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Numerical Simulation of Rod-Climbing Effect in Newtonian Fluids

Manojkumar Nimmakayala*, V. R. K Raju[a]

[*,a] *National Institute of Technology, Department of Thermal Engineering*
Warangal-506004, Telangana, India

Abstract

This paper discusses about the onset of instabilities and rod climbing effect due to rotation of inner cylinder in Taylor Couette flow with two immiscible Newtonian fluids. The rod climbing effect, also known as Weissenberg effect, occurs in non-Newtonian fluids due to imbalance in normal stresses which pull the liquid inwards and make it climb. In two immiscible Newtonian fluids it occurs due to the onset of flow instabilities causing Taylor vortices in less viscous fluid (bottom fluid) which makes the interface to climb. Two-dimensional numerical simulations have been performed for a cylinder of 55 mm diameter and 60 mm height (30 mm by water & 30 mm of silicon oil) by using the Volume of Fluid method (VoF) in FLUENT. The initial volume fraction of water is patched into the lower half domain containing water. The initial swirl velocity is patched for the whole domain. For higher values of rotational speed the rod climbing effect was observed with upper fluid as silicon oil (μ = 300 mPas) and bottom fluid as water (μ = 1 mPas).

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Keywords: Taylor vortices; Volume of Fluid (VoF) method; Weissenberg effect.

1. Introduction

The flow between two concentric cylinders with the inner cylinder rotating is known as Taylor Couette flow. There are many applications of fluid flow in confined annuli: emulsifiers, chemical, food processing and petroleum industry. In petroleum industry to obtain the desired flow rate during drilling one has to know the velocity distribution and pressure loss accompanying the flow.

The Taylor-Couette flow of two immiscible fluids between two coaxial cylinders has been studied (Joseph et al., [1]; Joseph and Renardy, [2]), where the two liquids were separated by a cylindrical interface. The inner cylinder was

* Corresponding author. Tel.: +0-0870-246-2316.
E-mail address: n.manojkumar24@gmail.com

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set in rotation while the outer cylinder was kept stationary. Taylor vortices for different parameters were found to develop.

Brady et al. [3] numerically investigated two fluid confined flow in a cylinder driven by end walls. They investigated the effects of disk rotation rate, viscosity ratio, Reynolds number (*Re*), Weber number (*We*) and Froude number (*Fr*) on steady state vortex lines, streamlines and azimuthal velocity. As Reynolds number increases (due to disk rotation) fluid in the bottom boundary layer centrifugally pushed sideward to form sidewall boundary layer by upward advection of angular momentum. Due to growth in the sidewall boundary layer near the interface causes an upward displacement of the interface. As *Re* is increased sidewall boundary layer grows stronger pushing the lower density fluid further upward. As the viscosity ratio decreases the deformation of the interface increases due to reduced viscous damping. As *We* increases, the deformation of the interface increases due to decrease in surface tension. As the *Fr* increases, the deformation of the interface increases due to decrease in gravity.

Wei et al. [4] studied liquid sloshing damping in the cylindrical container using VoF method. The factors determining the free surface wave damping are free surface, boundary layer, interior fluid and contact line. They are determined by no slip and slip wall boundary conditions on both side wall and bottom wall. The conclusions drawn are as the liquid height increases enough the damping contribution from bottom wall can be neglected. When surface tension and *Re* increases, damping contribution from wall boundary layer increases and that of interior fluid decreases. Hirt and Nicholas [5] presented volume of fluid method for capturing interfaces of free surface flows. Stability of two axially superposed fluids of different viscosities between two rotating cylinders was performed by Raju [6]. He observed that the variation of the critical Taylor number with viscosity ratio is small when the heights of the fluid columns are large compared to the gap between the cylinders. But it is significant when the heights are comparable to the gap. The present numerical simulation is validation of experimental work which was performed by Bonn et al. [7] with silicon oil μ = 300 mPas as upper fluid and water μ = 1 mPas as bottom fluid. The dimensions of cylinder: 55 mm diameter and 90 mm height. The inner wall with a stainless steel rod of 5 mm diameter and height of 100 mm is rotating at a frequency of 15 Hz.

