

## Optimal spot pricing in electricity market with inelastic load using constrained bat algorithm

M. Murali <sup>\*</sup>, M. Sailaja Kumari, M. Sydulu

Electrical Engineering Department, NIT Warangal, Warangal 506 004, India



### ARTICLE INFO

#### Article history:

Received 20 May 2013

Received in revised form 26 April 2014

Accepted 13 May 2014

Available online 21 June 2014

#### Keywords:

Bat algorithm

Concentrated loss

Distributed loss

Delivery factors

### ABSTRACT

In restructured electricity markets, an effective transmission pricing is required to address transmission issues and to generate correct economic signals. These prices depend on generator bids, load levels and transmission network constraints. A congestion charge is incurred when the system is constrained due to physical limitations. Spot pricing or Locational Marginal Pricing (LMP) or Nodal pricing is a popular method in restructured power markets to address these issues. This paper presents a DC optimal power flow (DCOPF) based spot pricing approach in single auction model with fuel cost minimization as objective function. This is solved with a heuristic technique called Bat algorithm and the results are compared with Linear Programming (LP) and Genetic algorithm (GA) approaches in a constrained pool based restructured electricity market. The developed models have been tested on IEEE 14 bus system, New England 39 bus system and 75 bus Indian practical power system. Different cases such as without loss, concentrated loss and distributed loss are considered for this problem. Two types of generator bids i.e., fixed bids and linear bids are considered for generators. Load is assumed to be inelastic. Generator profit, ISO profit and Social surplus during congestion have been computed in all the cases. In most of the cases studied, Bat algorithm is proven to be better than LP and GA algorithms for fuel cost minimization and social welfare (Social surplus) improvement.

© 2014 Elsevier Ltd. All rights reserved.

### Introduction

In April 2003 in a White Paper, the U.S. Federal Energy Regulatory Commission (FERC) proposed a market design for common adoption by U.S. wholesale power markets. The electric power industry has undergone deregulation around the world, a core tenet of which is to build an open-access, unambiguous and fair electricity markets. Proper and fair pricing of real power is an important issue in this competitive market. Core features of a market design include; a two settlement system consisting of a day-ahead market supported by a real-time market to ensure continual balancing of supply and demand for power; and grid congestion management by means of Spot pricing also called Locational Marginal Pricing (LMP) or Nodal pricing.

Under the deregulated electricity market environment, transmission networks play a vital role in supporting the transaction between producers and consumers. One limitation on transmission flow is congestion. Congestion occurs when transmission lines or transformers operate at or above its thermal limits and this

prevents the system operators from dispatching additional power from a specific generator. Congestion can result in an overall increase in the cost of power delivery. Presently there are two pricing structures [10] that are being used in a competitive energy market to account for congestion: the uniform pricing based market clearing price (MCP) and the non-uniform pricing method (LMP). In the first method, all generators are paid the same price (MCP) based on the bid of the marginal generator that would be dispatched in the absence of congestion. The second method (LMP) has been the basic approach in power markets to calculate nodal prices and to manage transmission congestion. The theory of spot price, which was first proposed by Schweppe et al. [1], is increasingly being employed in the form of (LMP) within an OPF framework. The LMP at a location is defined as the marginal cost to supply an additional increment of power to the location without violating any system security limits [15]. Because of the effects of both transmission losses and transmission system congestion, LMP can vary significantly from one location to another. Mathematically, LMP at any node in the system is the dual variable (sometimes called a shadow price) for the equality constraint at that node (sum of injections and withdrawals is equal to zero). Or, LMP is the additional cost of providing one additional MW at a certain node [19]. Buyers pay ISO based on their price for

\* Corresponding author. Tel.: +91 9494842708.

E-mail addresses: [murali233.nitw@gmail.com](mailto:murali233.nitw@gmail.com) (M. Murali), [sailaja\\_matam@yahoo.com](mailto:sailaja_matam@yahoo.com) (M. Sailaja Kumari), [sydulumaheswarapu@yahoo.co.in](mailto:sydulumaheswarapu@yahoo.co.in) (M. Sydulu).

dispatched energy. The ISO pays sellers based on their respective prices. The LMP difference between two adjacent buses is the congestion cost which arises when the energy is transferred from one location (injection) to the other location (withdrawal). Marginal losses represent incremental changes in system losses due to incremental demand changes. Incremental losses yield additional costs which are referred to as the cost of marginal losses. Thus LMP is the summation of the costs of marginal energy, marginal loss and congestion. Therefore LMP is stated as follows:

$$\text{LMP} = \text{generation marginal cost} + \text{congestion cost} + \text{marginal loss cost} \quad (1)$$

In this decomposition model, LMP congestion component at Bus  $B$ , i.e.,  $\text{LMP}_B^{\text{cong}}$  remains invariant w.r.to different reference buses, and the combination of the other two components, i.e.,  $\text{LMP}_B^{\text{energy}} + \text{LMP}_B^{\text{loss}}$ , is also reference-independent. But each of  $\text{LMP}^{\text{energy}}$  or  $\text{LMP}^{\text{loss}}$  is still reference-dependent [36].

LMP's can be derived using either an ACOPF model or a DCOPF model [2–5,7–9].

The objective function of OPF is maximizing Social surplus while meeting the load in the power system and respecting operational constraints. In the absence of price elastic load (which is mostly the case in the real time market) maximizing social welfare is equivalent to minimizing the total production cost. In this paper load is assumed to be price inelastic. There are two approaches to calculate nodal prices in RTM: ex post and ex ante. NY ISO uses ex ante prices as the real time prices and penalises non-performing resources on the basis of reduced generation quantity [11], whereas ISO NE, PJM and MISO adopt the ex post pricing that provides dispatch incentives on the ground of rational prices [12,13]. Both ex ante and ex post approaches have their own merits and demerits. For example ex ante pricing does not have the capability to penalise non-performing units, whereas ex post pricing has some difficulties in implementing co-optimization of the energy and reserves [30]. In market planning and simulation, DC model is desired due to its robustness and speed [20]. DCOPF is broadly employed by a number of industrial spot price simulators, such as ABB's Gid View™, GE's MAPS™, Siemens' Promod IV™, and Power World [18,6]. Several papers have reported different models for price calculation. Ref. [14] describes components of nodal prices for electric power systems. Ref. [16,17] demonstrated the usefulness of DC power flow in calculating loss penalty factors, which has a significant impact on generation scheduling. It also pointed out that it is not advisable to apply predetermined loss penalty factors from a typical scenario to all cases. Ref. [21] presented a slack-bus-independent approach to calculate LMPs and its congestion components. Ref. [22] presented LMP simulation algorithms to address marginal loss pricing based on the dc model. Literature shows that dc model is acceptable in optimal power flow studies if the line flow is not very high, the voltage profile is sufficiently flat, and the  $R/X$  ratio is less than 0.25 [23]. Ref. [25] presented Nodal pricing with Genetic algorithm for congestion management with DCOPF for lossless system. Ref. [26] presented different methods and properties on LMP calculations based on DCOPF with and without loss. An LP based approach for LMP without loss case [33], with concentrated case and distributed loss case using DCOPF model is presented in [28] with piecewise linear cost curves but it does not give actual marginal cost of generation. Ref. [29] presented Cumulant and Gram Charlier (CGC) method for calculating LMP and compared the results with Monte Carlo and point estimation method. This method combines the concept of Cumulants and Gram-Charlier expansion theory to obtain Probabilistic Distribution Functions (PDF) and Cumulative Distribution Function (CDF) of LMP. This method is complex and time consuming. Ref. [30] gives a systematic description on how the LMP's are

