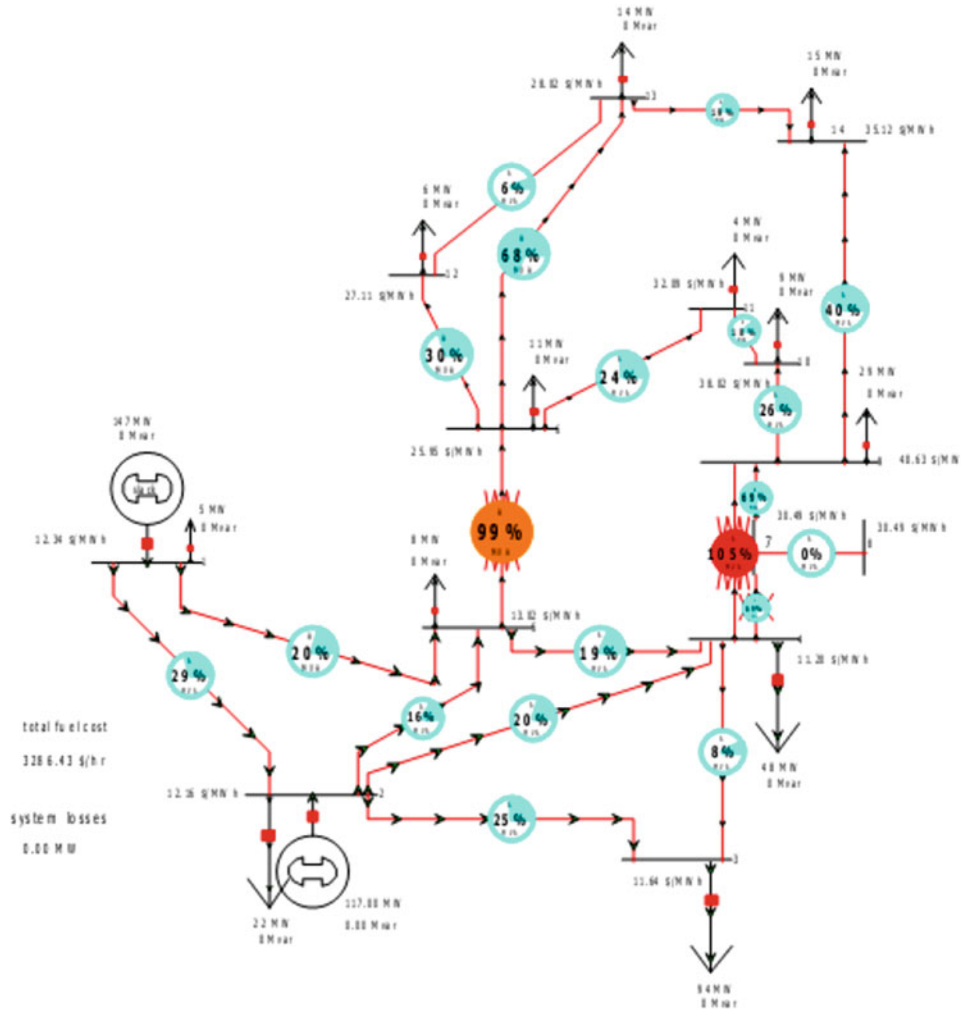


Fig. 13 Power world simulation diagram of IEEE 14 bus system for concentrated loss model with LP approach fixed bids

resistance (R) taken as 10 % of reactance (X). The problem is

$$\text{Minimize } J = \sum_{i=1}^{N} MC_i \times P_{G_i} \tag{13}$$

$$\sum_{i=1}^{N} DF_i \times (P_i) + P_{\text{loss}} = 0 \tag{14}$$

$$F_k \le \text{limit}_k, \quad k = 1, 2, \ldots, M \tag{15}$$

$$P_{G_i}^{\min} \le P_{G_i} \le P_{G_i}^{\max} \quad i = 1, 2, \ldots, N, \tag{16}$$

where P_{loss} is the total system loss. P_{loss} in (14) is used to offset the doubled average system loss caused by the marginal loss factor (LF) and the marginal delivery factor (DF).

After getting power outputs of generators for the above dispatch, slack bus power is calculated using (10) or (14) and the price at the reference (slack) bus has to be calculated by substituting slack bus power either in fixed bids or linear bids. At the reference bus both loss price and congestion price are always zero. Therefore, the price at the reference bus is equal to the energy component. Now the LMP formulation at a bus B can be written as

$$\text{LMP}_{\text{B}} = \text{LMP}^{\text{energy}} + \text{LMP}_{\text{B}}^{\text{cong}} + \text{LMP}_{\text{B}}^{\text{loss}} \tag{17}$$

The decomposition of LMP is shown here

$$\text{LMP}^{\text{energy}} = \lambda = \text{ price at the reference bus} \tag{18}$$

$$\text{LMP}_{\text{B}}^{\text{cong}} = -\sum_{k=1}^{M} \text{GSF}_{k-B} \times \mu_k \tag{19}$$

where μ_k is the constraint cost or shadow price of line k, defined as

$$\mu_k = \frac{\text{change in total cost}}{\text{change in constraint's flow}}$$

$$\text{LMP}_{\text{B}}^{\text{loss}} = \lambda \times (\text{DF}_{\text{B}} - 1) \tag{20}$$

($\text{LMP}_{\text{B}}^{\text{loss}} = 0$ for lossless power system)