Nomenclature

f	frequency of rotation of inner cylinder
F	fluid volume fraction
L	length of concentric cylinder
P	gauge pressure
r, θ, z	radial, azimuthal and axial directions respectively
R_1	radius of inner annulus of concentric cylinder
R_2	radius of outer annulus of concentric cylinder
u, v, w	velocities in axial, radial and azimuthal directions respectively
μ	absolute viscosity

2. Mathematical models

2.1. Problem definition

In the present problem two co-axial cylinders of radii R_1 and R_2 were considered. The space between the two cylinders contains two immiscible fluids. In the unperturbed state fluid 1 (silicone oil) occupies the region $0 < z < L$, $R_1 < r < R_2$. Whereas fluid 2 (water) occupies the region $-L < z < 0$, $R_1 < r < R_2$. The cylindrical coordinate system (r, θ, z) with the z-axis along the axis of the cylinders has been used. The plane $z = 0$ to coincide with the unperturbed interface between the two fluids. Numerical experiment of the present problem is carried out using VoF method.

2.2. Volume of fluid method

In the Lagrangian scheme same fluid element is present throughout the grid. In Eulerian scheme grid remains fixed, but fluid element is changed. In Lagrangian grid simply moves with the computed element velocities, while in an

Eulerian calculation, it is necessary to compute the flow of fluid through the mesh. It requires convective averaging, for updating the solution with time, which smears the discontinuities such as free surfaces. So, a special scheme, volume of fluid method for tracking the interface of two immiscible fluids is used. VoF technique requires less computational storage and fast to implement. Because it follows regions rather than surfaces, all logical problems associated with intersecting surfaces are avoided. Fluid fraction function F is defined such that unity at any point represents occupied by fluid and zero otherwise. The interface is defined between the cells that contain the value from 0 to 1. Normal direction to the boundary the value of F changes most rapidly as it is a step function. The values of the F field in the cells are updated by the governing equation as follows

$$\frac{\partial F}{\partial t} + u\frac{\partial F}{\partial z} + v\frac{\partial F}{\partial r} = 0 \tag{1}$$

Where r, z are variable coordinates in two dimensional computational space whose dependent variable F is updated with time t and thereby updating the interface position and shape.

2.3. Formulation

2.3.1. Geometry

The vertical half section of rotating rod and stationary cylinder arrangement of radii R_1 = 2.5 mm and R_2 = 27.5 mm, respectively and each of length L = 30 mm is shown in Fig.1. The 2D-Axisymmetric-Swirl including gravity case is modeled in FLUENT. The two fluid configuration and its ANSYS geometric modeler is shown in Fig. 1.

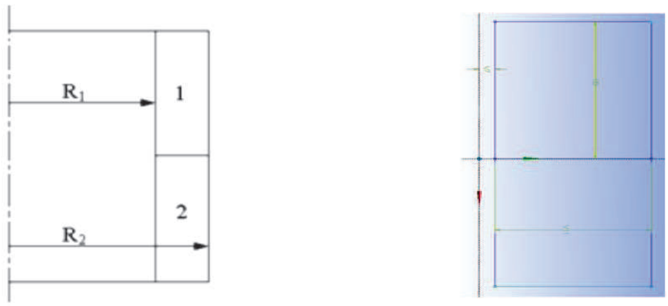


Fig.1. Schematic figure of two fluid configuration: (a) Experiment; (b) ANSYS geometric modeler.

2.3.2. Mesh

The mesh is made of 2225 quadrilateral elements and 2352 nodes. The boundary near inner cylinder and interface are inflated as shown in Fig. 2.

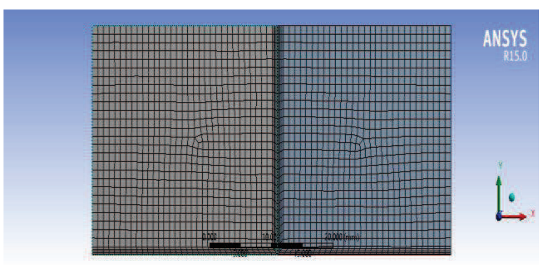


Fig.2. Schematic figure showing inflated boundaries of a quadrilateral mesh

2.4. Governing equations

The present case is studied under 2D axisymmetric swirl, laminar regime, incompressible and immiscible fluids. The following governing equations including eq. (1) are satisfied by the two fluids. Here *z*-axis is vertical and *r*-axis is radial. The *rz*-plane is rotated by an axisymmetric swirl, where the tangential or swirl momentum equation is solved. After obtaining the velocities from momentum equations volume fraction equation (1) is solved to obtain the position of interface. In the next step pressure equation (PISO algorithm) is solved and velocities are corrected according to the obtained pressure. Thus the boundary conditions are updated according to the new velocities and the time step is incremented to next time level.

