MOMENT CURVATURE CHARACTERISTICS OF HYBRID FERRO FIBER CONCRETE(HFFC) BEAM

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The characteristic equation of the stress-strain curve for Hybrid Ferro Fiber Concrete (HFFC) is used to study the M-C characteristics of HFFC sections. The theoretical procedure has been validated by conducting an experimental investigation on 23 reinforced concrete beams provided with HFFC at critical sections. The correlation between experimental and analytical values of ultimate moments and corresponding curvatures, arrived at based on the above procedure is found to be good.

NOTATION

D, b	=	*Lateral dimensions of beam*
d	=	*Effective depth*
f, ε	=	*Stress and corresponding strain*
C_c	=	*Compressive force in HFFC*
f_u'	=	*Ultimate strength of HFFC*
f_{ck}	=	*Concrete cube strength*
A_b	=	*Cross sectional area of ferrocement*
V_f	=	*Volume fraction*
f_{fs}	=	*Stress in ferrocement shell*
η	=	*Efficiency factor of mesh*
S_f	=	*Specific surface factor [5.6,14]*
M_{ue}	=	*Experimental ultimate moment*
M_{ut}	=	*Theoretical ultimate moment*
C_i	=	*Confinement index [15]*
M_c	=	*Moment of Cc about neutral axis*
ε_{fu}	=	*Strain at ultimate of HFFC*
M_s	=	*Moment of tensile force of tension steel about neutral axis*
M_f'	=	*Moment of tensile force in ferrocement shell in tension zone about neutral axis*
M_{fi}	=	*Moment of tensile force of fiber reinforced concrete in tension zone about neutral axis*
φ_{ue}	=	*Experimental curvature at ultimate*
φ_{ut}	=	*Theoretical curvature at ultimate*
RI	=	*Reinforcing index of fibers [8]*

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INTRODUCTION

In view of the insufficient rotation capacity of reinforced concrete sections, full redistribution of moments cannot be ensured in indeterminate structures. At present, it is known that the ductility of concrete can be improved by confining the concrete in steel binders and such concrete being called as confined concrete or ductile concrete. The spacing limitations on stirrups, limit the confinement offered by the stirrups [2]. To overcome this, the confinement due to HFFC has been suggested as one of the alternatives [5]. Further, the investigation on HFFC revealed that the additional confinement due to combination of ferrocement and fiber reinforcement improved the ultimate strength, strain at ultimate. The prediction equations for the same were proposed in authors earlier paper [5]. This paper presents a theoretical procedure based on characteristic stress-strain curve for HFFC, for the assessment of moments and curvatures of HFFC sections. The method has been validated with the experimental results on 24 reinforced concrete simply supported beams confined with HFFC at critical sections.

STRESS - STRAIN CURVE FOR HFFC

The stress-strain curve for HFFC as proposed by the author [6] is of the form:

$$f = \left[\frac{A\varepsilon}{1.0 + B\varepsilon + C\varepsilon^2} \right] \qquad ...(1)$$

The shape of the stress-strain curve is shown in Fig.1. The constants *A*, *B* and *C* that satisfy the boundary condition are:

$$A = A_1 \left(\frac{f_u}{\varepsilon_{fu}} \right) \qquad B = B_1 \left(\frac{1.0}{\varepsilon_{fu}} \right) \qquad C = C_1 \left(\frac{1.0}{\varepsilon_{fu}} \right)^2$$

Where, $A_1 = 6.852,\ B_1 = 4.852,\ C_1 = 1.0$ for ascending portion of stress-strain curve
$A_1 = 2.833,\ B_1 = 0.833,\ C_1 = 1.0$ for descending portion of stress-strain curve

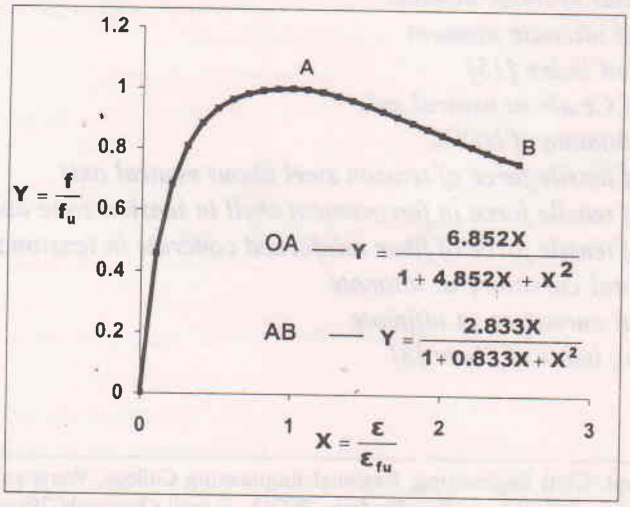

Fig. 1. Characteristic stress ratio vs. strain ratio

$$P = f_c'(1 + 0.55C_i)(1.0228 + 0.1024\,RI)(1.0 + 0.0166\,S_f)\,A_g + f_y A_s \qquad ...(2)$$

$$\varepsilon_{fu} = \varepsilon_c'(1.0 + 5.2\,C_i)(0.9899 + 0.2204\,RI)(1.0 + 0.1359\,S_f) \qquad ...(3)$$