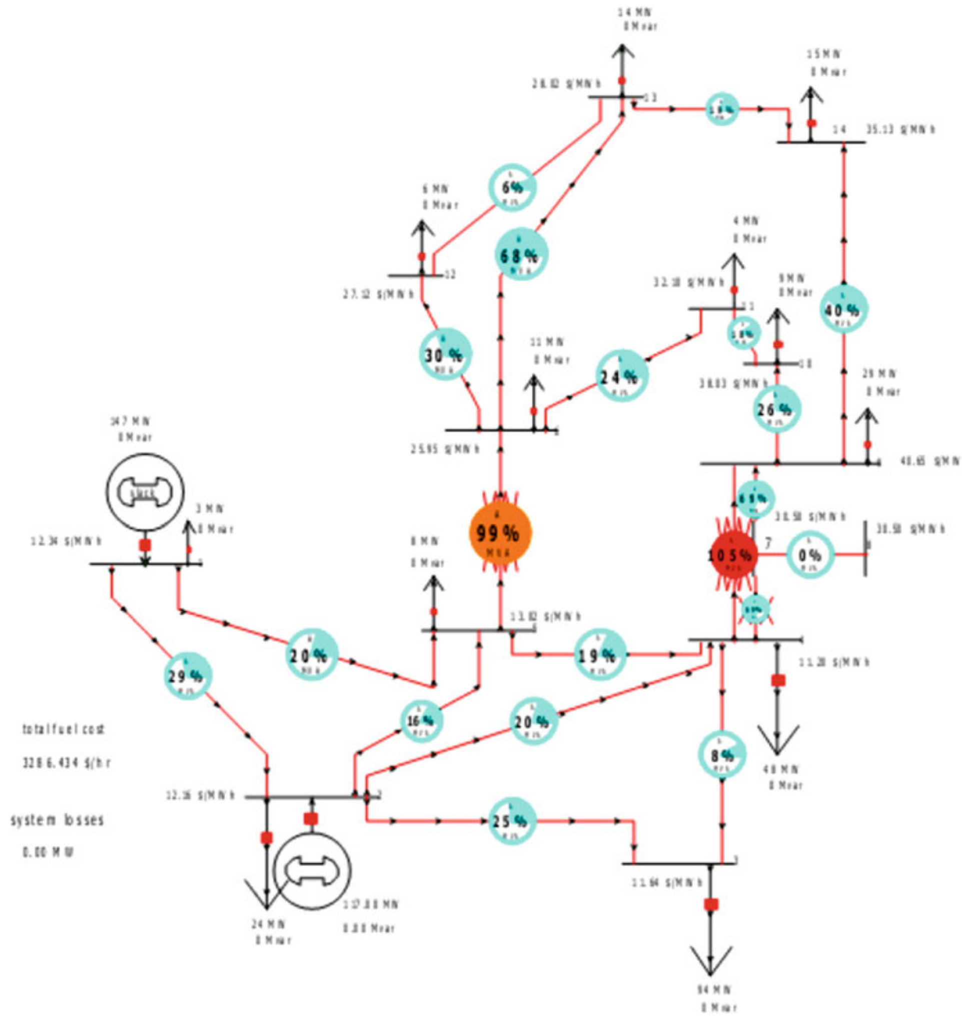


Fig. 14 Power world simulation diagram of IEEE 14 bus system for distributed loss model with LP approach fixed bids

Case 3. With Distributed Loss Using Seed GA-Based DCOPF

Concentrated loss model addresses the marginal loss price through the delivery factors. However, the line flow constraints in (15) still assume a lossless network. But the equality constraint in (14) gives total generation is greater than the total demand by the average system loss. This causes a mismatch at slack bus and this mismatch is absorbed by the system slack bus. If system demand is huge, e.g. a few GW, then the system loss may be of the order of hundreds of MW and it may not be possible to add whole of loss to slack bus. To address this issue, it is necessary that the line losses are shared among buses. This paper employs the concept of distributed loss to represent the losses of the lines connected to a bus. In this method system losses are distributed among all buses and eliminate the large mismatch at the reference bus. By this approach, loss in each transmission line is divided into two equal halves, and each half is added to bus end of the line as an extra load. So for each bus the total extra load is the sum of halves of line losses which are connected to that bus. The extra load at bus 'i' is assumed as E_i, and it is defined as follows:

$$E_i = \sum_{k=1}^{M_i} \frac{1}{2} \times F_k^2 \times R_k, \tag{21}$$

where M_{i} is number of lines connected to Bus i. The line flow F_{k} for this model is calculated as in (20).

$$F_k = \sum_{i=1}^{N} \mathrm{GSF}_{k-i} \times (G_i - D_i - E_i) \tag{22}$$

The algorithm for this problem is same as in section case 2. After getting power outputs of generators, LMPs at all buses are calculated using (17)–(20). In this approach, since loss is modelled as equivalent load ISO recovers loss cost from loads which it has to pay for generators and burden on the slack bus is eliminated. In LP approach, i.e. in Power world simulator it is modelled as, total system loss which is calculated using ac

Table 3 Active power generations of generators for lossless, concentrated and distributed loss models of New England 39 bus test system

Generator bus no.	Power generations in MW without loss			Power generations in MW with distributed loss			Power generations in MW with concentrated loss		
	LP approach	GA approach		LP approach	GA approach		LP approach	GA approach	
	Fixed bids	Fixed bids	Linear bids	Fixed bids	Fixed bids	Linear bids	Fixed bids	Fixed bids	Linear bids
30	220	164.17	164.17	220	164.17	164.17	220	303.66	303.66
31 (slack bus)	609.2	609.21	609.21	643.3	643.88	643.88	612.614	609.2	609.2
32	750	500.42	500.42	750	500.42	500.42	750	566.62	566.62
33	650	644.31	644.31	650	644.31	644.31	653.414	647.76	647.76
34	608	535.83	535.83	608	535.83	535.83	608	487.58	487.58
35	605	699.02	699.02	605	699.02	699.02	605	704.68	704.68
36	406	527.29	527.29	406	527.29	527.29	406	324.36	324.36
37	640	505.46	505.46	640	505.46	505.46	640	626.93	626.93
38	777.3	866.58	866.58	777.3	866.58	866.58	804.612	821.29	821.29
39	885	1,098.16	1,098.16	885	1,098.16	1,098.16	885	1,089.23	1,089.23
System loss (MW)				34.14	34.67	34.67	34.14	30.86	30.86
Fuel cost ($/h)	82,080.92	81,868.24	62,927.19	82,461.48	82,255.16	63,322.76	82,542.22	82,356.55	63,023.98
Generators paid ($/h)	83,023.089	52,569.83	53,463.26	83,403.645	51,707.59	53,853.16	83,484.39	51,823.9	52,708.47
Supplier surplus ($/h)	942.169	−29,298.4	−9,463.93	942.165	−30,547.57	−9,469.59	942.17	−31,306.05	−10,315.5
Load payment to ISO ($/h)	86,633.59	50,829.81	51,723.23	86,633.59	50,354.48	52,512.30	86,633.59	50,769.65	51,663.15
ISO profit ($/h)	3,610.50	−1,740.02	−1,740.02	3,229.945	−1,353.1	−1,340.85	3,149.2	−1,054.24	−1,045.32

