ISSN 0021-8944, Journal of Applied Mechanics and Technical Physics, 2015, Vol. 56, No. 4, pp. 590–600. © Pleiades Publishing, Ltd., 2015.
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NON-DARCY NATURAL CONVECTION FROM A VERTICAL PLATE WITH A UNIFORM WALL TEMPERATURE AND CONCENTRATION IN A DOUBLY STRATIFIED POROUS MEDIUM

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UDC 532.5

Abstract: In this paper, non-similarity solutions for natural convection heat and mass transfer along a vertical plate with a uniform wall temperature and concentration in a doubly stratified porous medium saturated by a fluid are obtained. The Darcy–Forchheimer-based model is employed to describe the flow in the porous medium. The nonlinear governing equations and their associated boundary conditions are initially cast into dimensionless forms by using pseudo-similarity variables. The resulting system of nonlinear partial differential equations is then solved numerically by using the Keller-box method. The effects of the buoyancy parameter, Forchheimer number, and thermal and solutal stratification parameters on the dimensionless velocity, temperature, concentration, and heat and mass transfer coefficients are studied.

Keywords: natural convection, stratified medium, Darcy–Forchheimer law.

DOI: 10.1134/S0021894415040069

INTRODUCTION

The study of transport phenomena in porous media has many important applications in geothermal systems, nuclear engineering, petroleum engineering, insulation technology, and many others.

The analysis of free convection heat and mass transfer in porous media has been an active field of research. These processes play a crucial role in versatile applications, such as thermal insulation, distributions of temperature and moisture over agricultural fields and groves of fruit trees, protection of crops from freezing and pollution of the environment, extraction of crude oil, chemical catalytic reactors, etc.

A fundamental study of the phenomenon of natural convection heat and mass transfer near a vertical surface embedded in a fluid saturated porous medium was reported by Bejan and Khairy [1].

Kim and Vafai [2] performed an analytical and numerical analysis of a buoyancy-driven fluid flow and heat transfer about a vertical plate embedded in a porous medium for a constant wall temperature. The effect of the heat flux is also taken into account.

Lai and Kulacki [3] reported similarity solutions for buoyancy-induced heat and mass transfer from a vertical plate embedded in a saturated porous medium for a constant wall temperature/concentration and constant wall heat/mass fluxes.

Bakier et al. [4] presented a non-similar boundary layer analysis for free convection along a vertical plate embedded in a fluid saturated porous medium in the presence of surface mass transfer and internal heat generation. Abbas et al. [5] considered a numerical study on the combined effect of thermal dispersion and thermal radiation

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 0021-8944/15/5604-0590 © 2015 by Pleiades Publishing, Ltd.

on the non-Darcy natural convection flow over a vertical plate kept at a high and constant temperature in a fluid saturated porous medium.

Cheng [6] studied double diffusive natural convection near an inclined wavy surface in a fluid saturated porous medium with a constant wall temperature and concentration.

Partha [7] examined natural convection in a non-Darcy porous medium by using a temperature-concentration-dependent density relation. Non-similarity solutions were presented for natural convection from a vertical wavy plate embedded in a saturated porous medium in the presence of surface mass transfer by Mahdy et al. [8]. Vajravelu et al. [9] obtained numerical solutions for free convection heat transfer in a viscous fluid at a permeable surface embedded in a saturated porous medium in the presence of viscous dissipation with temperature-dependent variable fluid properties. Shakeri et al. [10] analyzed free convection heat transfer over a vertical cylinder with variable surface temperature distributions in a porous medium.

A natural convection flow past a vertical porous plate in a porous medium was studied numerically, with the Dufour and Soret effects taken into account, by Aouachriaa et al. [11]. Khanafer [12] analyzed numerically non-Darcian effects on a natural convective flow and heat transfer in a square enclosure filled with a porous medium by using a fluid-structure interaction model.

Several attempts have been made in recent years to investigate the problem of natural convection over a vertical plate in a stratified medium due to its geophysical and industrial applications. These applications include heat rejection into the environment, such as lakes, rivers, and seas, thermal energy storage systems, such as solar ponds, and heat transfer from thermal sources, such as the condensers of power plants. However, the effect of double stratification on free convection in porous media has received very little attention.

Stratification of fluids arises due to temperature variations, concentration differences, or the presence of different fluids. The input of thermal energy into enclosed fluid regions often leads to generation of stable thermal stratification.

Murthy et al. [13] analyzed the effect of double stratification on double diffusive natural convection from a vertical impermeable plate in Darcian porous media.

A combined heat and mass transfer process by natural convection along a vertical wavy surface in a thermally and mass stratified fluid saturated porous enclosure was numerically studied by Rathish Kumar and Shalini [14]. Magyari et al. [15] considered the problem of unsteady free convection heat transfer from a one-dimensional (parallel) flow along an infinite vertical plate embedded in a thermally stratified fluid saturated porous medium. Lakshi Narayana and Murthy [16] analyzed numerically the problem of free convective heat and mass transfer from a vertical plate embedded in a doubly stratified Darcy porous medium with the Soret and Dufour effects. Beg et al. [17] presented a mathematical model for a two-dimensional steady incompressible laminar free convection flow over a continuously moving plate immersed in a thermally stratified high-porosity non-Darcian porous medium.

Lin et al. [18] investigated a transient natural convection boundary-layer flow adjacent to a vertical plate heated by a uniform flux in an initially linearly stratified ambient fluid with the Prandtl number smaller than unity by using a scaling analysis and direct numerical simulation.

Cheng [19] studied the coupled heat and mass transfer by natural convection near a vertical wavy surface in a non-Newtonian fluid saturated porous medium with thermal and mass stratification.

Srinivasacharya and RamReddy [20] considered natural convection heat and mass transfer along a vertical plate embedded in a doubly stratified micropolar fluid saturated non-Darcy porous medium.

A series solution for non-similarity natural convection boundary-layer flows over a permeable vertical surface was presented by Kousar and Liao [21]. Neagu [22] considered natural convection heat and mass transfer from a vertical wavy wall with constant surface heat and mass fluxes to a non-Darcy porous medium with thermal and mass stratification.

Srinivasacharya and RamReddy [23] studied the flow and heat and mass transfer characteristics of free convection on a vertical plate with uniform and constant heat and mass fluxes in a doubly stratified micropolar fluid saturated non-Darcy porous medium.

Early studies on convection transport focused on seeking similarity solutions because similarity variables can give a great physical insight with minimum efforts.

