Embedded Cost Allocation Methods using Game Theory

P. Sri Divya, M.Murali, *Student Member, IEEE*, M.Sailaja Kumari, *Member, IEEE,* and M. Sydulu, *Member, IEEE*
Electrical Engineering Department,
National Institute of Technology Warangal,
Andhra Pradesh, India-506004
sridivya.nitw@gmail.com
murali233.nitw@gmail.com

Abstract—**In deregulated electricity markets there is a strong need for effective allocation of fixed costs to market participants. The conventional usage based methods currently employed in market scenario may fail to send right economic signals. Hence in this paper, cooperative game theory is applied for power system fixed cost allocation. Increasing competition in the energy market can help maximize customers' payoffs. This can be achieved by applying game theory. In this regard, two solution methodologies such as Nucleolus and Shapley value are adopted in a Multi-lateral market. Both the methods have their pros and cons, while it can be inferred that Shapley value is a more preferable method when the solution is in the core of the game. In this paper, these methods are applied in case of IEEE 14 bus, New England 39 bus and Indian 75 bus power system and the results obtained are compared with the conventional usage based methods.**

Keywords- core, coalition, cooperative game, imputation, nucleolus, payoff, shapley.

I. Introduction

IN the present day power markets, the issue of allocating fixed costs to the market participants is of great significance. Fixed costs make up the largest part of transmission charges; hence there is a great demand for a fair and effective allocation of these costs to the market participants [1, 2, 3]. By adopting policies which could allocate fixed costs to participants depending on their network usage, the isonomy introduced by law, can be cancelled. Aiming at this, several policies have been proposed [4]. The conventional usage based methods are advantageous from an engineering point of view, but they may fail to send right economic signals. The fixed cost allocation is a typical case where cooperation between the agents produces incentives and economies of scale. These benefits can in turn be shared among the network participants. Thus the cooperative game theory suggests reasonable fixed cost allocations that may be economically efficient as well as advantageous from engineering point of view.

P. Sridivya, M.Murali, M.Sailaja kumari, M.Sydulu are with the Electrical Engineering Department, National Institute of Technology, Warangal 506004, INDIA.

II. Game Theory

Game Theory is the formal study of decision making where several players must make choices that potentially affects the choice of other players. Thus, game theory deals with any problem in which each player's strategy depends on what other players do. It is assumed that the rationality of all players is of common knowledge. A player is said to be rational if he seeks to play in a manner which maximizes his own payoff. Payoff is nothing but the payment received at the end of game. It is mainly employed in power systems to prevent collusion due to market power i.e. discourage collusions that could minimize payoff [5]. The game theory methodologies can be used to identify non-competitive situations (from market co-coordinator point of view) and minimize the risks in price decisions (from participant's point). In this paper, Cooperative game theory is employed to allocate power system fixed cost.

III. Solution Methodologies in Cooperative Game Theory

A. Terminology

Consider a game of N players with a characteristic function v. These players can form 2^N coalitions including the φ coalition. The characteristic function assigns to each coalition 'S' the minimum payoff under any adverse conditions. This can be found by applying max-min criteria to S and (N-S) players. An introduction to the cooperative game theory is given in [2, 6]. The set of all possible distribution of payoffs to the participants are called Imputations. A payoff vector $y = (y_1, y_2, ...y_n)$ is an imputation if it holds the following two conditions:

$$\sum_{i=1}^{n} y_i = v(N) \tag{1}$$

$$y_i \geq v(i),\ i = 1, 2, ... n \tag{2}$$

There are numerous methods for the allocation of benefits among the participants of a cooperative game. Some of them are briefly described below:

B. The Core

One of the first solutions suggested for cooperative game is the core concept [7]. It is based on domination of

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imputations. That is, the core of a game is the set of all the imputations that are not dominated over any coalition.

For an imputation to belong to the core, it must satisfy

$$\sum_{i=1}^{n} y_i = v(N) \tag{3}$$

$$\sum_{i=1}^{n_s} y_i \geq v(S) \qquad \forall S \subset N \tag{4}$$

It is clear that the core may include one or more than one imputation or may be even empty. Thus to choose a single solution whenever the core is non empty, Nucleolus concept was introduced in [6, 8].

