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Analytical solution of mixed convection flow of couple stress fluid between two circular cylinders with Hall and ion-slip effects

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Abstract

This paper presents the Hall and ion-slip effects on electrically conducting couple stress fluid flow between 2 circular cylinders in the presence of a temperature dependent heat source. The governing non-linear partial differential equations are transformed into a system of ordinary differential equations using similarity transformations and then solved using the homotopy analysis method (HAM). The effects of the magnetic parameter, Hall parameter, ion-slip parameter, and couple stress fluid parameter on velocity and temperature are discussed and shown graphically.

Key Words: Mixed convection, couple stress fluid, hall and ion-slip effects, circular cylinders, HAM

1. Introduction

Mixed convection heat transfer and fluid flow in an annulus between 2 vertical concentric cylinders have been the focus of investigation for many decades due to their wide range of practical applications such as electrical machineries where heat transfer occurs in the annular gap between the rotor and stator, growth of single silicon crystals, heat exchangers, cooling systems for electronic devices, solar collectors, and other rotating systems (Aung et al., 1987; Jackson et al., 1989). Maitra and Raju (1975) investigated the fully developed flow with the inner wall heated and outer wall being adiabatic. Rokerya and Iqbal (1971) investigated the effect of viscous dissipation on mixed convection through a vertical annulus. Kou and Huang (1997) solved the problem of fully developed laminar mixed convection through a vertical annular duct embedded in a porous medium.

In recent years, several simple flow problems associated with classical hydrodynamics have received new attention within the more general context of magnetohydrodynamics (MHD). Several investigators have extended many of the available hydrodynamic solutions to include the effects of magnetic fields for those cases when the fluid is electrically conducting. Interest in rotating hydro-magnetic flow in annular spaces was initiated in the late 1950s with an important analysis by Globe (1959), who considered fully developed laminar MHD flow in an annular channel. Jain and Mehta (1962) examined wall suction/injection effects on the Globe problem. Antimirov et al. (1976) studied unsteady MHD convection in a vertical channel and Borkakati et al. (1984)

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considered the MHD heat transfer in the flow between 2 coaxial cylinders. Analytic solutions for MHD flow in an annulus were investigated by Hayat et al. (2010). In most of the MHD flow problems, the Hall and ion-slip terms in Ohm's law were ignored. However, in the presence of a strong magnetic field, the influence of Hall current and ion-slip is important. Tani (1962) studied the Hall effects on the steady motion of electrically conducting viscous fluid in channels. Hall and ion-slip effects on MHD Couette flow with heat transfer have been considered by Soundelgekar et al. (1979). Attia (2005) considered the steady Couette flow of an electrically conducting viscous incompressible fluid between 2 parallel horizontal non-conducting porous plates with heat transfer, taking the ion-slip into consideration. An exact solution of the Hall effect on the pipe flow of a Burgers' fluid was presented by Hayat et al. (2009). Many of the problems in the literature deal with MHD flow between parallel plates/flow through circular pipes with Hall and ion-slip effects, but not much attention has been given to the flow through a closed rectangular channel and concentric cylinders.

In recent years the study of convection heat and mass transfer in non-Newtonian fluids has received much attention and this is because the traditional Newtonian fluids cannot precisely describe the characteristics of real fluids. In addition, considerable progress has been made in the study of heat and mass transfer in magnetohydrodynamic flow of non-Newtonian fluids due to its application in many devices, like the MHD power generator, aerodynamics heating, electrostatic precipitation, and Hall accelerator. A number of theories have been proposed to explain the behavior of non-Newtonian fluids. Among these, couple stress fluids introduced by Stokes (1966) have distinct features, such as the presence of couple stresses, body couples, and non-symmetric stress tensor. The couple stress fluid theory presents models for fluids whose microstructure is mechanically significant. The effect of a very small microstructure in a fluid can be felt if the characteristic geometric dimension of the problem considered is of the same order of magnitude as the size of the microstructure. The main feature of couple stresses is to introduce a size dependent effect. Classical continuum mechanics neglects the size effect of material particles within the continua. This is consistent with ignoring the rotational interaction among particles, which results in symmetry of the force-stress tensor. However, in some important cases such as fluid flow with suspended particles, this cannot be true and a size dependent couple-stress theory is needed. The spin field due to microrotation of freely suspended particles set up an antisymmetric stress, known as couple-stress, and thus forming couple-stress fluid. These fluids are capable of describing various types of lubricants, blood, suspension fluids etc. The study of couple stress fluids has applications in a number of processes that occur in industry such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, and colloidal solutions. Recently mixed convection in a couple stress fluid with Soret and Dufour effects was studied by Srinivascharya and Kaladhar (2011).

