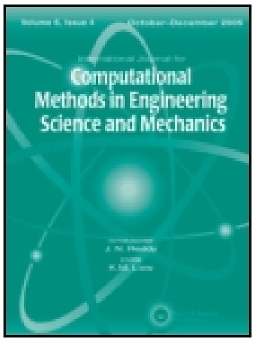

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Magnetic and Double Dispersion Effects on Free Convection in a Non-Darcy Porous Medium Saturated with Power-Law Fluid

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In this paper, effects of magnetic field and double dispersion on free convection heat and mass transfer along a vertical plate embedded in a doubly stratified non-Darcy porous medium saturated with power-law fluid is considered. The governing partial differential equations are transformed into ordinary differential equations using similarity transformations and then solved numerically. The numerical results are compared and found to be in good agreement with previously published results as special cases of the present investigation. The effects of magnetic parameter, dispersion parameters, and power-law index on the velocity, temperature, and concentration are illustrated graphically.

Keywords Hydromagnetic effects, Double dispersion, Free convection, Non-Darcy porous medium, Power-law fluid

NOMENCLATURE

A	Slope of ambient temperature
B	Slope of ambient concentration
B_0	Magnetic field strength
C	Concentration
$C_{\infty,0}$	Ambient concentration
c	Emperical constant
D	Solutal diffusivity
D_c	Solutal dispersion parameter
D_e	Effective solutal diffusivity
D_s	Thermal dispersion parameter
f	Reduced stream function
G	Modified Darcy parameter
g	Gravitational acceleration
K	Permeability constant
k	Thermal conductivity
L	Characteristic length
Le	Lewis number
M	Magnetic paramter
N	Buoyancy ratio
n	Power-law index
q_w, q_m	Heat, mass transfers from the plate
Ra_x	Darcy-Rayleigh number
T	Temperature
$T_{\infty,0}$	Ambient temperature
u, v	Darcian Velocity components in x and y directions
X	X-location
x, y	Coordinates along and normal to the plate
α	Thermal diffusivity
α_e	Effective thermal diffusivity
β_T, β_C	Coefficients of thermal, solutal expansion
η	Similarity variable
γ	Thermal dispersion coefficient
ν	Kinematic viscosity
ϕ	Dimensionless concentration
ψ	Stream function
ρ	Density of the fluid
σ	Electrical conductivity of the fluid
θ	Dimensionless temperature
$\varepsilon_1, \varepsilon_2$	Thermal and Solutal stratification parameters
ξ	Solutal dispersion coefficient

Superscripts

∞	Ambient condition

Superscript

$'$	Differentiation with respect to η

1. INTRODUCTION

Free and forced convection flows in a fluid saturated porous media are of great interest because of their various engineering, scientific, and industrial applications in heat and mass transfer, which occurs in the fields of design of chemical processing

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equipment, formation and dispersion of fog, distributions of temperature and moisture over agricultural fields and groves of fruit trees, and damage of crops due to freezing and pollution of the environment, grain storage systems, heat pipes, packed microsphere insulation, distillation towers, ion exchange columns, subterranean chemical waste migration, solar power absorbers, etc. A number of studies have been reported in the literature focusing on the problem of combined heat and mass transfer in porous media. The analysis of convective transport in a porous medium with the inclusion of non-Darcian effects has also been a matter of study in recent years. Due to its important applications in many fields, a full understanding of combined heat and mass transfer by non-Darcy natural convection from a heated flat surface embedded in fluid saturated porous medium is meaningful. The inertia effect is expected to be important at a higher flow rate and it can be accounted for through the addition of a velocity squared term in the momentum equation, which is known as the Forchheimers extension. A detailed review of convective heat transfer in Darcian and non-Darcian porous medium, including an exhaustive list of references, can be found in the book by Nield and Bejan [1].

The study of flow, heat, and mass transfer in non-Newtonian fluid flows has gained much attention from researchers because of its engineering and industrial applications, such as the thermal design of industrial equipment dealing with molten plastics, polymeric liquids, foodstuffs, or slurries. Also, the nonlinear behavior of non-Newtonian fluids in a porous matrix is quite different from that of Newtonian fluids in porous media. The prediction of heat or mass transfer characteristics about natural convection of non-Newtonian fluids in porous media is very important due to the practical engineering applications, such as oil recovery and food processing. Several investigators have extended the convection of heat and mass transfer problems to fluids exhibiting non-Newtonian rheology. Different models have been proposed to explain the behavior of non-Newtonian fluids. Among these, the power–law, the differential type, and the rate type models gained importance. Although this model is merely an empirical relationship between the stress and velocity gradients, it has been successfully applied to non-Newtonian fluids experimentally. Chen and Chen [2] have studied the natural convection of a non-Newtonian fluid about a horizontal cylinder and sphere in a porous medium. Pascal and Pascal [3] have considered the free convection in a non-Newtonian fluid saturated porous medium with lateral mass flux. Free convection heat and mass transfer of non-Newtonian power–law fluids with yield stress from a vertical flat plate in a saturated porous media was studied by Rami and Anna [4]. The flow of natural convection heat and mass transfer of non-Newtonian power–law fluids with yield stress in porous media from a vertical plate with variable wall heat and mass fluxes was considered by Cheng [5]. Buoyant convection of power-law fluid in an enclosure filled with heat-generating porous media was considered by Kim and Hyun [6]. The study of free convection in boundary layer flows of power–law fluids past a vertical flat plate with suction/injection was done by Sahu and Mathur [7]. Free convection from a horizontal line heat source in a power-law fluid-saturated porous medium was studied by Nakayama [8].

