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FLOW THROUGH NONUNIFORM CHANNEL WITH PERMEABLE WALL AND SLIP EFFECT

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An investigation of viscous incompressible fluid flow in a nonuniform rigid channel with permeable walls is presented by taking into consideration the influence of slip velocity at the walls. The effect of fluid absorption through permeable walls is accounted for by prescribing flux as a function of axial distance. The nonlinear governing equations of motion are linearized by the perturbation method by assuming δ *(the ratio of inlet width to the length of the channel) as a small parameter to get an approximate analytical solution. The effects of reabsorption coefficient* (α)*, slope parameter* (k)*, and slip parameter* (β) *on the velocity profiles, mean pressure drop, and wall shear stress are studied and presented graphically. The results indicate that the slip parameter influences the flow field considerably.*

KEY WORDS: *nonuniform channel, perturbation method, permeable wall, slip effect*

1. INTRODUCTION

The study of viscous fluid flow in a channel of varying cross section with permeable walls is significant because of its applications to both physiological and engineering flow problems. For example, mathematical models of the flow of fluid in a renal tubule have been studied by various authors. Macey (1963) formulated the problem as the flow of an incompressible viscous fluid through a circular tube with linear rate of reabsorption at the wall, whereas Kelman (1962) found that the bulk flow in the proximal tubule decays exponentially with the axial distance. Then, Macey (1965) used this condition to solve the equations of motion and mentioned that the longitudinal velocity profile is parabolic and the drop in mean pressure is proportional to the mean axial flow.

Marshall and Trowbridge (1974) and Palatt et al. (1974) used the physical conditions existing at the rigid permeable tube instead of prescribing the flux at the wall as a function of axial distance.

The above studies considered the linearized models of renal flow with the renal tubule as a cylindrical tube of uniform cross section. But, in general, renal tubules may not have uniform cross section throughout their length. Radhakrishnamacharya et al. (1981) made an attempt to understand the hydrodynamical aspects of an incompressible viscous fluid in a circular tube of varying cross section with reabsorption at the wall. Chandra and Prasad (1992) analyzed flow in rigid tubes of slowly varying cross section with absorbing walls. The effect of fluid absorption through permeable wall is accounted for by prescribing flux as an arbitrary function of axial distance. Chaturani and Ranganatha (1991) considered fluid flow through a diverging/converging tube with variable wall permeability. Recently, Muthu and Tesfahun (2010) considered the effects of slope parameter and reabsorption coefficient on the flow of fluid in a symmetric channel with varying cross section with no-slip velocity at the walls.

The concept of slowly varying flow forms the basis of a large class of problems in fluid mechanics in which viscous forces dominate the nonlinear inertial forces. For example, Manton (1971) obtained an asymptotic series solution for the low Reynolds number flow through an axisymmetric tube whose radius varies slowly in the axial direction.

In all the above studies, the boundary condition at the wall is taken as a no-slip condition. The no-slip boundary condition is one of the cornerstones on which the mechanics of the viscous liquids is built. However, there are situations where this assumption does not hold (Rao and Rajagopal, 1999). The effect of slip at the wall is significant, as illustrated by Moustafa (2004). The slip velocity at the boundary is proportional to the shear rate at the boundary. This slip velocity is connected with the presence of a thin layer of streamwise moving fluid in the boundary region just below the permeable surface. The fluid in this layer is considered to be pulled along by the flow above the porous surface (Singh and Laurence, 1979). Also, the slip would be most useful for certain problems in chemical engineering and other applications (Chu, 2000; Joseph and Ocando, 2002; Vasudeviah and Balamurugan, 1999; Wang, 2002).

The objective of the present paper is to understand the hydrodynamical aspect of an incompressible viscous fluid flow in a rigid channel of varying cross section with reabsorption and a slip velocity at the walls of the channel. The boundary of the channel walls is assumed to be symmetric about the x axis and vary with x. It is taken as

$$\eta(x) = \pm\left[d + \frac{k_1}{\lambda}x + a\sin\left(\frac{2\pi x}{\lambda}\right)\right], \qquad (1)$$

where d is the half width of the channel at the inlet (at x = 0), k_1 is a constant whose magnitude depends on the length of the channel exit and inlet dimensions and which is assumed to be $\ll$1, a is the amplitude, and λ is the length of the channel; see Fig. 1.

