An Application of Geometric Active Contour in Bio-medical Engineering

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***Abstract*—Segmentation and boundary detection is one of the most important task in image processing. In segmentation, an image is partitioned into a set of distinct regions. This is an intermediate step in various image processing applications like content based image and video retrieval, object recognition and tracking, and biomedical engineering etc. In this paper, we implemented an geometric active contour model which uses level set methods and an energy function, by minimizing it we can evolve the boundary of an object. This algorithm does not depend on the gradient of the image, so its performance is good for noisy images also. It can be used to detect lesion or tumor in an MRI image.**

***Keywords* - Image segmentation, geometric active contour, level set method, biomedical engineering.**

I. Introduction

Boundary detection and segmentation of objects has many applications in object tracking and object recognition, robotics, and biomedical engineering. The detection of object boundaries through active contours is an emerging new research topic in computer vision and pattern recognition. In general, most of the active contour models converge towards some desired contour by minimizing a sum of internal and external energy terms. For such type of active contours we define a contour as an initial segmentation in the image plane and then we solve this contour using mathematical equations. The goal is to stop the evolution of contour on the exact boundary of the object. The evolution equation can be defined in different ways such as the contour may move with a velocity that depends on the local curvature or the gradient of the image at the given point.

The active contours are classified into three types: parametric [1]–[4], non-parametric or geometric active contours [5]–[7] and physics inspired particle based [8]. Despite this large number of approaches, none of the parametric active contour models are able to handle the problems associated with image noise contamination, sensitivity to the algorithm parameters, complex high curvature boundaries, initialization sensitivity, ineffective stopping criteria and slow convergence rate. Non-parametric approaches are initialization independent and are able to handle high curvature regions and topology.

II. Parametric methods

Parametric contours are explicitly represented as parameterized curves [9] in a Lagrangian formulation. In classical parametric contours, we define an energy functional by minimizing it we can extract the object boundaries. This energy function represents a dynamic equation that consists both internal and external forces. In this, the external forces are conservative which can be expressed as gradients of scalar potential function. In general, a parametric active contour can be represented as a time varying curve [10] $\mathbf{X}(s,t) = [X(s,t), Y(s,t)]$ where s$\epsilon[0,1]$ is arc length and t is artificial time.

$$\gamma \mathbf{X}_t = \mathbf{F}_{int} + \mathbf{F}_{ext} \tag{1}$$

In the above equation $\mathbf{X}_t$ is partial derivative of $\mathbf{X}$, $-\gamma\mathbf{X}_t$ is the damping factor (γ is non negative), the external force($\mathbf{F}_{ext}$) attracts the contour towards the boundary of the object and the internal force($\mathbf{F}_{int}$) is the sum of elastic force and rigid force.

$$\mathbf{F}_{elastic} = [\alpha(s,t)\mathbf{X}_s(s,t)]_s \tag{2}$$

$$\mathbf{F}_{rigid} = [\beta(s,t)\mathbf{X}_{ss}(s,t)]_{ss} \tag{3}$$

where the parameters $\alpha(s,t), \beta(s,t)$ control the contour's elasticity and rigidity. Finally, the active contour comes to steady state when the sum of damping factor, internal and external forces becomes zero.

III. Geometric models and Level set methods

In geometric active contour models we define a contour in the image plane as an initial segmentation and then this contour is evolved using some mathematical equations. The main aim is to stop the evolution of contour on the boundary of the image foreground region. The evolution equation can be defined in different ways such as the contour may move with a velocity that depends on gradient of the image at a given point.

