DOA Estimation of IR-UWB Signals using Coherent Signal Processing

Santhosh Kumar D., I. S. Hinduja and V. V. Mani
Dept. of Electronics and Communication Engineering
National Institute of Technology
Warangal, India
Email: *skumar@nitw.ac.in* *ivshinduja@gmail.com* *vvmani@nitw.ac.in*

***Abstract*—In this paper coherent signal subspace method (CSM) is used to estimate the direction of arrival (DOA) of ultra wide band (UWB) signals in a multipath, non line of sight (NLOS) environment. As the name suggests it focuses the signal space of the wideband signal into a narrow subspace and the pseudospectrum of the resultant focused signal is calculated. The focusing frequency is calculated by minimizing the subspace fitting error. The frequency hence obtained gives better estimation results as compared to the case where the mid frequency of the spectrum is taken as the focusing frequency. This method proves to be the best choice for a wideband signal as it involves the multiple signal classification (MUSIC) algorithm, which has this innate ability to seperate multipath channels.**

Keywords: DOA, focusing matrix, weighted signal subspace, MUSIC, optimum focusing frequency

I. INTRODUCTION

UWB has been generating widespread interest because unlike the carrier based modulation, it is based on pulsed radio. This above mentioned characterstic of UWB makes it less complex. Its enhanced ability of signal propogation [1], through walls and high temporal resolution can be attributed to high frequency and large bandwidth respectively. Due to the aforementioned characteristics UWB is also capable of multipath mitigation, making it suitable for indoor localization and radar sensing applications. UWB has an added advantage in terms of interference mitigation. The spreading of the pulses over a large bandwidth and the subsequent lowering of the power spectral density (PSD) reduces interference to other systems. So far work has been done in the field of indoor positioning of UWB nodes using the direction of arrival (DOA) techniques [2] under LOS multipath channel conditions. Joint TOA and DOA estimation has been proposed in [3]. The work was extended in [4] where in the author has shown the effect of the number of recieving elements on estimation accuracy, with the help of ranging estimation error graphs. DOA plays a crucial role in beamforming and its estimation technique under ideal channel single path conditions is proposed in [5]- [6]. The following paper is organized as follows. Section II mathematically explains the transmitted time hopped-pulse position modulated (TH-PPM) UWB signal. In addition to that it also explains the nature of channels that constitute the IEEE 802.15.3a channel model based on Saleh-Valenzuela model. Section III will take you through the CSM that has been adopted to estimate the DOA. The DOA estimation graphs are given in Section IV. The paper is concluded in Section V.

II. SYSTEM AND CHANNEL MODEL

Impulse radio ultra wideband (IR-UWB) pulse (Fig.1) transmitted is of sub nanosecond duration. The UWB monocycle [7] is the second order derivative of a Gaussian pulse. We consider a TH-PPM version of the UWB monocycle [8] given by

$$s_q(t) = \sum_j P(t - jT_f - c_j^q * T_c - \delta * d^q) \tag{1}$$

j is the frame number
q is the user number
T_f is the duration of a frame
T_c is the chip duration
δ duration by which the pulse is to be shifted if the bit transmitted is 1
d^q is the symbol of the q^{th} user.
c_j^q is the code corresponding the j^{th} frame and q^{th} user.
The IEEE 802.15.3a channel model is a stochastic channel model based on the S-V model [9] where multipath components arrive in clusters. The S-V model representation of the channel is given by

$$h(t) = X \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} \alpha_{k,m} exp(j\theta_{k,m}) \delta(t - T_m - \tau_{k,m}) \tag{2}$$

where T_m :arrival time of the m^{th} cluster.
$\tau_{k,m}$:time delay of the k^{th} ray in the m^{th} cluster.
$\theta_{k,m}$:uniformly distributed phase in the range of [0,2π].
$\alpha_{k,m}$:multipath gain coefficients of the k^{th} ray in the m^{th} cluster.
X:log-normal shadow fading of the total multipath.