2. MATHEMATICAL FORMULATION

Consider an incompressible Newtonian fluid flow through a channel with slowly varying cross section as given by Eq. (1). The motion of the fluid is assumed to be laminar, steady, and symmetric. The channel is assumed to be long enough to neglect both the entrance and end effects. The effect of gravity is neglected. The governing equations of such fluid motion are given by

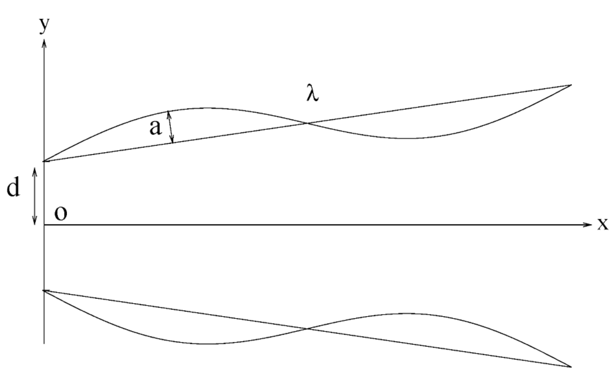

FIG. 1: Geometry of two-dimensional renal tubule.

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, \qquad (2)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \qquad (3)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right), \qquad (4)$$

where u and v are the velocity components along the x and y axes, respectively, p is the pressure, ρ is density, and v (= μ/ρ) is kinematic viscosity of the fluid.

The boundary conditions are taken as follows: The tangential velocity at the wall is not zero (Dulal et al., 1988; Moustafa, 2004). That is,

$$u + \frac{\partial\eta}{\partial x}v = -\frac{\sqrt{\gamma}}{\beta}\left(\frac{\partial u}{\partial y} + \frac{\partial\eta}{\partial x}\frac{\partial v}{\partial y}\right) \text{ at } y = \eta(x), \qquad (5)$$

where β is slip parameter and γ is the specific permeability of the porous medium.

The regularity condition requires

$$v = 0 \text{ and } \frac{\partial u}{\partial y} = 0 \text{ at } y = 0. \qquad (6)$$

The reabsorption has been accounted for by considering the bulk flow as a decreasing function of x. That is, the flux across a cross section is given as

$$Q(x) = \int_0^{\eta(x)} u(x,y)dy = Q_0 F(\alpha x), \qquad (7)$$

where $F(\alpha x)$ = 1 when α = 0 and decreases with x. Further, $\alpha \geq 0$ is the reabsorption coefficient, a constant and Q_0 is the flux across the cross section at x = 0. The boundary condition (5) is the well-known Beavers and Joseph (1967) condition when applied to tangential velocity. Further, reabsorption is assumed to be independent of the absorption area.

Eliminating pressure p from Eqs. (3) and (4) and introducing stream function ψ by

$$u = \frac{\partial\psi}{\partial y} \text{ and } v = -\frac{\partial\psi}{\partial x}, \qquad (8)$$

and using the following nondimensional quantities:

$$x' = \frac{x}{\lambda},\ y' = \frac{y}{d},\ \eta' = \frac{\eta}{d},$$

$$\psi' = \frac{\psi}{Q_0},\ \alpha' = \alpha\lambda,\ p' = \frac{d^2}{\mu Q_0}p,$$

Eqs. (2)–(4) are transformed to the nondimensional form as (after dropping the primes):

$$\left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi = \delta R_e\left[\frac{\partial\psi}{\partial y}\left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\frac{\partial\psi}{\partial x} - \frac{\partial\psi}{\partial x}\left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\frac{\partial\psi}{\partial y}\right], \quad (9)$$

where $d = d/\lambda$ and $R_e = Q_0/v$.

Further, the boundary conditions (5)–(7) become

$$\frac{\partial\psi}{\partial y} - \delta^2\frac{\partial\eta}{\partial x}\frac{\partial\psi}{\partial x} = -\xi\left[\frac{\partial^2\psi}{\partial y^2} - \delta^2\frac{\partial\eta}{\partial x}\frac{\partial^2\psi}{\partial x\partial y}\right]$$
$$\text{at } y = \eta(x) = 1 + kx + \varepsilon\sin(2\pi x), \quad (10)$$

$$\psi = 0 \text{ and } \frac{\partial^2\psi}{\partial y^2} = 0 \text{ at } y = 0, \quad (11)$$

and

$$\psi = F(\alpha x) \text{ at } y = \eta(x) = 1 + kx + \varepsilon\sin(2\pi x), \quad (12)$$

where $\xi = \sqrt{\gamma}/\beta d$, $\varepsilon = a/d$, and $k = (k_1\lambda)/d$.