There has been a renewed interest in Magnetohydrodynamics (MHD) flow and heat transfer in porous and clear domains due to the important effect of magnetic field on the boundary layer flow control and on the performance of many systems using electrically conducting fluid, such as MHD power generators, the cooling of nuclear reactors, plasma studies, purification of molten metals from non-metallic inclusion, geothermal energy extractions, etc. Many problems of MHD Darcian and non-Darcian flow of Newtonian as well as non-Newtonian fluid in porous media have been analyzed and reported in the literature. Non-Darcy mixed convection in power-law fluids along a non-isothermal horizontal surface in a porous medium has been analyzed by Kumari and Nath [9]. The effect of magnetic field on non-Darcy axisymmetric free convection in a power-law fluid saturated porous medium with variable permeability has been considered by Mansour and El-Shaer [10]. A numerical approach has been used to study the heat and mass transfer from a vertical plate embedded in a porous medium experiencing a first-order chemical reaction and exposed to a transverse magnetic field by Makinde and Aziz [11]. Makinde [12] investigated the hydromagnetic mixed convection heat and mass transfer flow of an incompressible Boussinesq fluid past a vertical porous plate with constant heat flux in the presence of radiative heat transfer in an optically thin environment, viscous dissipation, and an nth order homogeneous chemical reaction between the fluid and the diffusing species. Beg and Makinde [13] studied the two-dimensional steady, laminar flow of an incompressible, viscoelastic fluid with species diffusion in a parallel plate channel with porous walls containing a homogenous, isotropic porous medium with high permeability.

Yet for all the simplifications that it made, the continuum hypothesis as applied to transport phenomena in porous media has confronted some difficulties that entailed the introduction of some constitutive relationships to account for the apparent differences between the upscaled and the actual variables. To give an example, the actual velocity of fluid particles within the pore structures changes significantly between zero at the interface and different than zero within the pores, whereas within the continuum hypothesis the upscaled velocity may be constant or, at most changes, comparatively slowly. This would result in the existence of additional mass and/or energy fluxes. It has thus been hypothesized that these additional fluxes may be accounted for by adding terms to their respective flux terms that may be assumed to depend on the upscaled velocity. Thus, in terms of solute transport, the usual diffusion mechanism has been augmented by another mechanism that is called mass dispersion term, which depends on the upscaled velocity. Likewise, energy transport (like heat transport) in porous media required the addition of a thermal dispersion mechanism to the usual thermal diffusion mechanism. Also, it is known from the literature that in a non-Darcy medium where the inertial effects are prevalent, the

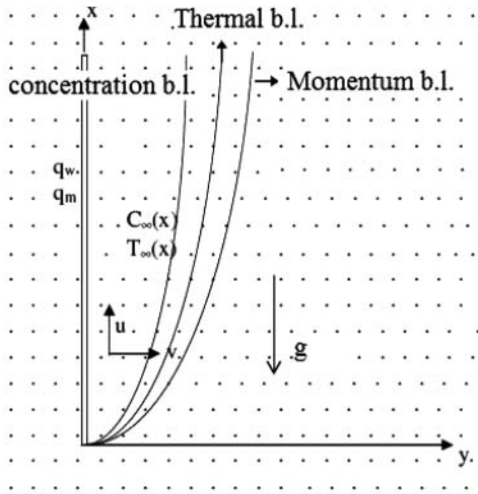

FIG. 1. Physical model and coordinate system.

thermal and solutal dispersion effects will become significant [1]. These effects are very significant in vigorous natural convection and mixed convection flows. The effect of double dispersion on mixed convection heat and mass transfer in Newtonian fluid saturated non-Darcy porous medium has been investigated by Murthy [15]. Mohammadien and El-Amin [16] analyzed double dispersion effects on natural convection heat and mass transfer in a non-Darcy porous medium.

From the literature survey, it seems that the problem of natural convection heat and mass transfer from a vertical plate in non-Darcy porous media saturated with power-law fluids with double dispersion and magnetic effects has not been investigated so far. Thus this work aims to study the effects of double dispersion and magnetic field on natural convection in a power-law fluid saturated non-Darcy porous medium with uniform heat and mass flux. The results are compared with relevant results in the existing literature and are found to be in good agreement.

2. MATHEMATICAL FORMULATION

Consider the two–dimensional free convection flow of an electrically conducting fluid from the vertical flat plate in a doubly stratified, non-Newtonian, power-law fluid saturated non-Darcy porous medium as shown in Fig. 1. The x-axis is taken along the plate and y- axis normal to it. A uniform magnetic field is applied normal to the plate. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. The fluid and the porous structure are everywhere in local thermodynamic equilibrium. The temperature and the mass concentration of the ambient medium are assumed to be in the form $T_\infty(x) = T_{\infty,0} + Ax^m$ and $C_\infty(x) = C_{\infty,0} + Bx^l$, respectively. The plate is maintained at constant heat flux q_w and constant mass flux q_m. In addition, thermal and solutal dispersion effects are considered.

Using the Boussinesq and boundary layer approximations, the governing equations for the power-law fluid are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u^n + \frac{\sigma \mu_e^2 B_0^2 K}{\rho \nu} u + \frac{c\sqrt{K}}{\nu} u^2 = \frac{Kg}{\nu}\{\beta_T (T - T_\infty) + \beta_C (C - C_\infty)\} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left[\alpha_e \frac{\partial T}{\partial y}\right] \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \frac{\partial}{\partial y}\left[D_e \frac{\partial C}{\partial y}\right], \tag{4}$$

where u and v are the Darcian velocity components along x and y directions, T is the temperature, C is the concentration, K is the permeability constant, c is an empirical constant, n is the power-law index, ν is the kinematic viscosity, g is the acceleration due to gravity, ρ is the density, σ is the electrical conductivity of the fluid, μ_e is the magnetic permeability, B_0 is the strength of the magnetic field, β_T is the coefficient of thermal expansion and β_C is the coefficient of solutal expansion,

TABLE 1
Comparison of local Nusselt and Sherwood numbers for free convection with a vertical flat plate in Newtonian fluids of Murthy et al. [17] with $Le = 1.0$.

$\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.5$			$\varepsilon_1 = 0.5$ and $\varepsilon_2 = 0.1$		
$\frac{Nu_x}{(Nu_x)_0} = \frac{\theta(0)}{\theta(0,\varepsilon_1)}$			$\frac{Sh_x}{(Sh_x)_0} = \frac{\phi(0)}{\phi(0,\varepsilon_2)}$		
N	Murthy et al. [17]	Present	N	Murthy et al. [17]	Present
−0.5	0.9375	0.9375	−0.5	1.0888	1.0888
0.0	0.9821	0.9821	0.0	1.0469	1.0469
0.5	1.0041	1.0041	0.5	1.0290	1.0290
1.0	1.0184	1.0184	1.0	1.0184	1.0184
3.0	1.0507	1.0507	3.0	0.9978	0.9978
5.0	1.0694	1.0694	5.0	0.9878	0.9878

$\alpha_e = \alpha + \gamma du$ and $D_e = D + \xi du$ are the effective thermal and solutal diffusivities, respectively, α and D are the thermal diffusivity and solutal diffusivity constants, respectively, and γ and ξ are the coefficients of the thermal and solutal dispersion, respectively. The values of these quantities lie between $1/7$ and $1/3$. The subscript ∞ indicates the condition at the outer edge of the boundary layer.

