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Brinkman Model for Unsteady MHD Free Convection Flow of a Chemical Reacting Doubly Stratified Fluid with Heat Generation and Radiation

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Abstract

In this investigation, the effects of magneto-hydrodynamics, heat generation and thermal radiation have been discussed on unsteady natural convection flow in a Brinkman porous medium saturated with a chemically reacting doubly stratified fluid. Initially, the flow is described through time-dependent differential equations. These are converted into a set of coupled non-dimensional differential equations. Crank-Nicolson type scheme is employed to analyze the solution of the model. In order to analyze the usefulness of the investigation the dimensionless local skin-friction, heat and mass transfer rates have been presented graphically for different combinations of flow governing parameters.

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Keywords: Radiation, Heat generation and Chemical reaction, Double stratification, Brinkman porous medium, Crank-Nicolson method;

1. Introduction

In heat transfer problems, radiation plays a significant role in both the free and forced convection processes [1]. The electrical power generation, nuclear power plants, gas turbines, solar power technology, missiles etc. are some examples where radiation effects are quite considerable. A chemical reaction which depends on whether it occurs as a single-phase volume reaction or at an interface by treating it as either a homogeneous or heterogeneous is also a situation worth citing in this context.

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With the emerging applications of industrial and engineering problems, several researchers are working to explore the significance of chemical reaction in fluid flow problems. The steady MHD natural free convection flow has been discussed by Srinivasacharya and Upender [2] in the presence of first order chemical reaction and thermal radiation and Jain [3] presented these similar effects with unsteady double diffusive free convective flow. Further, an extensive research has been carried out on the effect of heat generation on free convection boundary layer problems with different surface geometries such as vertical and horizontal wavy surfaces, vertical and inclined plates, rotating and stationary cylinders etc. Initially, Martin [4] analyzed the free convection flow in a vertical cylinder with internal heat generation. Effect of internal heat generation on free convection flow along a wavy surface, by taking into account variable thermal conductivity and MHD, has been investigated by Alim *et al*. [5].

The fluid saturated porous medium with the free and mixed convective transport are of enormous curiosity (For more application of porous medium, one may refer Nield and Bejan[6]). Rout *et al*. [7] considered the radiation and chemical reaction whereas Chamkha *et al*. [8] analysed the effect of internal heat absorption or generation on free convective flow through a porous medium. Shateyi and Marewo [9] obtained numerical solution of unsteady flow over a porous body with MHD, thermal radiation and chemical reaction. Recently, the problem of natural convection in a nanofluid saturated non-Darcy porous medium with the effects of suction/injection and internal heat generation has been studied by Chamka *et al*.[10].

Several researchers explored the importance of convective flow in a doubly stratified porous medium using Brinkman and Darcy–Forchheimer models, since the stratification of fluid arises due to concentration differences, temperature variations or the presence of different fluids. The free convection within a porous medium in the presence of thermal stratification has been discussed by Nakayama and Koyama [11]. Then significance of stratifications (thermal and solutal) on natural convection in a Darcian and non-Darcy porous medium has been discussed noticeably by quite a few researchers (for example see Murthy et al. [12], Lakshmi Narayana and Murthy [13], Srinivasacharya and RamReddy [14]; Ibrahim and Makinde [15] etc).

The aim of this paper is to consider the combined effects of MHD, thermal radiation and internal heat generation on unsteady free convection flow in a chemically reacting doubly stratified fluid saturated Brinkman porous medium using the Crank-Nicolson scheme. The effects of governing parameters on skin-friction, Nusselt number and Sherwood number are analyzed and shown graphically.