The parameter R_e is the Reynolds number, δ is the wall variation parameter (the ratio of inlet width to the length of the channel), ε is the amplitude ratio (the ratio of amplitude to the inlet width), k is slope parameter, and ξ is the slip coefficient. In this problem, we consider exponentially decaying bulk flow (Radhakrishnamacharya et al., 1981). That is, in Eq. (7), F is taken as

$$F(\alpha x) = e^{-\alpha x} \quad (13)$$

In the following, we consider the case of low Reynolds number flow when R_e is such that $R_e\delta \cong O(\delta)$ (Chandra and Prasad, 1992; Manton, 1971).

3. METHOD OF SOLUTION

Note that the flow is complex because of the nonlinearity of the governing equation and the boundary conditions (9)–(12). Thus, to solve Eq. (9) for velocity components in the present analysis, assuming the wall variation parameter $\delta \ll 1$, we shall seek a solution for stream function $\psi(x, y)$ in the form of a power series in terms of δ as

$$\psi(x,y) = \psi_0(x,y) + \delta\psi_1(x,y) + \cdots. \quad (14)$$

Substituting Eq. (14) in Eqs. (9)–(12) and collecting coefficients of various like powers of δ, we get the following sets of equations for $\psi_0(x,y)$, $\psi_1(x,y)$, ...

δ^0 case:

$$\frac{\partial^4\psi_0}{\partial y^4} = 0. \quad (15)$$

The boundary conditions are

$$\frac{\partial\psi_0}{\partial y} = -\xi\frac{\partial^2\psi_0}{\partial y^2} \text{ at } y = \eta(x), \quad (16)$$

$$\psi_0 \text{ and } \frac{\partial^2\psi_0}{\partial y^2} = 0 \text{ at } y = 0 \quad (17)$$

$$\psi_0 = F(\alpha x) = e^{-\alpha x} \text{ at } y = \eta(x) \quad (18)$$

δ^1 case:

$$\frac{\partial^4\psi_1}{\partial y^4} = R_e\left[\frac{\partial\psi_0}{\partial y}\frac{\partial^3\psi_0}{\partial y^2\partial x} - \frac{\partial\psi_0}{\partial x}\frac{\partial^3\psi_0}{\partial y^3}\right]. \quad (19)$$

The boundary conditions are

$$\frac{\partial\psi_1}{\partial y} = -\xi\frac{\partial^2\psi_1}{\partial y^2} \text{ at } y = \eta(x), \quad (20)$$

$$\psi_1 = 0 \text{ and } \frac{\partial^2\psi_1}{\partial y^2} = 0 \text{ at } y = 0, \quad (21)$$

$$\psi_1 = 0 \text{ at } y = \eta(x) \quad (22)$$

Similar expressions can be written for higher orders of δ. However, since we are looking for an approximate analytical solution for the problem, we consider equations up to order of δ^1

The solution of Eq. (15) together with boundary conditions (16)–(18) is

$$\psi_0(x,y) = A_1(x)y^3 + A_2(x)y, \quad (23)$$

where $A_1 = (-e^{-\alpha x})/(2\eta^3 + 6\xi\eta^2)$ and $A_2 = [(3\eta^2 + 6\xi\eta)e^{-\alpha x}]/[2\eta^3 + 6\xi\eta^2]$.

The solution of Eq. (19) together with boundary conditions (20)–(22) is

$$\psi_1(x,y) = A_8(x)y^7 + A_9(x)y^5 + A_{10}(x)y^3 + A_{11}(x)y, \quad (24)$$

where

$$A_3(x) = \frac{1}{10} A_1 \frac{dA_1}{dx} \eta^6 + \frac{1}{4}\left(A_2 \frac{dA_1}{dx} - A_1 \frac{dA_2}{dx}\right)\eta^4,$$

$$A_4(x) = \frac{3}{5} \xi A_1 \frac{dA_1}{dx} \eta^5 + \xi \left(A_2 \frac{dA_1}{dx} - A_1 \frac{dA_2}{dx}\right)\eta^3,$$