The boundary conditions are

$$v = 0, \quad q_w = -k\frac{\partial T}{\partial y}, \quad q_m = -D\frac{\partial C}{\partial y} \quad at \quad y = 0 \tag{5a}$$

$$u = 0, \quad T = T_\infty(x), \quad C = C_\infty(x) \quad as \quad y \to \infty, \tag{5b}$$

where k is the thermal conductivity of the fluid.

3. SOLUTION OF THE PROBLEM

In view of the continuity Eq. (1), we introduce the stream function ψ by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{6}$$

Substituting Eq. (6) in Eqs. (2)–(4) and then using the following local-similarity transformations

$$\left.\begin{aligned} \eta &= \frac{y}{x}Ra_x^{\frac{n}{2n+1}}, \quad \psi = \alpha Ra_x^{\frac{n}{2n+1}} f(\eta), \\ \theta(\eta) &= \frac{k(T - T_\infty(x))}{q_w x} Ra_x^{\frac{n}{2n+1}}, \\ \phi(\eta) &= \frac{D(C - C_\infty(x))}{q_m x} Ra_x^{\frac{n}{2n+1}} \end{aligned}\right\} \tag{7}$$

where $Ra_x = \frac{x}{\alpha}(\frac{Kg\beta_T q_w x}{k\nu})^{1/n}$ is the modified Reyleigh number, we get the following system of equations

$$(f')^n + X^{\frac{1-n}{2n+1}} M f' + X^{\frac{2-n}{2n+1}} G (f')^2 = \theta + N\phi \tag{8}$$

$$\left(1 + D_s X^{\frac{-1}{2n+1}} f'\right)\theta'' + D_s X^{\frac{-1}{2n+1}} f''\theta' = \frac{1}{2n+1}[nf'\theta - (n+1)f\theta' + \varepsilon_1 f'] \tag{9}$$

$$\left(\frac{1}{Le} + D_c X^{\frac{-1}{2n+1}} f'\right)\phi'' + D_c X^{\frac{-1}{2n+1}} f''\phi' = \frac{1}{2n+1}[nf'\phi - (n+1)f\phi' + \varepsilon_2 f']. \tag{10}$$

In Eqs. (8)–(10), n is the viscosity index parameter. The power–law fluids with $n < 1$ are called pseudoplastics, while those with $n > 1$ are termed dilatants. L is the characteristic length, $X = \frac{x}{L}$ is the X-location, $M = \frac{\sigma\mu_e^2 B_0^2 K}{\rho\nu}(\frac{L}{\alpha})^{(n-1)} Ra^{\frac{-2n(n-1)}{2n+1}}$ is the magnetic parameter, $Ra = \frac{L}{\alpha}(\frac{Kg\beta_T q_w L}{k\nu})^{1/n}$ is the Darcy-Reyleigh number, $G = \frac{c\sqrt{K}}{\nu}(\frac{L}{\alpha})^{(n-2)} Ra^{\frac{-2n(n-2)}{2n+1}}$ is the modified non-Darcy parameter, $N = \frac{\beta_C q_m k}{\beta_T q_w D}$ is the buoyancy ratio, $Le = \frac{\alpha}{D}$ is the Lewis number, $D_s = \frac{\gamma d}{L} Ra^{\frac{2n}{2n+1}}$ is the thermal dispersion parameter, $D_c = \frac{\xi d}{L} Ra^{\frac{2n}{2n+1}}$ is solutal dispersion parameter. It is noteworthy that $G = 0$ corresponds to the Darcian free convection whereas $\gamma = 0$ and $\xi = 0$, respectively, are the cases where the thermal and solutal dispersion effect are neglected. $\varepsilon_1 = \frac{(2n+1)k}{q_w} Ra_x^{\frac{n}{2n+1}} \frac{\partial T_\infty(x)}{\partial x}$ is the thermal stratification parameter, $\varepsilon_2 = \frac{(2n+1)D}{q_m} Ra_x^{\frac{n}{2n+1}} \frac{\partial C_\infty(x)}{\partial x}$ is the solutal stratification parameter. These parameters, ε_1 and ε_2, will be independent of x only when $l = \frac{n}{2n+1}$ and $m = \frac{n}{2n+1}$, i.e. when $T_\infty(x) = T_{\infty,0} + Ax^{\frac{n}{2n+1}}$ and $C_\infty(x) = C_{\infty,0} + Bx^{\frac{n}{2n+1}}$.

The boundary conditions in Eq. (5) in terms of f, θ, and ϕ become

$$f(0) = 0, \ \theta'(0) = -1, \ \phi'(0) = -1 \tag{11a}$$

$$f'(\infty) = 0, \ \theta(\infty) = 0, \ \phi(\infty) = 0. \tag{11b}$$

A close look at Eqs. (8)–(10) reveals that, in free convection due to power-law fluid saturated non-Darcy porous medium, the velocity, temperature, and concentration profiles are not similar because the x-coordinate cannot be eliminated from these equations. Although local non-similarity solutions have been found for some boundary layer flows dealing with porous medium, the technique is hard to extend to power-law fluids. Thus, for ease of analysis, it was decided to proceed with finding local-similarity solutions for the governing equation, Eqs. (8)–(10). That is, taking $X = \frac{x}{L}$ and then varying the X-location, one can still study the effects of various parameters on different profiles at any given X-location, where L is the characteristic length.

The heat and mass transfer from the plate into the doubly stratified non-Darcy porous medium are expressed in terms of the local Nusselt and Sherwood numbers and are defined as

$$\frac{Nu_x}{(Nu_x)_0} = \frac{\theta(0)}{\theta(0, \varepsilon_1)} \quad \text{where} \quad Nu_x = \frac{q_w x}{T_w - T_{\infty,0}} \tag{12a}$$

$$\frac{Sh_x}{(Sh_x)_0} = \frac{\phi(0)}{\phi(0, \varepsilon_2)} \quad \text{where} \quad Sh_x = \frac{q_m x}{C_w - C_{\infty,0}}, \tag{12b}$$

with $(Nu_x)_0$ and $(Sh_x)_0$ as the values of Nu_x and Sh_x at $\varepsilon_1 = 0$ and $\varepsilon_2 = 0$, respectively.