Keeping in view the practical applications mentioned above, it is the objective of this paper to investigate the Hall and ion-slip effects on steady mixed convective heat transfer flow between 2 concentric cylinders in couple stress fluid. The homotopy analysis method is employed to solve the nonlinear problem. The homotopy analysis method (HAM), introduced by Liao (2003), is one of the most efficient methods in solving different types of nonlinear equations such as coupled, decoupled, homogeneous, and non-homogeneous. HAM also provides us with great freedom to choose different base functions to express solutions of a nonlinear problem (Liao, 2004).

2. Formulation of the problem

Consider a steady, laminar, incompressible, and electrically conducting couple stress fluid between 2 coaxial concentric circular cylinders of radii a and b $(a < b)$. Choose the cylindrical polar coordinate system $(r, \varphi,$

z) with z-axis as the common axis for both cylinders. The inner cylinder is at rest and the outer cylinder is rotating with constant angular velocity. The flow is generated due to the rotation of the outer cylinder. Since the flow is fully developed and the cylinders are of infinite length, the flow depends only on r. The inner and outer cylinders are held at different temperatures, T_a and T_b respectively ($T_a < T_b$). A uniform magnetic field (B_0) is applied in the axial direction. Assume that the magnetic Reynolds number is very small so that the induced magnetic field can be neglected in comparison with the applied magnetic field. The electron-atom collision frequency is assumed to be relatively high, so that the Hall effect and the ion-slip cannot be neglected. Further, assume that all the fluid properties are constant except the density in the buoyancy term of the balance of momentum equation. The flow is a mixed convection flow taking place under thermal buoyancy and uniform pressure gradient in azimuthal direction. With the above assumptions, the equations governing the mixed convection flow of an couple stress fluid under usual MHD approximations are

$$\frac{\partial p}{\partial r} = \frac{u}{r^2} - \frac{\sigma B_0^2 \beta_h}{\alpha_e^2 + \beta_h^2} u \tag{1}$$

$$\eta_1 \nabla_1^4 u - \mu \nabla_1^2 u - \rho g \beta_T (T - T_a) + \frac{\sigma B_0^2 \alpha_e}{\alpha_e^2 + \beta_h^2} u + \frac{1}{r}\frac{\partial p}{\partial \phi} = 0 \tag{2}$$

$$K_f \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] + \mu \left[\left(\frac{\partial u}{\partial r}\right)^2 - 2\frac{u}{r}\frac{\partial u}{\partial r} + \left(\frac{u}{r}\right)^2\right] + \eta_1 \left(\nabla_1^2 u\right)^2 + \gamma_0 \Omega (T - T_a) = 0 \tag{3}$$

where u is the velocity component of the fluid in the direction of φ, p is the pressure, ρ is the density, μ is the coefficient of viscosity, σ is the electrical conductivity, β_h is the Hall parameter, β_i is the ion-slip parameter, $\alpha_e = 1 + \beta_i \beta_h$, β_T is the coefficient of thermal expansion, K_f is the coefficient of thermal conductivity, η_1 is the couple stress fluid parameter, γ_0 is the constant of proportionality, $\gamma_0 \Omega(T - T_a)$ is the amount of heat generated per unit volume in unit time, which is assumed to be a linear function of temperature, and $\nabla_1^2 u = \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru)\right]$

The boundary conditions are given by

$$u = 0 \text{ at } r = a, u = b\Omega \text{ at } r = b \tag{4a}$$

$$\nabla_1^2 u = 0 \text{ at } r = a \text{ and } r = b \tag{4b}$$

$$T = T_a \text{ at } r = a \text{ and } T = T_b \text{ at } r = b \tag{4c}$$

The boundary condition (4a) corresponds to the classical no-slip condition from viscous fluid dynamics. The boundary condition (4b) implies that the couple stresses are zero at the surfaces.