$$A_6(x) = R_e(A_3 + A_4),$$

$$A_7(x) = -R_e\left[\frac{1}{10} A_1 \frac{dA_1}{dx} \eta^7 + \frac{1}{20}\left(A_2 \frac{dA_1}{dx} - A_1 \frac{dA_2}{dx}\right)\eta^5\right],$$

$$A_8(x) = \frac{1}{70} R_e A_1 \frac{dA_1}{dx},$$

$$A_9(x) = \frac{1}{20} R_e \left(A_2 \frac{dA_1}{dx} - A_1 \frac{dA_2}{dx}\right),$$

$$A_{10} = \frac{\eta A_6 - A_7}{2\eta^3 + 6\xi\eta^2},$$

$$A_{11} = \frac{-\eta^3 A_6 + (3\eta^2 + 6\xi\eta) A_7}{2\eta^3 + 6\xi\eta^2}.$$

Hence, substituting ψ_0 and ψ_1 in Eq. (14), we get that

$$\psi(x,y) = A_1(x)y^3 + A_2(x)y + \delta[A_8(x)y^7 + A_9(x)y^5 + A_{10}(x)y^3 + A_{11}(x)y]. \tag{25}$$

Now, the nondimensional pressure $p(x,y)$ can be obtained using Eqs. (25), (8), and (3).

It is given as

$$p(x,y) = \delta \frac{\partial u}{\partial x} + \frac{1}{\delta}\int \frac{\partial^2 u}{\partial y^2} dx - R_e\left(\int u \frac{\partial u}{\partial x} dx + \int v \frac{\partial u}{\partial y} dx\right) + g(y), \tag{26}$$

where $g(y)$ is constant of integration. The mean pressure is given as

$$\bar{p}(x) = \frac{1}{\eta(x)} \int_0^{\eta(x)} p(x,y)dy + g(y). \tag{27}$$

Further, the mean pressure drop between $x = 0$ and $x = x_0$ is

$$\Delta\bar{p}(x_0) = \bar{p}(0) - \bar{p}(x_0). \tag{28}$$

The wall shear stress $\tau_w(x)$ is defined as

$$\tau_w = \frac{(\sigma_{yy} - \sigma_{xx})\dfrac{d\eta}{dx} + \sigma_{xy}\left[1 - \left(\dfrac{d\eta}{dx}\right)^2\right]}{1 - \left(\dfrac{d\eta}{dx}\right)^2} \quad \text{at } y = \eta(x), \tag{29}$$

where $\sigma_{xx} = 2\mu(\partial u)/(\partial x)$, $\sigma_{yy} = 2\mu(\partial v)/(\partial y)$, and $\sigma_{xy} = \mu[(\partial u)/(\partial y) + (\partial v)/(\partial x)]$.

Using the nondimensional quantity $\tau'_w = (d^2/\mu Q_0)\tau_w$, the wall shear τ_w becomes (after dropping the prime),

$$\tau_w = \left\{2\delta^2\left(\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x}\right)\frac{\partial \eta}{\partial x} + \left(\frac{\partial u}{\partial y} + \delta^2 \frac{\partial v}{\partial x}\right) \times \left[1 - \delta^2\left(\frac{d\eta}{dx}\right)^2\right]\right\} \Big/ \left\{1 + \delta^2\left(\frac{d\eta}{dx}\right)^2\right\}. \tag{30}$$

Noted that in Eq. (26), the integrals are difficult to evaluate analytically to get a closed-form expression for $p(x,y)$. Therefore, they are calculated by numerical integration.

4. RESULTS AND DISCUSSION

The aim of this analysis is to study the behavior of an incompressible fluid flow through a channel of converging/diverging and slowly varying cross section with absorbing walls by considering a slip velocity at the walls.

It may be recalled that k characterizes the slope of the converging/diverging wavy walls. Here, k = 0.1 represents a diverging channel, k = 0 represents a normal (sinusoidal channel), and $k = -0.1$ represents a converging channel. The ε and a represent amplitude and permeability parameters of slowly varying walls, respectively. Note that as $\xi \to 0$, the solutions coincide with the results of Muthu and Tesfahun (2010).

We discuss the effects of these parameters on the transverse velocity $[v(x,y)]$, mean pressure drop $(\Delta\bar{p})$, and wall shear stress (τ_w) quantities. In all our numerical calculations, the following parameters are fixed as ε = 0.1 and δ = 0.1. We take R_e = 1.0 to consider the flow with small values of Reynolds number.