2. Mathematical Formulation

Consider the physical model and the coordinate system such that the *x*-axis is along the vertical plate and the *y*-axis is normal to the plate. A two-dimensional unsteady laminar incompressible free convective flow past a vertical plate in an electrically conducting doubly stratified fluid saturated porous medium is considered. A magnetic field of uniform strength B_0 is introduced normal to the direction of the flow. The fluid and the plate are assumed to be at the constant temperature and constant concentration initially at $t'=0$ whereas the temperature and concentration of the plate are changed to T_w and C_w respectively for the time $t'>0$. The ambient medium is assumed to be vertically linearly stratified with respect to both temperature and concentration $T_\infty(x) = T_{\infty,0}+Ax$ and $C_\infty(x) = C_{\infty,0}+Bx$ respectively (See Ref [13]-[14]).

Under the usual Boussinesq's and Brinkman porous medium approximation, under above assumptions, the governing boundary layer equations for the flow are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\varepsilon\frac{\partial u}{\partial t'}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\varepsilon\nu\frac{\partial^2 u}{\partial y^2}+\varepsilon^2 g\left[\beta_T(T'-T_\infty(x))+\beta_C(C'-C_\infty(x))\right]-\frac{\varepsilon^2\mu}{k}u-\frac{\sigma B_0^{\,2}}{\rho}\varepsilon u \tag{2}$$

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=\alpha\frac{\partial^2 T'}{\partial y^2}+\frac{Q_0}{\rho C_P}\left(T'-T_\infty(x)\right)+\frac{16\sigma^* T'^3_{\infty,0}}{3\rho C_P k_0}\frac{\partial^2 T'}{\partial y^2} \tag{3}$$

$$\frac{\partial C'}{\partial t'}+u\frac{\partial C'}{\partial x}+v\frac{\partial C'}{\partial y}=D\frac{\partial^2 C'}{\partial y^2}-R(C'-C_\infty(x)) \tag{4}$$

where u and v are Darcy velocity components along the x and y directions respectively, ρ is the density, g is the acceleration due to gravity, C_p is the specific heat, μ is the coefficient of viscosity, σ is the electrical conductivity, k is the permeability, ε is the porosity, T' is the temperature, C' is the concentration, β_T and β_C are the coefficients of thermal and solutal expansions, α is the thermal diffusivity, D is the mass diffusivity. The third term on RHS of Eq. (3) is obtained by using Rosseland approximation [1] and $T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty$.

The boundary conditions are

$$\begin{aligned} &t' \le 0: u(x,y,t') = 0,\ v(x,y,t') = 0,\ T'(x,y,t') = T_\infty(x),\ C'(x,y,t') = C_\infty(x) \\ &t' > 0: u(x,0,t') = 0,\ v(x,0,t') = 0,\ T'(x,0,t') = T_w,\ C'(x,0,t') = C_w \\ &\quad u(0,y,t') = 0,\ v(0,y,t') = 0,\ T'(0,y,t') = T_{\infty,0},\ C'(0,y,t') = C_{\infty,0} \\ &\quad u(x,\infty,t') \to 0,\ T'(x,\infty,t') \to T_\infty(x),\ C'(x,\infty,t') \to C_\infty(x) \end{aligned} \tag{5}$$

Introducing the following non-dimensional variables

$$X = \frac{x}{L},\ Y = \frac{y}{L}Gr^{1/4},\ U = \frac{uL}{\upsilon}Gr^{-1/2},\ V = \frac{vL}{\upsilon}Gr^{-1/4},\ t = \frac{t'\upsilon}{L^2}Gr^{1/2},\ T = \frac{T' - T_\infty(x)}{T_w - T_{\infty,0}},\ C = \frac{C' - C_\infty(x)}{C_w - C_{\infty,0}}$$

into equations (1)-(4), we obtain

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{6}$$

$$\varepsilon\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \varepsilon\frac{\partial^2 U}{\partial Y^2} + \varepsilon^2(T + NC) - \frac{\varepsilon^2}{DaGr^{1/2}}U - \frac{\varepsilon M}{Gr^{1/2}}U \tag{7}$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{1}{\Pr}(1 + N_R)\frac{\partial^2 T}{\partial Y^2} + QT - \varepsilon_1 U \tag{8}$$

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - R'C - \varepsilon_2 U \tag{9}$$