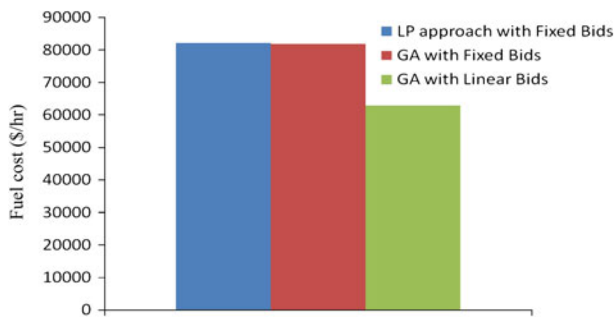


Fig. 15 Fuel cost comparison graph for without loss case of New England 39 bus system

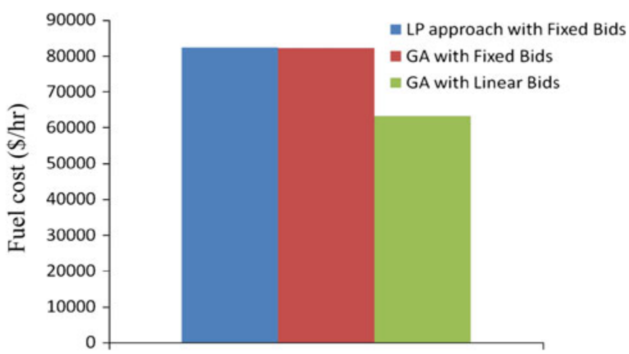


Fig. 16 Fuel cost comparison graph for concentrated loss case of New England 39 bus system

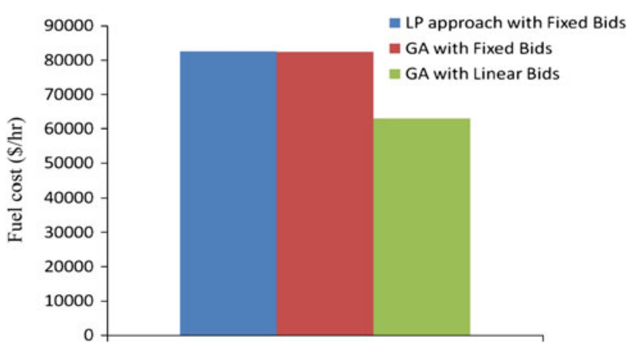


Fig. 17 Fuel cost comparison graph for distributed loss case of New England 39 bus system

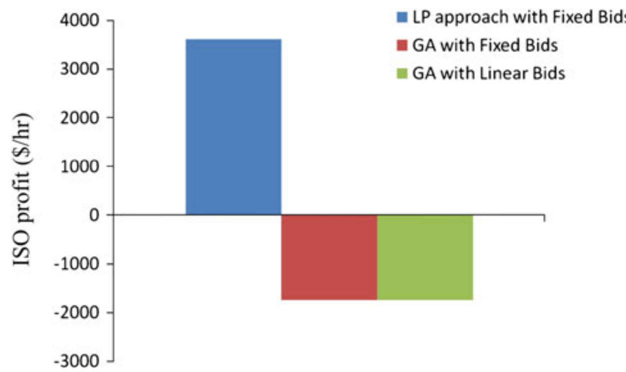


Fig. 18 ISO profit comparison graph for without loss case of New England 39 bus system

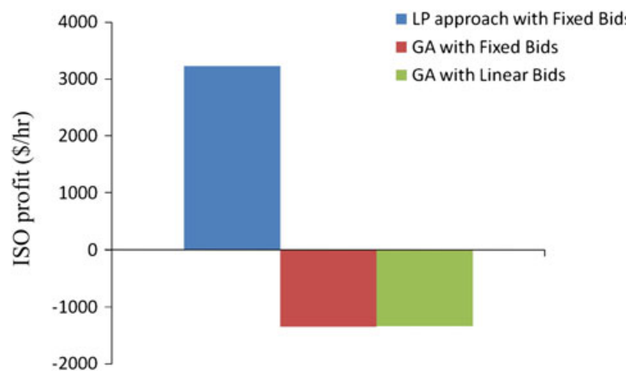


Fig. 19 ISO profit comparison graph for concentrated loss case of New England 39 bus system

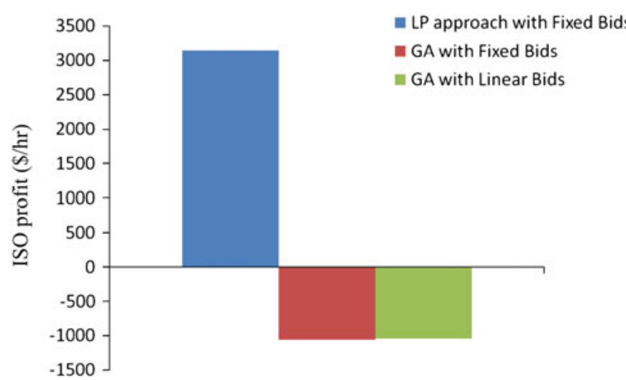


Fig. 20 ISO profit comparison graph for distributed loss case of New England 39 bus system

load flow is shared equally with all generators in the system as extra loads.

4 Implementation of Seed Genetic Algorithm Approach for LMP Calculation

Genetic algorithms are implemented as a computer simulation in which a population of abstract representations (called chromosomes) of candidate solutions (called individuals, creatures, or phenotypes) to an optimization problem evolves toward better solutions. GAs start with random generation of initial population and then the selection, crossover and mutation are performed until the best population is found. A major problem with genetic algorithms (GAs) is the length of time it can take for the algorithm to converge on a good solution. This may be addressed by seeding the initial population with reasonable solutions so as to start the search in promising regions of the solution space. The present work employed roulette wheel parent selection technique, Single point Crossover and bitwise mutation.