produced; it also described both the modeling and implementation challenges and solutions. Ref. [31] described ACOPF based LMP calculation considering distributed loss. In [37] the authors described ACOPF based probabilistic calculation of LMP with Point Estimation method. In [38] LMP calculation with GA based DCOPF is presented in two cases of losses i.e., without loss and concentrated loss for IEEE 14 bus, New England 39 bus and 75 bus Indian power systems.

The present paper proposes a metaheuristic algorithm called Bat algorithm (BA) [35] introduced for the first time in power system application area for solving DCOPF problem of LMP calculation without loss, with concentrated loss and with distributed loss models with linear bid. All these three loss models have been attempted for fixed generation bids and linear generation bids in LP (Power World simulation) approach and GA. Then Bat algorithm results are compared with LP and GA approaches. LMP components are derived for all the three models of losses and presented in this paper. In concentrated loss DCOPF model total system loss is supplied by the slack bus which creates a burden on the slack bus. To eliminate a large mismatch at the slack bus, loss is distributed among the buses as an extra load.

'Types of generator bids' describes different types of bids used for generators. The problem formulation for LMP calculation using delivery factors is discussed in 'Mathematical formulation for nodal price calculation' for all the loss models. 'Implementation of Bat algorithm for spot price calculation' presents Bat algorithm implementation. 'Results and discussion' gives results and discussion for IEEE 14 bus system, New England 39 bus system and 75 bus Indian power system. 'Conclusion' concludes the paper.

## Types of generator bids

In general, generators bids depend on many factors, some of which (e.g. strategic behavior) are difficult to model. To avoid excessive complexity, generators bids are assumed to be equal to their incremental costs for perfect competition. Two bidding models are generally used, namely; fixed generator bids (corresponding to piecewise-linear heat rates) and linear bids (corresponding to quadratic heat rates).

### Fixed bids

The piecewise linear heat rate curve is converted to an approximate fixed incremental heat rate for each unit through Linear regression method. The cost changes in steps with respect to generation. Nordic pool uses such type of bids. The main drawback of

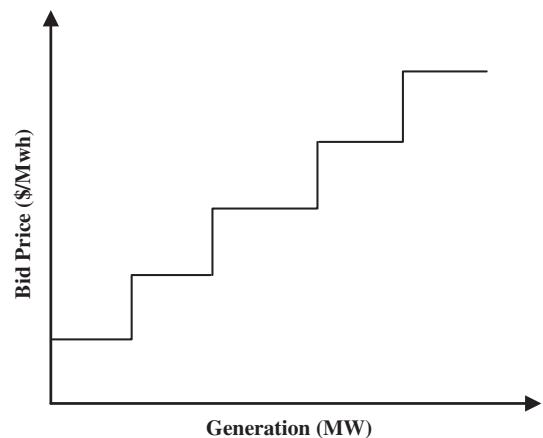


Fig. 1. Fixed bids.

this bid is it does not give true marginal cost of generator. The fixed bid curve for piecewise linear cost characteristics is shown in Fig. 1. These types of bids are under practice in PJM market.

#### Linear bids

The non-smooth nature of the fixed bid shown in Fig. 1 may result in step changes in prices at certain load levels. One way to mitigate this is to use linear bids for the generating units. This will result in a much smoother supply curve as shown in Fig. 2 and also gives the actual marginal cost of a generator. These types of bids are under practice in Nordic Pool market. The generator cost curve is given by (2).

$$C_i(P_{Gi}) = a_i + b_i P_{Gi} + c_i P_{Gi}^2 (\$/hr) \quad (2)$$

where  $C_i(P_{Gi})$  is the cost of unit  $i$  generating  $P_{Gi}$  MW,  $a_i$  is the no-load cost,  $b_i$  is the linear cost coefficient and  $c_i$  is the quadratic cost coefficient of unit  $i$ . These  $a$ ,  $b$ ,  $c$  coefficients of generators are given by manufacturer of the generator. The incremental costs of the units can be modeled as fixed quantities (Fig. 1) or can be expressed as linear function with slope  $m$  of the unit outputs (Fig. 2) as in (3). The linear price curve introduces non-linearities in the problem; however, it is a more realistic representation of price than that of a fixed price.

$$dC_i(P_{Gi})/dP_{Gi} = b_i + 2c_i P_{Gi} (\$/MW h) \quad (3)$$

#### Mathematical formulation for nodal price calculation

Nodal prices using DCOPF with and without considering line losses for generator fixed bids is simulated with Linear Programming (LP) approach [28] using Power World Simulator (12th version). Then GA with generator fixed bids is attempted to minimize the fuel cost, and following that the GA with generator linear bids is attempted for further optimization. It was observed that using generator linear bids gives minimal fuel cost than fixed bids of all loss cases for all the test systems discussed in this paper. Finally, Bat algorithm with generator linear bids is proposed for further optimization. Active power generations of the generators except slack generation are considered as variables for optimization problem. The obtained power generations are used in calculation of spot prices with and without loss for the congested transmission system. Generation Shift factors (GSF) have been used for the calculation of transmission line flows. Delivery factors (DF) at all buses have been used to consider the impact of marginal losses on nodal prices.