The non-dimensional conditions associated with the reduced equations are

$$\begin{aligned} &t \le 0: U = 0,\ V = 0,\ T = 0,\ C = 0 \\ &t > 0: U(X,0,t) = 0,\ V(X,0,t) = 0,\ T(X,0,t) - 1 + \varepsilon_1 X = 0,\ C(X,0,t) - 1 + \varepsilon_2 X = 0 \\ &\quad U(0,Y,t) = 0,\ V(0,Y,t) = 0,\ T(0,Y,t) = 0,\ C(0,Y,t) = 0 \\ &\quad U(X,\infty,t) \to 0,\ T(X,\infty,t) \to 0,\ C(X,\infty,t) \to 0 \end{aligned} \tag{10}$$

where $Gr = g\beta_T L^3(T_w - T_{\infty,0})/\upsilon^2$ is the Grashof number, $N = \beta_C(C_w - C_{\infty,0})/(\beta_T(T_w - T_{\infty,0}))$ is the buoyancy ratio, $Da = k\upsilon/(\mu L^2)$ is the Darcy number, $M = \sigma B_0^2 L^2/(\rho\upsilon)$ is the magnetic parameter, $N_R = 16\sigma^* T'^3_\infty/(3kk_0)$ is the thermal radiation parameter, $Q = Q_0 L^2 Gr^{-1/2}/(\rho C_P \upsilon)$ is heat generation parameter, $Pr = \upsilon/\alpha$ and $Sc = \upsilon/D$ are the Prandtl and Schmidt numbers, $R' = RL^2 Gr^{-1/2}/\upsilon$ is dimensionless chemical reaction parameter, $\varepsilon_1 = AL/(T_w - T_{\infty,0})$ and $\varepsilon_2 = BL/(C_w - C_{\infty,0})$ are the thermal and solutal stratification parameters.

The non-dimensional forms of physical parameters of interest such as local skin friction, Nusselt number and Sherwood number are obtained as

$$\tau_X = Gr^{3/4}\left(\frac{\partial U}{\partial Y}\right)_{Y=0},\ Nu_X = -Gr^{1/4}(1 + N_R)\frac{X\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{1 - \varepsilon_1 X} \text{ and } Sh_X = -Gr^{1/4}\frac{X\left(\frac{\partial C}{\partial Y}\right)_{Y=0}}{1 - \varepsilon_2 X} \tag{11}$$

The non-dimensional forms of average skin friction, Nusselt number and Sherwood number are obtained as

$$\bar{\tau} = Gr^{3/4}\int_0^1 \left(\frac{\partial U}{\partial Y}\right)_{Y=0} dX \ , \ \overline{Nu} = -Gr^{1/4}(1+N_R)\int_0^1 \frac{\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{1-\varepsilon_1 X} dX \ \text{and} \ \overline{Sh} = -Gr^{1/4}\int_0^1 \frac{\left(\frac{\partial C}{\partial Y}\right)_{Y=0}}{1-\varepsilon_2 X} dX \qquad (12)$$

3. Results and Discussion

The implicit finite difference scheme known as Crank-Nicolson type scheme (see Loganathan *et al.* [16] and citations therein) is used to solve the Eqs. (6)-(9) along with B.C.(10) and the results of non-dimensional local and average values of skin friction, heat and mass transfer rates are analyzed. To test the accuracy of the present results, the velocity profiles for $Pr=0.71$ $Sc=0.94$, $N=1.0$, $Q = 0.0$, $N_R = 0.0$, $R' = 0.0$ and $M=0.0$ of the present result are compared with the existing solution of Gebhart and Pera [17] in the absence of steady doubly stratified porous medium. In order to analyze the combined effects of physical parameters, the computations are taken for fixed values of $Pr=0.71$, $Sc=0.22$, $Gr=5.0$, $\varepsilon=0.6$, $M=1.0$ and $N=1.0$. These computations are carried out at $t=2.0$.