C. The Nucleolus

It is based on the idea of minimizing the dissatisfaction of the most dissatisfied groups. For a coalition S, measure of its dissatisfaction is the excess e(S):

$$e(S) = v(S) - y(S) \tag{5}$$

where $y(S) = \sum_{i=1}^{n_s} y_i$

Thus the larger the excess, the more dissatisfied the coalition is with this Imputation. Thus it reduces to the following optimization problem.

$$\min e \tag{6}$$

$$y(S) + e \geq v(S) \tag{7}$$

$$y(N) = v(N) \tag{8}$$

One main drawback of nucleolus is that it is not monotonic that is even though the characteristic function v(s) of a coalition is increased, the payoff to the members of this coalition is not affected.

D. The Shapley Value

For the foundation of Shapley value [8], three axioms have been settled.

i. Symmetry: $\phi_i(v)$ is independent of the labeling of the players.

$$\phi_{\Pi(i)}(v) = \phi_i(v) \tag{9}$$

ii. Efficiency: The sum of the expectations must be equal to the characteristic functional value for the grand coalition N.

$$\sum_{i=1}^{n} \phi_i(v) = v(N) \tag{10}$$

iii. Additivity: The sum of expectations, for a player, by playing two games with characteristic values v_1 and v_2 must be equal to the value if he played both games together.

$$\phi_i(v_1 + v_2) = \phi_i(v_1) + \phi_i(v_2) \tag{11}$$

Thus the Shapley value which satisfies three axioms is given by

$$\phi_i(v) = \sum_{S, i \in S} \frac{(n_s - 1)!(n - n_s)!}{n!} [v(S) - v(S - \{i\})] \tag{12}$$

Its main advantage is it exhibits monotonicity. However its main disadvantage is that it may or may not lie inside the core.

IV. Fixed Cost Allocation Using Usage based Methods

A. The Postage Stamp Method

The Postage Stamp method (Rolled in method) [2] allocates the fixed cost to the participants based on their magnitude of transaction whereas the power flows in the network are neglected. For a transaction 'i', the cost allocation is given by

$$PS_i = K * \left(P_i \Big/ \sum_{j=1}^{n} P_j\right) \tag{13}$$

where, PS_i= Amount charged to participant 'i'

K = Total fixed cost to be allocated

P_i = Transaction amount

n = Number of participants

B. The MW-Mile Method

Since the Postage Stamp method does not take power flows into account, it leads to cross-subsidization of long-distances transactions by short-distance transactions. Thus to overcome this problem, the MW-Mile method was implemented. In this method, the operator runs the power flow for each transaction and then calculates the actual power flows in all lines.

Thus for each transaction 'i'

$$f_{i,l} = C_l \left| P_{i,l} \right| \tag{14}$$

where, C_l = specific transfer cost of branch 'l'

$f_{i,l}$ =use of branch 'l' by participant 'i'

Thus the total network usage for 'nl' number of lines is given by

$$f_i = \sum_{l=1}^{nl} f_{i.,l} \tag{15}$$

Thus the cost allocation by MW-Mile method is given by

$$MWM_i = K * \left(f_i \Big/ \sum_{j=1}^{n} f_j\right) \tag{16}$$

C. The Zero Counter Flow Method

In this method, the direction of flow is also considered. Thus the change for each transaction 'i' is given as

$$f_{i,l} = \begin{cases} C_l P_{i,l} & P_{i,l} > 0 \\ 0 & P_{i,l} \le 0 \end{cases} \quad (17)$$

Thus the total network usage is given by

$$f_i = \sum_{l=1}^{nl} f_{i.,l} \quad (18)$$

Thus the cost allocation by ZCF is given by

$$ZCF_i = K * (f_i \Big/ \sum_{j=1}^{n} f_j) \quad (19)$$

But this method may fail to send right economic signals, i.e. is it is well established from engineering point of view but subsidizes the largest network users with comparatively smaller users due to the counter flows of former.

V. FIXED COST ALLOCATION USING COOPERATIVE GAME THEORY

A. Game definition

Many of the fixed cost allocation methods are based on the network usage from the side of market participants. The payment R_i allocated to each participant 'i' may be given by one of the following forms:

$$R_i = K * (f_i \Big/ \sum_j f_j) \quad (20)$$

$$R_i = f_i b \quad (21)$$

where b= cost of 1 MW power flow

B. General Algorithm

Step 1: Consider the number of possible coalitions 'S' that can be formed using the players of the game.

Step 2: Calculate the total fixed cost to be allocated by running a load flow with all the players present.

Step 3: Now calculate the characteristic function v(S), of each coalition by running a power flow and calculating 'f' values as in (15)

$$v(S) = \sum_{i=1}^{n_s} f_i - f_s \quad (22)$$

Step 4: Then using either Shapley or Nucleolus, allocate the savings to all the players to calculate y_i.