4.1 Transverse Velocity

The velocity field can be obtained from Eqs. (14) and (8). In this section, we discuss the effects of reabsorption coefficient (α), the slope parameter (k) in the presence of nonzero slip coefficient (ξ) on the transverse velocity. Also, we look into the behavior of the velocity at different cross sections of the channel.

The effect of reabsorption coefficient (α) , with ξ = 0.15, is presented in the Fig. 2(a). It can be Oobserved from the figure that as α increases, the transverse velocity of the flow increases. Figure 2(b) illustrates the effect of slip coefficient (ξ) on the transverse velocity. Note that the increment of slip coefficient (ξ = 0.0 to ξ = 0.4) is to increase the transverse velocity at the boundary. However, the solution reduces to no-slip case when $\xi \rightarrow 0$, coinciding with the results of Muthu and Tesfahun (2010).

The effect of slope parameter (k), with ξ = 0.15, on the transverse velocity is shown in Fig. 2(c). As k decreases, that is, as the channel changes from diverging to normal and then to converging channels, the velocity decreases. Figure 2(d) shows the behavior of the velocity at the different cross sections of the channel, when k = 0.1. As the fluid moves from the entrance to the exit, the transverse velocity decreases. Naturally, since the outflow of the fluid decreases with x, the transverse velocity has a lesser value at the exit than at the entrance. Moreover, it attains the maximum at the point $\cong$ 0.8 at the entrance and it shifts towards the boundary at the exit.

4.2 Mean pressure Pressure drop Drop ($\Delta\bar{p}$)

The values of the mean pressure drop over the length of the channel are calculated for different values of k, ξ, and a. Figure 3(a) displays the effect of slope parameter k on mean pressure drop. Note that $\Delta\bar{p}$ is less for the divergent channel than for the normal or convergent channels. The slip coefficient ξ has an influence on the mean pressure drop as illustrated in Fig. 3(b). It can be Aobserved that as the slip coefficient increases, the mean pressure drop decreases, because an increase in ξ increases the ve-