The first set of Figs. 1(a)-1(f) are prepared to show the variation of local and average skin friction, Nusselt and Sherwood numbers in the presence and/or absence of stratification (both thermal and solutal) parameters for fixed values of other parameters. Figs. 1(a)-1(b) imply that the local and average skin friction decreases with the increase in both thermal and solutal stratification parameters. Figs.1(c)-1(d) indicate that the local and average Nusselt number not show any considerable effect with an enhancement in solutal stratification parameter but, they enhances with a rise in thermal stratification parameter. The local and average Sherwood number increase with increase in solutal stratification parameter as shown in Figs. 1(e)-1(f). It is evident from these figures that average skin friction is more while the average heat and mass transfer rates are less in the presence of double stratification in comparison with the absence of double stratification. Finally, the heat and mass transfers reaches the asymptotic steady state when time increases.

The variation of local and average skin friction, Nusselt and Sherwood numbers for different values of chemical reaction and internal heat generation parameters, are plotted in the second set of Figs. 2(a)-2(f). From Figs. 2(a)-2(b), it can be noted that local and average Nusselt number diminishes with sequential increase in heat generation and chemical reaction parameter. From Figs. 2(c)-2(d), it is found that the local and average Nusselt number increases with increasing values of heat generation and chemical reaction parameter, but initially (ie., at $Q=0.0$) there is no considerable effect. From Figs. 2(e) - 2(f) explores that the local and average Sherwood number increases with rise in chemical reaction parameter and in the absence of heat generation parameter, whereas they decreases with rise in heat generation parameter and the absence of chemical reaction parameter.

The third set of Figs. 3(a)-3(f) are prepared to study the variation of local and average skin friction, Nusselt number and Sherwood number for different values of Darcy and radiation parameters. From Figs. 3(a) - 3(b) we notice that with the sequential increase in Darcy and radiation parameters, the local and average skin friction increases. Figs. 3(c) - 3(d) depict that, the local and average Nusselt number increases with increasing values of radiation parameter, but there is no considerable effect with increasing values of Darcy parameter. Figs. 3(e) - 3(f) show that the local and average Sherwood number enhance as Darcy parameter increases while they do not reveal any significant effect with rise in radiation parameter. The absence of these parameters is evident from the Figs. 3(a)-3(f) and hence are not discussed specially for the sake of brevity.

4. Conclusion

In this paper, the combined effects of radiation, heat generation and chemical reaction on unsteady MHD free convective flow in a doubly porous medium are discussed. The governing equations are solved numerically by the implicit finite difference method of Crank-Nicolson type. The main findings are summarized as follows: (a) as the thermal and solutal stratification parameters increase, the average skin friction increases whereas an average Nusselt number and Sherwood number decrease. Further, heat and mass transfer attained the steady state after certain time, (b) the average skin friction is more but the Nusselt and Sherwood numbers are less in the presence of heat generation and chemical reaction parameters, and (c) with an increasing values of Darcy and radiation parameters, the average skin friction increases but average heat and mass transfer rates increase.