Step 5: The new calculated values will be

$$f_i' = \begin{cases} f_i - y_i & f_i > y_i \\ 0 & f_i < y_i \end{cases} \quad (23)$$

Step 6: Then the cost allocation R_i is done using (24)

$$R_i = K * (f_i' \Big/ \sum_{j=1}^{n} f_j') \quad (24)$$

When the electricity market operates in an environment of multilateral transactions then each transaction agent is responsible to pay a part of power system fixed cost. The formulation of a coalition between some players can be profitable by the existence of counter flows.

Case Study 1: The above algorithm is implemented in case of IEEE14 bus system [16]. The loads are aggregated based on their locational marginal prices (LMP) and then the generator outputs are obtained by running an OPF and thus the obtained transactions (Players) are as given in Table I.

TABLE I. TRANSACTION DATA OF IEEE14 BUS SYSTEM

Player	Power(MW)	S(j,k)	B(i)
1	29.3	(1,16.9419),(2,12.3803)	2,5
2	142	(1,75.24),(2,66.75)	3,4
3	30.8	(1,17.277),(2,13.522)	6,12,13
4	56.9	(1,27.06),(2,29.83)	9,10,11,14

where S(j,k)=Bus 'j' supplying load 'k' for transaction 'i'. B(i)=Load Buses. In the above table, row 1, the first transaction comprises of a total load of 29.3 MW (buses 2 and 5 are grouped together based on their LMP's evaluated using Power World software). This load is met by both generators with 1st generator generating 16.94 MW where as 2nd generator 12.38 MW.

By running a load flow for each transaction, the characteristic functional values are obtained and presented in table II. Table II shows the minimum amount which the coalition can assure itself. The last row shows the grand coalition in which all players are present, which assures maximum savings.

Table III shows the allocation of savings (171.72€) to the participants using nucleolus and Shapley. The nucleolus method favors a few players, as can be seen from table III that it favors second and third players, this can be overcome in Shapley method.

TABLE II. CHARACTERISTIC FUNCTIONAL VALUES OF IEEE 14 BUS SYSTEM

Coalition(S)	Characteristic function (v(S))
1	0
2	0
3	0
4	0
(1,2)	9.95
(1,3)	5.72
(1,4)	7.59
(2,3)	83.384
(2,4)	67.813
(3,4)	83.32
(1,2,3)	91.32
(1,2,4)	76.38
(1,3,4)	90.68
(2,3,4)	163.47
(1,2,3,4)	171.72

TABLE III. SAVINGS ALLOCATION USING VARIOUS METHODS FOR IEEE 14 BUS SYSTEM

Player	Nucleolus	Shapley
1	4.17	6.013
2	76.96	63.26
3	83.09	59.90
4	7.60	52.64

TABLE IV. COST ALLOCATION USING VARIOUS METHODS IN IEEE 14 BUS SYSTEM

Player	PS(€)	MWM(€)	ZCF(€)	Nucleolus(€)	Shapley Value(€)
1	119.00	27.04	29.685	24.297	25.42
2	576.73	416.73	371.99	407.67	431.802
3	125.09	220.93	223.88	173.07	196.68
4	231.10	387.23	428.02	443.88	398.03

Table IV shows the allocation of 1051.9371 € to four transactions using various methods. This cost is calculated by multiplying the power flows with their corresponding line lengths and line costs. Table IV shows that players 1, 3 and 4 are benefitted from cost allocation using Game Theory.

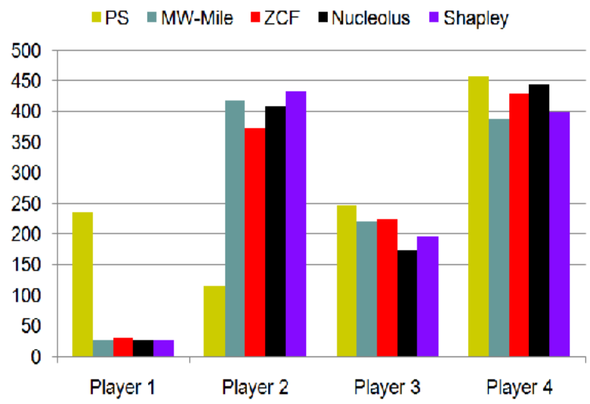

Fig. 1. Cost allocation using different methodologies for IEEE 14 bus system

From Fig.1, we can infer that the cost allocated using Game Theory is in tune with other fixed cost allocation methods.