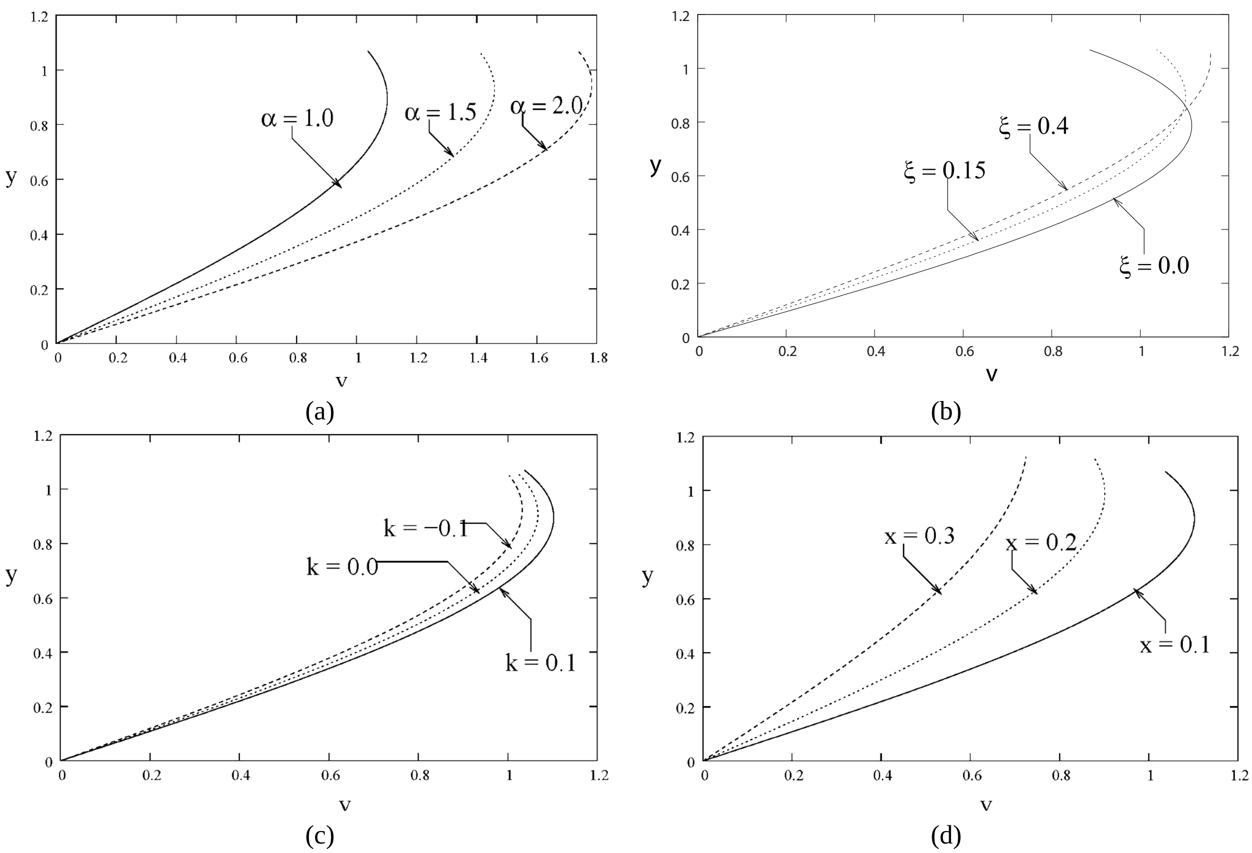

FIG. 2: (a) Distribution of transverse velocity (v) with y (k = 0.1, ξ = 0.15, x = 0.1); **(b)** Distribution of transverse velocity (v) with y (k = 0, α = 1.0, x = 0.1); **(c)** Distribution of transverse velocity (v) with y (k = 1.0, ξ = 0.15, x = 0.1); **(d)** Distribution of transverse velocity (v) with y (k = 0.1, ξ = 0.15, α = 1.0).

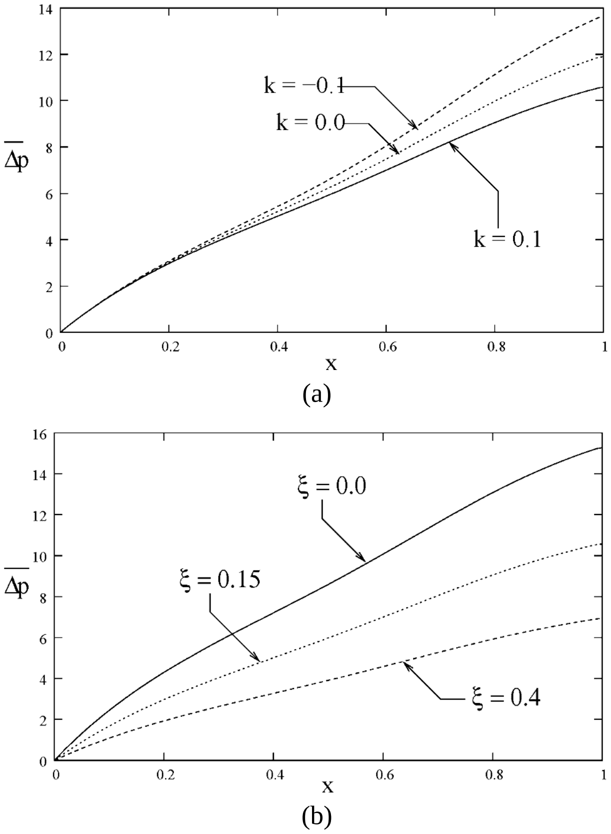

FIG. 3: (a) Distribution of mean pressure drop ($\Delta\bar{p}$) with x (ξ = 0.15, α = 1.0); **(b)** Distribution of mean pressure drop ($\Delta\bar{p}$) with x (k = 0.1, α = 1.0).