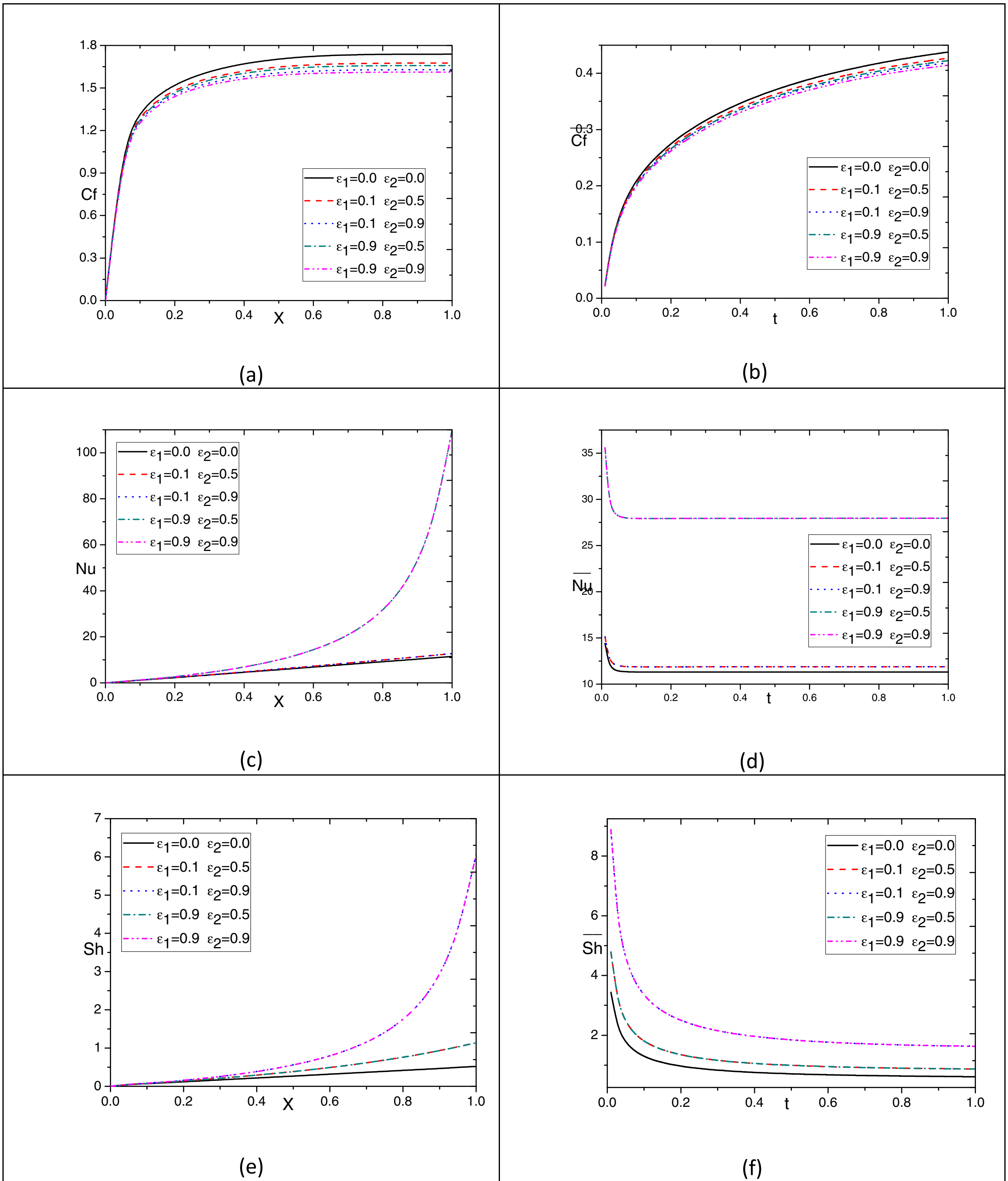

Fig-1: Effect of thermal and solutal stratification parameter with $Q=1.0$, $R'=0.5$, $Da=0.5$, $N_R=1.0$ on (a) local skin friction, (b) average skin friction, (c) local Nusselt number, (d) average Nusselt number, (e) local Sherwood number and (f) average Sherwood number.