The strain at 0.85 times ultimate stress is:

$$\frac{\varepsilon_{0.85fu}}{\varepsilon_u} = 2.1127 + 0.0338\,S_f \qquad ...(4)$$

DEVELOPMENT OF MOMENT-CURVATURE DIAGRAMS FOR HFFC SECTIONS

It becomes necessary to compute the total compressive force developed in a HFFC section and moment of compressive force about the neutral axis for any extreme fiber concrete strain, in order to generate the moment-curvature diagram of any cross-section. It can be seen that for any concrete strain (ε_c), in the extreme fiber (Fig.2).

$$C_c = f_a bnd \qquad ...(5)$$

Where, $f_a = \left(\frac{1}{\varepsilon_c}\right)\int_0^{\varepsilon_c} f d\varepsilon$

$$M_c = b\left[\frac{nd}{\varepsilon_c}\right]^2 \int_0^{\varepsilon_c} \varepsilon f\, d\varepsilon \qquad ...(6)$$

The corresponding curvature can be obtained by: $\varphi_c = \frac{\varepsilon_c}{nd}$

The evaluation of integrals $\int_0^{\varepsilon_c} f d\varepsilon$ and $\int_0^{\varepsilon_c} \varepsilon f d\varepsilon$ leads to the expressions:

$$C_c = \frac{bnd}{\varepsilon_c}\left(\frac{A}{2C}K_1 - \frac{AB}{2C^2}K_2\right) \qquad ...(7)$$

$$M_c = b\left(\frac{nd}{\varepsilon_c}\right)^2\left[\frac{A}{2C^3}\left(2C^2\varepsilon_c - BCK_1 - \left(2C - B^2\right)K_2\right)\right] \qquad ...(8)$$

Where, $K_1 = \ln\left(1 + B\varepsilon_c + C\varepsilon_c^2\right)$

K_2 will have three expressions depending on $(4C - B^2)$, as follows:

For $4C - B^2 < 0.0$ and $Q = \sqrt{(B^2 - 4C)}$;

$$K_2 = \frac{C}{Q} ln\left\{\frac{(2C\varepsilon_c + B - Q)(B+Q)}{(2C\varepsilon_c + B + Q)(B-Q)}\right\} \quad ...(9)$$

For $4C - B^2 = 0.0$;

$$K_2 = \frac{2A\varepsilon_c}{2C\varepsilon_c + B} \quad ...(10)$$

For $4C - B^2 > 0.0$ and $R = \sqrt{4C - B^2}$;

$$K_2 = \frac{2C}{R} Tan^{-1}\left(\frac{R\varepsilon_c}{2 + B\varepsilon_c}\right) \quad ...(11)$$

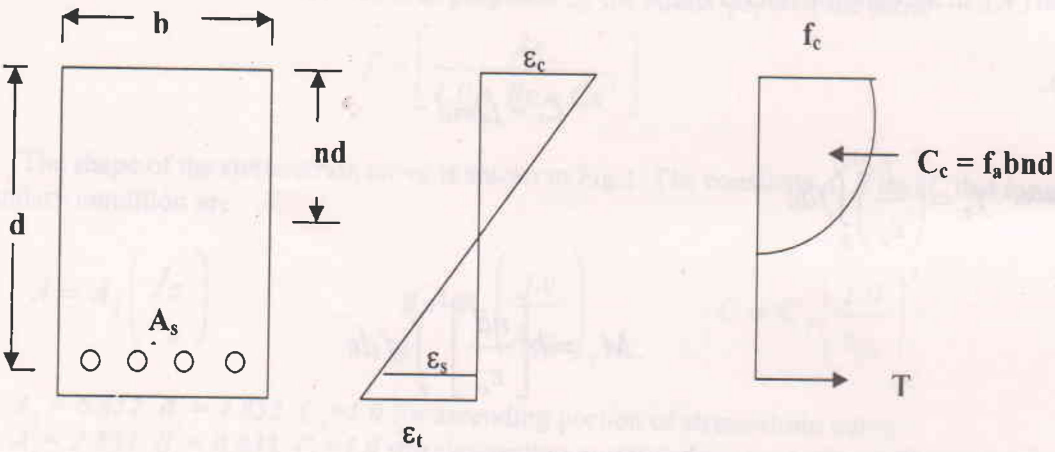

Fig. 2. Compressive forces and moment of compressive forces in HFFC

For obtaining the complete moment curvature relationship for any cross-section, discrete values of extreme fiber concrete strains (ε_c) were selected such that even distribution of points on the plot, both before and after the maximum moment, were obtained. The procedure used in computation is as follows:

1. For the selected value of ε_c, the extreme fiber concrete strain, the neutral axis depth, '*nd*' is assumed initially at a value of 0.5 d.
2. For the assumed value of '*nd*', the compressive force C_c and the value of Moment M_c of this resultant compressive force about neutral axis are calculated.
3. The strain in tension steel ε_s is calculated based on strain compatibility.
4. The tensile force T_s in the tension steel is arrived at by taking the corresponding stress from the stress-strain diagram of steel and multiplying the stress with cross sectional area of steel. The corresponding moment M_s about neutral axis is:

$$M_s = T_s (d\text{-}nd) \quad ...(12)$$

5. The force in ferrocement shell in the tension zone T_f is calculated using the methodology as detailed below [9].
   i. Knowing the volume fraction V_f of the mesh reinforcement the effective cross-Sectional area of the mesh reinforcement in ferrocement shell in the tension zone is calculated using $A_{cf} = \eta A_b V_f$
   ii. For known value of ε_t, the strain in the bottom fiber, the stress f_{fs} in the Ferrocement is obtained using corresponding stress-strain diagram of mesh steel. The stress when multiplied with effective cross sectional area of the ferrocement gives the force in the ferrocement T_f in the tension zone and moment of this force about the neutral axis is obtained.
6. The force in fiber reinforced concrete in the tension zone T_{fi} is calculated using the Methodology detailed in reference [11,12,13].
7. The total tensile force is calculated as $T = T_s + T_f + T_{fi}$
8. The values of C_c and T are now compared. If C_c and T are the same, then the assumed position of neutral axis is correct. Then, the moment M and the curvature for that particular fiber strain in concrete are calculated as $M = M_c + M_s + M_f + M_{fi}$ and curvature $\varphi_c = \dfrac{\varepsilon_c}{nd}$
9. If C_c and T are not equal, a new value of the neutral axis depth is assumed based on judgment, whether C_c is greater of smaller than T and the above procedure is repeated until equilibrium condition $C_c = T$ is satisfied.

The above analytical procedure enables the assessment of flexural strength of HFFC sections. The assumptions made in deriving the flexural response are (i) mortar contribution towards the strength of ferrocement in tension is neglected and (ii) variation of strain across the section is linear up to failure.

In addition to the above assumptions, the three basic relationships viz., (i) equilibrium forces, (ii) compatibility of strains and (iii) stress-strain relationships of the material have to be satisfied.

PRESENT WORK

Results derived from the above-proposed analytical procedure are compared with the experimental data documented by the authors [7]. Details of comparison are shown in Table.2. The experimental programme included casting and testing of 24 reinforced concrete beams of size 120 x 200 x 2100mm with HFFC at critical zone, i.e., the zone in which the plastic hinge forms under flexure. The length of critical zone was arrived based on plastic hinge length criteria for confined concrete. Fig.3 shows the zone of HFFC in the beam. The 24 beams consisted of two groups of 12 beams each. In each group two types of reinforcing index viz., RI = 1.23 and 2.46 or (V_f= 0.5% and 1.0%) were used and designated as 'A' and 'B'. The longitudinal reinforcement with each type reinforcing index is varied to give two sets of beams viz., under reinforced (U) and over reinforced (O) beams. In each set, the specific surface factor is the only variable. The number of layers of mesh in each set is varied to get the varied specific surface factor of ferrocement shell. Thus, each specimen is designated by type of failure, type of reinforcing index of the fiber and the number of layers of mesh, i.e., the specimen whose designation UA4 stands for 'U' under reinforced beam type failure, reinforcing index of fiber RI=1.23 and '4' indicates the serial number of the value of the specific surface factor (4 layers of mesh). Table.1

gives the details of the tested simply supported reinforced concrete beams provided with HFFC at critical sections.