A. Level Set Methods

Level set methods [11] are one of the most important tool used to perform contour evolution. We define some function $\phi(i,j,t)$ (the level-set function) as shown in Fig. 1, where (i, j) are coordinates of the image plane and t is the time. At any given time t, the level set function simultaneously represents an edge contour and image segmentation. The edge contour is nothing but the zero level set (i,j) s.t. $\phi(i,j,t) = 0$, and the two segmented regions are given by $\phi > 0$ and $\phi < 0$ as shown in Fig. 1. The LSF is subjected to various mathematical equations such that it will reach the steady state ϕ that gives a useful segmentation of the image. If we define the foreground

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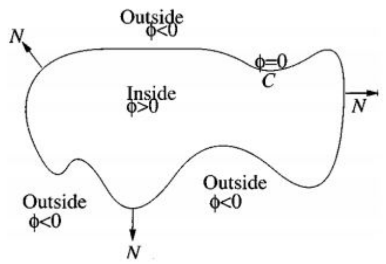

Fig. 1. Example level set function

to be the region where $\phi > 0$, then the foreground found by this segmentation algorithm would be the region inside the circle. Level set methods are especially useful because they can easily handle topological changes in the edge contour that would be difficult to handle with a model that directly evolves the contour. Since this contour is computed indirectly as the zero contour of a surface, there are no computational issues when this contour splits from, say a single circle into two distinct closed curves; the level set function is represented by a smooth surface in either case.

The most important and difficult step is to actually determine a LSF which locates the boundaries in a meaningful way. The simplest example is to define the level set function to be value of a gray level image at each pixel minus threshold value, i.e. set $\phi(i,j) = I(i,j) - t$. Then the LSF is positive in regions where the gray level is above the threshold, and negative in regions where the gray level is below the threshold. The level set formalism is not necessary for this example but it illustrates the basic idea.

B. Variational Level Set Methods

This geometric active contour algorithm uses variational calculus methods to evolve the level set function. Variational methods work by defining a level set function that minimizes some functional energy. In this context, by functional we mean a mapping that takes a level set function ϕ as input, and returns a real number. The problem is then to seek a function ϕ that is a critical point (minimum or maximum) of this functional. The hope is that we can come up with a functional whose critical points are level sets that give useful segmentations for a given problem.

For example, if we have a bi-level image I with domain Ω that takes on only the values - 1 and 1. A function F is defined such that, for any function $\phi : \Omega \rightarrow$IR

$$F[\phi] = \int_{\Omega} H(\phi) I, \tag{4}$$

where H is the Heaviside function $H(x) = \{1, x \geq 0; 0, x = 0\}$. In case of discrete images, the integral is represented as sum of all the image pixels in the given region. The value of above equation is the integration of I over all the pixels where ϕ is positive. Ignoring for a second how we can determine a minimizer for the above function computationally, we came to know that the minimizers are those functions ϕ which are positive only where I = -1.

It is very difficult to define an appropriate functional and determining a method to minimize that functional. In this paper we will focus on such type of functional which performs well for different types of images and describe how it performs image segmentation.

IV. The geometric active contour Algorithm

A. Energy Functional formulation

In this active contour model, we define a fitting energy functional. The goal of this active contour algorithm is to minimize this fitting energy functional for a given image, and the minimizing LSF ϕ will define the segmentation. The general form of the fitting energy [5] is

$$F(\phi) = \mu(\textstyle\int_{\Omega} | \bigtriangledown H(\phi)|dx)^p + v \int_{\Omega} H(\phi)dx +$$

$$\lambda_1 \int_{\Omega} |I - c_1|^2 H(\phi)dx + \lambda_2 \int_{\Omega} |I - c_2|^2 (1 - H(\phi)dx) \tag{5}$$

and

$$H(\phi) = \frac{1}{2}(1 + \frac{2}{\pi}\arctan(\frac{\phi}{\epsilon})) \tag{6}$$