Continuity equation:

$$\nabla.\overline{V} = 0 \tag{2}$$

z- momentum equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial r} = -\frac{\partial p}{\partial z} + g_z + \nu\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{z}\frac{\partial u}{\partial z} - \frac{u}{z^2}\right) \tag{3}$$

r- momentum equation:

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial z} + v\frac{\partial v}{\partial r} = -\frac{\partial p}{\partial r} + \nu\left(\frac{\partial^2 v}{\partial z^2} + \frac{\partial^2 v}{\partial r^2} + \frac{1}{z}\frac{\partial v}{\partial z}\right) \tag{4}$$

Swirl momentum equation:

$$\frac{\partial(\rho \mathrm{w})}{\partial t} + \frac{1}{r}\frac{\partial(\rho r u \mathrm{w})}{\partial z} + \frac{1}{r}\frac{\partial(\rho r v \mathrm{w})}{\partial z} = \frac{1}{r}\frac{\partial(r\mu\frac{\partial w}{\partial z})}{\partial z} + \frac{1}{r^2}\frac{\partial(r^3\mu\frac{\partial(\frac{w}{r})}{\partial r})}{\partial z} - \frac{\rho v w}{r} \tag{5}$$

Here *u*, *v* and *w* are axial, radial and swirl components of velocities respectively and *F* is fluid volume fraction.

2. 5. Boundary conditions

The boundary conditions required to be satisfied by any two phase flow configuration are continuity of velocity and normal stress at the interface. No slip boundary conditions on the wall. Constant normal pressure boundary condition is given at the inlet i.e. free surface. The inner wall of the cylinder is given a rotating wall boundary condition. The outer wall and bottom wall of the cylinder have stationary wall boundary condition.

Continuity of velocity at the interface requires

$$u_1 = u_2,\ v_1 = v_2,\ w_1 = w_2 \text{ at } z = 0, \quad (6)$$

Continuity of normal stress at the interface requires

$$P_1+2\mu_1\,(\delta w_1/\delta z) = P_2+2\mu_2\,(\delta w_2/\delta z) \text{ at } z = 0, \quad (7)$$

The no slip boundary conditions on the walls are

$$u = v = w = 0 \quad \text{at } r = R_1,\ R_2 \text{ and } -L < z < L, \quad (8)$$

Constant normal pressure at inlet which is open to atmospheric should be satisfied

$$P_{z=L} = P_{atm}. \quad (9)$$

3. Methodology

The pressure implicit with splitting of operator (PISO) scheme is selected for solver setting. VoF implicit formulations are selected and gravity force has been included. Momentum equations are solved by using Quadratic upwind differencing (QUICK) scheme. Silicon oil is chosen as the primary phase and water as the secondary phase. The value of surface tension between silicon oil and water was taken as σ = 24.5 dyne/cm2 (Than et al. [8]). Operating conditions of pressure and density are chosen to be that of primary phase, so that rapid convergence can be achieved. The secondary phase fluid is patched into the cells containing water. The initial swirl velocity condition is patched into the entire flow field by defining a custom field function as follows:

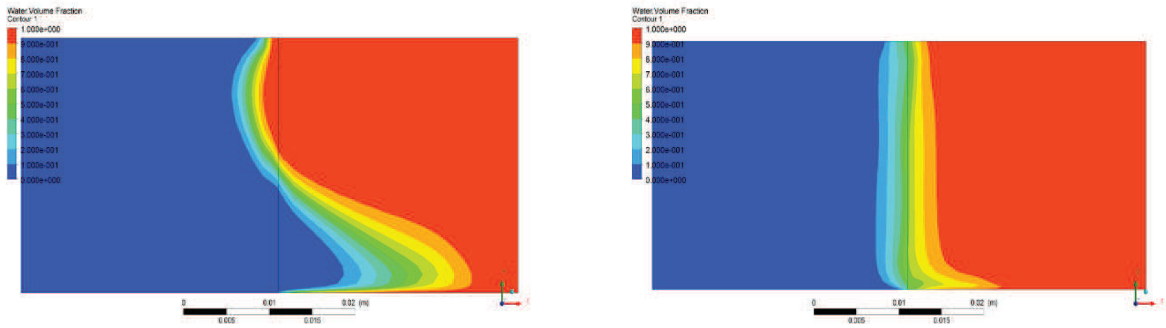
Swirl velocity = $2\pi f \times$ abs(y-0.0275), where y is radial position in mm and f is the frequency in Hz.