Case Study 2: The developed algorithms are tested on New England 39 bus system [17]. Loads are aggregated to form 2 transactions as shown in table V.

TABLE V. TRANSACTIONS IN CASE OF NEW ENGLAND 39 BUS SYSTEM

Player	Power (MW)	S(j,k)	B(i)
1	4794.8	(30,137.5), (31,584.43), (32,565.0), (33,647.91),(34,607.85),(35,565.0), (36,538.12),(37,87.98),(38,642.50), (39,418.48).	3,4,7,8,12, 15,16,18,20, 21,23,24,27
2	1965.7	(30,63.20),(31,180.48),(32,194.81), (33,279.7),(34,205.63),(35,195),(36 ,172.5),(37,187.80),(38,204.06) (39,282.5)	25,26,28,29, 31,39

where, S(j,k)=Bus 'j' supplying load 'k' for transaction 'i', B(i)=Load Buses

TABLE VI. CHARACTERISTIC FUNCTIONAL VALUES FOR NEW ENGLAND 39 BUS SYSTEM

Coalition(S)	Characteristic function (v(S))
1	0
2	0
(1,2)	12174.16

Table VI shows the characteristic functional values in case of New England 39 bus system. These values are calculated using power flow method. Table VII shows the allocation of 64298.03 € to two transactions using various methods. This is calculated in a similar manner by running a load flow with all the players present.

TABLE VII. COST ALLOCATION USING VARIOUS METHODS IN NEW ENGLAND 39 BUS SYSTEM

Player	PS(€)	MWM(€)	ZCF(€)	Nuc(€)	Shap(€)
1	45602.5	37559	24909	39841.6	39841.6
2	18695.4	26739	39388	24456.3	24456.3

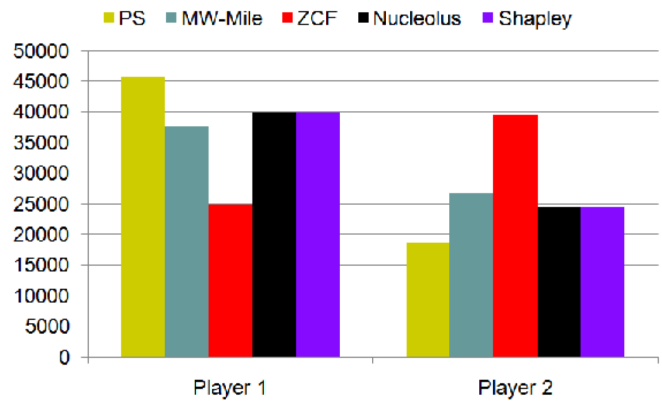

Fig. 2. Cost allocation using different methodologies for NEW ENGLAND 39 bus system

From the above results shown in Fig.2, we can see that by adopting Game theory we can allocate higher fixed cost to the largest network users, thus this allocation also mitigates market power. From table VII, the first player with higher transacted power is charged with a higher cost by adopting Game Theory. Hence the drawback of Zero Counter Flow method can be overcome.

Case Study 3: The algorithms are tested on a practical Indian 75 bus power system [18]. Loads are aggregated into 2 groups based on LMPs to form 2 transactions and are listed in table VIII. For each transaction the table also presents the respective generation allocations (S (j,k)). Table IX presents the corresponding characteristic functional values.

Table X shows the allocation of 14545.317584€ to two transactions using various methods. Here also we can see that the cost allocation is more rational (larger network user is charged higher Fixed Cost).

TABLE VIII. TRANSACTIONS IN CASE OF PRACTICAL INDIAN 75 BUS POWER SYSTEM

Player	Power (MW)	S(j,k)	B(i)
1	5199.26	(1,847.74),(2,331.63),(3,258.77),(4,25.91),(5,93.98),(6,205.75),(7,90.60),(8,68.56),(9,296.25),(10,62.00),(11,19.52),(12,1704.82),(13,806.88),(14,216.66),(15,170.1)	16,20,24,25,27,28 ,30,32,34 ,37,39,42,46 ,47 ,48,49 50 ,51,52,53,54 ,55,56,60,61, 62, 63,64 65 ,66,67,68,69, 70,71,72,73 ,74 ,75
2	590.23	(1, 64.57) ,(2, 21.02), (3, 25.89) (4, 77.86) ,(5,86.79), (6,64.86) (7,40.91) ,(8,16.60) ,(9,36.72) (10,10) ,(11, 53.09) , (12,56.14) (13,11.45) , (14, 12.93) (15,11.59)	57,58,59

where S(j,k)=Bus 'j' supplying load 'k' for transaction 'i', B(i)=Load Buses.