Paths in CM1 and CM2 tend to concentrate in two or three clusters with a smaller delay, whereas in CM3 and CM4 paths tend to spread over more clusters and have bigger delay. This is due to their different cluster arrival rates. The gains of paths in CM3 and CM4 decay faster in comparison to CM2 (Fig.2). The TH-PPM UWB pulse given in (1) convolved with the channel impulse response in (2) gives the following equation.

$$x(t) = \sum_{m=0}^{M-1} \sum_{q=1}^{Q} h_m s_q(t - \tau_m) + n(t) \tag{3}$$

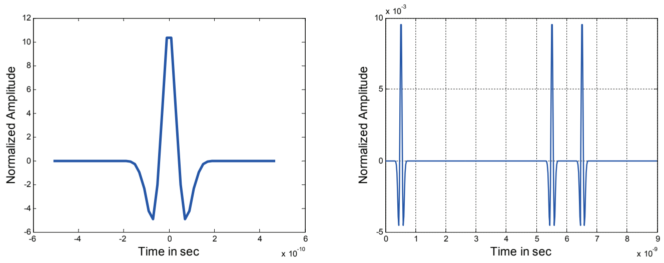

Fig. 1: UWB pulse and its PPM version

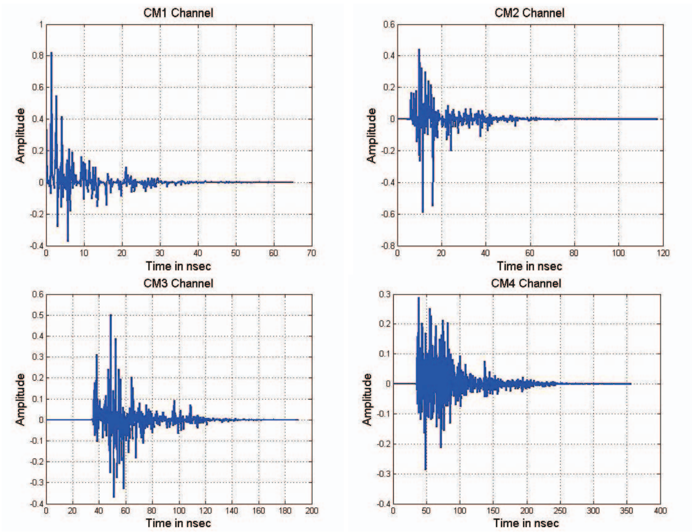

Fig. 2: IEEE 802.15.3a channels

where h_m is the gain coefficient of the m^{th} path and M is the number of paths in the multipath channel. Representation of (3) in the frequency domain is given as follows.

$$X(\omega) = \sum_{m=0}^{M-1} \sum_{q=1}^{Q} h_m S_q(\omega_m) e^{-j\omega_m \tau_m} + N(\omega) \qquad (4)$$

The above equation in matrix form can be expressed as

$$\mathbf{X}_q = \sum_{m=0}^{M-1} h_m \mathbf{S}_q e_{\tau_{mq}} + \mathbf{N}_q \qquad (5)$$

where $e_{\tau_{mq}}$ is the delay signature vector given by

$$e_{\tau_{mq}} = [1 \quad e^{-j\omega_o \tau_{mq}} \quad e^{-2j\omega_o \tau_{mq}} .... e^{-(N-1)j\omega_o \tau_{mq}}] \qquad (6)$$

where τ_{mq} denotes the delay associated with the m^{th} path and the q^{th} source.

III. DOA Estimation Using Coherent Signal Subspace Processing

In this method we condense the wide subspace into a narrow subspace by using the focusing matrix. DOA estimation system using an antenna array and five-port circuit is described in [4] for detecting arrival angles in the azimuth plane of signals pitched by the antenna array. Another technique for DOA estimation using weighted signal subspace has been proposed in [10]. The entire procedure can be divided in the following steps

- Calculation of the location vectors for the entire range of frequencies observed
- Calculation of focusing matrix
- Calculation of optimum focusing frequency
- The newly obtained subspace is subjected to MUSIC algorithm and DOA is estimated

A. *Location vector*

A location vector gives us the information about the DOA of the wideband signals at different elements of the antenna array.

$$A(\omega_k, \phi) = [a(\omega_k, \phi_1)..........a(\omega_k, \phi_Q)] \qquad (7)$$

$$a(\omega_k, \phi_q) = [1 \ e^{-j\omega_k \frac{d_1}{c} sin(\phi_q)} .... \ e^{-j\omega_k \frac{d_L}{c} sin(\phi_q)}]^T \qquad (8)$$

$$\phi = [\phi_1.........\phi_Q] \qquad (9)$$

where Q is the number of sources and L is the number of antenna array elements.