The location of reference bus or slack bus will not impact nodal prices, when ignoring system losses. But, the individual components

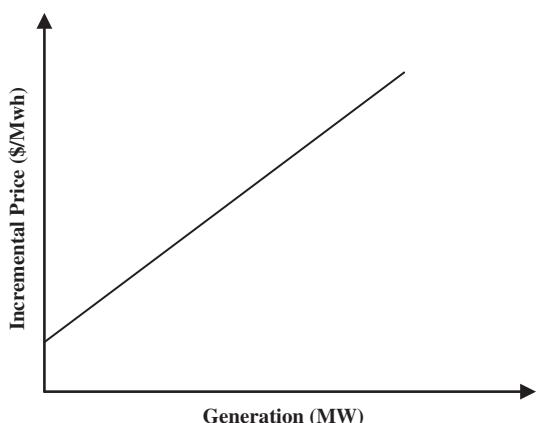


Fig. 2. Linear bids.

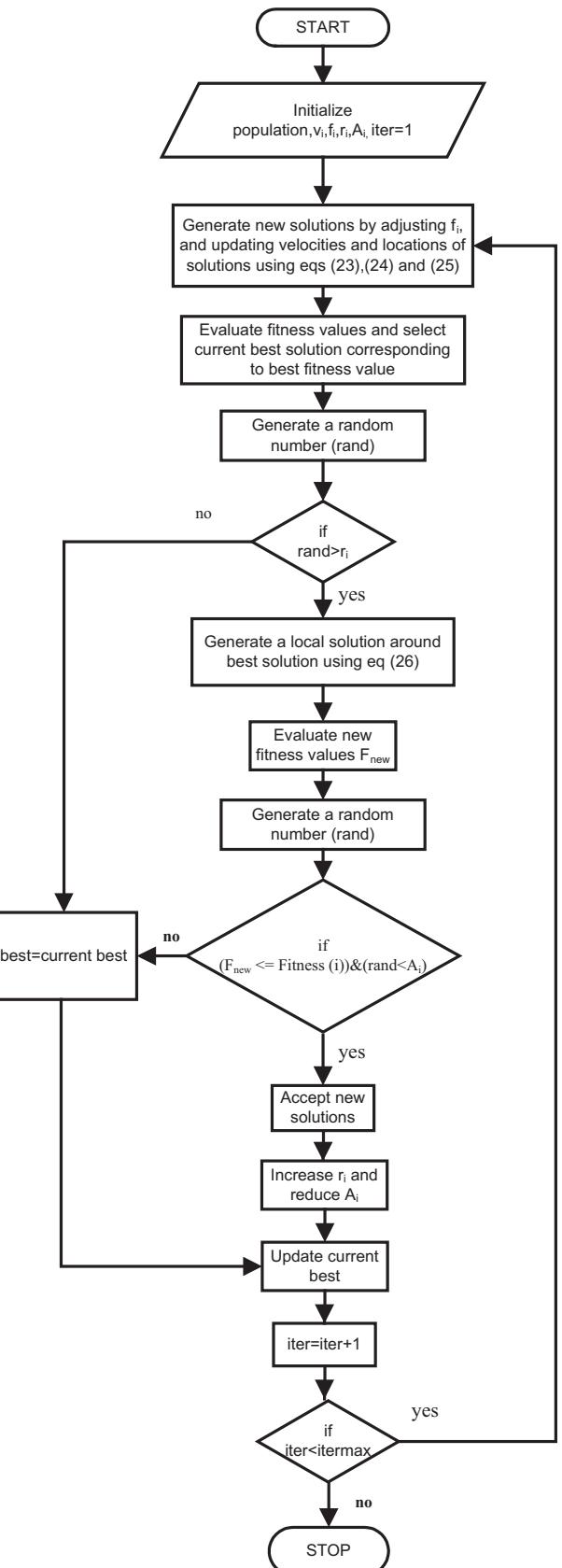


Fig. 3. Flow chart of Bat algorithm.

of nodal price depend on the location of reference bus. If transmission losses are balanced at reference bus, i.e., in concentrated loss model, the bus spot prices depend on the location of reference

bus. In distributed loss model, the bus nodal prices are independent of the choice of reference bus. It should also be noted that although the line flow based on GSF is the same with different references buses, actual GSF values depend on the choice of slack bus.

#### Generation shift factor

Generation shift factor is the ratio of change in power flow of line 'k' to change in power injection at bus 'i'. GSF can be computed using (4), where  $B^{-1}$  is the inverse of B matrix,  $x_k$  is reactance of line  $k$ , 'a' and 'b' are sending and receiving end buses of line  $k$ .

$$GSF_{k-i} = (B_{(a,i)}^{-1} - B_{(b,i)}^{-1})/x_k \quad (4)$$

#### Delivery factor

The delivery factor ( $DF_i$ ) at the  $i$ th bus represents the effective MW delivered to the customers to serve the load at that bus. It is defined as (5)

$$DF_i = 1 - LF_i = 1 - \partial P_{loss} / \partial P_i \quad (5)$$

$$P_{loss} = \sum_{k=1}^M F_k^2 \times R_k \quad (6)$$

$$F_k = \sum_{i=1}^N GSF_{k-i} \times P_i \quad (7)$$

$$\begin{aligned} \frac{\partial P_{loss}}{\partial P_i} &= \sum_{k=1}^M \frac{\partial}{\partial P_i} (F_k^2 \times R_k) = \sum_{k=1}^M R_k \times 2F_k \times \frac{\partial F_k}{\partial P_i} \\ &= \sum_{k=1}^M 2 \times R_k \times GSF_{k-i} \times \left( \sum_{j=1}^N GSF_{k-j} \times P_j \right) \end{aligned} \quad (8)$$

In (5)–(7),  $LF_i$  represents the loss factor at bus 'i' which is calculated using (8),  $F_k$  is the power flow in line  $k$ ,  $R_k$  is the resistance of line  $k$ ,  $P_i$  is the injected power at bus  $i$ ,  $GSF_{k-i}$  is the generation shift factor to line 'k' from bus 'i'.  $LF_i$  may be viewed as the change of total system loss with respect to 1 MW increase in injection at that bus. Interestingly, the loss factor at a bus may be positive or negative. When it is positive, it implies that an increase of injection at the bus may increase the total system loss. If it is negative, it implies that an increase of injection at that bus reduce the total loss.

#### Spot price calculation with different loss cases

##### Case 1: Without losses using DCOPF

In this method the objective function is minimization of total marginal production cost subjected to power balance and line flow constraints [32]. Spot prices are calculated from the obtained generator power outputs. ISO payments to generators, Generator profit, load payment to ISO, total ISO profit and system Social Welfare are also calculated.