4. RESULTS AND DISCUSSION

The flow Eq. (8) coupled with the energy and concentration Eqs. (9) and (10) constitute a set of nonlinear, non-homogeneous differential equation for which closed-form solution cannot be obtained and hence the problem has to be solved numerically. The boundary value problem given by Eqs. (8)–(10) along with the boundary conditions (11) are integrated using the fourth order Runge-Kutta method by giving appropriate initial guess values for $f'(0)$, $\theta(0)$, and $\phi(0)$ to match the values with the corresponding boundary conditions at $f'(\infty)$, $\theta(\infty)$, and $\phi(\infty)$, respectively. In the present study, the boundary conditions for

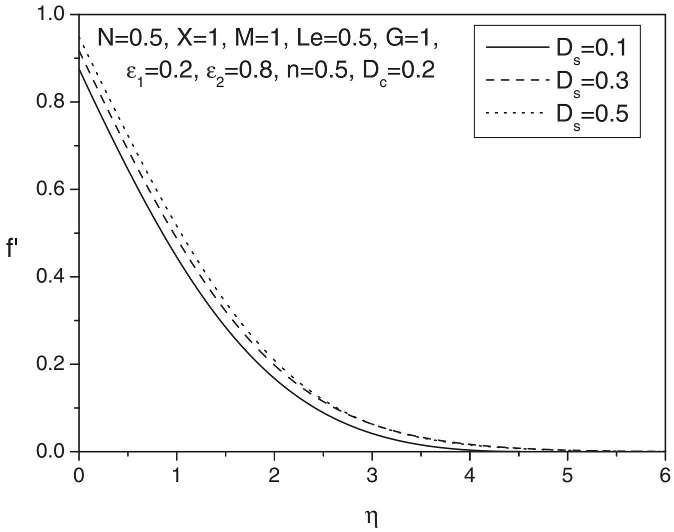

FIG. 2. Velocity profiles for various values of D_s.

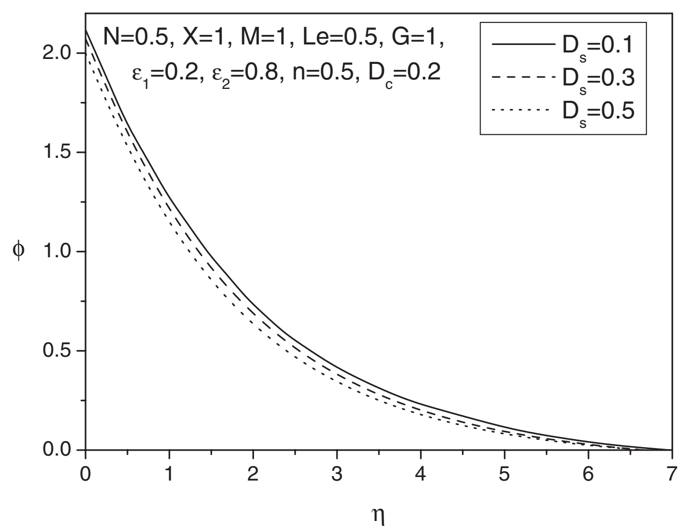

FIG. 4. Concentration profiles for various values of D_s.

η at ∞ varies with parameter values and it has been suitably chosen at each time such that the velocity, temperature, and concentration profiles approach zero at the outer edge of the boundary layer. Extensive calculations have been performed to obtain the wall velocity, temperature, and concentration fields for a wide range of parameters. The effect of thermal dispersion parameter, solutal dispersion parameter, magnetic parameter, and power-law index parameter is studied on the velocity, temperature, and concentration fields for uniform wall heat and mass flux conditions is plotted for some selected combinations of parameter values.

In order to assess the accuracy of the solution, Eqs. (1)–(4) governing the power-law fluid flow reduce to those limited case Newtonian fluid along the vertical plate of Murthy et al. [17], as shown in Table 1 in the absence of thermal dispersion parameter D_s, solutal dispersion parameter D_c, non-Darcy parameter G, magnetic parameter M, and power-law index $n = 1$, who investigated the effect of double stratification on free convection in Darcian porous medium. Also, the results have been compared and it is found that they are in good agreement.

The non-dimensional velocity $f'(\eta)$, temperature $\theta(\eta)$, and concentration $\phi(\eta)$ are plotted for $N = 0.5, X = 1, M = 1$, $Le = 0.5, G = 1, n = 0.5, D_c = 0.2, \varepsilon_1 = 0.2$, and $\varepsilon_2 = 0.8$ in Figs. 2–4 with varying thermal dispersion parameter. It can be observed from Fig. 2 that the velocity of the fluid increased with increasing the value of the thermal dispersion parameter. It is notable from this figure that as D_s increases the momentum boundary layer thickness increases. The effect of thermal dispersion on the thermal characteristics of the non-Darcy porous medium saturated with power-law fluid is analyzed in Fig. 3. Introducing the effect of thermal dispersion in the energy equation in general favors conduction

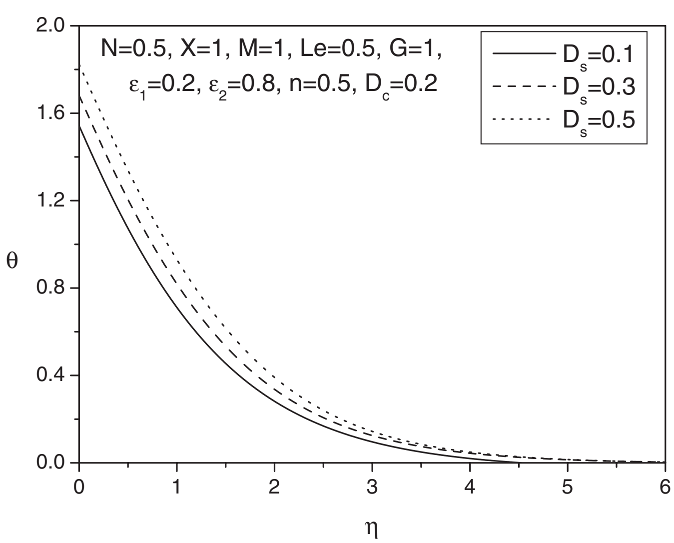

FIG. 3. Temperature profiles for various values of D_s.