However, non-similarity boundary layer flows are more general in nature and in our everyday life; thus, they are more important than similarity flows. Hence, in the present study, we made an attempt to obtain non-similar

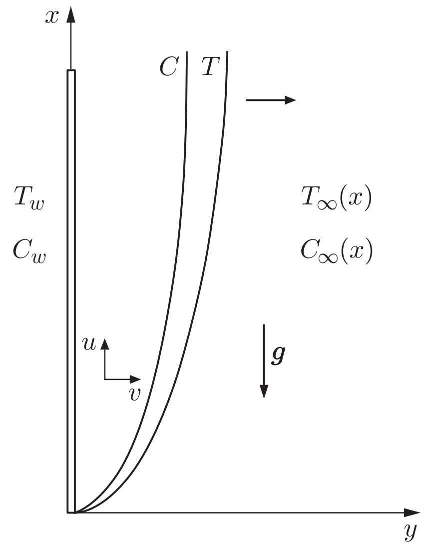

Fig. 1. Physical model and coordinate system.

solutions for the problem of natural convection on a vertical plate with a constant and uniform wall temperature and concentration in a stable doubly stratified non-Darcian fluid saturated porous medium in which the ambient temperature and concentration vary linearly. An implicit finite difference scheme given by Cebeci and Bradshaw [24] is employed to investigate the nonlinear system of this particular problem. The effects of thermal and mass stratification parameters, Lewis number, Forchheimer number, and buoyancy parameter are examined.

1. FORMULATION OF THE PROBLEM

We consider non-Darcian natural convective heat and mass transfer along a semi-infinite vertical plate in a stable, doubly stratified viscous fluid saturated porous medium. The x coordinate is taken along the plate, in the ascending direction, and the y coordinate is measured normal to the plate, while the origin of the reference system is considered at the leading edge of the vertical plate. The physical model and the coordinate system are shown in Fig. 1. The plate is maintained at a uniform and constant wall temperature T_w and concentration C_w. The ambient medium is assumed to be vertically linearly stratified with respect to both temperature and concentration in the form $T_\infty(x) = T_{\infty,0} + A_1 x$, $C_\infty(x) = C_{\infty,0} + B_1 x$ (A_1 and B_1 are constants characterizing the intensity of stratification; $T_{\infty,0}$ and $C_{\infty,0}$ are the ambient temperature and concentration, respectively). The temperature T_w and concentration C_w are assumed to be greater than the ambient temperature $T_{\infty,0}$ and concentration $C_{\infty,0}$ at any arbitrary reference point in the medium.

In formulating the present problem, the following assumptions are made:

(1) the flow is considered to be steady, laminar, incompressible, and two-dimensional;

(2) the porous medium is considered to be homogeneous and isotropic (constant porosity and permeability);

(3) the fluid has constant properties, except for the density in the buoyancy term of the balance of the momentum equation;

(4) the Forchheimer relationship between the pressure gradient and the filtration rate is valid (Forchheimer flow model);

(5) the Boussinesq and boundary layer approximations are applicable.

With the above-made assumptions, the governing equations for the flow are written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0; \tag{1}$$

$$\frac{\partial p}{\partial y} = 0; \tag{2}$$

$$u + \frac{c\sqrt{K}}{\nu} u^2 = -\frac{\partial p}{\partial x} + \frac{Kg}{\nu} [\beta_T(T - T_\infty) + \beta_C(C - C_\infty)]; \tag{3}$$
$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}, \qquad u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}.$$

Differentiating Eqs. (2) and (3), we can eliminate p; thus, we obtain

$$\frac{\partial u}{\partial y} + \frac{2c\sqrt{K}}{\nu} u \frac{\partial u}{\partial y} = \frac{Kg\beta_T}{\nu} \frac{\partial T}{\partial y} + \frac{Kg\beta_C}{\nu} \frac{\partial C}{\partial y},$$
$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}, \qquad u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}. \tag{4}$$

In Eqs. (1)–(4), u and v are the average velocity components in the x and y directions, respectively, T is the temperature, C is the concentration, β_T and β_C are the thermal and solutal expansion coefficients, respectively, ν is the kinematic viscosity of the fluid, c is an empirical constant associated with the Forchheimer inertia term, K is the permeability, g is the acceleration due to gravity, and α and D are the thermal and solutal diffusivities of the porous medium.

The following boundary conditions are imposed:

$$y = 0: \qquad v = 0, \quad T = T_w, \quad C = C_w,$$
$$y \to \infty: \qquad u = 0, \quad T = T_\infty(x), \quad C = C_\infty(x). \tag{5}$$

In view of Eq. (1), we introduce the stream function ψ as

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}. \tag{6}$$

Substituting Eqs. (6) into Eqs. (4) and then using the dimensionless variables

$$\xi = \frac{x}{L}, \qquad \eta = \frac{\mathrm{Ra}^{1/2}}{L\xi^{1/2}} y, \qquad \psi = \alpha \, \mathrm{Ra}^{1/2} \xi^{1/2} f(\xi, \eta),$$
$$T - T_\infty(x) = (T_w - T_{\infty,0})\theta(\xi, \eta), \qquad C - C_\infty(x) = (C_w - C_{\infty,0})\varphi(\xi, \eta),$$

we obtain the system of differential equations

$$f'' + 2F_c f' f'' = \theta' + B\varphi',$$
$$\theta'' + \frac{1}{2} f\theta' - \varepsilon_1 \xi f' = \xi \Big(f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \Big), \tag{7}$$
$$\frac{1}{\mathrm{Le}} \varphi'' + \frac{1}{2} f\varphi' - \varepsilon_2 \xi f' = \xi \Big(f' \frac{\partial \varphi}{\partial \xi} - \varphi' \frac{\partial f}{\partial \xi} \Big),$$

where L is the plate length, the prime denotes differentiation with respect to η, $\mathrm{Ra} = Kg\beta_T(T_w - T_{\infty,0})L/(\alpha\nu)$ is the Rayleigh number, $F_c = c\sqrt{K}\,\mathrm{Ra}/(L\mathrm{Pr})$ is the Forchheimer number, $\mathrm{Pr} = \nu/\alpha$ is the Prandtl number, $\mathrm{Le} = \alpha/D$ is the diffusivity ratio, $B = \beta_C(C_w - C_{\infty,0})/[\beta_T(T_w - T_{\infty,0})]$ is the buoyancy ratio, and $\varepsilon_1 = A_1 L/(T_w - T_{\infty,0})$ and $\varepsilon_2 = B_1 L/(C_w - C_{\infty,0})$ are the thermal and solutal stratification parameters, respectively.