Introducing the following similarity transformations

$$r = b\sqrt{\lambda}, u = \frac{\Omega}{\sqrt{\lambda}}f(\lambda), T - T_a = (T_b - T_a)\theta, p = \frac{\Omega\mu}{b}P \tag{5}$$

in Eqs. (2)-(3), we get the following system of nonlinear differential equations:

$$\alpha^2 \left[\lambda^2 f^{(iv)} + 2\lambda f'''\right] - \frac{1}{4}\lambda f'' - \frac{1}{16}\frac{Gr}{Re}\sqrt{\lambda}\theta + \frac{1}{16}\frac{Ha^2\alpha_e}{\alpha_e^2 + \beta_h^2}f + A = 0 \tag{6}$$

$$\left[\lambda^3\theta'' + \lambda^2\theta'\right] + Br\left[\lambda^2\left(f'\right)^2 - \frac{3}{2}\lambda ff' + \frac{3}{4}f^2\right] + 4Br\alpha^2\lambda^3\left(f''\right)^2 + \frac{1}{4}\gamma_1 Re\,Pr\,\lambda^2\theta = 0 \tag{7}$$

where primes denote differentiation with respect to λ alone, $Re = \frac{\Omega b}{\nu}$ is the Reynolds number, $G = \frac{g\beta_T(T_b - T_a)\,b^3}{\nu^2}$ is the Grashof number, $Pr = \frac{\mu C_P}{K_f}$ is the Prandtl number, $Ha = B_0 b\sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number, $Br = \frac{\mu\,\Omega^2}{K_f(T_b - T_a)}$ is the Brinkman number, $\gamma_1 = \frac{\gamma_0 b}{\rho C_P}$ is the dimensionless vertical distance, $A = \frac{\partial P}{\partial \phi}$ is the constant pressure gradient, $\alpha = \frac{1}{b}\sqrt{\frac{\eta_1}{\mu}}$ is the couple stress parameter, and the effects of couple-stress are significant for large values of $\alpha\,(= l/d)$, where $l = \sqrt{\frac{\eta_1}{\mu}}$ is the material constant. If l is a function of the molecular dimensions of the liquid, it will vary greatly for different liquids. For example, the length of a polymer chain may be a million times the diameter of water molecule (Stokes, 1966).

Boundary conditions (4) in terms of f and θ become

$$
\begin{aligned}
f = 0,\ f'' = 0,\ \theta = 0 \quad &\text{at} \quad \lambda = \lambda_0 \\
f = b,\ f'' = 0,\ \theta = 1 \quad &\text{at} \quad \lambda = 1
\end{aligned}
\tag{8}
$$

where $\lambda_0 = \left(\frac{a}{b}\right)^2$

3. The HAM solution of the problem

For HAM solutions, we choose the initial approximations of $f(\eta)$ and $\theta(\eta)$ as follows:

$$f_0(\lambda) = \frac{b}{1 - \lambda_0}\left(\lambda - \lambda_0\right), \quad \theta_0(\lambda) = \frac{\lambda - \lambda_0}{1 - \lambda_0} \tag{9}$$

and choose the auxiliary linear operators:

$$L_1(f) = f^{(iv)},\ L_2(\theta) = \theta'' \tag{10}$$

such that

$$L_1\left(c_1 + c_2\lambda + c_3\lambda^2 + c_4\lambda^3\right) = 0,\ L_2\left(c_5 + c_6\lambda\right) = 0 \tag{11}$$

where $c_i\ (i = 1, 2, ..., 6)$ are constants. Introducing non-zero auxiliary parameters h_1 and h_2, we develop the zeroth-order deformation problems as follows:

$$(1 - p)\,L_1\left[f(\lambda; p) - f_0(\lambda)\right] = p h_1 N_1\left[f(\lambda; p)\right] \tag{12}$$

$$(1 - p)\,L_2\left[\theta(\lambda; p) - \theta_0(\lambda)\right] = p h_2 N_2\left[\theta(\lambda; p)\right] \tag{13}$$

subject to the boundary conditions

$$
\begin{aligned}
f(\lambda_0; p) = 0,\ f''(\lambda_0; p) = 0,\ f(1; p) = b \\
f''(1; p) = 0,\ \theta(\lambda_0; p) = 0,\ \theta(1; p) = 1
\end{aligned}
\tag{14}
$$

where $p\varepsilon\ [0,\ 1]$ is the embedding parameter and the non-linear operators N_1 and N_2 are defined as:

$$N_1\left[f(\lambda; p), \theta(\lambda; p)\right] = \alpha^2\left[\lambda^2 f^{(iv)} + 2\lambda f'''\right] - \frac{1}{4}\lambda f'' - \frac{1}{16}\frac{Gr}{Re}\sqrt{\lambda}\theta + \frac{1}{16}\frac{Ha^2\alpha_e}{\alpha_e^2 + \beta_h^2}f + A \tag{15}$$