Where $\mu, v, \lambda_1, \lambda_2$ and p are the user defined parameters for a particular class of images. The equation (5) is a generalization of the Mumford-Shah functional [12]. The Mumford-Shah functional is obtained by setting p = 1, v = 0, and $\lambda_1 = \lambda_2$ = 1. In the above equation (6), H is the Heavyside function and ϵ is the smallest non zero value used to avoid division by zero problem, I represents the image to be segmented, and Ω is the image domain. c_1 is the average intensity in the region where $\phi > 0$ and similarly, c_2 is the average intensity in the region where $\phi < 0$, given by

$$c_1 = \frac{\int_{\Omega} I.H(\phi)dxdy}{\int_{\Omega} H(\phi)dxdy}, c_2 = \frac{\int_{\Omega} I.(1 - H(\phi))dxdy}{\int_{\Omega}(1 - H(\phi))dxdy} \tag{7}$$

The first term in the equation(5), $\mu(\int_{\Omega} | \bigtriangledown H(\phi)|dx)^p$, represents the edge contours length for a given segmentation. If we expect a region with a smooth boundary, we might weight this term more heavily to avoid finding a complex (and therefore long) perimeter. Similarly, the second term $v \int_{\Omega} H(\phi)dx$ is a penalty on the total area of the foreground region found by the segmentaion.

The third term, $\lambda_1 \int_{\Omega} |I - c_1|^2 H(\phi)dx$, represents the variance of the image gray level [13] in the foreground region which measures how uniform the region is in terms of pixel intensity. Similarly the fourth term $\lambda_2 \int_{\Omega} |I - c_2|^2 (1 - H(\phi))dx$ measures for the background region. Minimizing the sum of these last two terms, leads to a segmentation into a foreground and background region that are each as uniform as possible. For example, in a bi-level image where pixels takes only two values, the sum of these terms is minimized by taking a segmentation that includes all pixels of the first value in the foreground region, and no pixels of the second value. Usually, we take $\lambda_1 = \lambda_2 = 1$, but we can adjust them as necessary to weight one term more heavily. If we set λ_1=2 and λ_2=1, then our final segmentation will have a more uniform foreground region (since the energy contributed by the variance in the foreground region is weighted more heavily), at the expense of allowing more variation in the background. In applications where we expect an approximately black background and foreground objects with varying gray levels, we have to set $\lambda_1 < \lambda_2$.

B. Fitting energy minimization

Euler-Lagrange equations and the gradient-descent method [14] are used to derive the following evolution equation for the level set function ϕ which will minimize the fitting energy F(ϕ).

$$\phi_t = \delta(\phi)[\mu p(\int_\Omega \delta(\phi)| \bigtriangledown \phi|)^{p-1}$$

$$div(\frac{\bigtriangledown \phi}{| \bigtriangledown \phi|}) - v - \lambda_1(I - c_1)^2 p + \lambda_2(I - c_2)^2] \quad (8)$$

C. Reinitializing the Level Set Function

It is necessary at each iteration to rescale the level set function to keep it from becoming too flat. This is necessary because of blurring that occurs due to use of the smoothed delta function δ_ϕ which is the derivative of $H(\phi)$.

$$\psi_t = sign(\phi(t))(1 - | \bigtriangledown \psi|) \quad (9)$$

The steady state of this evolution will be the signed distance function to the zero level contour of ϕ; that is, it will have the same sign as ϕ at each point, and the magnitude at that point will be the distance from that point to the contour ϕ=0. We then reinitialize ϕ to ψ.

In practice we have to evolve this partial differential equation(8) until the zero level set function achieves steady state; we have to compute Q at each time step as shown below

$$Q = \frac{\Sigma_{|\phi_{i,j}^m|} |\phi_{i,j}^{m+1} - \phi_{i,j}^m|}{\sharp[|\phi_{i,j}^m| < h]} \quad (10)$$

and stop the iteration when $Q < \Delta t.h^2$ where Δt is time step and h is a constant.

V. Simulation results

In this section, we discuss simulation results of the proposed method for real and synthetic images. The default parameters used in this algorithm are v = 1900, $\bigtriangledown t$ = 0.5 and h = 0.18. The parmeter μ depends on the size of the objects to be segmented i.e., we have to chose small value for μ to segment small objects in the image and large value for big objects. λ_1 and λ_2 are scaling factors used to control foreground and back ground intensity levels.