Inner wall boundary condition is given to be $2\pi f$ for each run of f = 1, 5, and 7 Hz respectively. Outer and bottom walls are stationary. Standard atmospheric pressure is given at free surface. The time step of 0.002 sec is taken, so that solution may not diverge. The volume fraction of water and stream function plots are auto saved for every 0.1 sec and are plotted till significant rise of the interface is observed.

4. Results and discussion

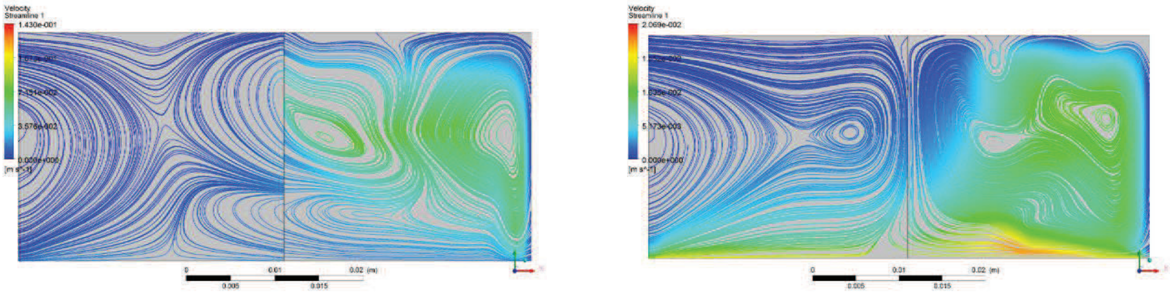
The numerical calculations for the present configuration with water as lower fluid and silicon oil (300 mPas) as the upper fluid has been carried out for different values of rotational speed of the inner cylinder. In this section, the bottom and top boundaries of the figure represents inner and outer cylinder walls respectively. The left and right boundaries of the figure represents pressure inlet at the free surface and bottom wall of concentric annuli respectively. The fluids on left and right of interface represents top (silicon oil) and bottom fluids (water) in the vertical plane of the apparatus respectively.

It was observed that for lower values of rotational speed (1 Hz) there is a displacement of the interface. However the rod climbing effect is not clearly visible as shown in the volume fraction plots in Fig 3. The corresponding stream function plots for the both fluids are shown in Fig 4. In Fig 4 (b), stream function contours are confined to their respective regions. This confirms there is no rise of interface for rotational speed of 1 Hz.



(a) (b)

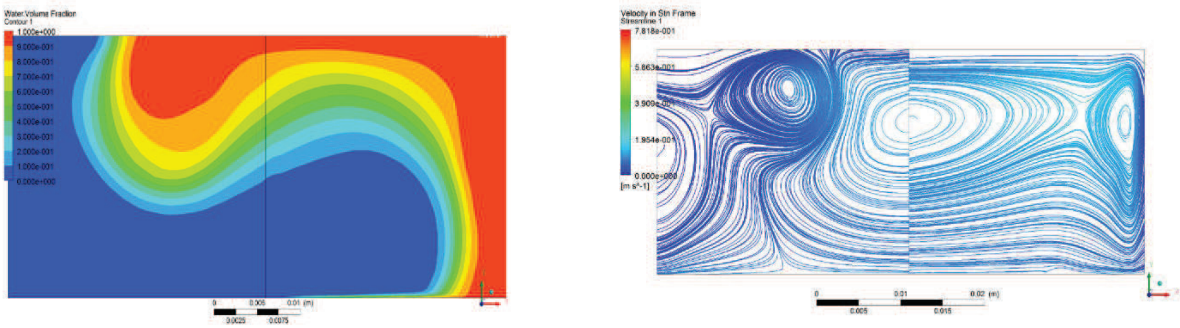
Fig 3. Volume fraction of water for f = 1Hz at (a) t = 1 sec and (b) t = 16 sec.



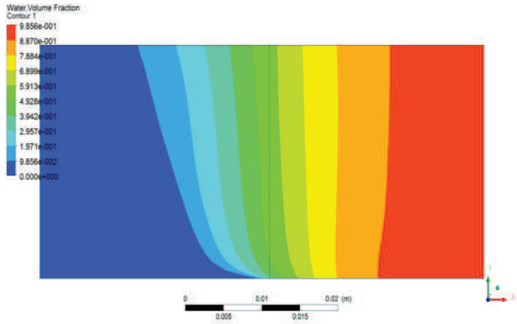
(a) (b)

Fig 4. Stream function of for the entire fluid domain for f = 1Hz at (a) t = 1 sec and (b) t = 16 sec.