229

$$N_2\left[f(\lambda;p),\theta(\lambda;p)\right] = \left[\lambda^3\theta'' + \lambda^2\theta'\right] + Br\left[\lambda^2\left(f'\right)^2 - \frac{3}{2}\lambda ff' + \frac{3}{4}f^2\right] + 4Br\alpha^2\lambda^3\left(f''\right)^2 + \frac{1}{4}\gamma_1 Re\,Pr\,\lambda^2\theta \quad (16)$$

For $p = 0$ we have the initial guess approximations

$$f(\lambda;0) = f_0(\lambda),\ \theta(\lambda;0) = \theta_0(\lambda) \tag{17}$$

When $p = 1$, Eqs. (12)-(13) are same as (6)-(7), respectively; therefore at $p = 1$ we get the final solutions

$$f(\lambda;1) = f(\lambda),\quad \theta(\lambda;1) = \theta(\lambda) \tag{18}$$

The initial guess approximations $f_0\ (\eta)$ and $\theta_0\ (\eta)$, the linear operators L_1, L_2 and the auxiliary parameters h_1 and h_2 are assumed to be selected such that Eqs. (12)-(14) have a solution at each point $p\varepsilon\ [0,\ 1]$ and also with the help of Taylor's series and due to Eq. (17), $f(\lambda;p)$ and $\theta\ (\lambda;p)$ can be expressed as

$$f(\lambda;p) = f_0(\lambda) + \sum_{m=1}^{\infty} f_m(\lambda)p^m \tag{19}$$

$$\theta(\lambda;p) = \theta_0(\lambda) + \sum_{m=1}^{\infty} \theta_m(\lambda)p^m \tag{20}$$

where

$$f_m(\lambda) = \frac{1}{m!}\frac{\partial^m f(\lambda;p)}{\partial p^m},\ \ \theta_m(\lambda) = \frac{1}{m!}\frac{\partial^m \theta(\lambda;p)}{\partial p^m} \tag{21}$$

in which h_1 and h_2 are chosen in such a way that the series (19)-(20) are convergent at $p = 1$. Therefore we have from (18) that

$$f(\lambda) = f_0(\lambda) + \sum_{m=1}^{\infty} f_m(\lambda) \tag{22}$$

$$\theta(\lambda) = \theta_0(\lambda) + \sum_{m=1}^{\infty} \theta_m(\lambda) \tag{23}$$

Differentiating the zeroth-order deformation Eqs. (12)-(13), m-times with respect to p and then dividing them by $m!$ and finally setting $p = 0$, we obtain the following m^{th}-order deformation problem:

$$L_1\left[f_m(\lambda) - \chi_m f_{m-1}(\lambda)\right] = h_1 R_m^f(\lambda) \tag{24}$$

$$L_2\left[\theta_m(\lambda) - \chi_m\theta_{m-1}(\lambda)\right] = h_2 R_m^\theta(\lambda) \tag{25}$$

with the boundary conditions

$$f_m(\lambda_0) = 0,\ f_m(1) = 0,\ f_m''(\lambda_0) = 0,\ f_m''(1) = 0,\ \theta_m(\lambda_0) = 0,\ \theta_m(1) = 0 \tag{26}$$

where

$$R_m^f = \alpha^2\left[\lambda^2 f^{(iv)} + 2\lambda f'''\right] - \frac{1}{4}\lambda f'' - \frac{1}{16}\frac{Gr}{Re}\sqrt{\lambda}\theta + \frac{1}{16}\frac{Ha^2\alpha_e}{\alpha_e^2 + \beta_h^2}f + A \tag{27}$$

$$R_m^\theta = \left[\lambda^3\theta'' + \lambda^2\theta'\right] + Br\left[\lambda^2\sum_{n=0}^{m-1}f'_{m-1-n}f'_n - \frac{3}{2}\lambda\sum_{n=0}^{m-1}f_{m-1-n}f'_n + \frac{3}{4}\sum_{n=0}^{m-1}f_{m-1-n}f_n\right]$$
$$+4Br\alpha^2\lambda^3\sum_{n=0}^{m-1}f''_{m-1-n}f''_n + \frac{1}{4}\gamma_1 Re\,Pr\,\lambda^2\theta \tag{28}$$

and, for m being an integer

$$\begin{aligned}\chi_m &= 0\,for\,m \le 1\\ &= 1\,for\,m > 1\end{aligned} \tag{29}$$

We emphasize here that Eqs. (24)-(25) are linear for all $m \ge 1$. These linear equations are solved using MATHEMATICA for the first 15 values of m and the expressions for $f(\lambda)$ and $\theta(\lambda)$ are calculated. As the expressions for $f(\lambda)$ and $\theta(\lambda)$ are too long, they are not presented here.