Table 4 LMPs at all buses for all models of New England 39 bus test system

Bus no.	LMPs at all buses ($/MWh)								
	Without loss model			Concentrated loss model			Distributed loss model		
	LP approach	GA approach		LP approach	GA approach		LP approach	GA approach	
	Fixed bids	Fixed bids	Linear bids	Fixed bids	Fixed bids	Linear bids	Fixed bids	Fixed bids	Linear bids
1	14.09	8.26	8.4	14.09	8.23	8.58	14.09	8.19	8.34
2	14.09	8.26	8.4	14.09	8.17	8.52	14.09	8.11	8.25
3	14.09	8.26	8.4	14.09	8.29	8.64	14.09	8.27	8.42
4	14.09	8.26	8.4	14.09	8.33	8.68	14.09	8.33	8.48
5	14.09	8.26	8.4	14.09	8.28	8.64	14.09	8.28	8.43
6	14.09	8.26	8.4	14.09	8.26	8.61	14.09	8.26	8.4
7	14.09	8.26	8.4	14.09	8.34	8.69	14.09	8.34	8.48
8	14.09	8.26	8.4	14.09	8.35	8.71	14.09	8.35	8.5
9	14.09	8.26	8.4	14.09	8.3	8.66	14.09	8.29	8.43
10	14.09	8.26	8.4	14.09	8.18	8.53	14.09	8.18	8.32
11	14.09	8.26	8.4	14.09	8.21	8.56	14.09	8.2	8.35
12	14.09	8.26	8.4	14.09	8.21	8.56	14.09	8.2	8.35
13	14.09	8.26	8.4	14.09	8.21	8.56	14.09	8.2	8.35
14	14.09	8.26	8.4	14.09	8.26	8.61	14.09	8.27	8.41
15	14.09	8.26	8.4	14.09	8.25	8.6	14.09	8.3	8.45
16	14.09	8.26	8.4	14.09	8.18	8.53	14.09	8.25	8.4
17	14.09	8.26	8.4	14.09	8.23	8.58	14.09	8.26	8.41
18	14.09	8.26	8.4	14.09	8.27	8.62	14.09	8.29	8.43
19	14.09	8.26	8.4	14.09	7.96	8.31	14.09	8.05	8.19
20	14.09	8.26	8.4	14.09	7.96	8.31	14.09	8.05	8.19
21	14.09	8.26	8.4	14.09	8.08	8.42	14.09	8.18	8.32
22	14.09	8.26	8.4	14.09	7.88	8.22	14.09	8.01	8.15
23	14.09	8.26	8.4	14.09	7.9	8.24	14.09	8.05	8.19
24	14.09	8.26	8.4	14.09	8.18	8.53	14.09	8.26	8.4
25	14.09	8.26	8.4	14.09	8.12	8.47	14.09	8.05	8.2
26	14.09	8.26	8.4	14.09	8.14	8.49	14.09	8.15	8.29
27	14.09	8.26	8.4	14.09	8.23	8.58	14.09	8.25	8.4
28	14.09	8.26	8.4	14.09	7.97	8.32	14.09	8.0	8.14
29	14.09	8.26	8.4	14.09	7.85	8.19	14.09	7.88	8.02
30	14.09	8.26	8.4	14.09	8.17	8.52	14.09	8.11	8.25
31	11.16	11.16	11.3	11.16	11.16	11.51	11.16	11.16	11.3
32	14.09	8.26	8.4	14.09	8.18	8.53	14.09	8.18	8.32
33	11.24	8.26	8.4	11.24	7.96	8.31	11.24	8.05	8.19
34	14.09	8.26	8.4	14.09	7.96	8.31	14.09	8.05	8.19
35	14.09	8.26	8.4	14.09	7.88	8.22	14.09	8.01	8.15
36	14.09	8.26	8.4	14.09	7.9	8.24	14.09	8.05	8.19
37	14.09	8.26	8.4	14.09	8.12	8.47	14.09	8.05	8.2
38	14.09	8.26	8.4	14.09	7.85	8.19	14.09	7.88	8.02
39	14.09	8.26	8.4	14.09	8.27	8.62	14.09	8.25	8.39