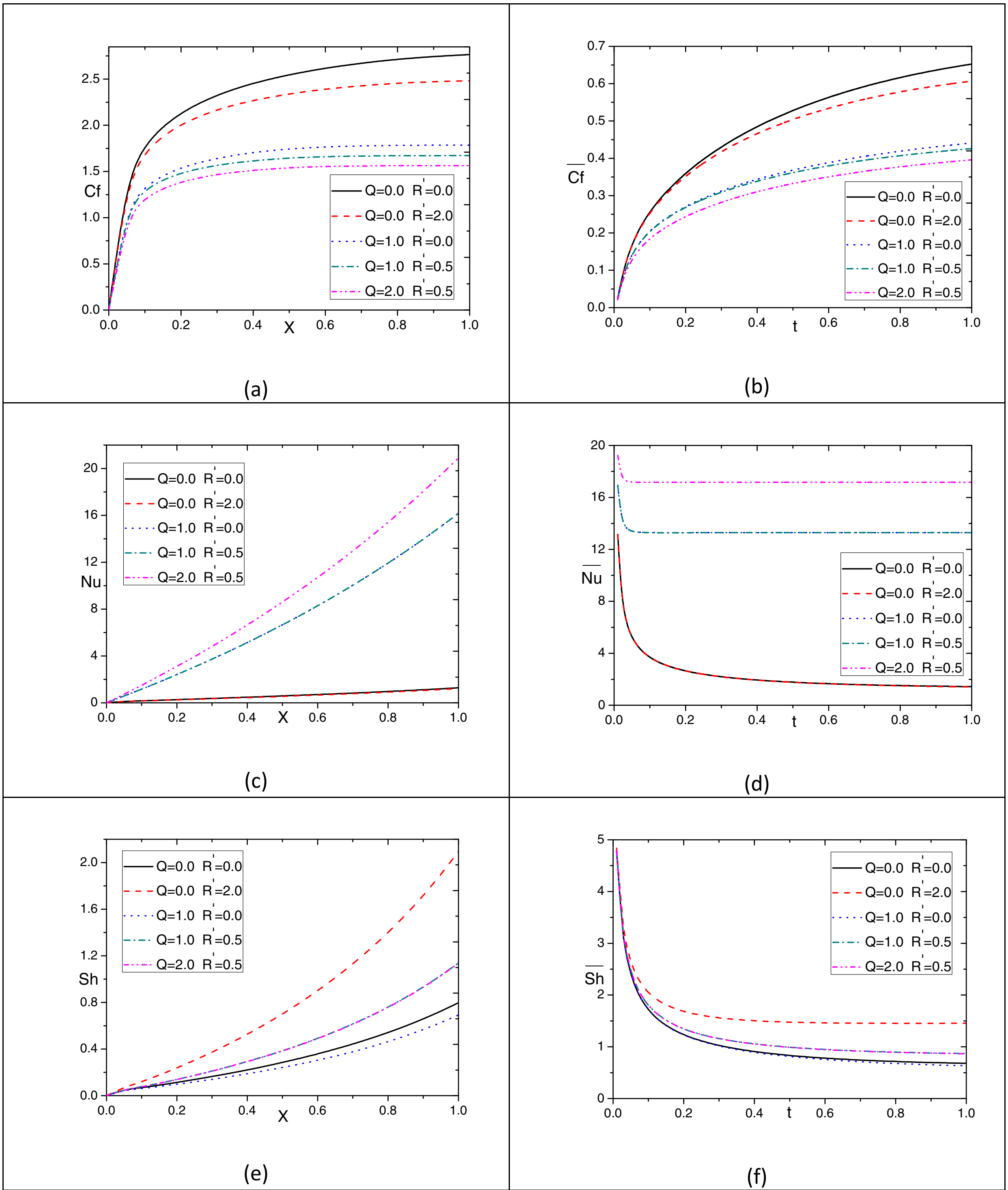

Fig-2: Effect of heat generation and chemical reaction parameter with $\varepsilon_1 = 0.3$, $\varepsilon_2 = 0.5$, $Da=0.5$, $N_R=1$ on (a) local skin friction, (b) average skin friction, (c) local Nusselt number, (d) average Nusselt number, (e) local Sherwood number and (f) average Sherwood number.