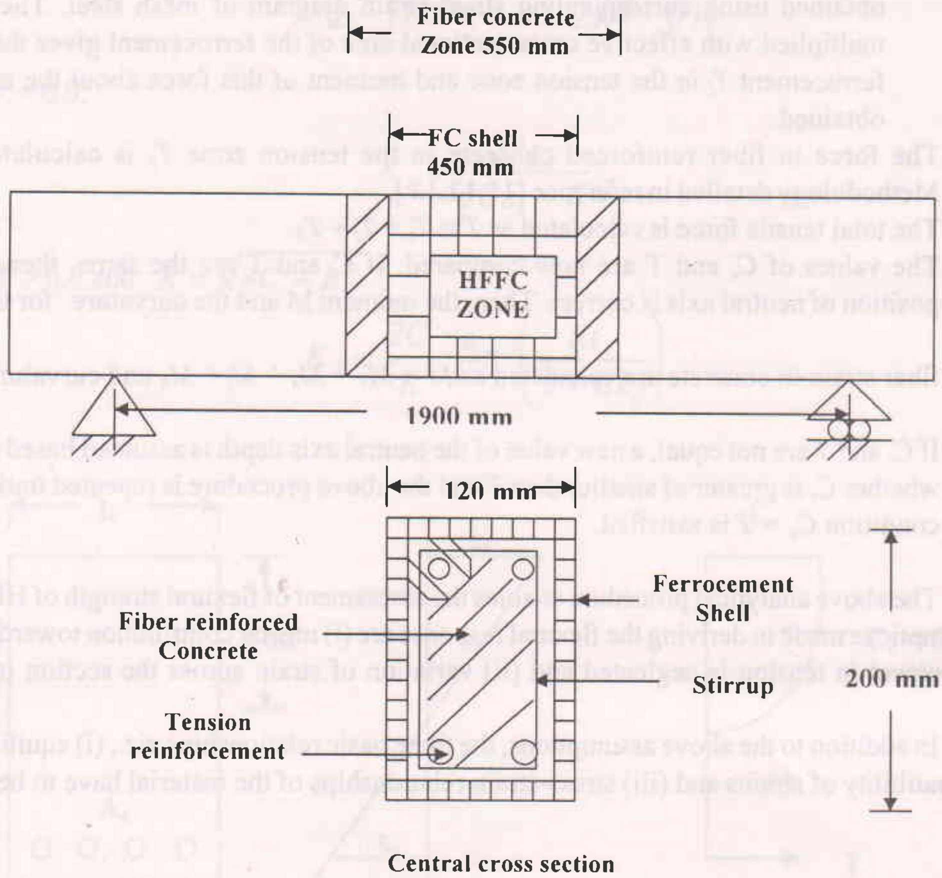

Fig. 3. Detail of mesh zone and fiber concrete in the beam

All the beams were tested under symmetrical two points loading on a simply supported span of 1900mm. Fig.4 shows the test arrangement of simply supported beams. Specially fabricated curvature meters were used to measure the curvatures in the central zone of 600mm of the beam, in three gage lengths of 200 mm each. Fig.5 shows the closer view of the curvature meter arrangement to the beam. Strain rate control was used to obtain the complete profile of moment - curvature behavior, especially in the post ultimate region. Moment - curvature diagrams generated for all the beams based on the characteristic stress - strain diagram of HFFC are shown by firm lines in Figs.6 (a) - 6(b). The experimental values of moments and curvatures are plotted as discrete points on the above moment - curvature diagrams. The experimental ultimate moments and theoretical moments computed based on characteristic stress - strain curve of HFFC are represented on a correlation diagram shown in Fig.7.

CORRELATION

It can be seen from Fig.6 (a) - 6(b), that the procedures developed for obtaining the complete profile of moment - curvature diagram of HFFC sections, based on the characteristic stress - strain curve for HFFC predict the experimental behavior satisfactorily. There is a good agreement between the analytical and experimental ultimate moments, as can be seen from the correlation diagram shown in Fig. 7. The average ratio of the experimental to analytical ultimate moments is 1.024with a standard deviation of 0.033 and coefficient of variation of 3.27%. The correlation between experimental and analytical ultimate curvatures is not so good as that of ultimate moments. The average ratio of the experimental to analytical curvature is 0.905, with a standard deviation of 0.175 and a coefficient of variation of 19.36%. The lack of very good correlation in curvatures may be attributed to the fact that the analytical curvature at a section, computed to satisfy the equilibrium and compatibility conditions and material properties. The experimental curvature is the curvature measured over a gage length of 200 mm and hence represents the average curvature over gage length including localized high curvatures at cracks. Hence, the average curvature depends upon the number of cracks occurring in the gage length, their width, distribution and location. The occurrence and location of cracks once again depends upon a number of factors, prominent among them being, uniformity of strength of concrete in the critical zone and local variation of bond between steel and concrete. The provision of HFFC transforms the brittle behavior of over reinforced RC sections into ductile ones (Figs.6 (a) - 6(b)) by developing moment - curvature diagrams with more horizontal plateau in post ultimate regions.