B. *Calculation of focusing matrix*

The focusing matrix satisfies the following property (10) and at each frequency ω_k it has a dimension of $[LXL]$.

$$T^H(\omega_k) T(\omega_k) = I \qquad (10)$$

$$T(\omega_k) = V(\omega_k) U(\omega_k)^H \qquad (11)$$

where V and U are the left and right singular vectors.

$$A(\omega_k)(A_o)^H = U(\omega_k) \Sigma V(\omega_k)^H \qquad (12)$$

where A_o is the location vector calculated at an optimum frequency.

C. *Calculation of optimum focusing frequency*

The fundamental principle behind the calculation of focusing frequency is to minimize the subspace fitting error using least squares minimization [11]. The matrix (13) denoting the subspace fitting error is observed iteratively for frequencies being incremented at the rate of $\Delta_f = (1/N) * f_s$ and the frequency that results in the minimum value gives the optimal focusing frequency. The focusing frequency hence obtained should lie within the spectrum range of the wideband signal [12]. A_o is intially calculated at f_{mid} (center frequency) of the wideband signal. The optimum frequency obtained after several iterations may or may not be equal to the center frequency. In the following equations we have denoted $A(\omega_k)$ as A_{ω_k}. Subspace fitting error can be mathematically represented as

$$\sum_{k=0}^{N-1} \| A_o - T_{\omega_k} A_{\omega_k} \|^2 \qquad (13)$$

The optimum frequency is given as

$$\omega_{opt} = \arg \min_{\omega_k} \|A_o - T_{\omega_k} A_{\omega_k}\|^2 \qquad (14)$$

We follow certain lemmas given in [11] to expand the Frobenious form and calculate the optimum focusing frequency.

- Lemma 1: The frobenious matrix (13) is expanded.

$$\sum_{k=0}^{N-1}[\|A_o\|^2 + \|A_{\omega_k}\|^2 - 2Retr(A_o A_{\omega_k} T_{\omega_k})] \quad (15)$$

$$= 2J_{pq} - 2\sum_{k=0}^{N-1}\sum_{i=0}^{q}\sigma_i(A_o A_{\omega_k}^H) \quad (16)$$

J_{pq} is constant so the variable part is

$$2\sum_{k=0}^{N-1}\sum_{i=0}^{q}\sigma_i(A_o A_{\omega_k}^H) \quad (17)$$

Therefore the minimization problem changes to the maximization of (17) which can be furthur expanded as per lemma 2.

- Lemma 2: It states that

$$\sum_{k=0}^{N-1}\sum_{i=0}^{q}\sigma_i(A_o A_{\omega_k}^H) \leq \sum_{k=0}^{N-1}\sum_{i=0}^{q}\sigma_i(A_o)\sigma_i(A_{\omega_k})^H \quad (18)$$

where N is the length of the DFT and

$$\sigma_i(A_o), i = 1, 2, ...q \quad (19)$$

denotes the singular values of A_o and q is the number of singular values. The value of ω_k for which the RHS of (18) obtains maximum value is said to be the optimum focusing frequency. Using the vector given in (5) we formulate a matrix holding the received signal at all the components of the antenna array, denoted by **Y**. The correlation of the received signal is P_y.

$$P_y = E[\mathbf{YY}'] \quad (20)$$

where P_y turns out to be a $[LXL]$ matrix. The correlation matrix given in (20) is multiplied by the focusing matrix and this is multiplied with the power spectral density P_s of the original TH-PPM signal. The weighted correlation matrix is subjected to MUSIC algorithm.

$$G = \sum_{k=0}^{N-1} P_s(\omega_k)T(\omega_k)P_y(\omega_k)T(\omega_k)^H \quad (21)$$