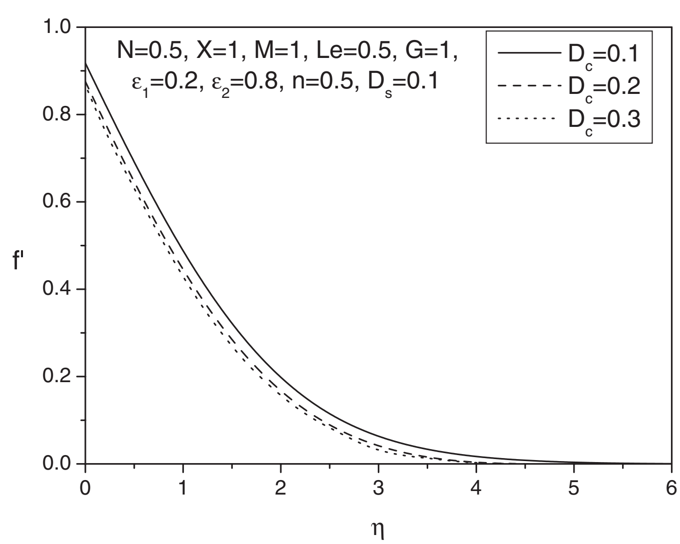

FIG. 5. Velocity profiles for various values of D_c.

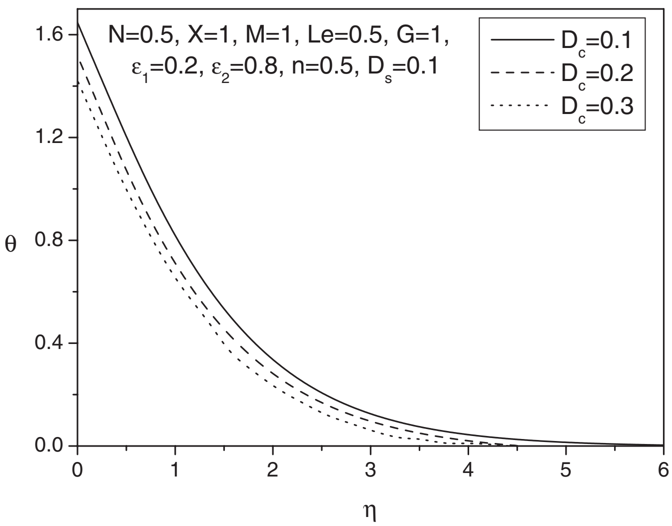

FIG. 6. Temperature profiles for various values of D_c.

over convection. In other words, supplementing dispersion effects to the energy equation gives thermal conduction more dominance. One can see from Fig. 3 that an increase in the mechanical thermal dispersion coefficient increases the thermal boundary layer thickness. That is, thermal dispersion enhances the transport of heat along the normal direction to the wall as compared with the case where dispersion is neglected (i.e., $D_s = 0$). In general, this work may be useful in showing that the use of porous media with better heat dispersion properties may result in better heat transfer characteristics that may be required in many industrial applications (like those concerned with packed bed reactors, nuclear waste disposal, etc.). It can be found from Fig. 4 that the concentration of the fluid is decreased with increase in the value of the thermal dispersion parameter.

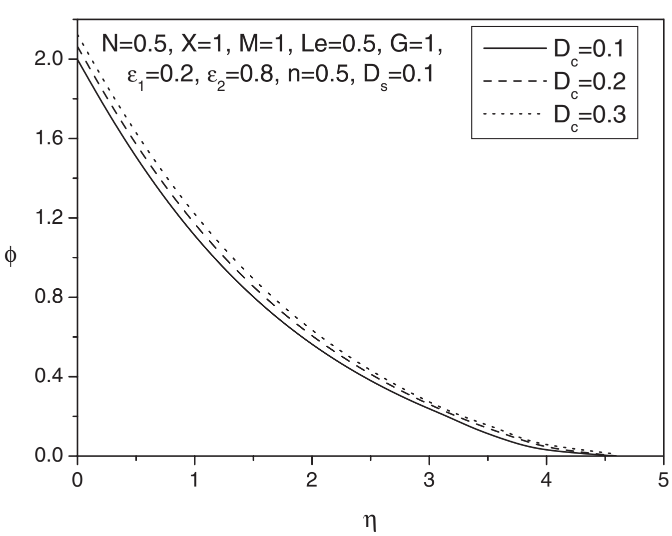

FIG. 7. Concentration profiles for various values of D_c.

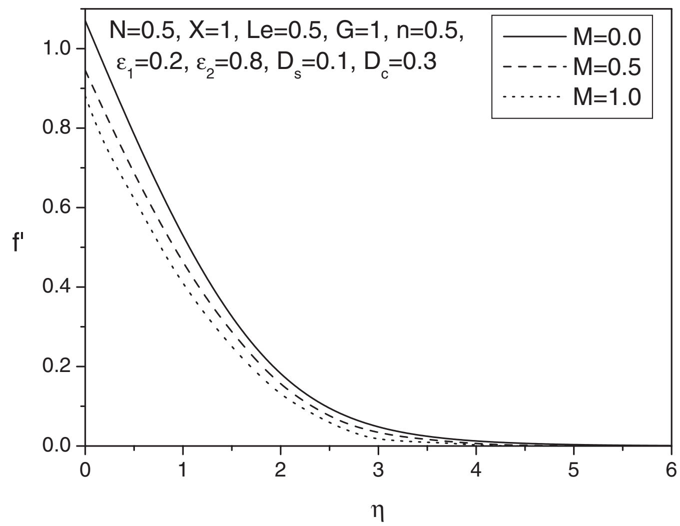

FIG. 8. Velocity profiles for various values of M.