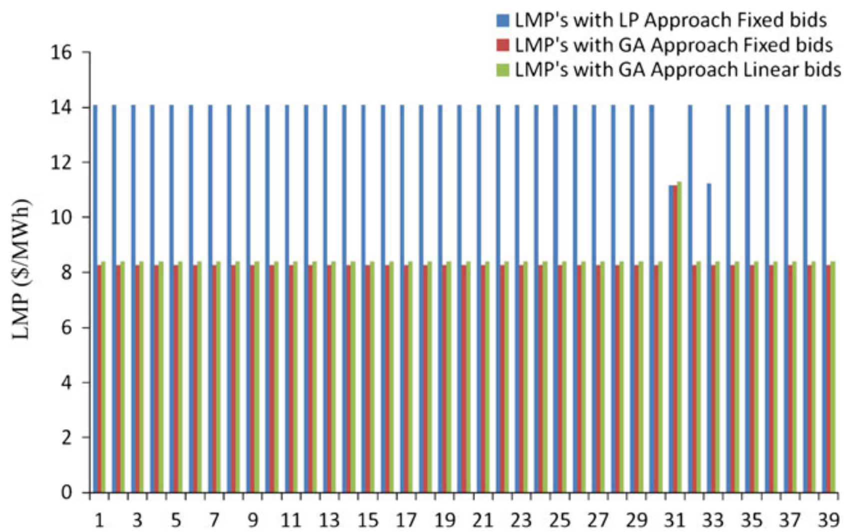


Fig. 21 LMP comparison graph for without loss case of New England 39 bus system

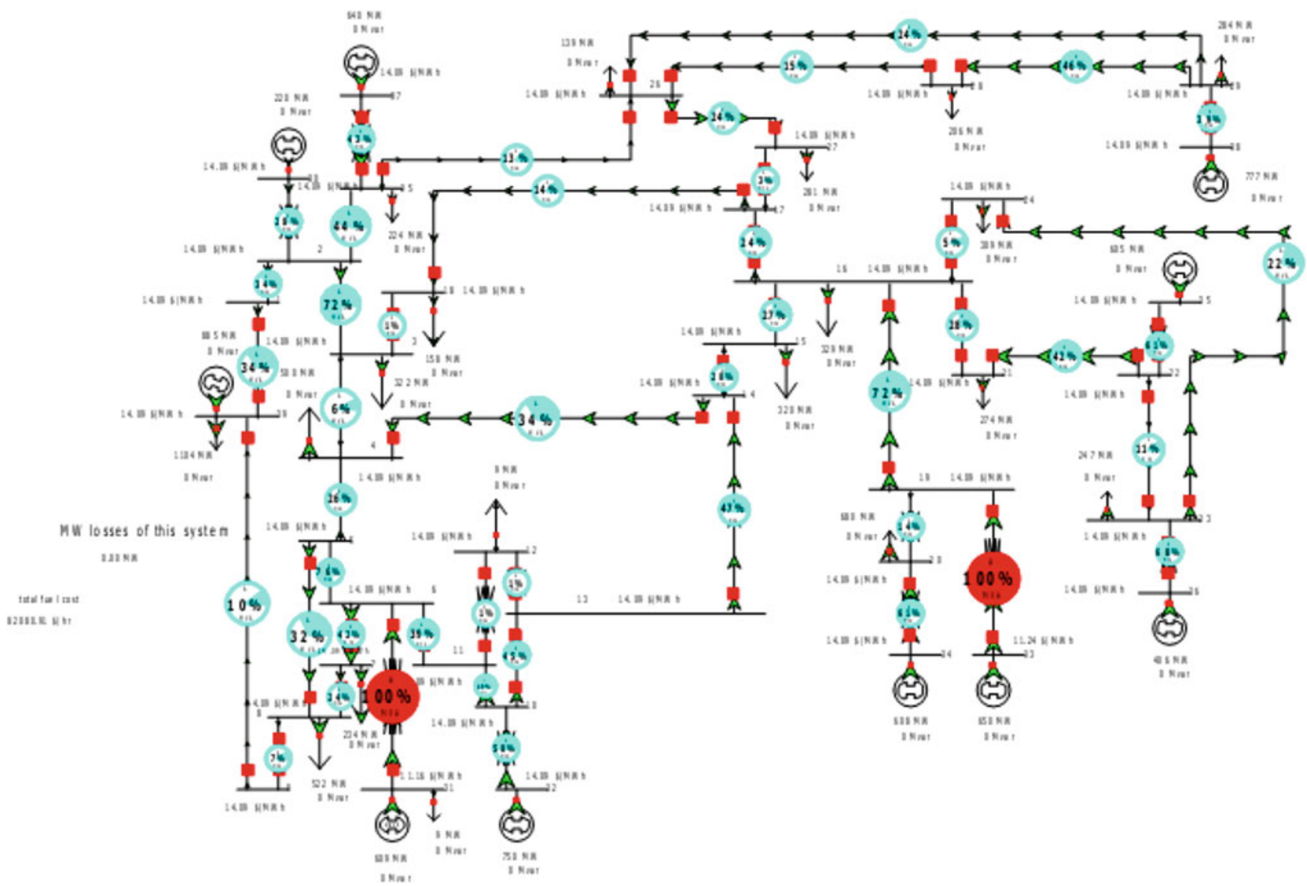


Fig. 22 Power world simulation diagram of New England 39 bus system for without loss model with LP approach fixed bids

4.1 Seeding

Seeding will speed up the GA by starting the search in promising regions of the search space. In order to find best optimal solution repeatedly with GA, we propose to seed the initial population from a case-base of previously solved problems instead of using the traditional random selection. In this approach the initial population is saved where the best optimal solution is obtained and this population is called as a seed. With this seeding approach of genetic algorithm, the

Table 5 Active power generations of generators for lossless, concentrated and distributed loss models of 75 bus Indian power system

Generator bus no.	Power generations in MW without loss			Power generations in MW with concentrated loss			Power generations in MW with distributed loss		
	LP approach	GA approach		LP approach	GA approach		LP approach	GA approach	
	Fixed bids	Fixed bids	Linear bids	Fixed bids	Fixed bids	Linear bids	Fixed bids	Fixed bids	Linear bids
1 (slack bus)	684.2	730.05	730.05	795.367	793.64	793.64	786.682	785.98	785.98
2	360	282.52	282.52	360	282.52	282.52	360	270.41	270.41
3	280	279.93	279.93	280	279.93	279.93	280	279.93	279.93
4	185	189.87	189.87	185	189.87	189.87	193.683	199.02	199.02
5	25	25.0	25.0	25	25.0	25.0	25	75.95	75.95
6	220	219.95	219.95	220	219.95	219.95	220	219.95	219.95
7	160	159.96	159.96	160	159.96	159.96	160	159.96	159.96
8	180	179.96	179.96	180	179.96	179.96	180	179.96	179.96
9	505.92	201.59	201.59	525	201.59	201.59	525	305.5	305.5
10	180	179.96	179.96	180	179.96	179.96	180	179.96	179.96
11	209	208.95	208.95	209	208.95	208.95	209	208.95	208.95
12	775	1106.8	1106.8	775	1106.8	1106.8	775	962.5	962.5
13	1,000	999.76	999.76	1,000	999.76	999.76	1,000	999.76	999.76
14	250	249.94	249.94	250	249.94	249.94	250	249.94	249.94
15	554	553.87	553.87	554	553.87	553.87	554	553.87	553.87
System loss (MW)				130.247	63.59	63.59	130.247	63.58	63.58
Fuel cost ($/h)	56,097.0356	55,981.2	48,528.21	57,347.664	56,696.62	49,237.91	58,070.319	57,376.59	49,569.53
Generators paid ($/h)	60,239.80	63,277.41	61,718.34	63,427.678	65,089.39	65,660.11	63,395.79	65,166.35	65,469.86
Supplier surplus ($/h)	4,142.7644	7,296.2	13,190.12	6,080.014	8,392.77	16,422.2	5,325.471	7,789.75	15,900.32
Load payment to ISO ($/h)	60,859.55	62,641.35	61,093.03	62,641.35	65,243.39	65,814.16	62,641.35	65,976.63	66,287.59
ISO profit ($/h)	619.75	−636.06	−625.3	−786.328	153.99	153.94	−754.44	810.28	817.72

probability of getting the best optimal solution repeatedly would be high because the search process always starts with same initial population or seed.