The objective function is

$$\text{Minimize } J = \sum_{i=1}^N C_i(s) \quad (9)$$

$$\text{s.t. } \sum_{i=1}^N P_{G_i} = \sum_{i=1}^N P_{D_i} \quad (10)$$

$$F_k \leq \text{limit}_k, \quad k = 1, 2, \dots, M \quad (11)$$

$$P_{G_i}^{\min} \leq P_{G_i} \leq P_{G_i}^{\max}, \quad i = 1, 2, \dots, N \quad (12)$$

where  $N$  is number of buses,  $M$  is number of lines,  $C(s)$  is cost function of the generator i.e.  $(C_i(s) = b_i P_{G_i} + c_i P_{G_i}^2)$  in \$/hr,  $P_{G_i}$  is output power of generator at bus  $i$  (MW),  $P_{D_i}$  is the demand at bus  $i$  (MW),  $F_k$  is line flow of line  $k$ ,  $\text{limit}_k$  is thermal limit of line  $k$ .

##### Case 2: With concentrated loss using DCOPF

In this method also the main objective is minimization of total marginal production cost subjected to energy balance and line flow constraints. However, in nodal price based electricity markets, system marginal losses have significant impact on the economics of power system operation. So, system marginal losses have to be taken into account for obtaining accurate prices. In this model it is assumed that total system loss is supplied by slack bus generator. The problem is to

$$\text{Minimize } J = \sum_{i=1}^N C_i(s) \quad (13)$$

$$\sum_{i=1}^N DF_i \times (P_i) + P_{loss} = 0 \quad (14)$$

$$F_k \leq \text{limit}_k, \quad k = 1, 2, \dots, M \quad (15)$$

$$P_{G_i}^{\min} \leq P_{G_i} \leq P_{G_i}^{\max}, \quad i = 1, 2, \dots, N \quad (16)$$

**Table 1**

Comparison of results with different methods for three loss models of 437 IEEE 14 bus system.

Generator bus no.	Power generations in MW without loss case 1				Power generations in MW with concentrated loss case 2				Power generations in MW with distributed loss case 3			
	LP approach fixed bids	GA approach		Bat linear bids	LP approach fixed bids	GA approach		Bat linear bids	LP approach fixed bids	GA approach		Bat linear bids
		Fixed bids	Linear bids			Fixed bids	Linear bids			Fixed bids	Linear bids	
1 (Slack bus)	142	141.37	141.37	141.23	147.143	144.9	144.9	144.75	147.143	140.48	140.48	145.78
2	117	117.62	117.62	117.77	117	117.62	117.62	117.77	117	122.00	122.00	116.81
System loss (MW)				5.143	3.527	3.527	3.527	3.52	5.143	3.485	3.485	3.604
Fuel cost (\$/hr)	3222.97	3223.11	2985.01	2985.015	3286.434	3266.64	3063.56	3063.32	3286.434	3267.13	3063.33	3060.01
ISO payment to Generator (\$/hr)	3178.51	3178.17	5681.85	5676.213	3238.464	3232.49	5919.08	5915.588	3238.464	3231.06	5744.59	5950.123
Generator profit (\$/hr)	-44.46	-44.94	2696.84	2691.198	-47.97	-34.14	2855.51	2852.268	-47.97	-36.06	2681.26	2890.113
Load payment to ISO (\$/hr)	4716.081	4715.85	7219.53	7213.915	5005.321	5107.1	7829.63	7834.127	5006.246	5104.78	7652.58	7867.24
ISO profit (\$/hr)	1537.571	1537.67	1537.67	1537.702	1766.861	1874.61	1910.55	1918.539	1767.782	1873.72	1907.99	1917.12
Social surplus (\$/hr)	1493.11	1492.73	4234.51	4228.9	1718.891	1840.47	4766.06	4770.807	1719.812	1837.65	4589.25	4807.233
Iterations			29	35			29	35		68	10	
CPU time (sec)			4.091	2.793			4.091	2.793		7.51	0.3996	

where  $P_{\text{loss}}$  is the total system loss.  $P_{\text{loss}}$  in (14) is used to offset the doubled average system loss caused by the marginal loss factor (LF) and the marginal delivery factor (DF). The derivation of (14) is given in [Appendix A](#).

After getting power outputs of generators for the above dispatch, slack bus power is calculated using (10) or (14) and the price at the reference (slack) bus needs to be calculated by substituting slack bus power either in fixed bids or linear bids. At the reference bus, both loss price and congestion price are always zero. Therefore, the price at the reference bus is equal to the energy component. Now the LMP formulation at a bus  $B$  can be written as

$$\text{LMP}_B = \text{LMP}^{\text{energy}} + \text{LMP}_B^{\text{cong}} + \text{LMP}_B^{\text{loss}} \quad (17)$$

The decomposition of LMP is shown here

$$\text{LMP}^{\text{energy}} = \lambda = \text{price at the reference bus} \quad (18)$$

$$\text{LMP}_B^{\text{cong}} = -\sum_{k=1}^M \text{GSF}_{k-B} \times \mu_k \quad (19)$$

where  $\mu_k$  is the constraint cost or shadow price of line  $k$ , defined as:

$$\mu_k = \frac{\text{change in total cost}}{\text{change in constraint's flow}} \quad (20)$$

$$\text{LMP}_B^{\text{loss}} = \lambda \times (\text{DF}_B - 1)$$

( $\text{LMP}_B^{\text{loss}} = 0$  for lossless power system)

### Case 3: With distributed loss using DCOPF

Concentrated loss model addresses the marginal loss price through the delivery factors. However, the line flow constraint in (15) still assumes a lossless network. But the equality constraint in (14) informs that total generation is greater than the total demand by the average system loss. This causes a mismatch at slack bus and this mismatch is absorbed by the system slack bus. If system demand is huge e.g., a few GW then the system loss may be of the order of hundreds of MW and it may not be possible to add all the losses to slack bus. To address this issue, it is necessary that the line losses are shared among buses. This paper employs the concept of distributed loss to represent the losses of the lines connected to a bus. In this method system losses are distributed among all the buses and eliminate the large mismatch at the reference bus. By this approach, loss in each transmission line is divided into two equal halves, and each half is added to respective end buses of the line as an extra load. So for each bus, the total

extra load is the sum of halves of line losses which are connected to that bus. The extra load at bus ' $i$ ' is assumed as  $E_i$ , and it is defined as follows:

$$E_i = \sum_{k=1}^{M_i} \frac{1}{2} \times F_k^2 \times R_k \quad (21)$$

where  $M_i$  is number of lines connected to bus  $i$ . The line flow  $F_k$  for this model is calculated as in (22).

$$F_k = \sum_{i=1}^N \text{GSF}_{k-i} \times (G_i - D_i - E_i) \quad (22)$$

The algorithm for this problem is the same as in case 2. After getting power outputs of generators, spot prices at all buses are calculated using (17)–(20). With this approach, the fuel cost is further reduced than the concentrated loss model and the burden on the slack bus is eliminated.