The effect of solutal dispersion parameter on the non-dimensional velocity $f'(\eta)$, temperature $\theta(\eta)$, and concentration $\phi(\eta)$ for $N = 0.5$, $X = 1$, $M = 1$, $Le = 0.5$, $n = 0.5$, $G = 1$, $D_s = 0.1$, $\varepsilon_1 = 0.2$, and $\varepsilon_2 = 0.8$ is depicted in Figs. 5–7. It is observed from Fig. 5 that the fluid velocity is decreased with increase in the value of solutal dispersion parameter. It can be evident from this figure that as D_c increases the momentum boundary layer thickness decreases. It can be seen from Fig. 6 that the temperature of the fluid in the medium is decreased with increase in the value of the solutal dispersion parameter. It can be found from Fig. 7 that the concentration of the fluid is increased by increasing the value of the solutal dispersion parameter. Hence the concentration boundary layer thickness enhances with an enhance in the solutal dispersion parameter D_c.

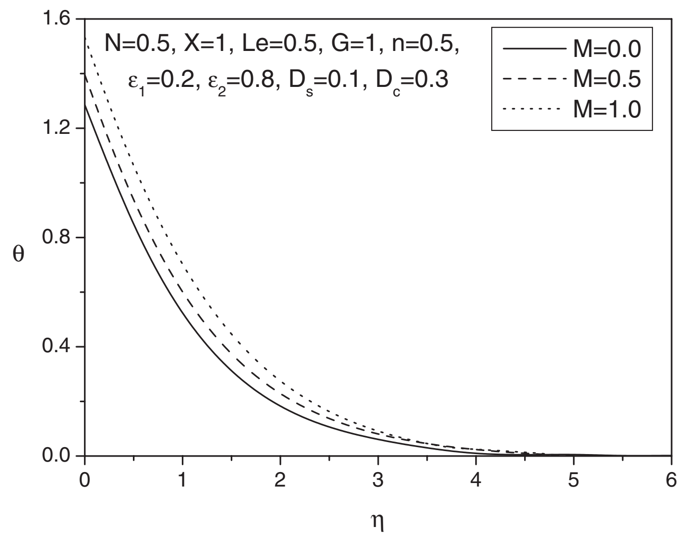

FIG. 9. Temperature profiles for various values of M.

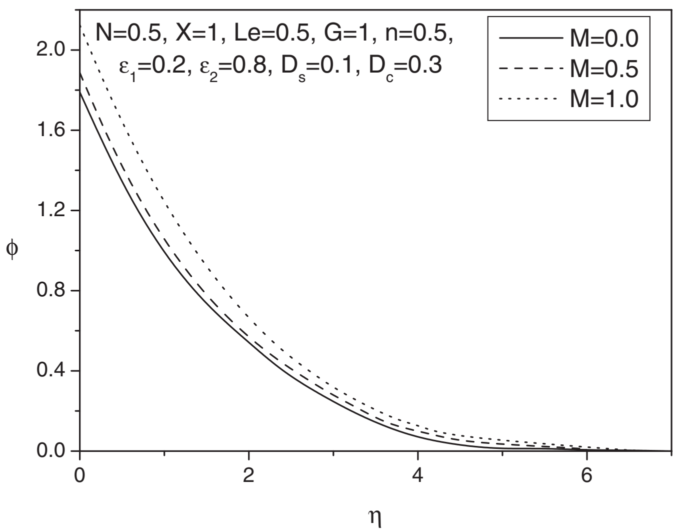

FIG. 10. Concentration profiles for various values of M.

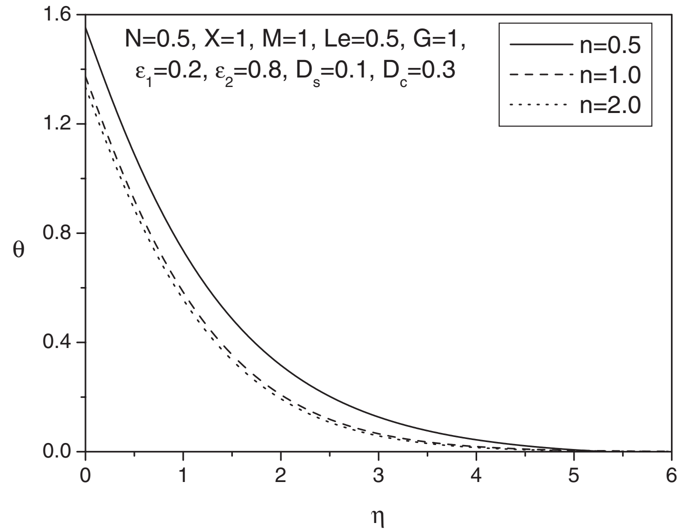

FIG. 12. Temperature profiles for various values of n.

The variation of the non-dimensional velocity $f'(\eta)$, temperature $\theta(\eta)$, and concentration $\phi(\eta)$ for $N = 0.5, X = 1, Le = 0.5, n = 0.5, G = 1, \varepsilon_1 = 0.2, \varepsilon_1 = 0.8, D_s = 0.1$, and $D_c = 0.3$ with magnetic parameter is shown in Figs. 8–10. It can be observed from Fig. 8 that the velocity of the fluid is decreased with increase in the value of the magnetic parameter. This is due to the fact that the introduction of a transverse magnetic field, normal to the flow direction, has a tendency to create the drag known as the Lorentz force, which tends to resist the flow. Hence the horizontal velocity profiles decrease as the magnetic parameter M increases. It can be found from Fig. 9 that increase in the value of the magnetic parameter increases the temperature of the fluid in the medium. It can be seen from Fig. 10 that the concentration of the fluid is increased by increasing the value of the magnetic parameter. As explained above, the transverse magnetic field gives rise to a resistive force known as the Lorentz force of an electrically conducting fluid. This force makes the fluid experience a resistance by increasing the friction between its layers and thus increases its temperature and concentration.