Fig. 2 shows the simulation of magnetic resonance image(MRI) of brain with the parameters μ = 2, λ_1 = 0.98 and λ_2 = 1.0. Fig. 2(a) is the MRI of brain which has a tumor. Fig. 2(b) is the initial levelset contour which is selected automatically.

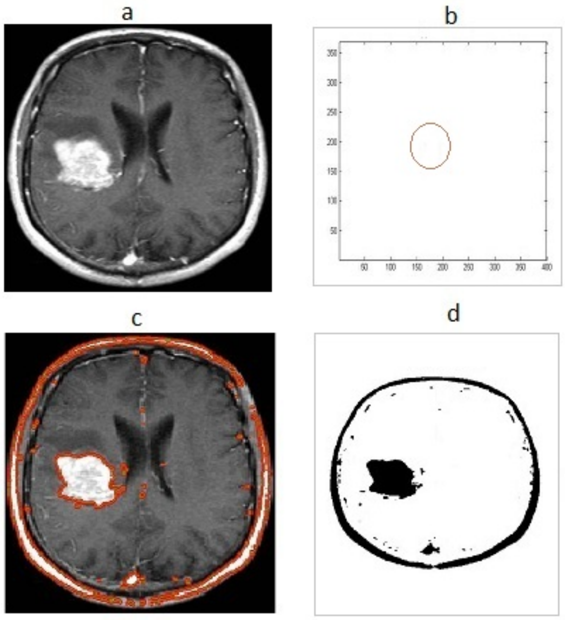

Fig. 2. Segmentation of brain tumor in MRI image (a) original image (b) initial levelset contour (c) intermediate result (d) final output

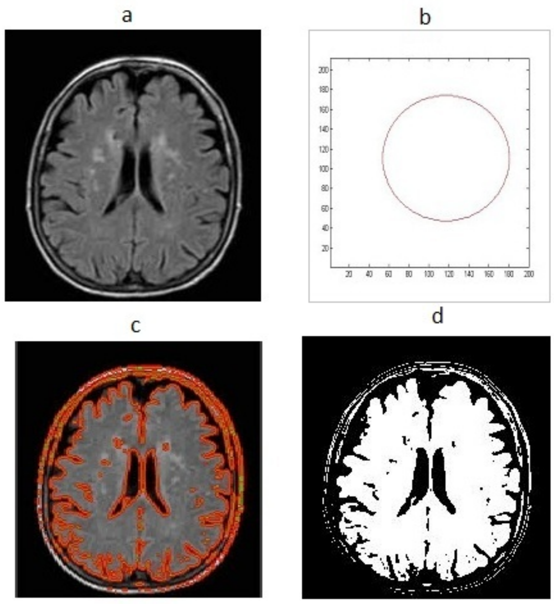

Fig. 3. White matter detection in brain (a) MRI image of brain (b) levelset function (c) intermediate segmentation result (d) final output

Fig. 2(c,d) are intermediate and final segmentation results.The black spot in the Fig. 2(d) represents the lesion in the brain.

In Fig. 3, we have shown that our proposed method can detect white matter in the brain. The parameters used for this image are λ_1 = 1.0 , λ_2 = 8.0 and μ = 5. In this method, the initial levelset contour is defined automatically and its location does not effect the segmentation of objects.