The MUSIC algorithm eigen decomposes the weighted correlation matrix given in (21) giving L eigen values and an eigen vector matrix of dimension $[LXL]$. The eigen vector subspace is divided into signal subspace which is of dimension $[LXQ]$ and a noise subspace given by **Un** of dimension $[LX(L-Q)]$. It follows the simple explaination that the noise subspace is orthogonal to the signal subspace. The steering vector is given by the following equation.

$$Steer(\phi) = [1 \quad e^{-j\omega_{opt}(d/c)sin(\phi)} ... e^{-j\omega_{opt}(L-1)(d/c)sin(\phi)}] \quad (22)$$

where d is the inter element spacing and c the speed of light. The pseudospectrum calculated at different values of ϕ is given by

$$H(\phi) = \frac{1}{Steer(\phi) * \mathbf{Un} * \mathbf{Un}' * Steer(\phi)'} \quad (23)$$

The location of the first n peaks of the pseudospectrum H, represent the angle of arrival from n sources.

IV. Results

Two sources and three antenna elements with an interelement spacing of $\lambda/2$ were considered. The sources are assumed to be at angles $20°$ and $40°$. The channel considered was not ideal, white gaussian noise corresponding to a signal to noise ratio (SNR) of 10dB was added. The entire computations were done in the frequency domain of the signal because working in the time domain of a sub nanosecond duration pulse leads to errors. Moreover the channels under the IEEE 802.15.3a model are frequency selective in nature thereby making the frequency domain analysis the best possible choice. The following results show the clear cut variation in estimation error observed in two cases, **(a)** focusing frequency equal to the optimum frequency **(b)** focusing frequency equal to the center frequency. Table I shows the results. Fig.3(a) represents the estimated DOA for the case in which the wide subspace was focused to the optimal frequency. Fig.3(b) shows the estimated values when the space was focused to the central frequency. Fig.4 and Fig.5 represents the

TABLE I: DOA estimation error for focusing frequency in two cases

Channel type	Error in source 1 for optimum frequency	Error in source 1 for center frequency	Error in source 2 for optimum frequency	Error in source 2 for center frequency
CM2	2°	10°	2°	3°
CM3	4°	2°	2°	14°
CM4	1°	13°	7°	16°

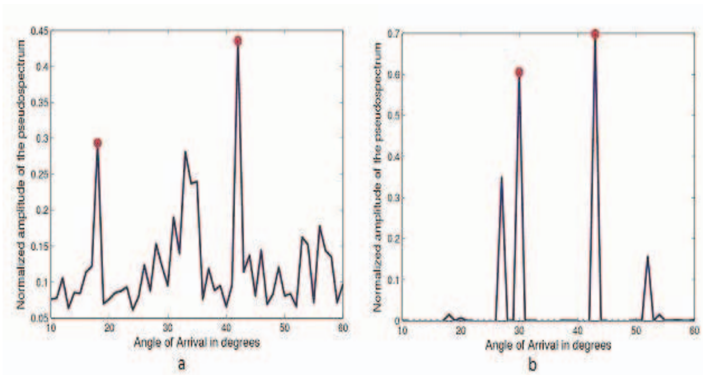

Fig. 3: CM2 channel(a) Focusing frequency equal to optimum frequency (b) Focusing frequency equal to center frequency

above mentioned cases observed under CM3 and CM4 channel conditions. The error in the estimation that we

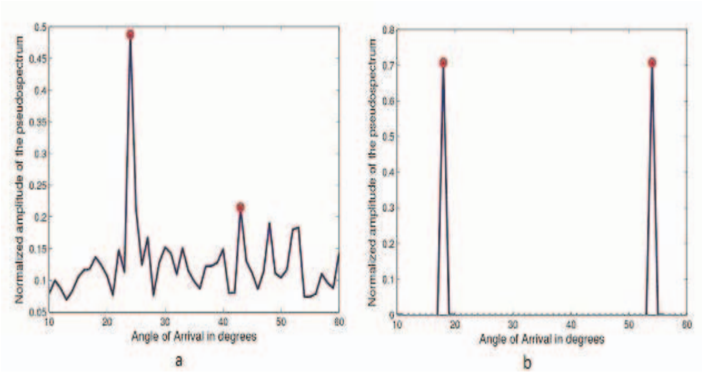

Fig. 4: CM3 channel(a) Focusing frequency equal to optimum frequency (b) Focusing frequency equal to center frequency