4. Results and discussion

The expressions for f and θ contain the auxiliary parameters h_1 and h_2. As pointed out by Liao (2003), the convergence and the rate of approximation for the HAM solution strongly depend on the values of auxiliary parameter h. For this purpose, h-curves are plotted by choosing h_1 and h_2 in such a manner that the solutions (21)-(22) ensure convergence (Liao, 2003). Here to see the admissible values of h_1 and h_2, the h-curves are plotted for 15th-order of approximation in Figures 1 and 2 by taking the values of the parameters $Br = 0.5$, $Pr = 0.71$, $\gamma_1 = 1$, $Re = 2$, $Gr/Re = 5$, $a = 0.5$, $b = 1$, $\beta_h = 2$, $\beta_i = 2$, $\alpha = 0.5$. It is clearly noted from Figure 1 that the range for the admissible values of h_1 is $-0.55 < h_1 < 0$. From Figure 2, it can be seen that the h-curve has a parallel line segment that corresponds to a region $-0.25 < h_2 < -0.15$. It is found from computation that the series given by (21)-(22) converge in the whole region of λ when $h_1 = -0.5$ and $h_2 = -0.2$.
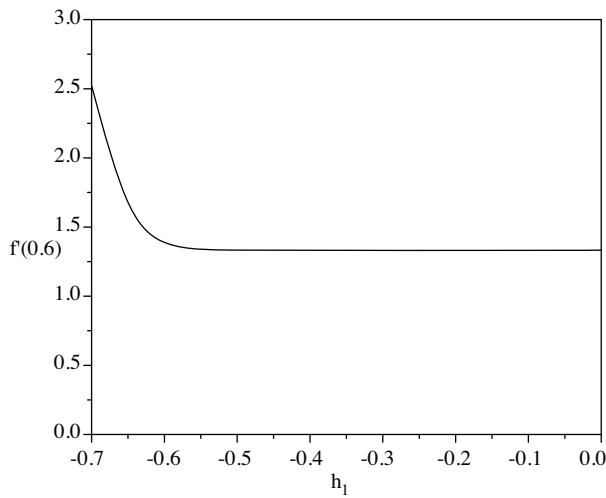
Figure 1. h curve for $f(\lambda)$ at $\beta_h = 2$, $\beta_i = 2$, $\gamma = 1.0$, $Ha = 20$.
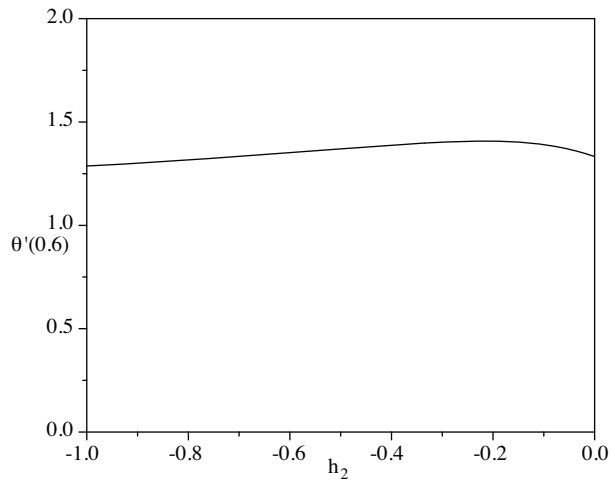
Figure 2. h curve for $\theta(\lambda)$ at $\beta_h = 2$, $\beta_i = 2$, $\gamma = 1.0$, $Ha = 20$.

The solutions for $f(\lambda)$ and $\theta(\lambda)$ have been computed and are shown graphically in Figures 3 to 10. The effects of magnetic parameter (Ha), Hall parameter (β_h), ion-slip parameter (β_i), and couple stress fluid parameter (α) have been discussed. To study the effect of Ha, β_h, β_i and α, computations were carried out

231

by taking $Br = 0.5$, $Pr = 0.71$, $\gamma_1 = 1$, $Re = 2$, $Gr/Re = 5$, $a = 0.5$, $b = 1$.