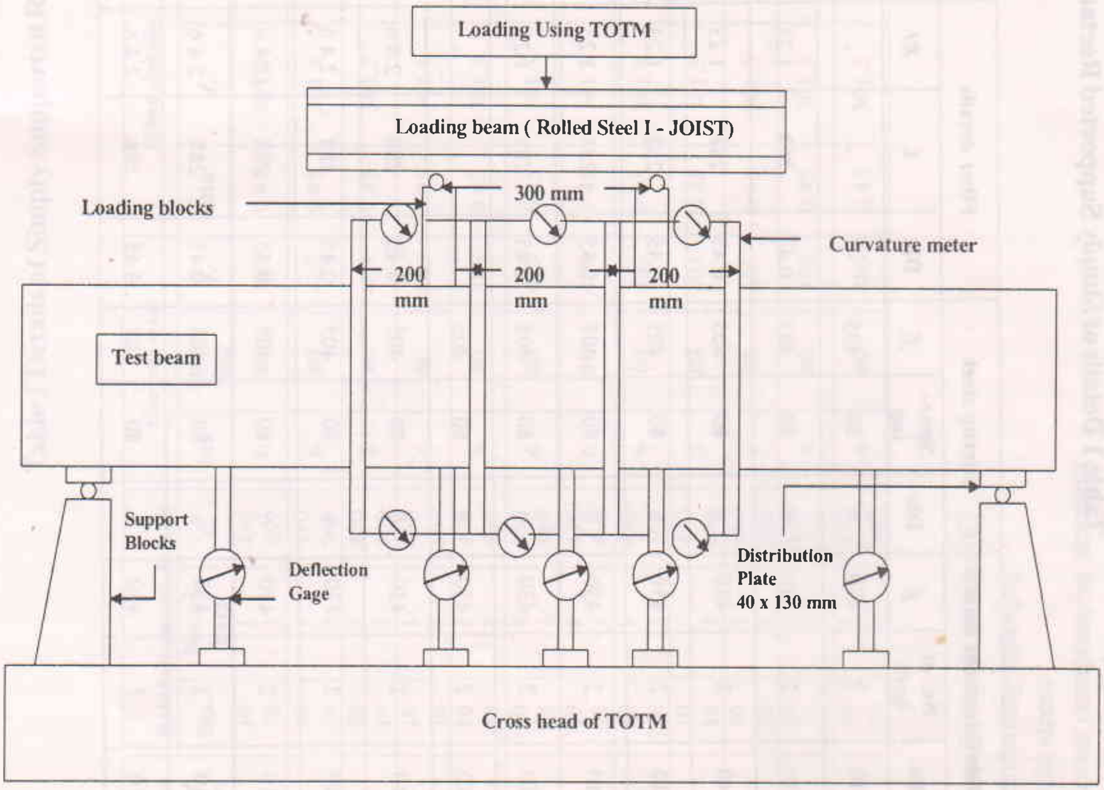

Fig. 4. Test Set-up of Simply Supported Beam

Table 1 Details of Simply Supported Rectangular Beams Tested

S.No	Designation of Beams	Reinforcement details			Stirrup steel			Fiber details			GI wire mesh details			f_{cm} (Mpa)	f_{ck} (Mpa)	S_f	Overall size of beams (mm X mm)
		Dia.	No. of bars	f_y	Dia	Spacing	f_y	Dia	f_y	RI	Dia.	Spacing	f_y				
1	U1	10	2	430	6	80	405	-	-	-	-	-	-	-	49.11	-	120 X 205
2	UA0	10	2	430	6	80	405	0.45	288	1.23	-	-	-	-	46.73	-	120 X 205
3	UA1	10	2	430	6	80	405	0.45	288	1.23	0.48	2.4	288	37.86	45.35	2.94	120 X 200
4	UA2	10	2	430	6	80	405	0.45	288	1.23	0.48	2.4	288	37.86	43.70	5.88	120 X 195
5	UA3	10	2	430	6	80	405	0.45	288	1.23	0.48	2.4	288	37.86	43.70	8.823	120 X 200
6	UA4	10	2	430	6	80	405	0.45	288	1.23	0.48	2.4	288	37.86	45.35	11.765	120 X 200
7	U2	10	2	430	6	80	405	-	-	-	-	-	-	-	49.11	-	120 X 200
8	UB0	10	2	430	6	80	405	0.45	288	2.4 6	0.48	-	-	-	47.06	-	120 X 200
9	UB1	10	2	430	6	80	405	0.45	288	2.4 6	0.48	2.4	288	20	47.06	5.56	120 X 200
10	UB2	10	2	430	6	80	405	0.45	288	2.4 6	0.48	2.4	288	20	50.69	11.13	120 X 200
11	UB3	10	2	430	6	80	405	0.45	288	2.4 6	0.48	2.4	288	20	46.40	16.70	120 X 200
12	UB4	10	2	430	6	80	405	0.45	288	2.4 6	0.48	2.4	288	20	43.70	22.27	120 X 195

Table 1 Details of Simply Supported Rectangular Beams Tested

Sl. No.	Designation of Beams	Reinforcement details			Stirrup steel			Fibre details			GI wire mesh details			f_{cm} (Mpa)	f_{ck} (Mpa)	S_f	Overall size of beams
		Dia.	No. of bars	f_y	Dia	Spacing	f_y	Dia	f_y	RI	Dia.	Spacing	f_y				
13	O1	10 16	1 3	430 569	6	80	405	0.45	301.5	0	0.48	2.4	288	-	43.00	-	120 X 200
14	OA0	10 16	1 3	430 569	6	80	405	0.45	301.5	1.23	0.48	2.4	288	-	48.0	0	119 X 201
15	OA1	10 16	1 3	430 569	6	80	405	0.45	301.5	1.23	0.48	2.4	288	23.85	48.51	4.66	120 X 200
16	OA2	10 16	1 3	430 569	6	80	405	0.45	301.5	1.23	0.48	2.4	288	23.85	38.76	9.32	119 X 202
17	OA3	10 16	1 3	430 569	6	80	405	0.45	301.5	1.23	0.48	2.4	288	23.85	43.90	13.98	120 X 201
18	OA4	10 16	1 3	430 569	6	80	405	0.45	301.5	1.23	0.48	2.4	288	23.85	45.28	18.64	120 X 200
19	O2	10 16	1 3	430 569	6	80	405	0.45	301.5	0	0.48	2.4	288	-	43.15	-	120 X 200
20	OBO	10 16	1 3	430 569	6	80	405	0.45	301.5	2.46	0.48	2.4	288	-	53.26	0	120 X 200
21	OB1	10 16	1 3	430 569	6	80	405	0.45	301.5	2.46	0.48	2.4	288	23.85	52.10	4.66	121 X 201
22	OB2	10 16	1 3	430 569	6	80	405	0.45	301.5	2.46	0.48	2.4	288	23.85	52.10	9.32	120 X 200
23	0B3	10 16	1 3	430 569	6	80	405	0.45	301.5	2.46	0.48	2.4	288	23.85	52.10	13.98	120 X 202
24	0B4	10 16	1 3	430 569	6	80	405	0.45	301.5	2.46	0.48	2.4	288	23.85	52.10	18.64	119 X 201