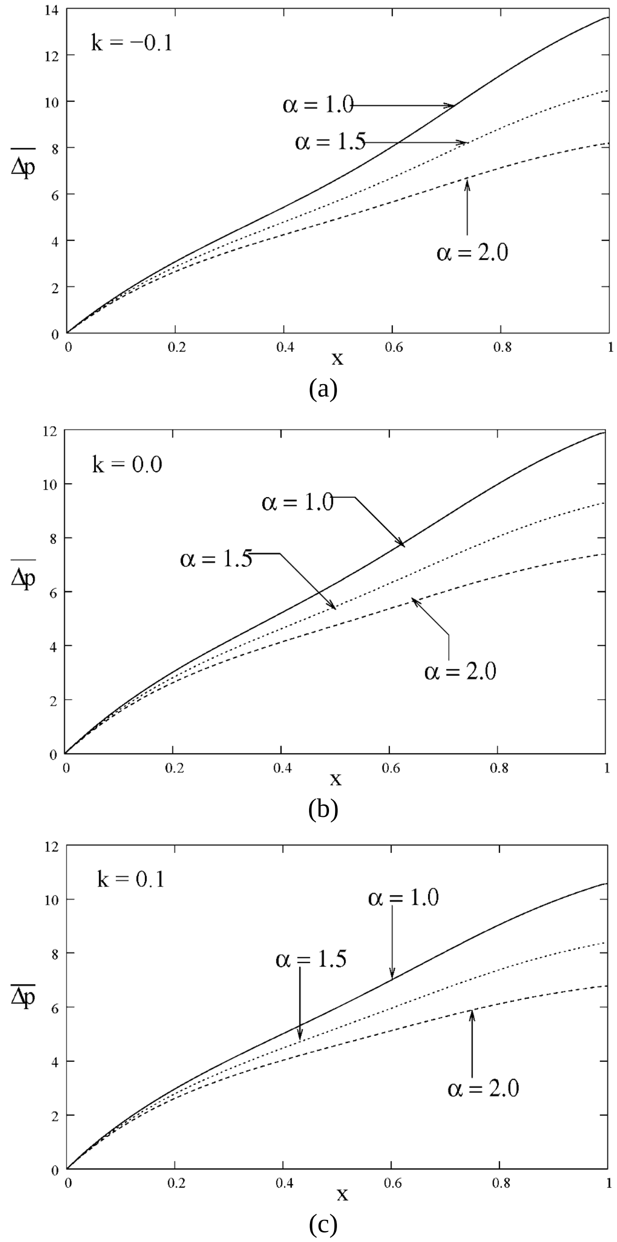

FIG. 4: (a) Distribution of mean pressure drop ($\Delta\bar{p}$) with x (ξ = 0.15, k = –0.1); **(b)** Distribution of mean pressure drop ($\Delta\bar{p}$) with x (ξ = 0.15, k = 0.0); **(c)** Distribution of mean pressure drop ($\Delta\bar{p}$) with x (ξ = 0.15, k = 0.1).

locity, which in turn decreases the mean pressure drop ($\Delta\bar{p}$).

The effect of reabsorption coefficient (a) is presented in the Figs. 4(a)–4(c). It can be Tobserved that the mean pressure drop ($\Delta\bar{p}$) decreases with an increase of a for all forms of the channel (converging, normal, and diverging channels).

4.3 Magnitude of Wall Shear Stress $|\tau_w|$

Figure 5(a) shows the effect of slope parameter k on the magnitude of wall shear stress. We can observe that $|\tau_w|$ is less for the divergent channel than for the normal or convergent channels. Figure 5(b) displays the influence of slip coefficient ξ on $|\tau_w|$. It shows that an increase in the slip coefficient decreases the wall shear stress considerably.

NoteIt may be noted from Figs. 6(a)–6(c) that the magnitude of wall shear stress $|\tau_w|$ decreases with an increase of reabsorption coefficient (a) for all forms of the channel (converging, normal, and diverging channels).

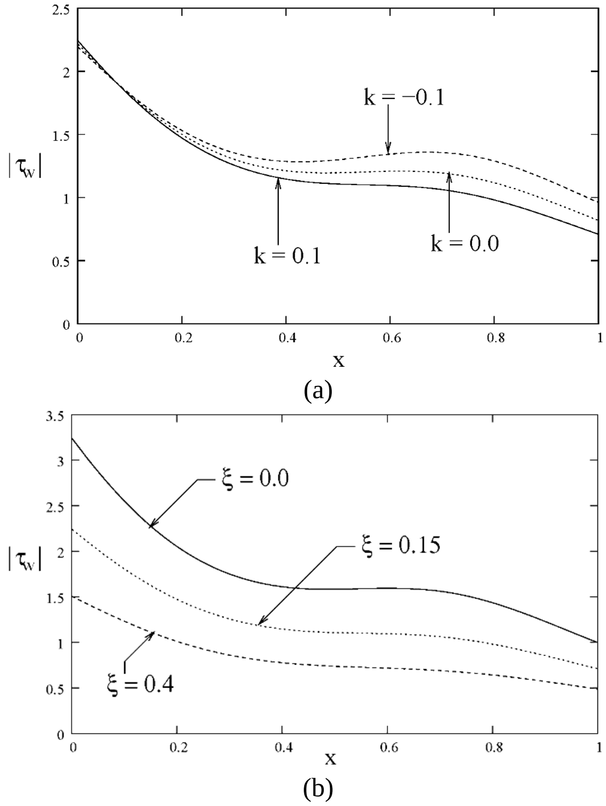

FIG. 5: (a) Distribution of wall shear stress ($|\tau_w|$) with x (ξ = 0.15, α = 1.0); **(b)** Distribution of wall shear stress ($|\tau_w|$) with x (k = 0.1, α = 1.0).