Figures 3 and 4 display the effect of the magnetic parameter Ha on $f(\lambda)$ and $\theta(\lambda)$. It can be observed from these figures that the velocity $f(\lambda)$ decreases and the temperature $\theta(\lambda)$ increases with an increase in the parameter Ha. This happens because of the imposing of a magnetic field normal to the flow direction. This magnetic field gives rise to a resistive force and slows down the movement of the fluid.
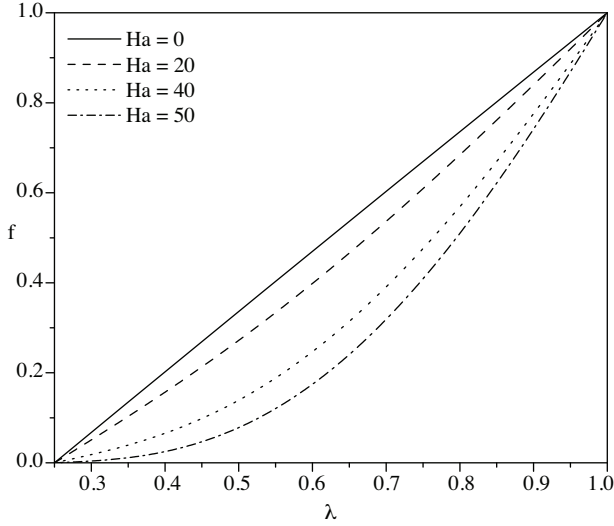
Figure 3. Effect of Ha on f at $\beta_h = 2$, $\beta_i = 2$.
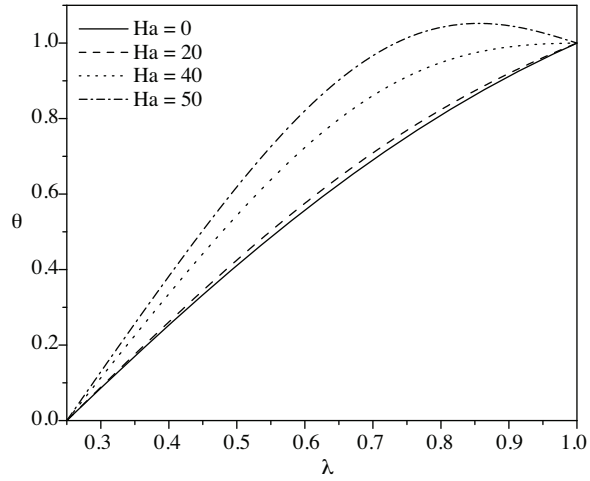
Figure 4. Effect of Ha on θ at $\beta_h = 2$, $\beta_i = 2$.

Figures 5 and 6 show the variation in velocity $f(\lambda)$ and temperature $\theta(\lambda)$ for several values of β_h. We see that the dimensionless velocity $f(\lambda)$ increases with increasing β_h. Figure 6 shows that the temperature $\theta(\lambda)$ decreases as β_h increases. The inclusion of the Hall parameter decreases the resistive force imposed by the magnetic field due to its effect in reducing the effective conductivity. Hence, the velocity component $f(\lambda)$ increases as the Hall parameter increases and the temperature $\theta(\lambda)$ decreases as β_h increases.
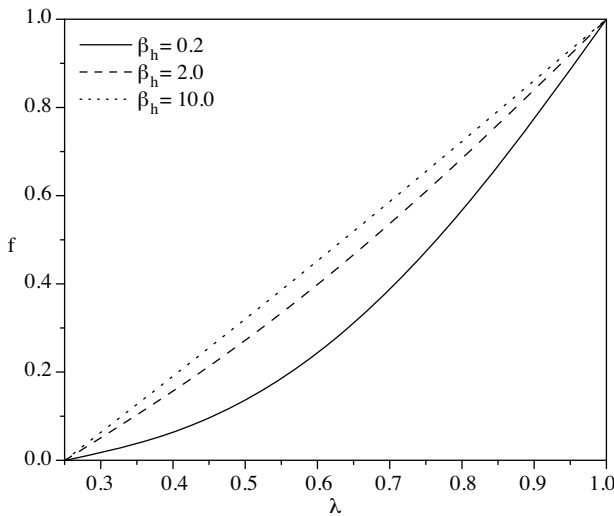