The parameter B measures the relative importance of mass and thermal diffusion in the buoyancy-driven flow. At $B = 0$, there is no mass diffusion and the flow is driven by thermal buoyancy alone, i.e., the buoyancy force arises solely from the temperature difference. At $B > 0$, the buoyancy forces induced by mass and thermal diffusion are combined to assist the flow; at $B < 0$, they oppose each other.

The boundary conditions (5) have the following form in the new variables:

$$f(\xi, 0) = -2\xi \Big(\frac{\partial f}{\partial \xi} \Big)\Big|_{\eta=0}, \qquad \theta(\xi, 0) = 1 - \varepsilon_1 \xi, \qquad \varphi(\xi, 0) = 1 - \varepsilon_2 \xi,$$
$$f'(\xi, \infty) = 0, \qquad \theta(\xi, \infty) = 0, \qquad \varphi(\xi, \infty) = 0. \tag{8}$$

Results of practical interest are both the heat and mass transfer rates. The local Nusselt number Nu_ξ and the local Sherwood number Sh_ξ are defined as

$$\frac{\mathrm{Nu}_\xi}{\mathrm{Ra}^{1/2}} = -\xi^{1/2}\theta'(\xi, 0), \qquad \frac{\mathrm{Sh}_\xi}{\mathrm{Ra}^{1/2}} = -\xi^{1/2}\varphi'(\xi, 0).$$

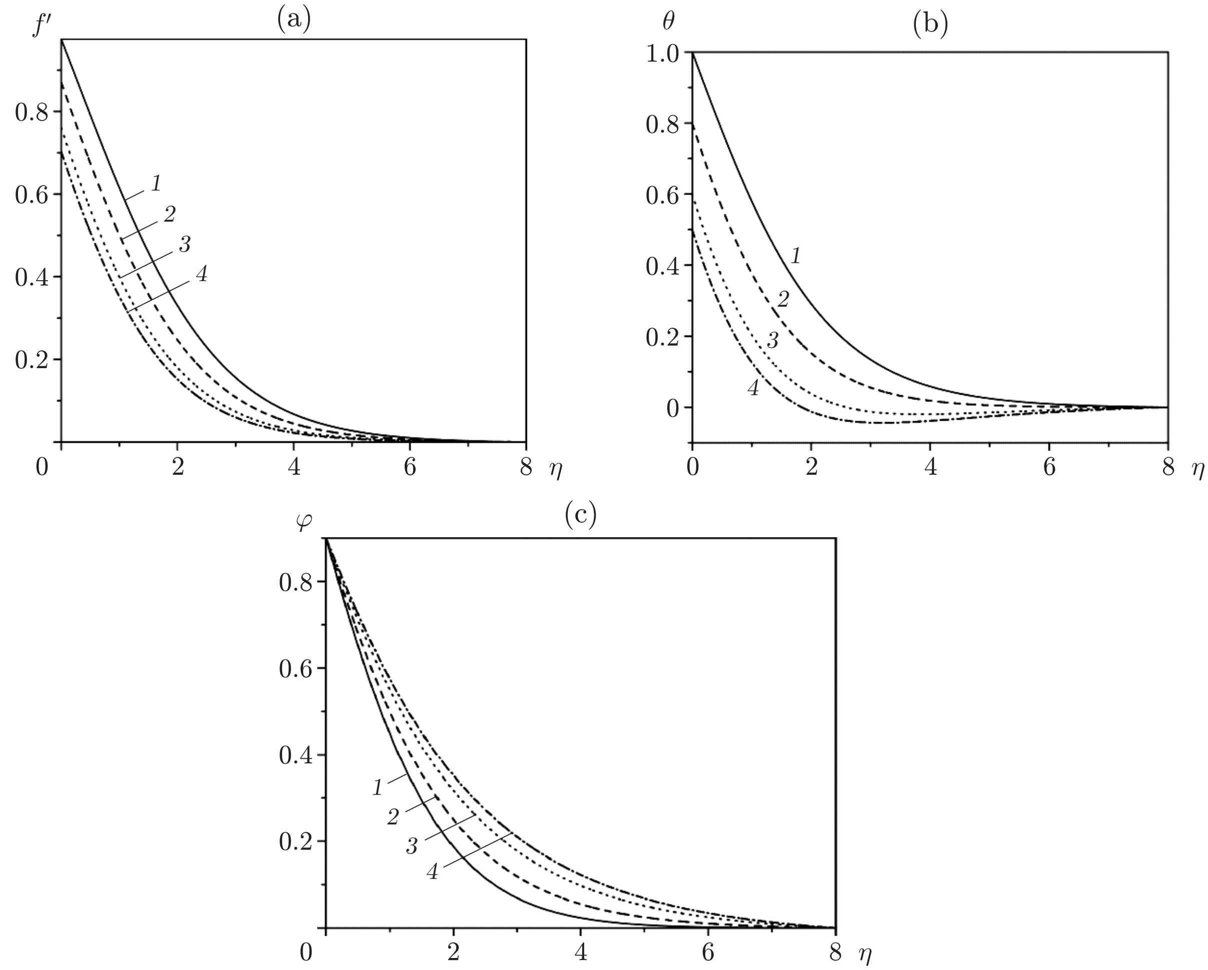

Fig. 2. Velocity (a), temperature (b), and concentration (c) versus η for $\varepsilon_2 = 0.2$, $F_c = 0.5$, $B = 0.5$, Le $= 1.0$, and different values of the thermal stratification parameter: $\varepsilon_1 = 0$ (*1*), 0.4 (*2*), 0.8 (*3*), and 1.0 (*4*).

2. RESULTS AND DISCUSSION

Equations (7) with the boundary conditions (8) constitute nonlinear non-homogeneous differential equations for which an analytic solution cannot be obtained. Hence, these equations are solved numerically by using a very efficient implicit finite-difference method known as the Keller-box scheme [24]. First, equations (7) are converted to a system of first-order equations by substituting $f' = F$, $\theta' = G$, and $\varphi' = P$. Then, the partial derivatives with respect to ξ and η are written in the difference form by using the central difference approximation averaged at the midpoints of net rectangles in the domain (ξ, η) to obtain finite difference equations. The resulting nonlinear algebraic equations are linearized by applying the Newton method and are cast as a block matrix system. This system is solved by using a block-tridiagonal-elimination technique.