All the 3 above mentioned cases have been initially attempted with Linear Programming approach with fixed bids (LP-FB), Genetic algorithm with fixed bids (GA-FB) and GA with Linear bids (GA-LB). Since, GA-LB results are proved to be better than GA-FB and LP-FB, in further optimization GA-LB results are compared with Bat with linear bids (BA-LB) approach. For case 2, in LP approach, loss is calculated using ac load flow and is added to the slack bus as load by modifying the line data with resistance ( $R$ ) taken as 10% of reactance ( $x$ ) to make the system linear [28]. For case 3, in LP approach to make an equivalent model of distributed loss; loss is calculated as in case 2 but it is distributed to all the generators as extra loads.

### Implementation of Bat algorithm for spot price calculation

For global optimization problems with a single objective, if the design functions are highly nonlinear, global optimality is not easy to achieve. Metaheuristic algorithms are very powerful in dealing with this kind of optimization. Furthermore, real-world optimization problems always involve a certain degree of uncertainty or noise. For example, material properties for a design product may vary significantly; an optimal design should be robust enough to allow homogeneity and also provide good choice for the decision-makers or designers.

In addition, metaheuristic algorithms start to emerge as a major player for global optimization; they often mimic the successful

**Table 2**

Spot prices at all buses for all models of IEEE 14 bus system.

Bus no.	LMP's at all buses (\$/MW h)											
	Without loss model case 1				Concentrated loss model case 2				Distributed loss model case 3			
	LP approach fixed bids	GA approach		Bat linear bids	LP approach fixed bids	GA approach		Bat linear bids	LP approach fixed bids	GA approach		
		Fixed bids	Linear bids			Fixed bids	Linear bids			Fixed bids	Linear bids	
1	12.34	12.34	22.00	21.99	12.34	12.34	22.52	22.51	12.34	12.34	21.87	22.65
2	12.19	12.18	21.85	21.83	12.16	12.27	22.56	22.57	12.16	12.27	21.89	22.70
3	11.76	11.75	21.42	21.40	11.64	12.11	22.69	22.72	11.64	12.11	22.00	22.84
4	11.38	11.38	21.05	21.03	11.20	11.58	22.08	22.10	11.20	11.57	21.40	22.22
5	12.91	12.91	22.58	22.56	13.02	13.34	23.8	23.81	13.02	13.34	23.12	23.94
6	23.77	23.76	33.43	33.41	25.95	26.28	36.74	36.76	25.95	26.27	36.06	36.88
7	27.58	27.57	37.24	37.22	30.49	30.86	41.36	41.38	30.50	30.85	40.67	41.50
8	27.58	27.57	37.24	37.22	30.49	30.86	41.36	41.38	30.50	30.85	40.67	41.50
9	36.10	36.09	45.76	45.74	40.63	41.00	51.50	51.52	40.65	41.00	50.81	51.64
10	33.91	33.9	43.57	43.55	38.02	38.40	48.91	48.93	38.03	38.4	48.23	49.05
11	28.93	28.92	38.59	38.57	32.09	32.45	42.95	42.96	32.10	32.45	42.26	43.09
12	24.74	24.74	34.4	34.39	27.11	27.48	37.99	38.01	27.12	27.48	37.31	38.13
13	25.50	25.5	35.17	35.15	28.02	28.40	38.91	38.93	28.02	28.39	38.23	39.05
14	31.47	31.46	41.13	41.11	35.12	35.55	46.09	46.12	35.13	35.54	45.41	46.24

characteristics in nature, especially biological systems. Many new algorithms are emerging with many important applications.

In this paper, a metaheuristic search algorithm, called Bat algorithm, which is a real coded algorithm has been proposed for solving DCOPF based spot price calculation with different loss cases for a congested system. Preliminary studies show that it is very promising and could outperform existing algorithms. The results of BA are validated with LP and GA and proved to be better.

#### Bat algorithm

The basic steps of Bat algorithm for single objective optimization are outlined here. The echolocation characteristics of microbats can

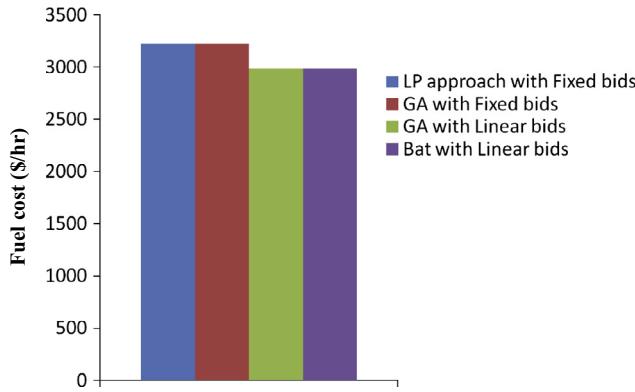


Fig. 4. Fuel cost comparison for case 1 of 458 IEEE 14 bus system.

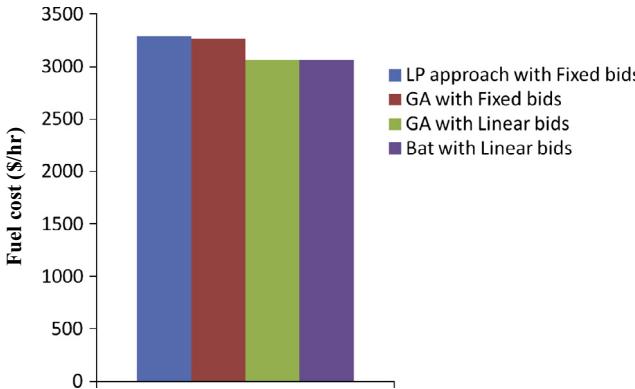


Fig. 5. Fuel cost comparison for case 2 of IEEE 14 bus system.

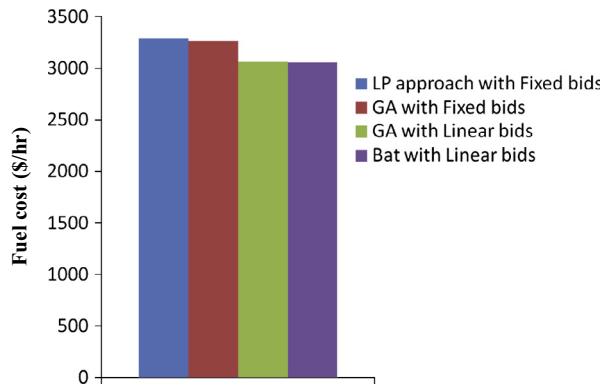


Fig. 6. Fuel cost comparison for case 3 of IEEE 14 bus system.

be idealized to develop various bat-inspired algorithms or bat algorithms. In the basic bat algorithm developed by Yang [35], the following approximate or idealized rules were used.