In this method power generations of generators (PG_i) except slack bus are taken as the control variables in the chromosomes. The problem is formulated as minimizing the objective function (9) subjected to (10) or (14) as equality and (15) as inequality constraints.

4.2 Constraint Handling

Constraints are handled using penalty function approach. If an individual S_j is a feasible solution and satisfies all constraints, its fitness will be measured by taking the reciprocal of the fuel cost function else it need to be penalized. Using the exterior penalty function approach, the violated operating constraints are incorporated as penalties in objective function.

Calculate the GA fitness function, FF = 100/(1 + J + penalties). The penalties are calculated for (10) or (14), (11) and slack bus power if they are violated as follows:

Penalty function for line flows
pcost_f = lambda_f(k) * df * (|pflow(k)| − limit)^2
Penalty function for power balance
pcost_error = lambda_error * (error)^2
Penalty function for slack bus power
pcost_s = lambda_s * ds * (pgen (nslack) − s_limit)^2

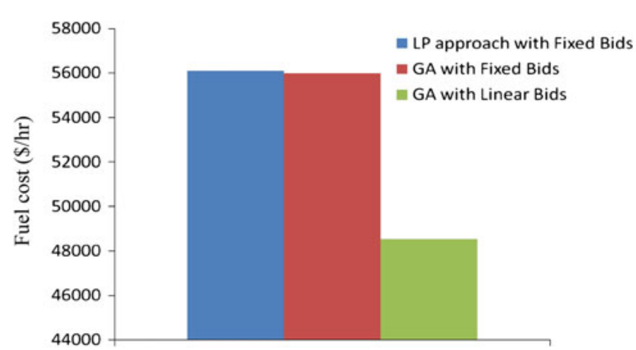


Fig. 23 Fuel cost comparison graph for without loss case of 75 bus Indian Power system

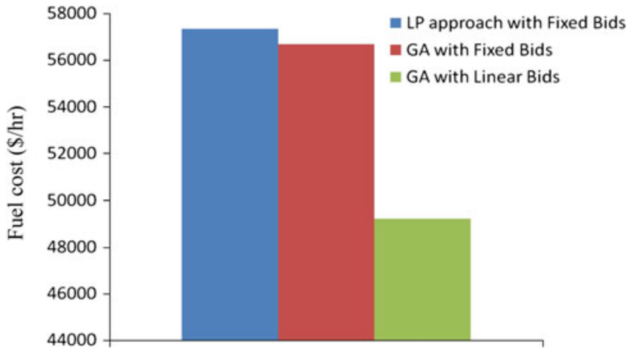


Fig. 24 Fuel cost comparison graph for concentrated loss case of 75 bus Indian Power system

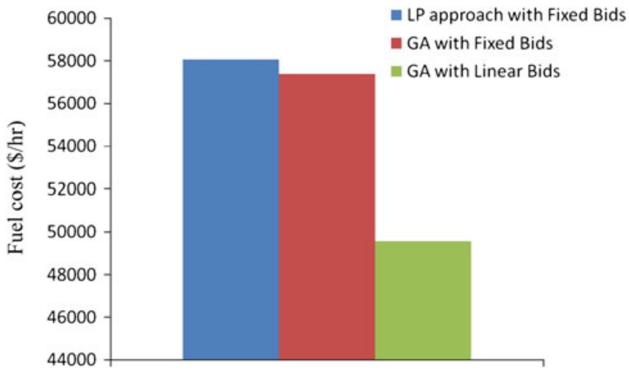


Fig. 25 Fuel cost comparison graph for distributed loss case of 75 bus Indian Power system

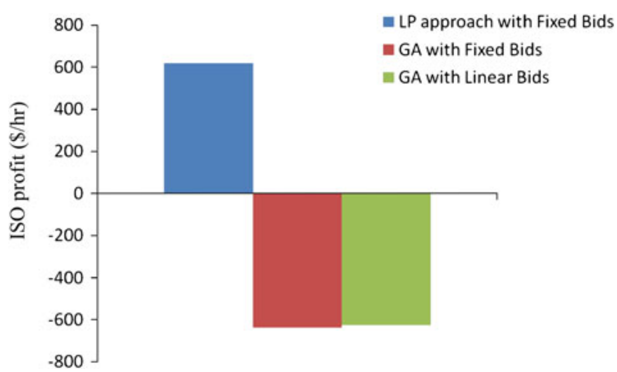


Fig. 26 ISO profit comparison graph for without loss case of 75 bus Indian Power system

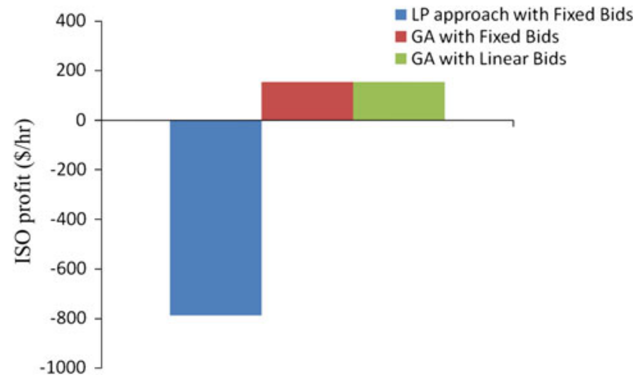


Fig. 27 ISO profit comparison graph for concentrated loss case of 75 bus Indian Power system

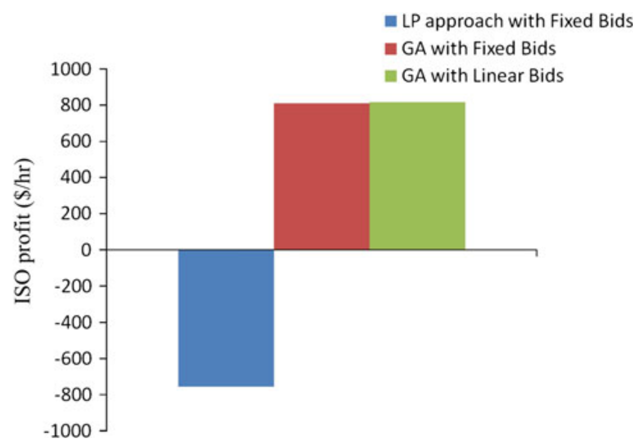


Fig. 28 ISO profit comparison graph for distributed loss case of 75 bus Indian Power system

where lambda_f(k), df, lambda_error, lambda_s, ds are all constants and take same value for all cases in each bus system.