The initial conditions are arbitrarily chosen so that they satisfy the boundary conditions. A convergence criterion based on the relative difference between the current and previous iterations is used. When this difference reaches 10^{-5}, the solutions are assumed to have converged and the iterative process is terminated.

This method has been proven to be adequate and gave accurate results for boundary layer equations. In the present study, the boundary conditions at infinity are replaced by a sufficiently large value of η at which the velocity, temperature, and concentration profiles approach zero. We take $\eta_\infty = 8$, and the grid size of η is 0.02. In order to study the effects of the stratification parameters ε_1 and ε_2, computations are carried out for different values of these parameters and fixed values $F_c = 0.5$, Le $= 1.0$, and $B = 0.5$.

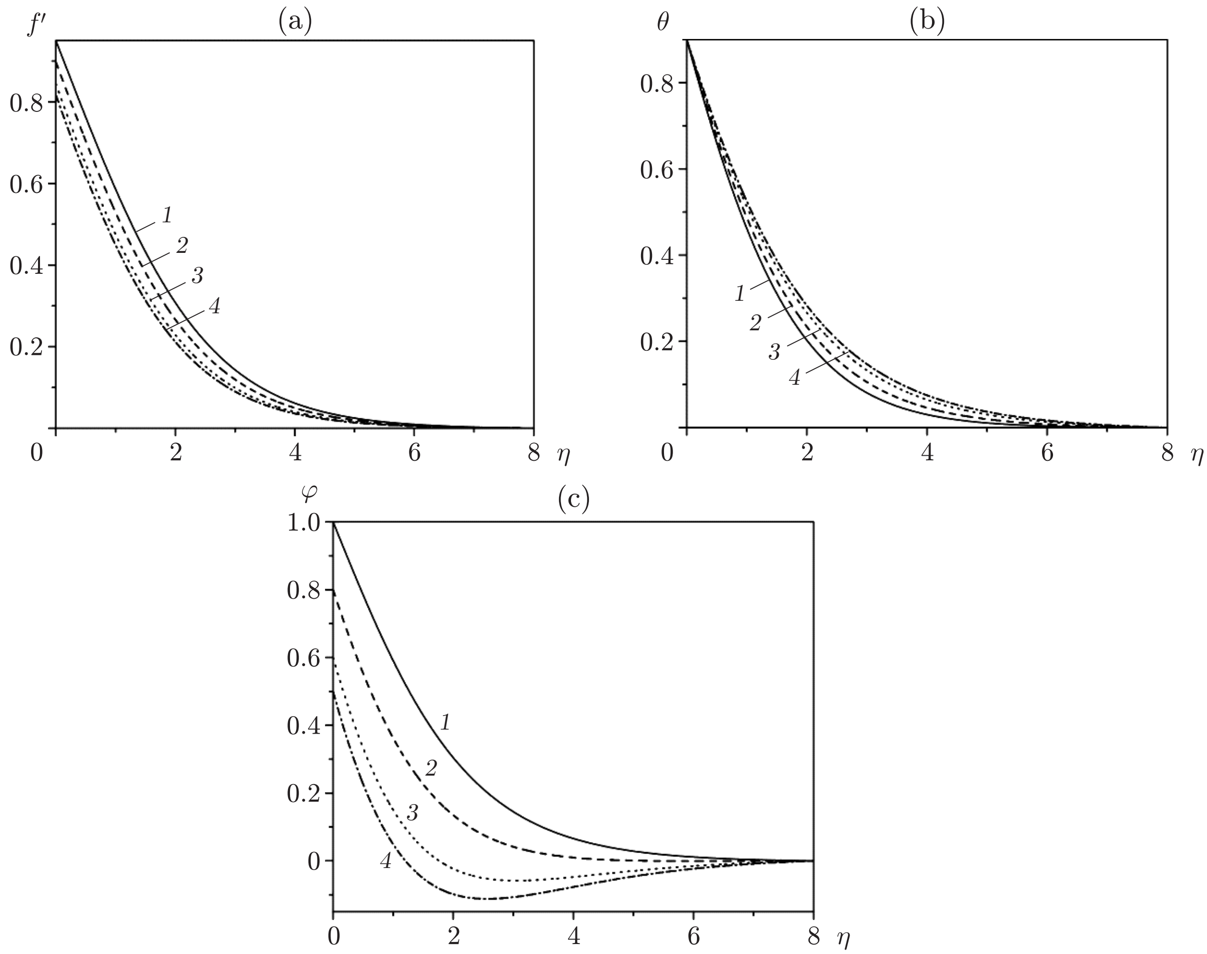

Fig. 3. Velocity (a), temperature (b), and concentration (c) versus η for $\varepsilon_1 = 0.2$, $F_c = 0.5$, $B = 0.5$, $\mathrm{Le} = 1.0$, and different values of the thermal stratification parameter: $\varepsilon_2 = 0$ (*1*), 0.4 (*2*), 0.8 (*3*), and 1.0 (*4*).

In order to assess the accuracy of our method, we compare our results with those of Plumb and Huenefeld [25] for $\varepsilon_1 = 0$, $\varepsilon_2 = 0$, $\xi = 0$, $B = 0$, $\mathrm{Le} = 1.0$, and different values of F_c (see the table).

The variations of the dimensionless velocity, temperature, and concentration profiles as functions of η for different values of the thermal stratification parameter ε_1 are illustrated in Fig. 2. It is observed from Fig. 2a that the fluid velocity decreases with an increase in the thermal stratification parameter. This is because thermal stratification reduces the effective convective potential between the heated plate and the ambient fluid. Hence, thermal stratification reduces the velocity in the boundary layer. From Fig. 2b, it is clear that the fluid temperature decreases with an increase in the thermal stratification parameter. If thermal stratification is taken into consideration, the effective temperature difference between the plate and the ambient fluid decreases; therefore, the thermal boundary layer is thickened and the temperature is reduced. It is noticed from Fig. 2c that the fluid concentration increases with an increase in the thermal stratification parameter.

Figure 3 depicts the effect of the solutal stratification parameter ε_2 on the dimensionless velocity, temperature, and concentration as functions of η. It is noticed from Figs. 3a and 3b that the fluid velocity decreases with an increase in the solutal stratification parameter ε_2, while the temperature increases. From Fig. 3c, it is observed that the fluid concentration decreases with an increase in the solutal stratification parameter ε_2. The dimensionless temperature and concentration values can become negative inside the boundary layer. The position and length of the interval with negative values depend on the values of other parameters. The reason is that the temperature and concentration near the plate are smaller than the corresponding values in the stratified ambient medium. This result is in tune with the observation made in [13, 14, 16, 26–28].