The non-dimensional velocity $f'(\eta)$, temperature $\theta(\eta)$, and concentration $\phi(\eta)$ for $N = 0.5, X = 1, M = 1, Le = 0.5, G = 1, \varepsilon_1 = 0.2, \varepsilon_1 = 0.8, D_s = 0.1$ and $D_c = 0.3$ with a variation in power–law index parameter is plotted in Figs. 11–13. It is observed from Fig. 11 that the fluid velocity is increased with increase in the value of the power–law index parameter. The effect of the increasing values of the power–law index n is to increase the horizontal boundary layer thickness. That is, the thickness is much smaller for shear thinning (pseudo plastic; $n < 1$) fluids than that of shear thickening (dilatants;

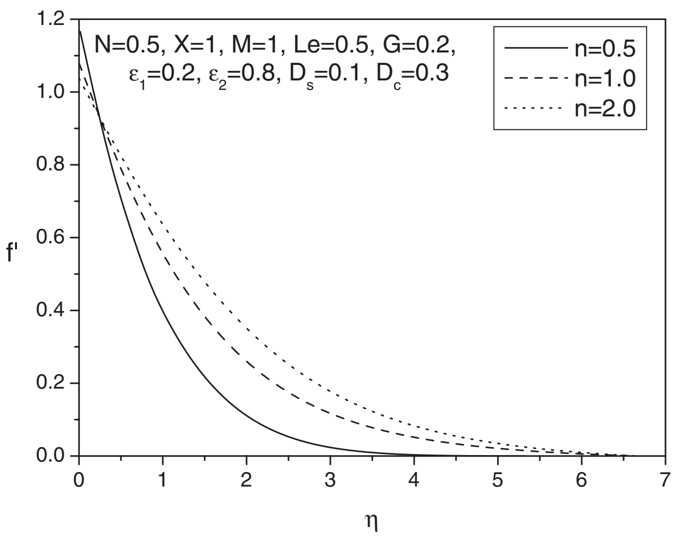

FIG. 11. Velocity profiles for various values of n.

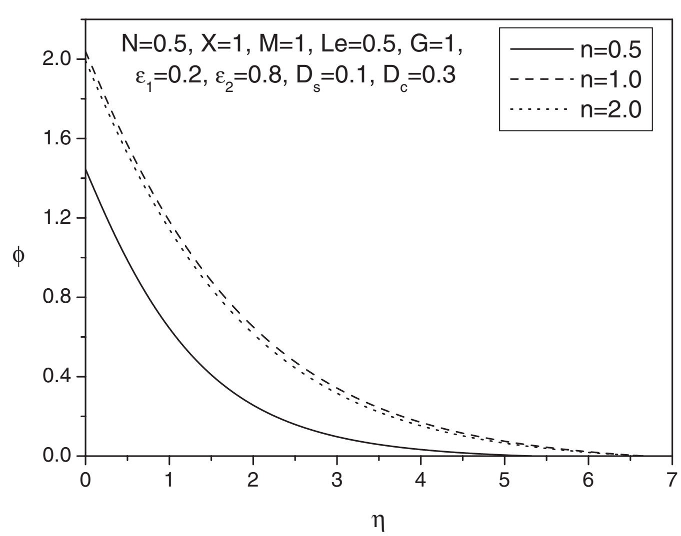

FIG. 13. Concentration profiles for various values of n.

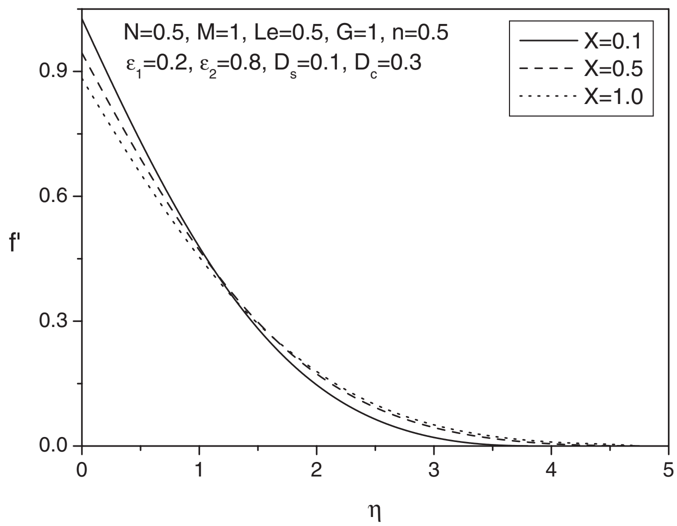

FIG. 14. Velocity profiles for various values of X.

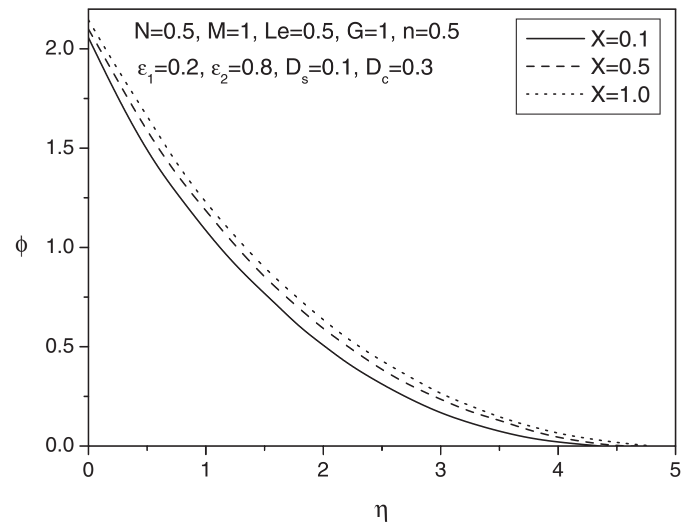

FIG. 16. Concentration profiles for various values of X.

$n > 1$) fluids. In the case of a shear thinning fluid ($n < 1$), the shear rates near the walls are higher than those for a Newtonian fluid. It can be seen from Fig. 12 that the temperature in the fluid is decreased with increase in the value of the power–law index parameter. Increasing the values of the power–law index leads to thinning of the thermal boundary layer thickness. It can be found from Fig. 13 that the concentration of the fluid increases with increase in the value of the power–law index parameter. Increasing the power-law index (n) tends to retard the flow and increasing the solutal boundary-layer thickness.

The variation of the non-dimensional velocity $f'(\eta)$, temperature $\theta(\eta)$, and concentration $\phi(\eta)$ are plotted for $N = 0.5$, $n = 0.5$, $M = 1$, $Le = 0.5$, $G = 1$, $\varepsilon_1 = 0.2$, $\varepsilon_1 = 0.8$, $D_s = 0.1$, and $D_c = 0.3$ with varying X-location is shown in Figs. 14–16. It can be seen from Fig. 14 that the velocity of the fluid decreased near the plate and the influence is reversed far from the plate with increasing the value of the X-location. It can be found from Fig. 15 increase in the value of the X-location increases the temperature of the fluid in the medium. It can be seen from Fig. 16 that the concentration of the fluid is increased by increasing the value of the X-location.