4.3 Algorithm for LMP Calculation Using GA-Based DCOPF Approach

Step 1: Read no. of buses, no. of lines, slack bus number and Bus data. Read GA parameters like population size, chromosome length, no. of units, maximum no. of generations, elitism probability, crossover probability, mutation probability and epsilon. Read a, b, c coefficients; min. and max. limits of generators. Read line data including line thermal limits.
Step 2: Generate randomly power generations of all generators except slack generator and decode them.
Step 3: Calculate Generation shift factors using (4).
Step 4: Calculate initial line flows using (7).
Step 5: Calculate the system loss i.e. P_{loss} in each line using (6) for case 2 and case 3.
Step 6: Calculate the extra load at each bus 'i' using (21) from initial line flows for case 3 and then calculate new line flows using (22).
Step 7: Calculate loss factors using (8) and then delivery factors at each bus using (5).
Step 8: Calculate P_{gen} of slack bus using (10) or (14).
Step 9: Check for line flow limits (11). If the line limits are violated add penalties to objective function.
Step 10: Check for slack bus power limits. If it violates the limits add penalties to objective function.
Step 11: Calculate the marginal fuel costs of all units with the randomly generated PG's; calculate the total cost (9) for all cases and then calculate the fitness function = 100/(1 + total cost + penalties).

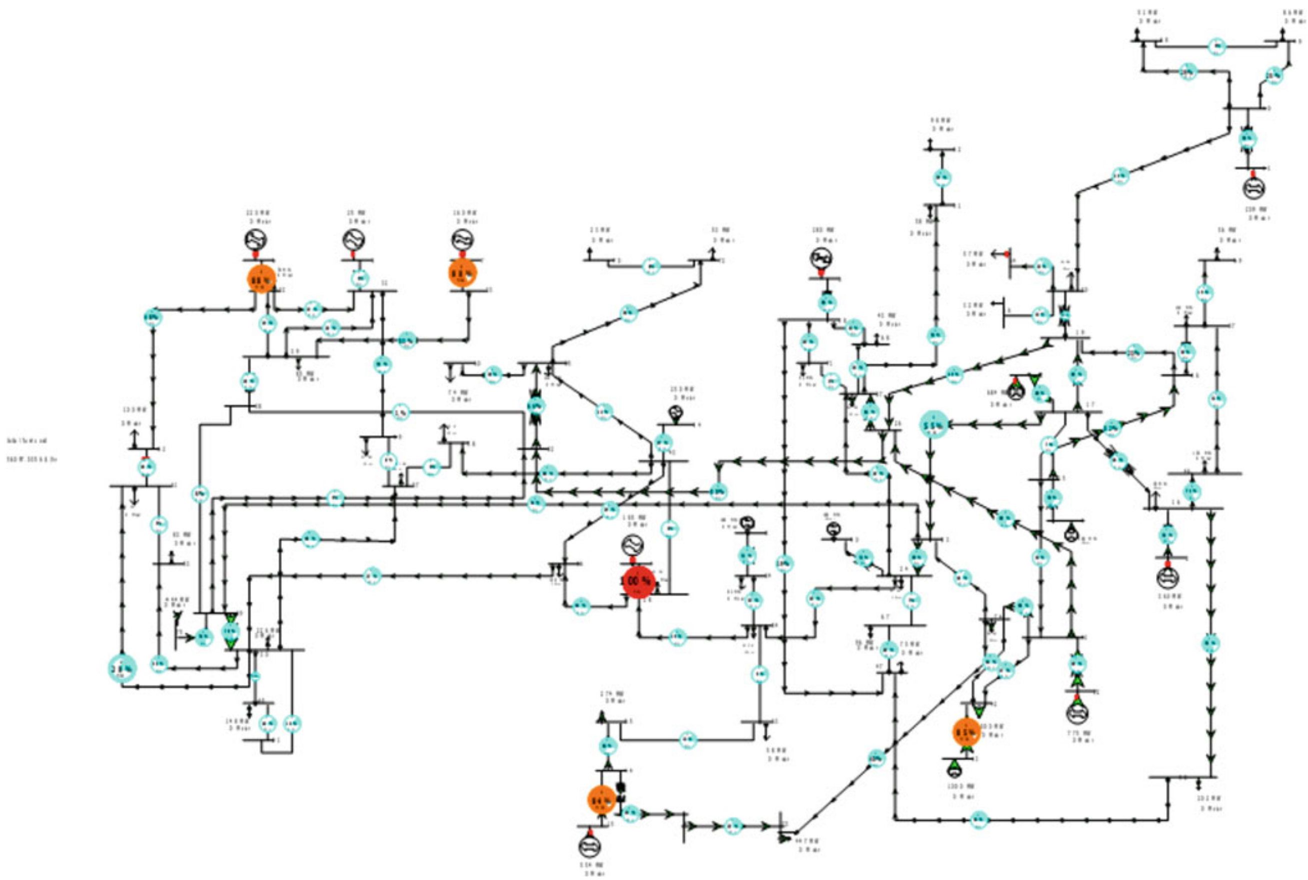


Fig. 29 Power world simulation diagram of 75 bus Indian power system for without loss model with LP approach fixed bids

Step 12: Sort the chromosomes in the descending order of fitness.
Calculate the energy price of the reference bus either with fixed bids or with linear bids and then calculate LMP at all buses using (17) and the decomposition of LMP using (18), (19), (20).
Step 15: Calculate generators paid by ISO ($\sum_i$ power generation at bus i * LMP at bus i) and then calculate supplier surplus for each generator 'i' (generator paid by ISO-fuel cost).
Step 16: Calculate load payment to ISO ($\sum_i$ load at bus i * LMP at bus i) and ISO profit (load payment to ISO − generators paid by ISO) for all the cases and STOP.
Step 17: Use selection, crossover and mutation operators. Generate new population.
iteration = iteration +1; Go to step 4.