1. All bats use echolocation to sense distance, and they also 'know' the difference between food/prey and background barriers in some magical way;

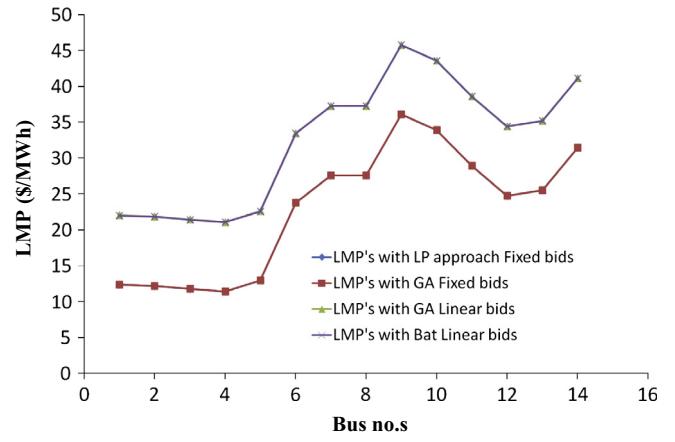


Fig. 7. LMP's comparison for case 1 of IEEE 14 bus system.

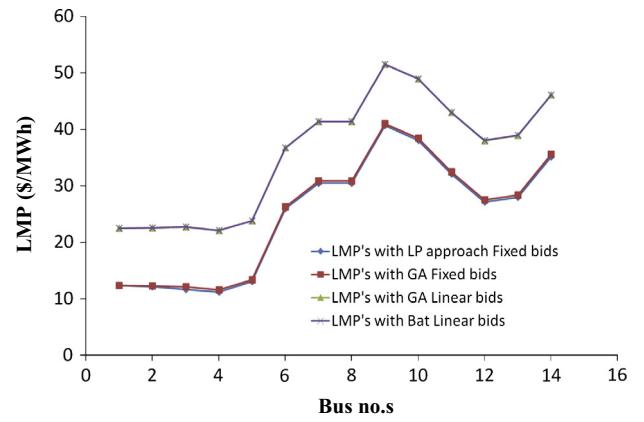


Fig. 8. LMP's comparison for case 2 of IEEE 14 bus system.

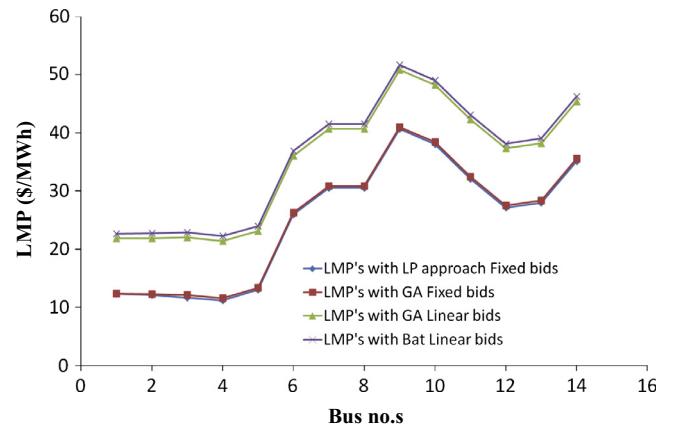
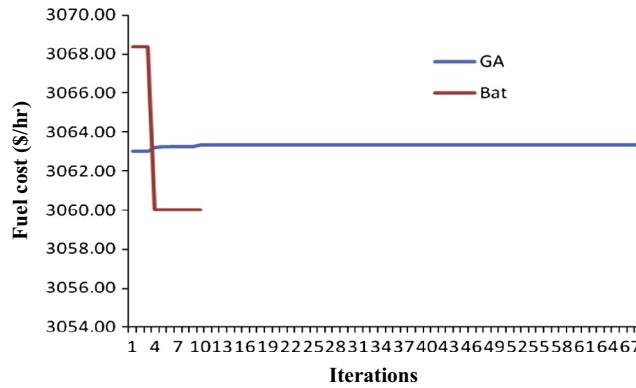


Fig. 9. LMP's comparison for case 3 of 500 IEEE 14 bus system.



**Fig. 10.** Convergence characteristics comparison of GA and BA for case 3 of IEEE 14 bus system.

2. Bats fly randomly with velocity  $v_i$  at position  $x_i$  with a frequency  $f_{\min}$ , with varying wavelength  $\lambda$  and loudness  $A_0$  to search for prey. They can automatically adjust the wavelength (or frequency) of their emitted pulses and adjust the rate of pulse emission  $r \in [0, 1]$ , depending on the proximity of their target;

3. Although the loudness can vary in many ways, we assume that the loudness varies from a large (positive)  $A_0$  to a minimum constant value  $A_{\min}$ .

Generally frequency  $f$  is selected in the range of  $[f_{\min}, f_{\max}]$  corresponding to the wavelength range of  $[\lambda_{\min}, \lambda_{\max}]$ . For example frequency in the range of  $[20 \text{ kHz}, 500 \text{ kHz}]$  corresponds to wavelengths of range of  $0.7\text{--}17 \text{ mm}$ . The ranges can be chosen freely to suit different applications.