Note:

S_f = Specific Surface factor[6]
RI = Reinforcing index[6]
f_y = Yield stress
f_{cm} = Mortar compressive strength
f_{ck} = Strength of the concrete

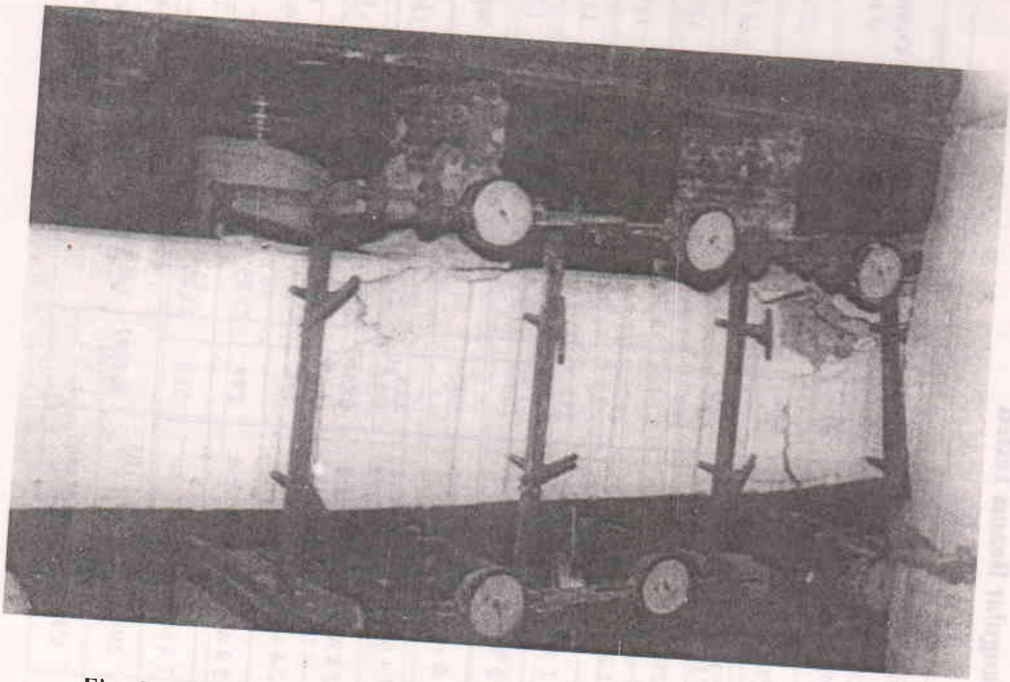

Fig. 5. Closer view of the curvature meter arrangement to the beam

IDEALISATION OF MOMENT - CURVATURE DIAGRAM

A critical appraisal of the moment - curvature diagrams of HFFC beams, shows that the moment - curvature diagrams can be idealized as a bilinear form consisting of two straight lines, one inclined raising straight line up to 90 % of the ultimate moment of that HFFC beam and other line is a horizontal straight line at the end of the raising straight line. This idealization leads to the assumption that up to 90% of the ultimate moment, the HFFC cross- section behaves elastically and beyond this value, the behavior is plastic. The assumption of elastic behavior up to such a high value of ultimate moment seems to be justified for reinforced concrete section confined with HFFC though not an ordinary RC section. The slope of the raising line in the idealized diagram is equal to flexural rigidity ' EI' of the cross-section. The value of ' I ', the moment of inertia of the cross-section is calculated based on uncracked transformed cross-section, taking the steel area also into consideration. The value of 'E' is the initial modulus of elasticity of concrete and can be calculated from the formula $E = 5000\sqrt{f_{ck}}$.

The limit for the horizontal straight line may be taken as that point where this idealized straight line cuts the actual moment-curvature diagram. If due to the hyper resistance of tor steel above 0.2% proof stress, for higher confinement indices, reinforcing index and specific surface factor values, the horizontal straight line may not intersect the actual moment - curvature diagram, in which the strain limits can be imposed.