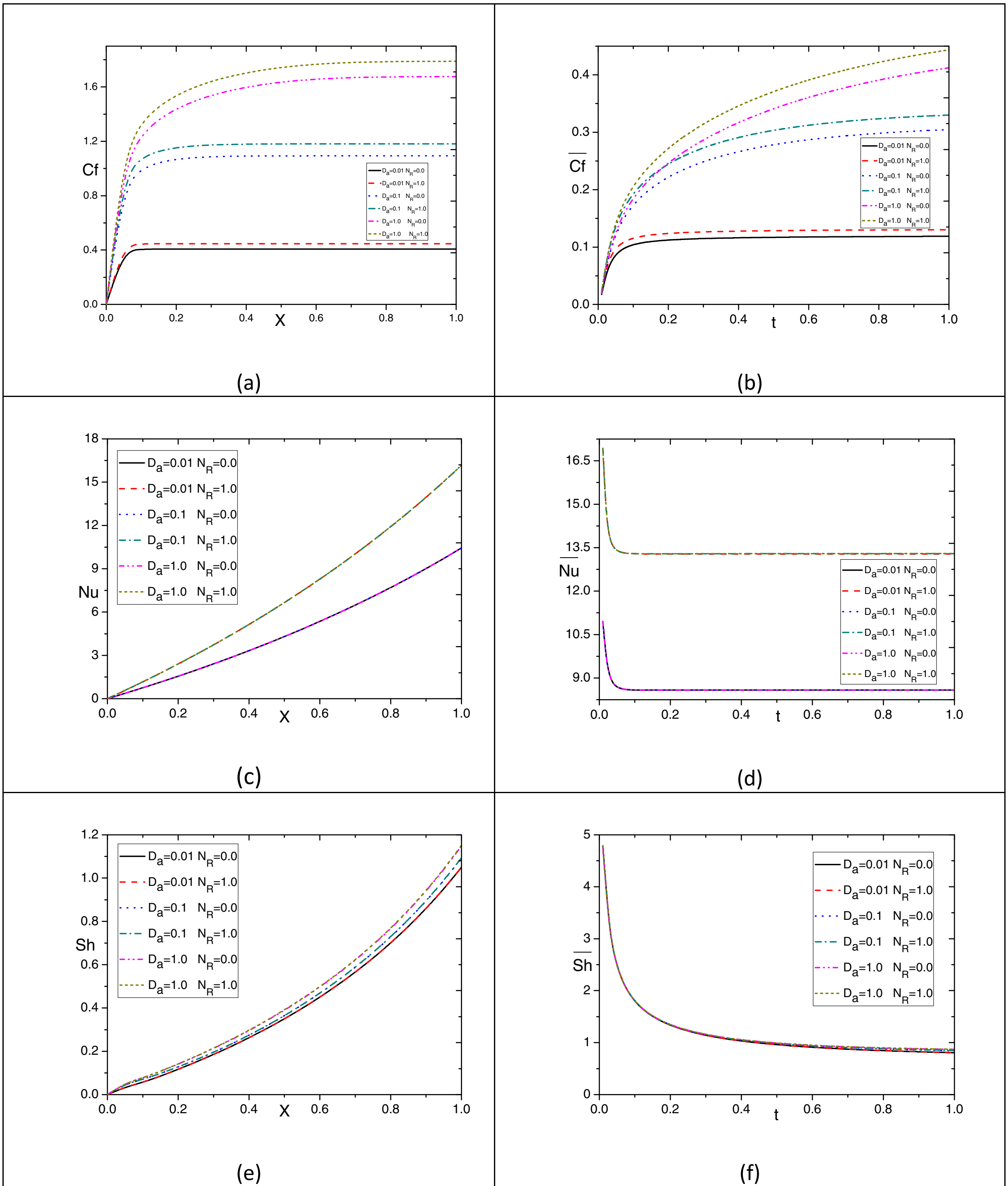

Fig-3: Effects of Darcy and radiation parameter with *$\varepsilon_1 = 0.3$, $\varepsilon_2 = 0.5$, $Q=1.0$, $R'=0.5$* on (a) local skin friction, (b)average skin friction, (c) local Nusselt number, (d) average Nusselt number, (e) local Sherwood number and (f) average Sherwood number.

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