#### Bat motion

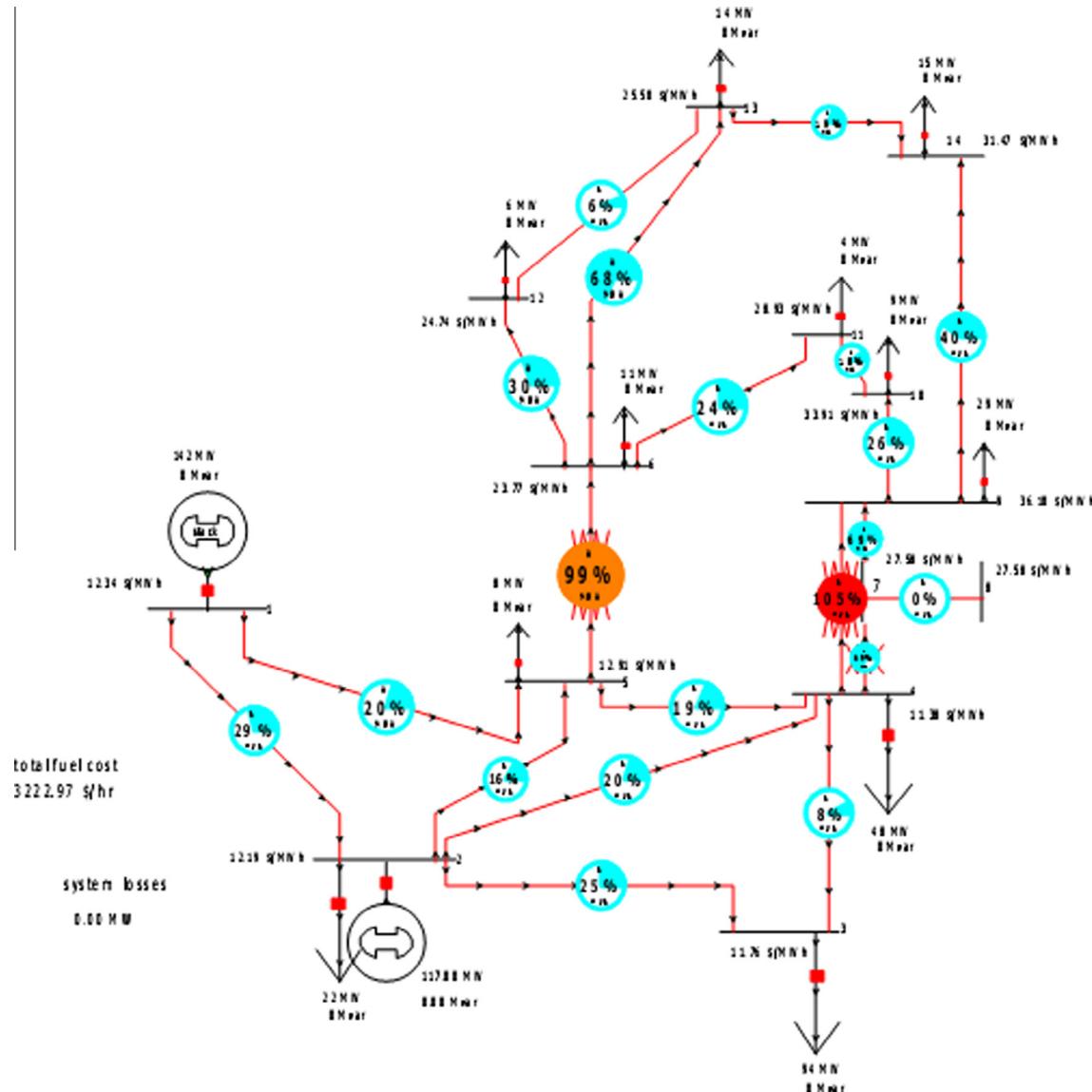
Bat position  $x_i$  and velocity  $v_i$  in a  $d$ -dimensional search space at a time step 't' are updated using (23)–(25).

$$f_i = f_{\min} + (f_{\max} - f_{\min})\beta \quad (23)$$

$$v_i^{t+1} = v_i^t + (x_i^t - x^*)f_i \quad (24)$$

$$x_i^{t+1} = x_i^t + v_i^t \quad (25)$$

where  $\beta \in [0, 1]$  is a random vector drawn from a uniform distribution. Here 'x' is the current global best location (solution) which is located after comparing all the solutions among all the 'n' bats at



**Fig. 11.** Power World simulation of IEEE 14 bus system for 536 case 1 with LP-FB.

**Table 3**

Comparison of results with different methods for three loss models of New England 39 bus system.

Generator bus no.	Power generations in MW without loss case 1				Power Generations in MW with concentrated loss case 2				Power generations in MW with distributed loss case 3			
	LP approach fixed bids	GA approach		Bat linear bids	LP approach fixed bids	GA approach		Bat linear bids	LP approach fixed bids	GA approach		Bat linear bids
		Fixed bids	Linear bids			Fixed bids	Linear bids			Fixed bids	Linear bids	
30	220	300.48	300.48	86.38	220	300.48	300.48	86.38	220	197.65	197.65	280.22
31 (Slack bus)	609.2	609.1	609.1	650.53	643.3	638.55	638.55	685.33	612.614	609.05	609.05	663.69
32	750	653.35	653.35	639.20	750	653.35	653.35	639.20	750	673.53	673.53	735.62
33	650	654.67	654.67	610.65	650	654.67	654.67	610.65	653.414	654.67	654.67	628.68
34	608	548.78	548.78	599.42	608	548.78	548.78	599.42	608	437.19	437.19	607.96
35	605	577.6	577.6	668.01	605	577.6	577.6	668.01	605	700.43	700.43	476.48
36	406	382.18	382.18	414.63	406	382.18	382.18	414.63	406	387.45	387.45	489.03
37	640	506.97	506.97	588.80	640	506.97	506.97	588.80	640	567.92	567.92	394.13
38	777.3	818.64	818.64	900.59	777.3	818.64	818.64	900.59	804.612	853.99	853.99	880.96
39	885	1098.68	1098.68	992.23	885	1098.68	1098.68	992.23	885	1099.73	1099.73	1024.07
System loss (MW)				34.14		29.45	29.45	34.79	34.14	31.17	31.17	30.37
Fuel cost (\$/hr)	82080.92	81086.69	62351.02	61930.4	82461.48	81415.38	622686.57	62336.00	82542.22	81569.19	62847.49	62792.68
ISO payment to generator (\$/hr)	83023.089	70538.12	71427.7	55108.18	83403.645	70098.32	72062.12	55792.75	83484.39	70024.2	70905.26	55105.19
Generator profit (\$/hr)	942.169	−10548.56	9076.68	−6822.21	942.165	−11317.06	9375.54	−6543.24	942.17	−11544.98	8057.77	−7687.49
Load payment to ISO (\$/hr)	86633.59	68639.58	69529.15	53248.3	86633.59	68528.46	70501.73	54342.14	86633.59	68473.59	69359.15	53700.44
ISO profit (\$/hr)	3610.50	−1898.54	−1898.54	−1859.87	3229.945	−1569.85	−1560.39	−1450.6	3149.2	−1550.61	−1546.11	−1404.74
Social surplus (\$/hr)	4552.669	−12447.11	7178.14	−8682.09	4172.11	−12886.92	7815.15	−7993.85	4091.37	−13095.6	6511.66	−9092.23
Iterations			267	188		267	188			188	50	
CPU time (sec)			151.179	109.936		113.642	9.138			115.506	10.827	