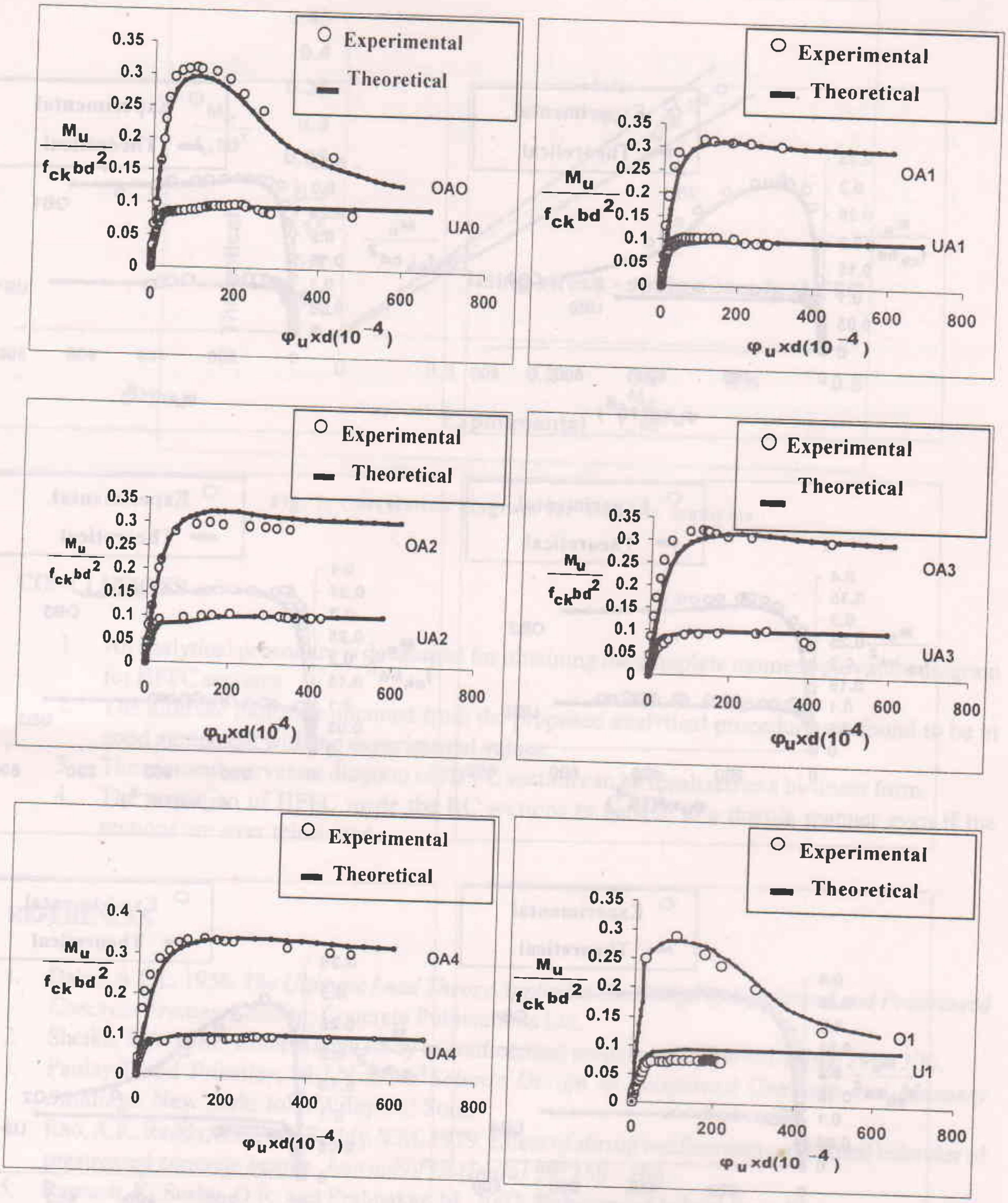

Fig. 6(a). Comparison of experimental and theoretical moment curvature diagrams

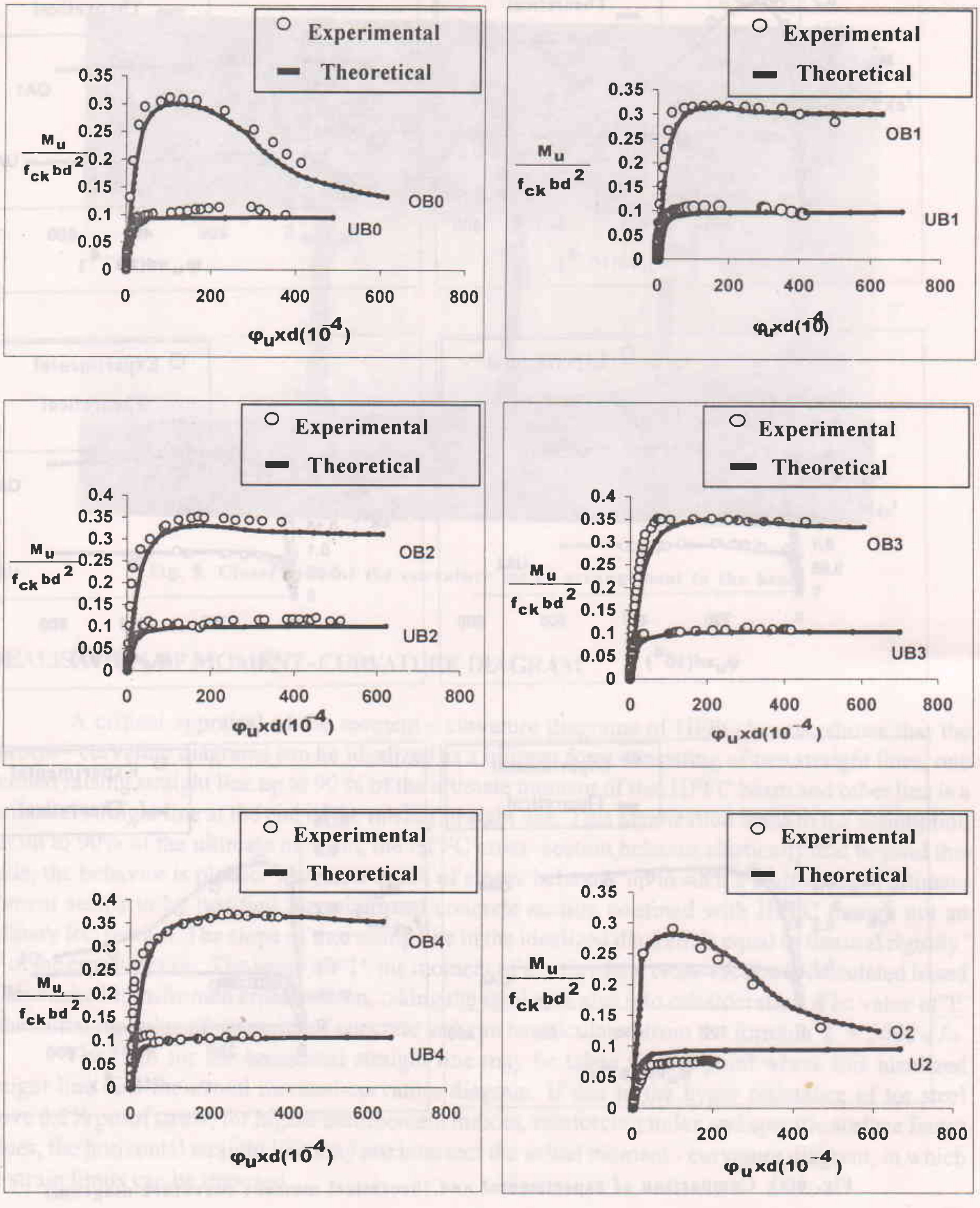

Fig. 6(b). Comparison of experimental and theoretical moment curvature diagrams

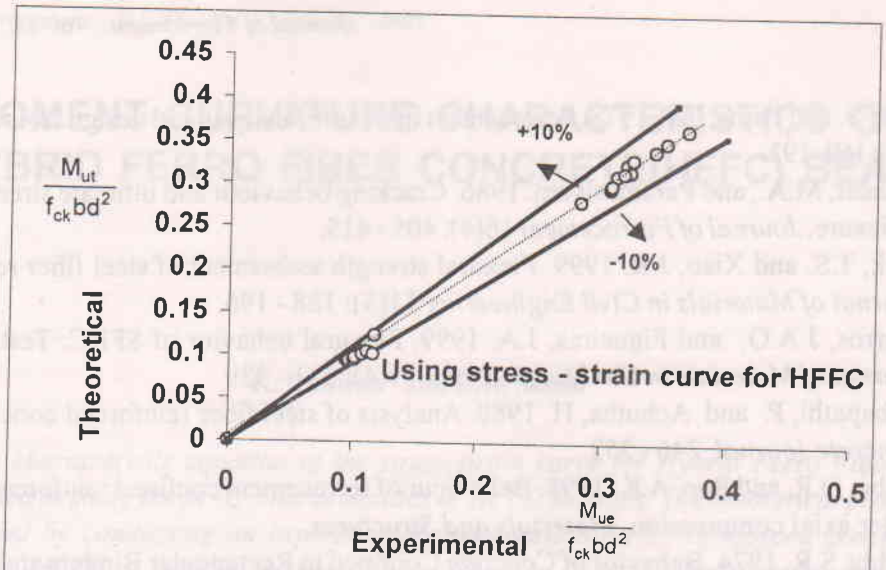

Fig. 7. Correlation diagram for ultimate moments

CONCLUSIONS

1. An analytical procedure is developed for obtaining the complete moment-curvature diagram for HFFC sections.
2. The ultimate moments obtained from the proposed analytical procedure are found to be in good agreement with the experimental values.
3. The moment-curvature diagram of HFFC section can be idealized as a bi-linear form.
4. The provision of HFFC made the RC sections to behave in a ductile manner even if the sections are over reinforced.

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