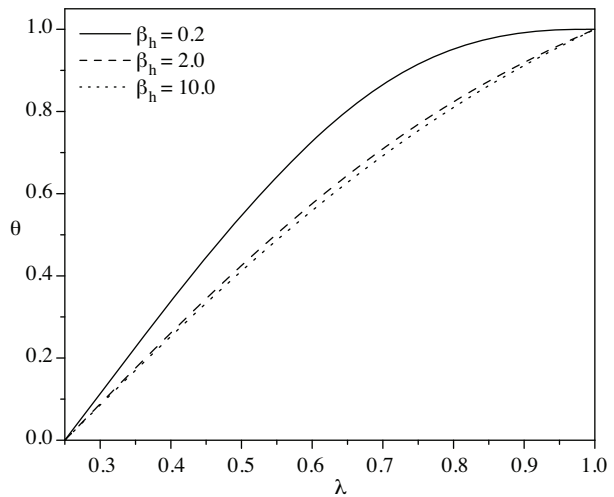**Figure 5.** Effect of β_h on f at $\beta_i = 2$, $\alpha = 0.5$, $Ha = 20$.

Figure 6. Effect of β_h on θ at $\beta_i = 2$, $\alpha = 0.5$, $Ha = 20$.

Figures 7 and 8 represent the effect of the ion-slip parameter β_i on $f(\lambda)$ and $\theta(\lambda)$. It can be seen from these figures that the velocity $f(\lambda)$ increases with an increase in the parameter β_i. The temperature $\theta(\lambda)$ decreases as β_i increases. As β_i increases the effective conductivity also increases, in turn decreasing the damping force on the velocity component in the direction of the flow and hence the velocity component in the flow direction increases.
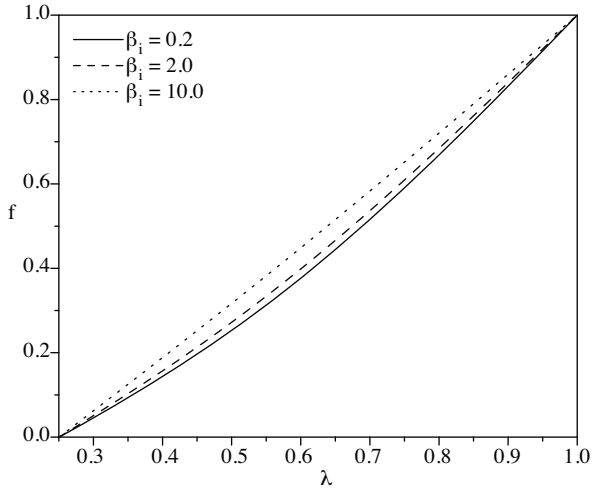

Figure 7. Effect of β_i on f at $\beta_h = 2.0$, $\alpha = 0.5$, $Ha = 20$.
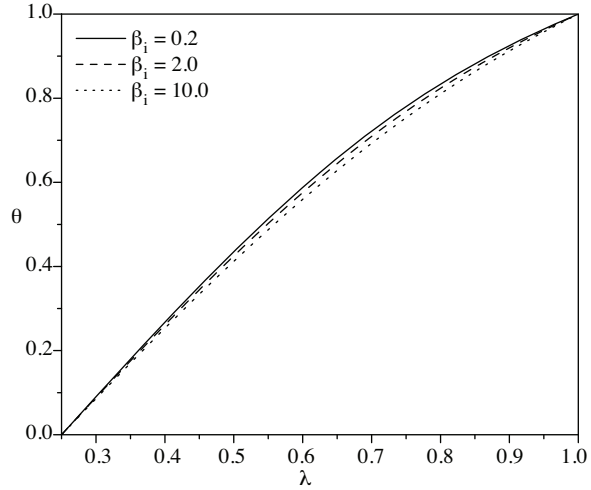

Figure 8. Effect of β_i on θ at $\beta_h = 2.0$, $\alpha = 0.5$, $Ha = 20$.