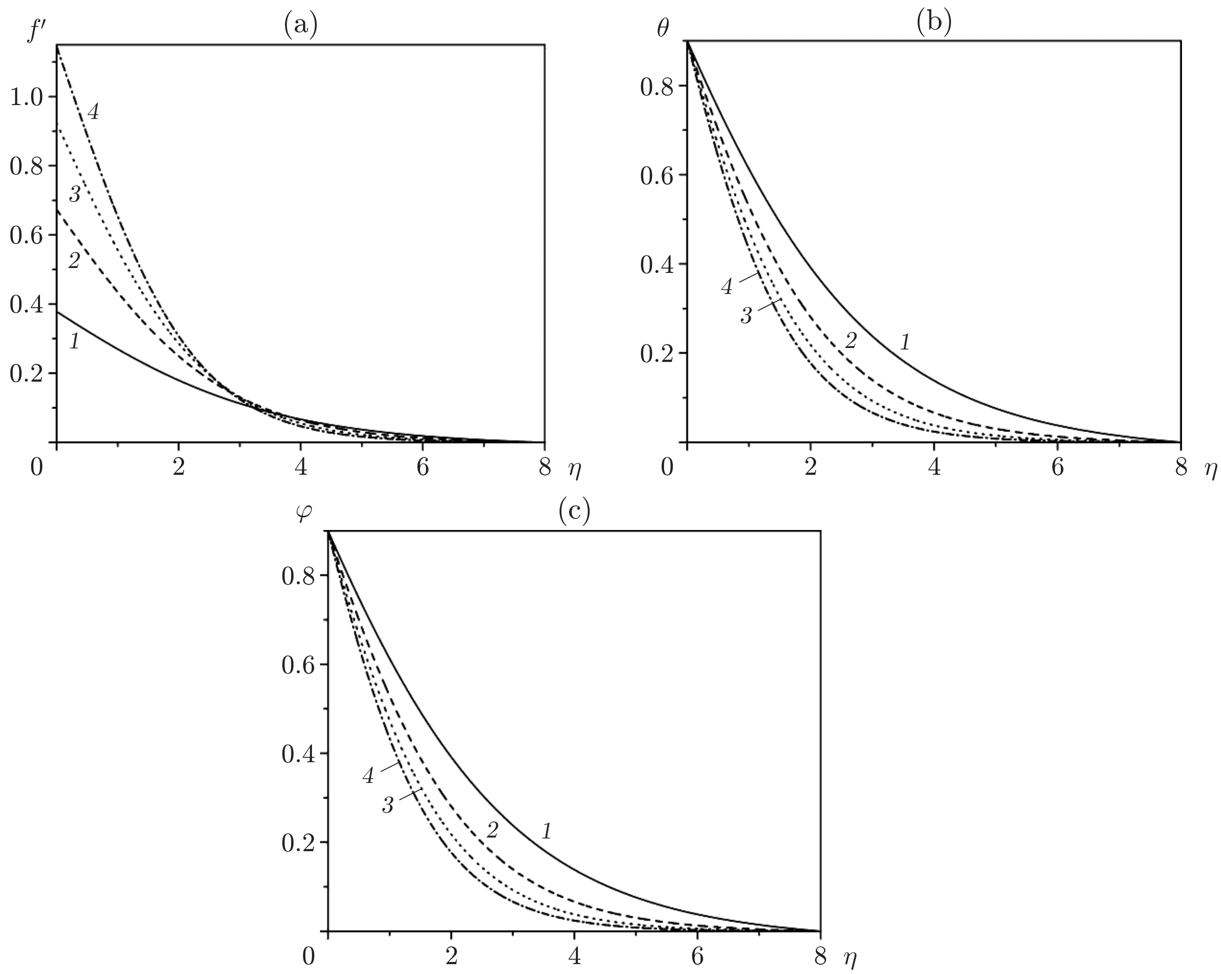

Fig. 4. Velocity (a), temperature (b), and concentration (c) versus η for $F_c = 0.5$, $\mathrm{Le} = 1.0$, $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.2$, and $B = -0.5$ (*1*), 0 (*2*), 0.5 (*3*), and 1.0 (*4*).

Values of $\theta'(0)$ and $f'(0)$ for $\varepsilon_1 = 0$, $\varepsilon_2 = 0$, $B = 0$, $\mathrm{Le} = 1.0$, and different values of F_c

F_c	$\theta'(0)$		$f'(0)$	
	Data of Plumb and Huenefeld [25]	Data of this work	Data of Plumb and Huenefeld [25]	Data of this work
0	−0.44390	−0.44391	1.00000	1.00000
0.01	−0.44232	−0.442 32	0.99020	0.99020
0.10	−0.42969	−0.429 69	0.91608	0.91608
1.00	−0.36617	−0.366 17	0.61803	0.61803
10.00	−0.25126	−0.251 26	0.27016	0.27016

The effect of the buoyancy parameter B on the dimensionless velocity, temperature, and concentration as functions of η is shown in Fig. 4. It is clear from Fig. 4a that the fluid velocity increases near the plate and decreases away from the plate with an increase in the buoyancy parameter. The fluid temperature and concentration decrease with an increase in the buoyancy parameter (see Figs. 4b and 4c). This is because increasing the buoyancy ratio B enhances the surface heat and mass transfer rates. An increase in the buoyancy parameter increases the velocity near the vertical plate, and the high-velocity flow near the surface carries more heat away from the surface. As a result, the thermal and solutal boundary layer thicknesses decrease.