TABLE IX. CHARACTERISTIC FUNCTIONAL VALUES OF PRACTICAL INDIAN 75 BUS POWER SYSTEM

Coalition(S)	Characteristic function (v(S))
1	0
2	0
(1,2)	479.289063

TABLE X. COST ALLOCATION BY DIFFERENT METHODS IN CASE OF PRACTICAL INDIAN 75 BUS POWER SYSTEM

Player	PS(€)	MWM(€)	ZCF (€)	Nuc(€)	Shap(€)
1	13063	11337	11171	11587	11587
2	1483	3208	3374	2958	2958

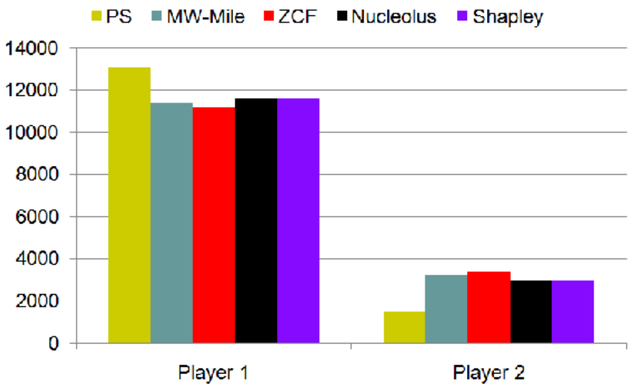

Fig. 3. Cost allocation using different methodologies for Indian 75 bus power system

Figure 3 shows the fixed cost allocated to the two transactions using different methodologies in Indian 75bus power system.

VI. FEATURES OF THE GAME

The two solution methodologies presented i.e. Nucleolus and Shapley have their own pros and cons. Greatest advantage of nucleolus is that when the core is non empty, the nucleolus is part of it and coalitional rationality is satisfied. However it is not monotonic as stated earlier. Consequently the adoption of nucleolus concept may fail to send right economic signals. However it has an added disadvantage, that it favors some players.

In order to overcome the above disadvantages, Shapley method is adopted. In contrast to nucleolus, the Shapley value is monotonic and there is an added disadvantage that it assigns a non zero payoff to every player due to the absence of dummy player. Due to the additivity axiom, the system operator may analyze the allocation process at each branch having at the end same result. However its main disadvantage is that it may or may not belong to the core. It all depends on the network topology, the number of players and their transaction patterns.

TABLE XI. COST ALLOCATION IN DIFFERENT CASES FOR IEEE 14 BUS SYSTEM

Player	C=0 S=0	C=0 S=1	C=1 S=0	C=1 S=1
1	27.045	25.421	244.19	221.09
2	416.73	431.80	418.71	432.45
3	220.93	196.68	D=3.828	D=1.009
4	387.232	398.029	389.07	398.38

$$C = \begin{cases} 0, \text{ No Coalition is formed} \\ 1, \text{ Coalition is formed} \end{cases}$$

$$S = \begin{cases} 0, \text{ Shapley value is not accepted} \\ 1, \text{ Shapley value is accepted} \end{cases}$$

From table XI we can see that players '1' and '3' benefit from the acceptance of a shapely value. Thus the other players cannot stop '1' and '3' from forming a coalition. The values obtained when the players '1, 3' form a coalition are tabulated as shown. In table XI $D = \sum_{i \in coal} R_i - R_{coal}$ shows how much each coalition saves within the framework if the allocation scheme used at each time.

From the above analysis we can draw the conclusion that no method can be said to be undoubtedly the best. Nucleolus has the great advantage of being part of the core but it lacks monotonicity. On the other hand, the Shapley value is monotonic but it need not be part of the core. However when it is part of the core, it is more preferable to nucleolus.

VII. CONCLUSIONS

Cooperative Game Theory is implemented for fixed cost allocation and the results obtained are compared with conventional usage methods. We can infer that ZCF method fails to send right economic signals by allocating larger part of the fixed cost to the smaller magnitude transaction. This can be overcome by using Nucleolus or Shapley value.

In case of a pool market, negative characteristic function values may arise if the game is played at each branch. However for a bilateral transaction, the fixed cost allocation can take place at the system branch or in the entire network.

Further, the paper presented the performance of different embedded cost based allocation methods. The cooperative Game Theory helps in market power mitigation and sends right economic signals. Thus this can be widely implemented in present power system scenario. The cooperative game theory can also be further extended for loss and congestion cost allocation.

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