5. CONCLUSIONS

In the present study, an analysis of the mathematical model of incompressible fluid flow in a rigid channel of slowly varying converging/diverging walls has been presented. The main contribution of this study is to show the effect of slip velocity at the boundary on the flow variables, as it is not discussed in the literature with non-zero Reynolds number. The reabsorption coefficient a and the slope parameter k have the same effect on transverse velocity. As they increase, the velocity also increases. The effect of slip coefficient ξ is to increase the transverse velocity at the boundary. When the reabsorption coefficient α increases, the mean pressure drop decreases. This can be justified because,that as most of the fluid flows out in larger amount, the pressure drops. Also, $\Delta\bar{p}$ and $|\tau_w|$ are less for the divergent channel than for the normal or convergent channels. Further, an increase in the slip coefficient ξ decreases both $\Delta\bar{p}$ and $|\tau_w|$ considerably. As $\xi \to 0$, the results are in agreement with the literature.

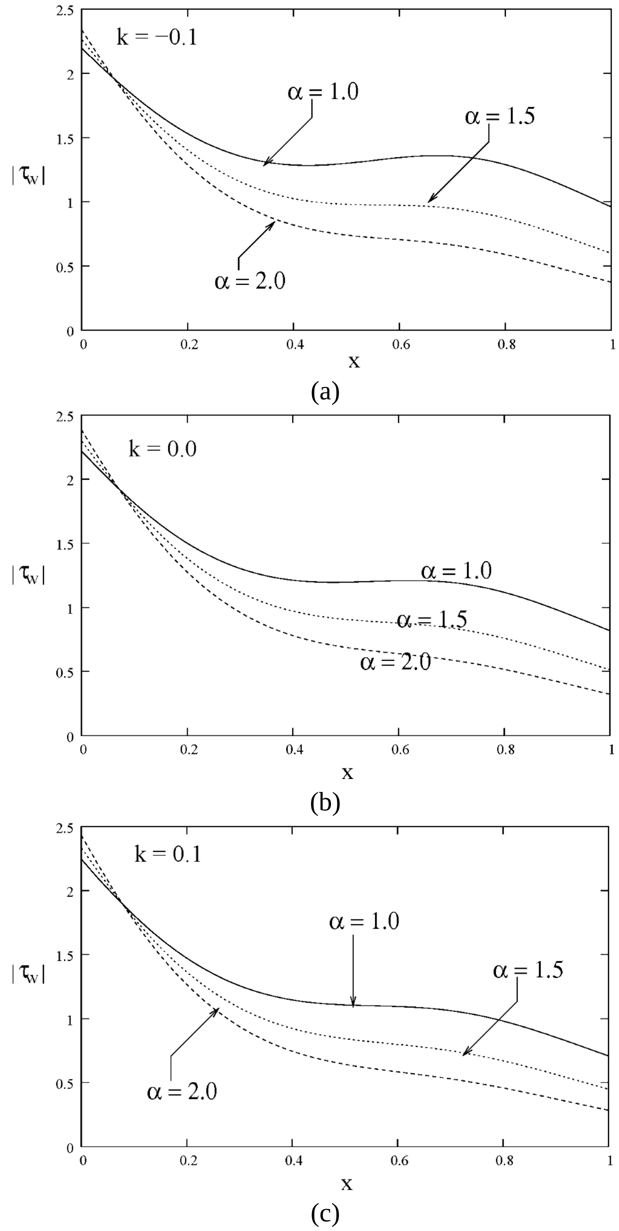

FIG. 6: (a) Distribution of wall shear stress ($|\tau_w|$) with x (ξ = 0.15, k = –0.1); **(b)** Distribution of wall shear stress ($|\tau_w|$) with x (ξ = 0.15, k = 0.0); **(c)** Distribution of wall shear stress ($|\tau_w|$) with x (ξ = 0.15, k = 0.1).

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