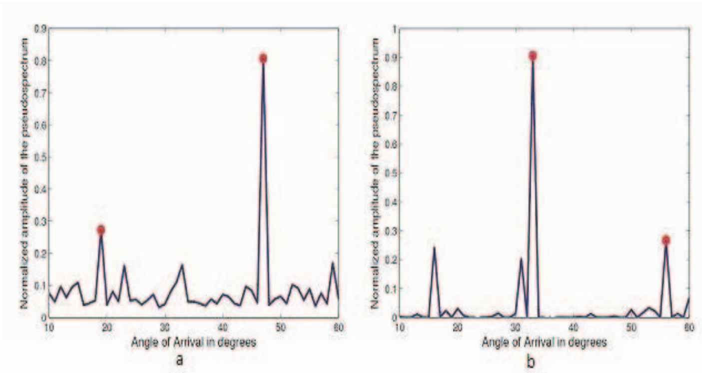

Fig. 5: CM4 channel(a) Focusing frequency equal to optimum frequency (b) Focusing frequency equal to center frequency

see is due to the frequency selective nature of the channel and length of the discrete fourier transform (DFT) taken, as it directly affects the value of the optimum frequency taken. Increasing the DFT length to a certain extent may improve the chances of arriving at a frequency that is closest to the ideal optimum focusing frequency but at the cost of longer computation time.

V. Conclusions

This method focuses a wideband subspace to a narrowband subspace space by scaling the signal with the focusing matrix. The focusing matrix is obtained by the constrained least squares minimization. The narrowband subspace chosen should be such that it minimizes the subspace fitting error. The received signal is then weighted with the power spectral density of the TH-PPM signal. The pseudospectrum of this resultant weighted signal in the narrowband subspace gives us the estimated DOA. Simulation results show that as we go from CM2 to CM4 the error in estimation increases. This is due to the fact that the paths are closely spaced in CM4 and CM3 as compared to CM2 thereby making the resolvability tedious. Moreover the paths experience a faster decay as we go from CM2-CM4, resulting in a weaker received signal. We can also conclude that the center frequency of the spectrum may necessarily not be the desired focusing frequency.

References

[1] W. P. Siriwongpairat and K. R. Liu, *Ultra-wideband communications systems: multiband OFDM approach*. John Wiley & Sons, 2007.

[2] E. Lagunas, M. Nájar, and M. Navarro, "Joint toa and doa estimation compliant with ieee 802.15. 4a standard," 2010, pp. 157–162.

[3] M. Nájar and M. Navarro, "Joint synchronization and demodulation for ir-uwb," vol. 2, 2008, pp. 55–58.

[4] L. Osman, I. Sfar, and A. Gharsallah, "An overview of doa estimation using a uniform linear antenna array with four elements," *American Journal of Applied Sciences*, 2012.

[5] V.V.Mani and R.Bose, "Smart antenna design for beamforming of uwb signals in gaussian noise," 2008, pp. 311–316.

[6] V.V.Mani and R. Bose, "Direction of arrival estimation and beamforming of multiple coherent uwb signals," 2010, pp. 1–5.

[7] P. Vial, B. J. Wysocki, and T. A. Wysocki, "An ultra wide band simulator using matlab/simulink," p. 46, 2005.

[8] C. M. Canadeo, "Ultra wide band multiple access performance using th-ppm and ds-bpsk modulations," DTIC Document, Tech. Rep., 2003.

[9] S. Yang and I. Darwazeh, "Uwb propagation channel modelling," *Department of Electronic & Electrical Engineering, University College London*.

[10] H. Keshavarz, "Weighted signal-subspace direction-finding of ultra-wideband sources," vol. 1, 2005, pp. 23–29.

[11] S. Valaee and P. Kabal, "The optimal focusing subspace for coherent signal subspace processing," *IEEE Transactions on Signal Processing*, vol. 44, no. 3, pp. 752–756, 1996.

[12] P. Kabal, "Selection of the focusing frequency in wideband array processing."

[13] D. Porcino and W. Hirt, "Ultra-wideband radio technology: potential and challenges ahead," *IEEE Communications Magazine*, vol. 41, no. 7, pp. 66–74, 2003.