Figure 5 depicts the effect of the Forchheimer number on the dimensionless velocity, temperature, and concentration as functions of η. It is observed from Fig. 5a that the fluid velocity decreases near the plate and increases away from the plate with an increase in the Forchheimer number. As F_c represents the inertial drag, an

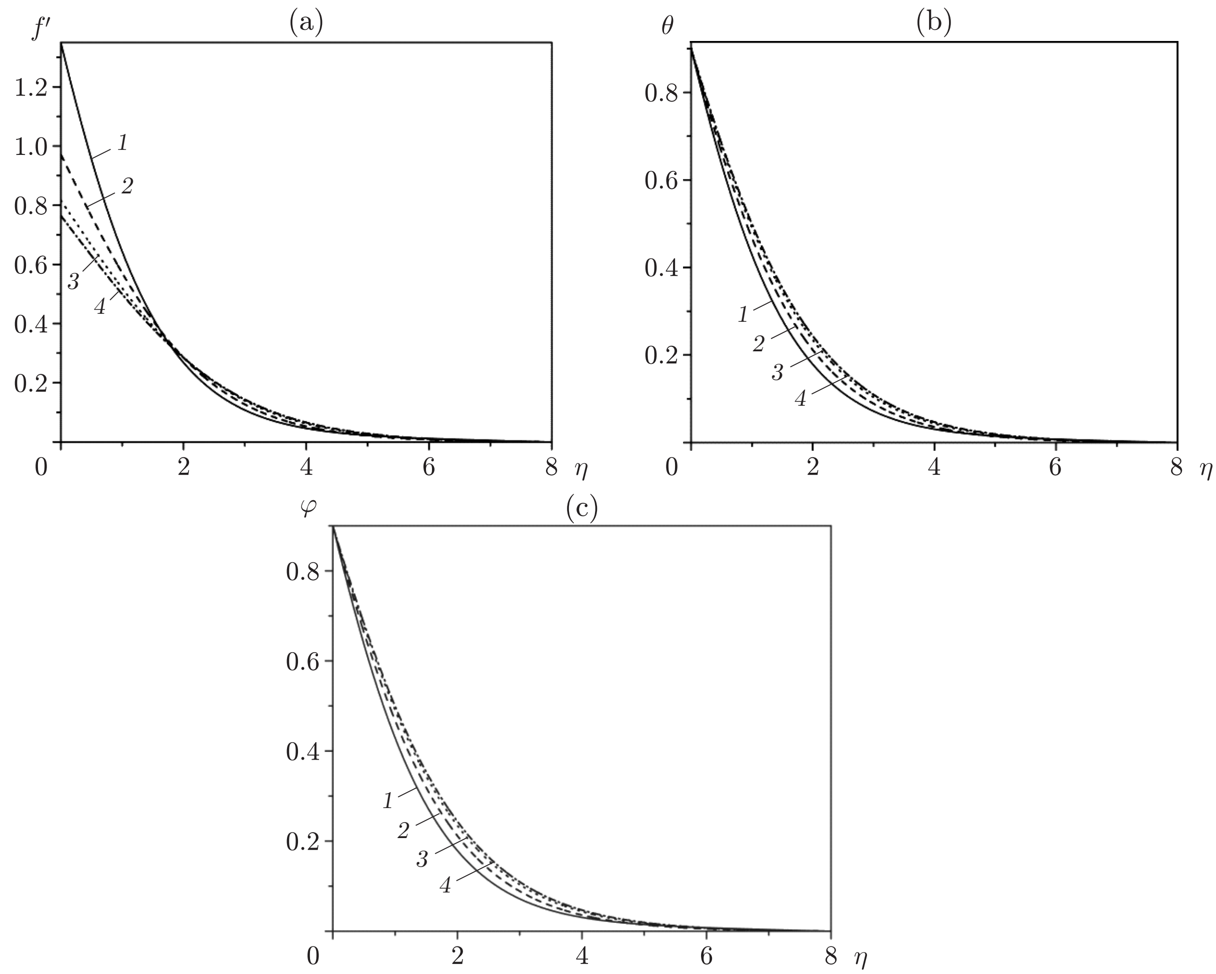

Fig. 5. Velocity (a), temperature (b), and concentration (c) versus η for $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.2$, $\mathrm{Le} = 1.0$, $B = 0.5$, and $F_c = 0$ (*1*), 0.4 (*2*), 0.8 (*3*), and 1.0 (*4*).

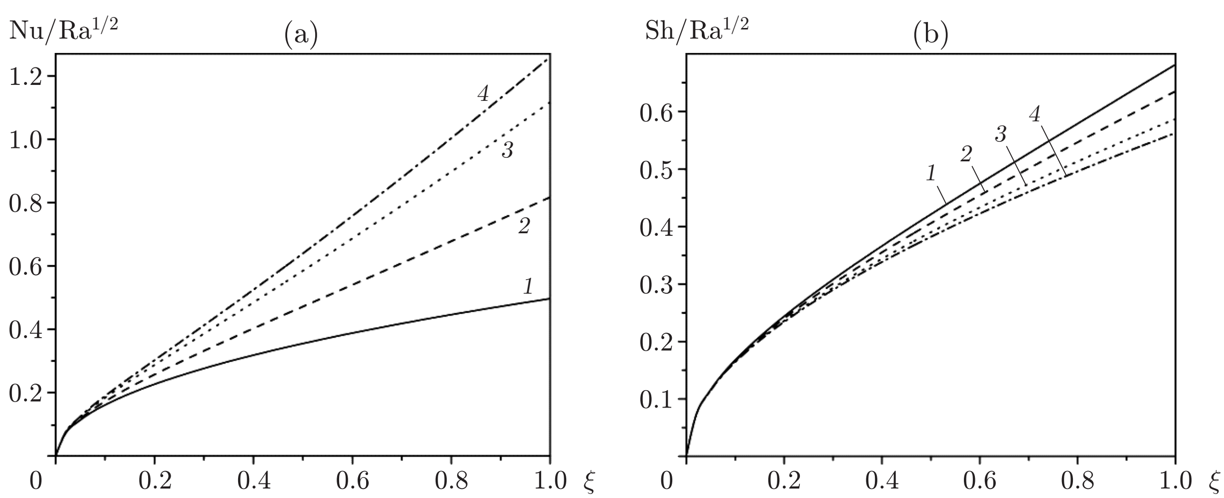

Fig. 6. Nusselt number (a) and Sherwood number (b) versus ξ for $\varepsilon_2 = 0.2$, $F_c = 0.5$, $B = 0.5$, $\mathrm{Le} = 1.0$, and $\varepsilon_1 = 0$ (*1*), 0.4 (*2*), 0.8 (*3*), and 1.0 (*4*).