Figures 9 and 10 indicate the effect of the couple stress fluid parameter α on $f(\lambda)$ and $\theta(\lambda)$. As the couple stress fluid parameter α increases, the velocity increases. It is also clear that the temperature $\theta(\lambda)$ increases with an increase in α. Thus, the presence of couple stresses in the fluid increases the velocity and temperature.
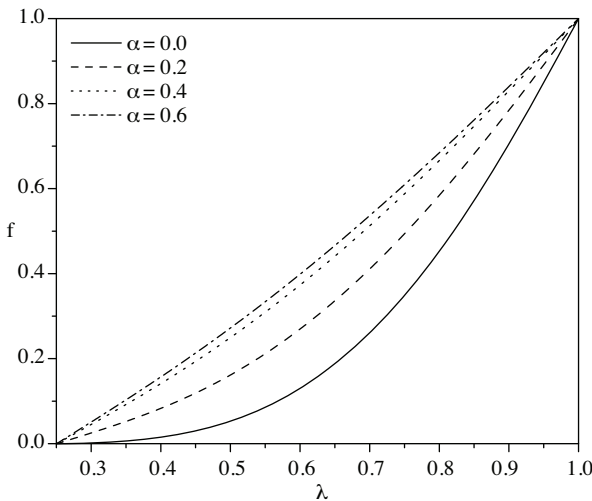
**Figure 9.** Effect of (α) on f at $\beta_h = 2$, $\beta_i = 2$, $Ha = 20$.
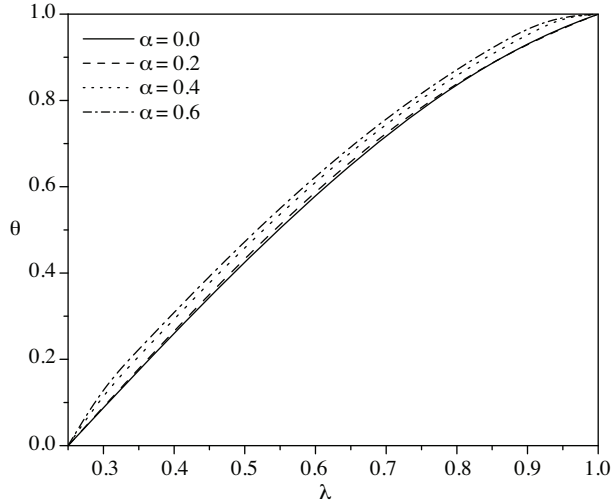
Figure 10. Effect of (α) on θ at $\beta_h = 2$, $\beta_i = 2$, $Ha = 20$.

Figure 11 depicts the special case of velocity distribution when the gap between 2 cylinders is very small. It is seen that narrow annulus yields a linear velocity distribution. Therefore, if the gap between the cylinders

is very small then the velocity distribution is similar to that of classical Couette flow (Schlichting et al., 2003).
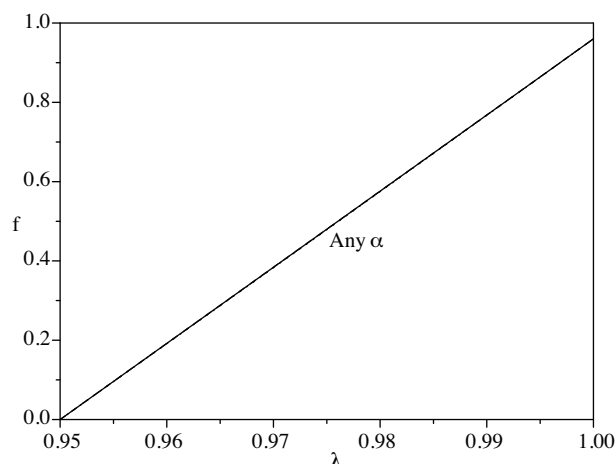
Figure 11. Velocity profile for the special case of (classical) Couette flow.

5. Conclusions

In this paper, the Hall and ion-slip effects on fully developed electrically conducting couple stress fluid flow between 2 concentric cylinders have been studied. Using similarity transformations, the governing equations have been transformed into non-linear ordinary differential equations. The similarity solutions are obtained numerically applying HAM (Liao, 2003). From the present study we see that the presence of a magnetic field decreases the velocity and increases the temperature. The inclusion of Hall and ion-slip currents in the flow increases the velocity and decreases the temperature. It is also noted that the presence of couple stresses in the fluid increases the velocity and temperature.

Nomenclature

A	constant pressure gradient.
Br	Brinkman number.
C_p	specific heat at constant pressure.
f	reduced stream function.
Gr	Grashof number.
Ha	Hartmann number.
K_f	thermal conductivity of the fluid.
P	pressure.
Pr	Prandtl number.
Re	Reynolds number.
T	temperature.
u	velocity components in the ϕ direction.

Greek symbols

α	couple stress fluid parameter.
β_i	ion-slip parameter.
β_h	Hall parameter.
β_T	coefficients of thermal expansion.
λ	similarity variable.
η_1	coupling material constant.
γ_0	proportionality constant.
γ_1	dimensionless vertical distance.
σ	electrical conductivity.
θ	dimensionless temperature.
μ	dynamic viscosity.
ν	kinematic viscosity.
ρ	density of the fluid.

Subscripts

C	concentration
T	temperature

Superscript

'	differentiation with respect to λ

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