Fig. 4 represents the simulation of CT scan image. In this,

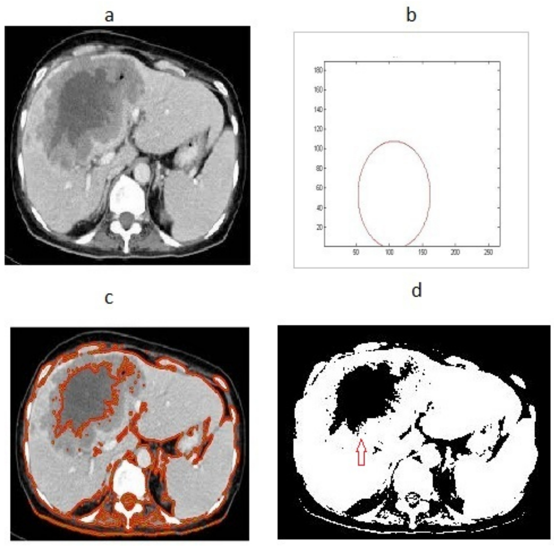

Fig. 4. Liver cancer detection (a) CT scan image of liver (b) Initial levelset contour (c) intermediate segmentation result (d) final segmented output(arrow mark indicating diseased area)

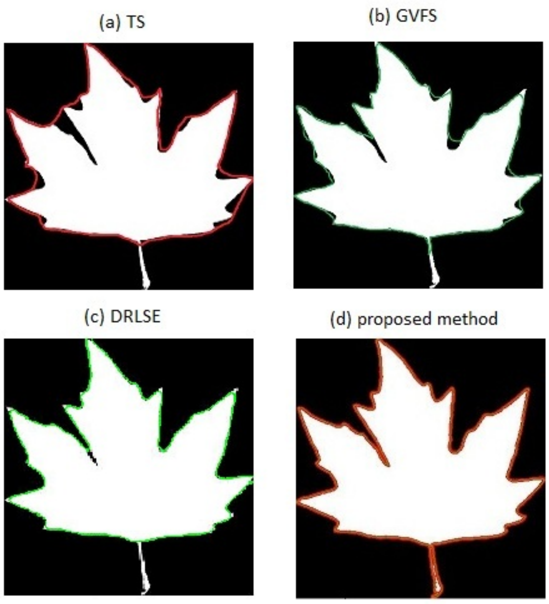

Fig. 5. Comparison of proposed method with TS, GVFS and DRLSE respectively

the algorithm is used to detect the cancer in the liver. The parameters used for this segmentation are λ_1=1.0, λ_2=4.0 and μ= 4.0. The black spot in Fig. 4(d) pointed by arrow mark indicates the liver cancer.

A. comparison with existing methods

The performance of proposed method is compared with traditional snake(TS), Distance regularized levelset evolution (DRLSE) and Gradient vector flow snake(GVFS) for an image shown in Fig. 5.

TABLE I
COMPARISON OF ACTIVE CONTOUR METHODS

Parameter	GVFS	TS	DRLSE	Proposed method
Iterations	720	560	800	450
accuracy	medium	less	medium	high
Execution time(sec)	86	289	60	37

TS [15] algorithm uses gradient descent optimization method to track object boundaries. It is highly sensitive to local minima and takes more time to converge towards boundary. Fig. 5(a) shows the result of TS algorithm which failed to detect the leaf true boundary.

GVFS [2] algorithm uses PDE and diffusion technique to spread the external energy so that it attracts the initial level set contour from large distances towards the object boundary. The problem with GVFS is, it fails to track high curvature object boundaries as shown in Fig. 5(b).

In case of DRLSE [16], the levelset function regularity is intrinsically maintained during its evolution towards boundary. It uses two terms to detect object boundaries. First, distance regularization term which maintains regularity of LSF. Second, external force which attracts LSF towards boundary. The Fig. 5(c) shows the segmentation of DRLSE image, its performance is better than TS and GVFS but fails to detect exact boundary.

Fig. 5(d) shows the result of proposed method which detects the exact leaf boundary when compared to other methods.

Table I compares the proposed method with existing active contour methods interms of accuracy, iterationas and execution time. From this table, we can say that the proposed method is faster than other algorithms and also its accuracy is high in case of boundary detection.