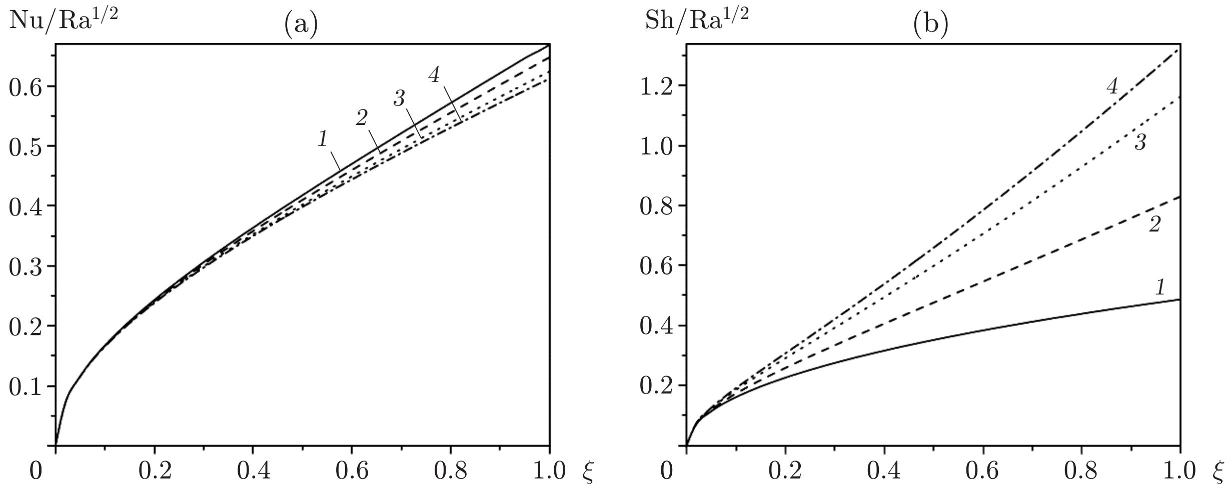

Fig. 7. Nusselt number (a) and Sherwood number (b) versus ξ for $\varepsilon_1 = 0.2$, $F_c = 0.5$, $B = 0.5$, $\mathrm{Le} = 1.0$, and $\varepsilon_2 = 0$ (*1*), 0.4 (*2*), 0.8 (*3*), and 1.0 (*4*).

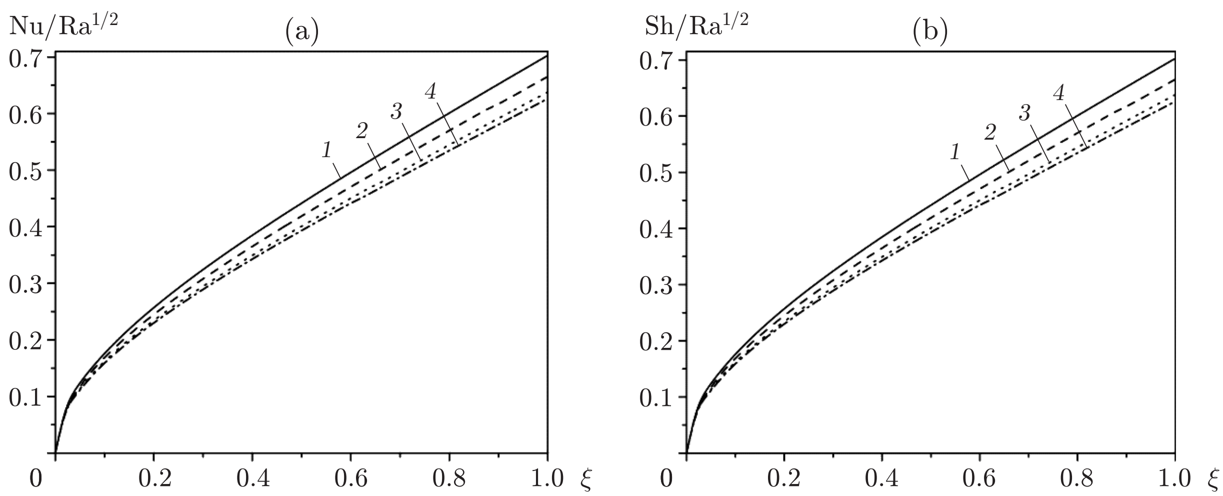

Fig. 8. Nusselt number (a) and Sherwood number (b) versus ξ for $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.2$, $B = 0.5$, $\mathrm{Le} = 1.0$, and $F_c = 0$ (*1*), 0.4 (*2*), 0.8 (*3*), and 1.0 (*4*).

increase in the Forchheimer number enhances the resistance to the flow and, as a consequence, the fluid velocity near the plate decreases. Here $F_c = 0$ represents the case where the flow is Darcian. The velocity has the maximum value in this case due to the total absence of inertial drag. It is noticed from Fig. 5b that the fluid temperature increases with an increase in the Forchheimer number because the fluid is decelerated and the kinetic energy transforms to heat. From Fig. 5c, it is observed that the fluid concentration increases with an increase in the Forchheimer number; the thermal and concentration boundary layer thicknesses increase thereby.

The variations of the local Nusselt number and Sherwood number as functions of ξ for different values of the thermal stratification parameter ε_1 are presented in Fig. 6. It is found from Fig. 6a that the local heat transfer rate increases with an increase in the thermal stratification parameter ε_1. Physically, positive values of the stratification parameter ε_1 have the tendency to decrease the boundary layer thickness due to reduction in the temperature difference between the plate and the free stream. This causes an increase in the Nusselt number. Figure 6b illustrates that the local Sherwood number decreases with an increase in the thermal stratification parameter.

The influence of the solutal stratification parameter ξ on the local Nusselt number Nu and Sherwood number Sh for different values of ε_2 is shown in Fig. 7. It is found from Fig. 7a that the local heat transfer rate slightly decreases with an increase in ε_2. The Sherwood number increases with an increase in the solutal stratification parameter (see Fig. 7b).

The effect of the Forchheimer number on the Nusselt number Nu and Sherwood number Sh as functions of ξ is illustrated in Fig. 8. It is seen that the Nusselt and Sherwood numbers decrease with an increase in F_c. As F_c represents the inertial drag, an increase in the Forchheimer number enhances the resistance to the flow.

CONCLUSIONS

Natural convection heat and mass transfer from a vertical surface embedded in a doubly stratified non-Darcy porous medium saturated with a Newtonian fluid is analyzed. The relationship between the fluid filtration rate and the pressure gradient is described by the Forchheimer model.

Numerical solutions are obtained for different values of the thermal stratification parameter, solutal stratification parameter, and buoyancy parameter. An increase in the thermal stratification parameter ε_1 decreases the velocity, temperature, and local mass transfer coefficient, but increases the concentration and local heat transfer coefficient. A higher value of the solutal stratification parameter ε_2 results in a lower velocity, concentration, and local heat transfer coefficient, but a higher temperature and local mass transfer coefficient. The effect of the buoyancy parameter is to increase the velocity near the plate and to decrease the velocity away from the plate, temperature, and concentration. The influence of the Forchheimer number is to decrease both the local heat and mass transfer coefficients.