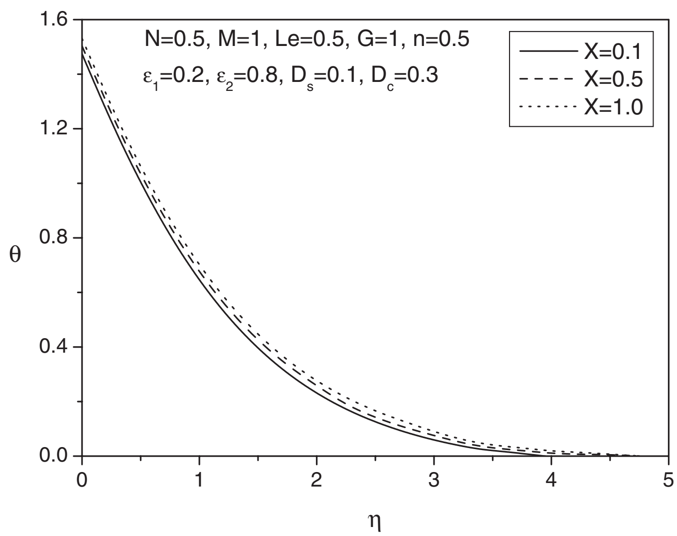

FIG. 15. Temperature profiles for various values of X.

5. CONCLUSIONS

In this paper, a boundary layer analysis for free convection heat and mass transfer along a vertical plate in a non-Darcy porous media saturated with power-law fluid with uniform heat and mass flux conditions in the presence of magnetic field, double dispersion, and double stratification is presented. Using the local-similarity variables, the governing equations are transformed into a set of ordinary differential equations where numerical solution has been presented for a wide range of parameters. The higher values of the thermal dispersion parameter result in higher velocity and temperature distributions but lower concentration distribution. The opposite behavior is seen for the solutal dispersion parameter. An increase in the values of the magnetic parameter result, in lower velocity distribution but higher temperature and concentration distributions. An increase in the values of the magnetic parameter results in lower velocity distribution but higher temperature and concentration distributions. Also, the higher values of the power-law index number result in lower velocity and temperature distributions but higher concentration distribution within the boundary layer.

REFERENCES

1. D.A. Nield and A. Bejan, Convection in Porous Media, 3rd ed., Springer-Verlag, NewYork, 2006.
2. H.T. Chen and C.K. Chen, Natural Convection of a Non-Newtonian Fluid about a Horizontal Cylinder and Sphere in a Porous Medium, Int. Commun. Heat Mass Trans., vol. 15, pp. 605–614, 1988.

3. J.P. Pascal and H. Pascal, Free Convection in a Non-Newtonian Fluid Saturated Porous Medium with Lateral Mass Flux, Int. J. Non-Linear Mech., vol. 32, pp. 471–482, 1997.
4. Y.J. Rami and S.M. Anna, Free convection Heat and Mass Transfer of Non-Newtonian Power Law Fluids with Yield Stress from a Vertical Flat Plate in a Saturated Porous Media, Int. Commun. Heat Mass Trans., vol. 27, pp. 485–494, 2000.
5. C.Y. Cheng, Natural Convection Heat and Mass Transfer of Non-Newtonian Power Law Fluids with Yield Stress in Porous Media from a Vertical Plate with Variable Wall Heat and Mass Fluxes, Int. Commun. Heat Mass Trans., vol. 33, pp. 1156–1164, 2006.
6. G.B. Kim and J.M. Hyun, Buoyant Convection of Power-law Fluid in an Enclosure Filled with Heat-generating Porous Media, Num. Heat Trans. Part A: Applications, vol. 45, pp. 569–582, 2004.
7. A.K. Sahu and M.N. Mathur, Free Convection in Boundary Layer Flows of Power Law Fluids Past a Vertical Flat Plate with Suction/injection, Indian J. Pure App. Math., vol. 27, p. 931, 1996.
8. A. Nakayama, Free Convection from a Horizontal Line Heat Source in a Power-law Fluid-saturated Porous Medium, Int. J. of Heat and Fluid Flow, vol. 14, pp. 279–283, 1993.
9. M. Kumari and G. Nath, Non-Darcy Mixed Convection in Power-law Fluids Along a Non-isothermal Horizontal Surface in a Porous Medium, Int. J. of Eng. Sci., vol. 42, pp. 353–369, 2004.
10. M.A. Mansour and N.A. El-Shaer, Effect of Magnetic Field on Non-Darcy Axisymmetric Free Convection in a Power-law Fluid Saturated Porous Medium with Variable Permeability, J. Magnet. Magnetic Materials, vol. 250, pp. 57–64, 2002.
11. O.D. Makinde and A.Aziz, MHD Mixed Convection from a Vertical Plate Embedded in a Porous Medium with a Convective Boundary Condition, Int. J. of Thermal Sci., vol. 49, pp. 1813–1820, 2010.
12. O.D. Makinde, MHD Mixed-convection Interaction with Thermal Radiation and nth Order Chemical Reaction Past a Vertical Porous Plate Embedded in a Porous Medium, Chem. Eng. Comm., vol. 198, pp. 590–608, 2011.
13. O. Anwar Beg and O.D. Makinde, Viscoelastic Flow and Species Transfer in a Darcian High-permeability Channel, J. Petroleum Sci. Eng., vol. 76, pp. 93–99, 2011.
14. R. Cortell, A note on Magnetohydrodynamic flow of a Power-law Fluid over a Stretching Sheet, Appl. Math. Comput., vol. 168, pp. 557–566, 2005.
15. P.V.S.N. Murthy, Effect of Double Dispersion on Mixed Convection Heat and Mass Transfer in Non-Darcy Porous Medium, Trans. ASME J. Heat Transf., vol. 122, pp. 476–484, 2000.
16. A.A. Mohammadien and M.F. El-Amin, Thermal Dispersion Radiation Effects on Non-Darcy Natural Convection in a Fluid Saturated Porous Medium, Transp. Porous Media, vol. 40, pp. 153–163, 2000.
17. P.V.S.N. Murthy, D. Srinivasacharya, and P.V.S.S.S.R. Krishna, Effect of Double Stratification on Free Convection in Darcian Porous Medium, Transactions of ASME, J. Heat Transf., vol. 126, pp. 297–300, 2004.