VI. CONCLUSION

The proposed algorithm for locating object boundaries is effective on various types of images. This is useful in cases where an edge based segmentation algorithm will not sufficient, since it depends on the global properties which includes graylevel intensities and contour lengths instead of local properties like gradients.

This algorithm takes several seconds to compute, so it is not suitable for real time video applications where frame rate is high. But it has a significant role in offline image analysis like lesion detection in MRI images.

Finally, this geometric active contour model represents an exciting trend in image analysis. Different mathematical concepts also played significant role in image analysis such as variational calculus and partial differential equations in detecting object boundaries.

REFERENCES

[1] A. A. Amini, T. E. Weymouth, and R. C. Jain, "Using dynamic programming for solving variational problems in vision," *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, vol. 12, no. 9, pp. 855–867, 1990.

[2] C. Xu and J. L. Prince, "Snakes, shapes, and gradient vector flow," *Image Processing, IEEE Transactions on*, vol. 7, no. 3, pp. 359–369, 1998.

[3] L. D. Cohen, "On active contour models and balloons," *CVGIP: Image understanding*, vol. 53, no. 2, pp. 211–218, 1991.

[4] B. Li and S. T. Acton, "Active contour external force using vector field convolution for image segmentation," *Image Processing, IEEE Transactions on*, vol. 16, no. 8, pp. 2096–2106, 2007.

[5] T. F. Chan and L. A. Vese, "Active contours without edges," *Image Processing, IEEE Transactions on*, vol. 10, no. 2, pp. 266–277, 2001.

[6] A. K. Mishra, P. W. Fieguth, and D. A. Clausi, "Decoupled active contour (dac) for boundary detection," *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, vol. 33, no. 2, pp. 310–324, 2011.

[7] V. Caselles, R. Kimmel, and G. Sapiro, "Geodesic active contours," *International journal of computer vision*, vol. 22, no. 1, pp. 61–79, 1997.

[8] A. C. Jalba, M. H. Wilkinson, and J. B. Roerdink, "Cpm: A deformable model for shape recovery and segmentation based on charged particles," *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, vol. 26, no. 10, pp. 1320–1335, 2004.

[9] M. P. Do Carmo and M. P. Do Carmo, *Differential geometry of curves and surfaces*. Prentice-Hall Englewood Cliffs, 1976, vol. 2.

[10] C. Xu, A. Yezzi Jr, and J. L. Prince, "On the relationship between parametric and geometric active contours," in *Signals, Systems and Computers, 2000. Conference Record of the Thirty-Fourth Asilomar Conference on*, vol. 1. IEEE, 2000, pp. 483–489.

[11] S. Osher and R. P. Fedkiw, "Level set methods," *CAM Report 00-07, UCLA, Mathematics Department*, 2000.

[12] A. Tsai, A. Yezzi Jr, and A. S. Willsky, "Curve evolution implementation of the mumford-shah functional for image segmentation, denoising, interpolation, and magnification," *Image Processing, IEEE Transactions on*, vol. 10, no. 8, pp. 1169–1186, 2001.

[13] M. S. VASANTHI, R. ANAND, G. A. BABU, K. DEEPA, and V. LEKHA, "Segmentation of optic disc in fundus images," *Indian Journal of Computer Science and Engineering*, vol. 3, no. 2, pp. 230–234, 2012.

[14] M. I. Miller, A. Trouvé, and L. Younes, "On the metrics and euler-lagrange equations of computational anatomy," *Annual review of biomedical engineering*, vol. 4, no. 1, pp. 375–405, 2002.

[15] M. Kass, A. Witkin, and D. Terzopoulos, "Snakes: Active contour models," *International journal of computer vision*, vol. 1, no. 4, pp. 321–331, 1988.

[16] C. Li, C. Xu, C. Gui, and M. D. Fox, "Distance regularized level set evolution and its application to image segmentation," *Image Processing, IEEE Transactions on*, vol. 19, no. 12, pp. 3243–3254, 2010.