5 Results and Discussion

The developed GA-based DCOPF for cases 1–3 for LMP estimation is applied on IEEE 14 bus system [26], New England 39 bus system [27] and 75 bus Indian power system [28]. GA parameters used are population size: 40, number of bits for each generator in the chromosome: 12, elitism probability: 0.15, crossover probability: 0.85, mutation probability: 0.01, tolerance: 0.0001. IEEE 14 bus system has 2 generators, 39 bus system has 10 generators and 75 bus system has 15 generators. The solution reported is the best solution over 20 different GA runs. The proposed GA approach with fixed bids is compared with LP approach with fixed bids for all the three cases. In this comparison GA with fixed bids gives less fuel cost than LP. Finally, fuel cost is further reduced in GA with linear bids than with fixed bids. It is observed that in all the three cases of losses, GA approach with linear bids gives best optimal fuel cost. The advantage of using linear bid over fixed bid is that the former recovers true marginal cost of slack generator or energy price for determining LMP. As mentioned in case 3 of Sect. 3.3, in concentrated loss model it is not feasible operation of adding total system loss to slack generator which may create burden on slack generator or slack generator may violate its limit. So distributed loss model is proposed in this paper, in which total system loss is distributed among all generators, burden on slack generator is removed and also total system loss cost paid to suppliers is secured from loads whereas in concentrated loss model loss cost paid to slack generator cannot be regained from loads which is a major drawback for ISO. It is observed that slack bus generation in distributed loss model is reduced than that in concentrated loss model for all the bus systems studied in this paper.

Step 13: Is iteration = max. no. of iterations. If yes stop else go to step 14.

Step 14: If fitness (1) == fitness (psize) → problem converged.

5.1 IEEE 14 Bus Test System

Results for all the cases of IEEE 14 bus system [26] are presented in Tables 1 and 2. For the base case loading, 9th line connecting 4–9 buses is congested with both LP and GA approaches whose shadow price is 91.747 \$/MWh for case 1 and 109.253 \$/MWh for cases 2 and 3. The corresponding generator power outputs are listed in Table 1 and LMPs at all buses are presented in Table 2. From Table 1, LP approach without loss using fixed price bids optimizes the fuel cost to 3,222.97 \$/h, while in GA using fixed bids the fuel cost obtained is 3,223.11 \$/h and it is further reduced to 2,985.01 \$/h with GA using linear bids, and the same phenomenon is observed in concentrated and distributed loss models also. From the LMP values it can be observed that loads away from generators have more locational marginal prices because of the addition of congestion costs and loss costs. Figures 3, 4, and 5 present a comparison of fuel cost for all the three loss cases. Figures 6, 7, and 8 show ISO profit comparison. Figures 9, 10, and 11 show the LMPs comparison for all the three models. Figures 12, 13, and 14 show the power world simulation diagrams.

5.2 New England 39 Bus Test System

For this system also congestion occurred for base case loading only. The generation dispatch for New England 39 bus system [27] are presented in Table 3. For all the three cases of loss in LP approach lines 37 and 39 connecting buses (6–31), (19–33), respectively, are congested, whereas in GA approach line 37 is only congested for all the three cases with both fixed bids and linear bids. Figures 15, 16 and 17 show the comparison of fuel cost. Figures 18, 19 and 20 show the comparison of ISO profit for all the loss cases studied. In this system also GA approach with linear bids gives most optimal fuel cost for without loss, concentrated loss and distributed loss models. LMPs are also calculated at all buses and are presented in Table 4. LMP comparison graph for without loss case only is shown in Fig. 21. Power world simulation diagram of only one model, i.e. without loss case is shown in Fig. 22. From Table 3 it is observed that slack bus generation in distributed loss model is reduced than in concentrated loss model and, therefore, burden on slack generator is removed. The shadow price for both the congested lines is 2.9 \$/MWh for cases 1, 2 and 3.

5.3 75 Bus Indian Power System

The developed algorithm is also tested on 75 bus Indian power system [28]. For base case loading of this system 9th line connecting 4–28 buses shows congestion for all the three cases with both LP and GA approaches. The corresponding generations of generators are listed in Table 5. The LMPs at all buses are also estimated but not reported here. The total fuel cost comparison graphs for lossless system and for all loss cases are shown in Figs. 23, 24 and 25, respectively. Figures 26, 27 and 28 show the ISO profit comparison for lossless case and for all the loss cases studied. Power world simulation diagram with LP approach using fixed bids of lossless model only is shown in Fig. 29. This case study also highlighted that GA approach with linear bids for generators minimizes the total fuel cost of generators, and slack generator power in distributed loss model is comparatively reduced than in concentrated loss model. The shadow price of congested line is 3.35 \$/MWh for case 1, 3.67 \$/MWh for case 2 and 3.7 \$/MWh for case 3.

6 Conclusion

This paper presented a transmission pricing scheme using seed Genetic algorithm-based DCOPF considering various generator bids and different loss cases during congestion in the system in a pool-type electricity market. LMPs have been estimated for without loss case, concentrated loss case and distributed loss case. Seed GA-based DCOPF evolves as a better optimization algorithm for LMP estimation in the power systems. This study also investigated the effect of generator bids and distributed loss model on the system operation. Distributing loss to all buses, rather than concentrating at slack bus removes burden on slack bus. Further, considering generator linear bids leads to true fuel cost of generators. LMPs with linear bids are calculated to avoid the non smooth nature of bid curve in fixed price bids. The developed GA-based DCOPF exhibits reliable convergence with reduced fuel cost in all cases studied and can assist the system operator in obtaining correct economic signals.

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