With further increase in rotational speed the rod climbing phenomenon was observed for rotational speed of 5 Hz and at time t = 20 sec as shown in Fig 5. However the rise is only marginal.

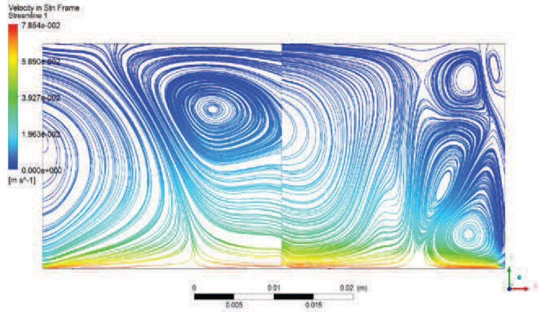


(a) (a)



(b)

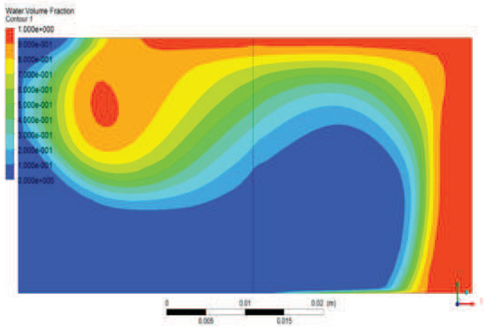
Fig 5. Volume fraction of water for f = 5Hz at (a) t = 1 sec and (b) t = 20 sec.



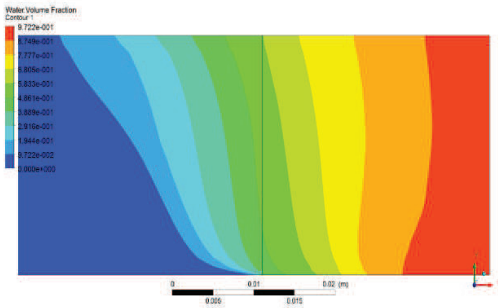
(b)

Fig 6. Stream function of the entire fluid domain for f = 5Hz at (a) t = 1 sec and (b) t = 20 sec.

In order to obtain a clear-cut rise of interface, rotational speed was increased further to 7 Hz. At this rotational speed there is a clear rise of the interface as the time increases from 1 sec to 20 sec as shown in Fig 7. Figure 8 shows the stream function variation of both the fluids at rotational speed of 7 Hz. As time increases the stream function of lower fluid penetrates into the upper fluid region further confirms the rise of lower liquid adjacent to the inner rotating cylinder.

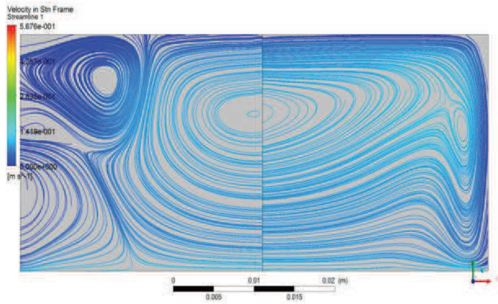


(a)

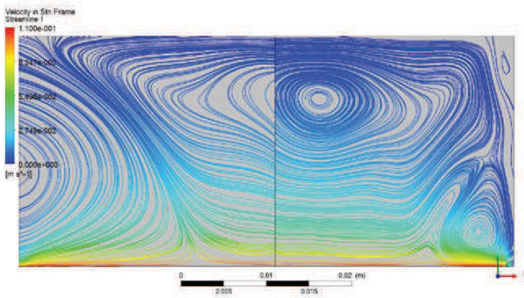


(b)

Fig 7. Volume fraction of water for f = 7Hz at (a) t = 1 sec and (b) t = 20 sec.



(a)



(b)

Fig 8. Stream function of the entire fluid domain for f = 7Hz at (a) t = 1 sec and (b) t = 20 sec.

5. Conclusion

In this paper effort has been made to study the rod climbing effect of two axially superposed immiscible Newtonian fluids between two coaxial cylinders when the outer cylinder is stationary and the inner cylinder rotates about its axis with constant angular speed. In the present configuration, it was observed that, initially at a frequency of 1 Hz there is no rod climbing effect. But as the angular velocity is increased to 5 Hz, the onset of Taylor vortices starts and, the rod climbing phenomenon is significant. This is in accordance with the experimental values of Bonn et al. (2004). This effect further enhanced as angular velocity increases to 7 Hz with a rise in the interface height.

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