REFERENCES

1. A. Bejan and R. K. Khairy, “Heat and Mass Transfer by Natural Convection in Porous Medium,” Int. J. Heat Mass Transfer **28**, 909–918 (1985).
2. S. J. Kim and K. Vafai, “Analysis of Natural Convection about a Vertical Plate Embedded in a Porous Medium,” Int. J. Heat Mass Transfer **32**, 665–677 (1989).
3. F. C. Lai and F. A. Kulacki, “Coupled Heat and Mass Transfer by Natural Convection from Vertical Surfaces in Porous Media,” Int. J. Heat Mass Transfer **34**, 1189–1194 (1991).
4. A. Y. Bakier, M. A. Mansour, R. S. R.Gorla, and A. B. Ebiana, “Nonsimilar Solutions for Free Convection from a Vertical Plate in Porous Media,” Heat Mass Transfer **33**, 145–148 (1997).
5. I. A. Abbas, M. F. El-Amin, and A. Salama, “Combined Effect of Thermal Dispersion and Radiation on Free Convection in a Fluid Saturated, Optically Thick Porous Medium,” Forsch. Ingenieurw. **72**, 135–144 (2008).
6. C. Y. Cheng, “Double Diffusive Natural Convection Along an Inclined Wavy Surface in a Porous Medium,” Int. Comm. Heat Mass Transfer **37**, 1471–1476 (2010).
7. M. K. Partha, “Nonlinear Convection in a Non-Darcy Porous Medium,” Appl. Math. Mech. **31**, 565–574 (2010).
8. A. Mahdy, R. A. Mohamed, and F. M. Hady, “Natural Convection Heat and Mass Transfer over a Vertical Wavy Surface with Variable Wall Temperature and Concentration in Porous Media,” Int. J. Appl. Math. Mech. **7**, 1–13 (2011).
9. K. Vajravelu, K. V. Prasad, R. A. Van Gorder, and J. Lee, “Free Convection Boundary Layer Flow Past a Vertical Surface in a Porous Medium with Temperature-Dependent Properties,” Transport Porous Media **90**, 977–992 (2011).
10. E. Shakeri, M. Nazari, and M. H. Kayhani, “Free Convection Heat Transfer Over a Vertical Cylinder in a Saturated Porous Medium using a Local Thermal Non-Equilibrium Model,” Transport Porous Media **93**, 453–460 (2012).
11. Z. Aouachriaa, F. Rouichia, and D. Haddadb, “Double Diffusion Effects on Convection in Flow on Vertical Plate Imbedded in Porous Media,” Frontiers Heat Mass Transfer **3**, 023004 (2012).
12. K. Khanafer, “Fluid-Structure Interaction Analysis of Non-Darcian Effects on Natural Convection in a Porous Enclosure,” Int. J. Heat Mass Transfer **58**, 382–394 (2013).
13. P. V. S. N. Murthy, D. Srinivasacharya, and P. V. S. S. S. R. Krishna, “Effect of Double Stratification on Free Convection in Darcian Porous Medium,” J. Heat Transfer **126**, 297–300 (2004).
14. B. V. Rathish Kumar and Shalini, “Double Diffusive Natural Convection in a Doubly Stratified Wavy Porous Enclosure,” Appl. Math. Comput. **171**, 180–202 (2005).

15. E. Magyari, I. Pop, and B. Keller, “Unsteady Free Convection along an Infinite Vertical Plate Embedded in a Stably Stratified Fluid-Saturated Porous Medium,” Transport Porous Media **62**, 233–249 (2006).
16. P. A. Lakshmi Narayana and P. V. S. N. Murthy, “Soret and Dufour Effects on Free Convection Heat and Mass Transfer in a Doubly Stratified Darcy Porous Medium,” J. Porous Media **10**, 613–623 (2007).
17. A. O. Beg, B. J. Zueco, and H. S. Takhar, “Laminar Free Convection from a Continuously-Moving Vertical Surface in Thermally-Stratified Non-Darcian High-Porosity Medium. Network Numerical Study,” Int. Comm. Heat Mass Transfer **35**, 810–816 (2008).
18. W. Lin, S. W. Armfield, and J. C. Patterson, “Unsteady Natural Convection Boundary-Layer Flow of a Linearly-Stratified Fluid with Pr < 1 on an Evenly Heated Semi-Infinite Vertical Plate,” Int. J. Heat Mass Transfer **51**, 327–343 (2008).
19. C. Y. Cheng, “Combined Heat and Mass Transfer in Natural Convection Flow from a Vertical Wavy Surface in a Power-Law Fluid Saturated Porous Medium with Thermal and Mass Stratification,” Int. Comm. Heat Mass Transfer **36**, 351–356 (2009).
20. D. Srinivasacharya and Ch. RamReddy, “Heat and Mass Transfer by Natural Convection in a Doubly Stratified Non-Darcy Micropolar Fluid,” Int. Comm. Heat Mass Transfer **37**, 873–880 (2010).
21. N. Kousar and S. Liao, “Series Solution of Non-Similarity Natural Convection Boundary-Layer Flows over Permeable Vertical Surface,” Sci. China Phys. Mech. Astron **53**, 360–368 (2010).
22. M. Neagu, “Free Convective Heat and Mass Transfer Induced by a Constant Heat and Mass Fluxes Vertical Wavy Wall in a Non-Darcy Double Stratified Porous Medium,” Int. J. Heat Mass Transfer **54**, 2310–2318 (2011).
23. D. Srinivasacharya and Ch. RamReddy, “Free Convective Heat and Mass Transfer in a Doubly Stratified Non-Darcy Micropolar Fluid,” Korean J. Chem. Eng. **28**, 1824–1832 (2011).
24. T. Cebeci and P. Bradshaw, *Physical and Computational Aspects of Convective Heat Transfer* (Springer-Verlag, New York, 1984).
25. O. A. Plumb and J. C. Huenefeld, “Non-Darcy Natural Convection from Heated Surfaces in Saturated Porous Media,” Int. J. Heat Mass Transfer **24**, 765–768 (2011).
26. B. Gebhart, Y. Jaluria, R. Mahajan, and B. Sammakia *Buoyancy Induced Flows and Transport* (Hemisphere, New York, 1988).
27. L. Prandtl, *Essentials of Fluid Dynamics* (Blackie and Son, London, 1952).
28. Y. Jaluria and K. Himasekhar, “Buoyancy Induced Two Dimensional Vertical Flows in a Thermally Stratified Environment,” Comput. Fluids **11**, 39–49 (1983).