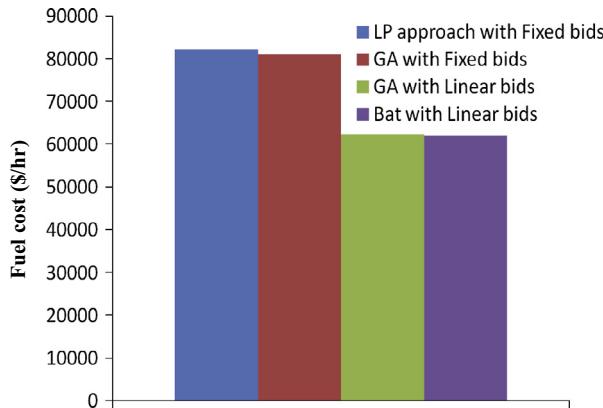


Fig. 12. Fuel cost comparison for case 1 of New England 39 bus system.

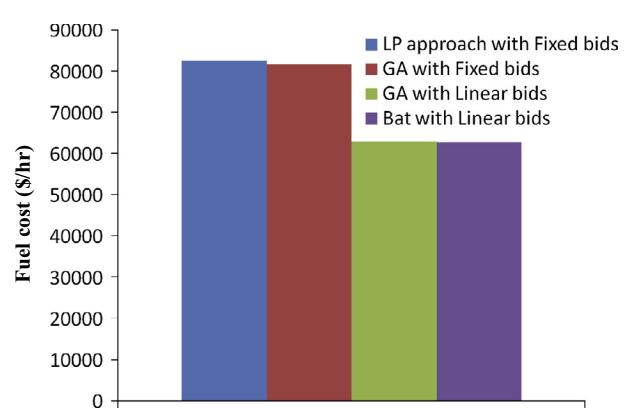


Fig. 14. Fuel cost comparison for case 3 of New England 39 bus system.

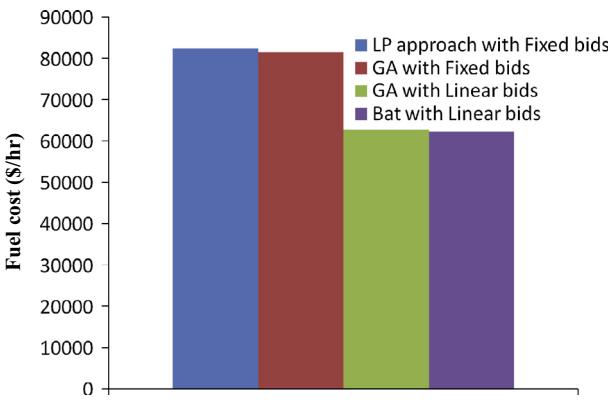


Fig. 13. Fuel cost comparison for case 2 of New England 39 bus system.

each iteration 'i'. As the product  $\lambda_i f_i$  is the velocity increment, we can use  $f_i$  (or  $\lambda_i$ ) to adjust the velocity change while fixing the other factor  $\lambda_i$  (or  $f_i$ ), depending on the type of the problem of interest. In this

paper  $f_{\min} = 0$  and  $f_{\max} = 1$  are used. Initially, each bat is randomly assigned a frequency which is drawn uniformly from  $[f_{\min}, f_{\max}]$ .

For the local search part, once a solution is selected from among the current best solutions, a new solution for each bat is generated locally using random walk

$$x_{\text{new}} = x_{\text{old}} + \varepsilon A^t \quad (26)$$

where  $\varepsilon$  is a random number vector drawn from  $[-1, 1]$ , while  $A^t = \langle A_i^t \rangle$  is the average loudness of all the bats at this time step.

The update of the velocities and positions of bats have some similarity to the procedure in the standard particle swarm optimization, as  $f_i$  essentially controls the pace and range of the movement of the swarming particles. To a degree, BA can be considered as a balanced combination of the standard particle swarm optimization and the intensive local search controlled by the loudness and pulse rate.

In this method power generations of generators ( $PG_i$ ) except slack bus are taken as the control variables in the chromosomes. The problem is formulated as minimizing the objective function

(9) subjected to (10) or (14) as equality and (15) as inequality constraints.

#### Loudness and pulse emission

Furthermore, the loudness  $A_i$  and the rate  $r_i$  of pulse emission have to be updated accordingly as the iterations proceed. As the loudness usually decreases once a bat has found its prey, while the rate of pulse emission increases, the loudness can be chosen as any value of convenience. For example, we can use  $A_0 = 100$  and  $A_{\min} = 1$ . For simplicity, we can also use  $A_0 = 1$  and  $A_{\min} = 0$ , assuming  $A_{\min} = 0$  means that a bat has just found the prey and temporarily stop emitting any sound. Now we have

$$A_i^{t+1} = \alpha A_i^t, r_i^t = r_i^0 [1 - \exp(-\gamma t)] \quad (27)$$

where  $\alpha$  and  $\gamma$  are constants. For any  $0 < \alpha < 1$  and  $\gamma > 0$ , we have

$$A_i^t \rightarrow 0, r_i^t \rightarrow r_i^0, \text{ as } t \rightarrow \infty \quad (28)$$

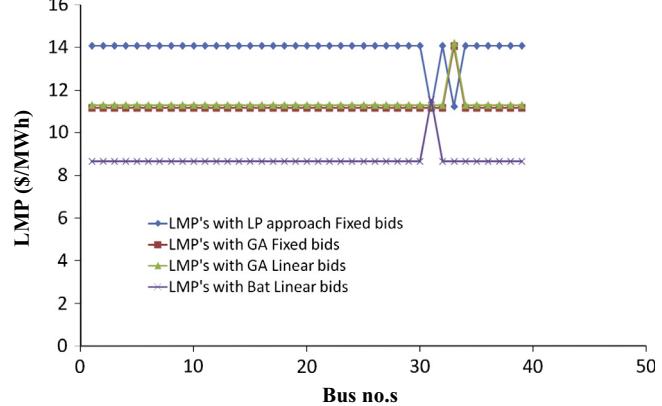
In the simplest case, we can use  $\alpha = \gamma$ , and we have used  $\alpha = \gamma = 0.9$  in our simulations. Bat algorithm is very promising for solving nonlinear global optimization problems.

#### Constraints handling

Constraints are handled by using penalty function approach. If an individual  $S_j$  is a feasible solution and satisfies all constraints,

**Table 4**  
LMP's at all buses for all models of New England 39 bus system.

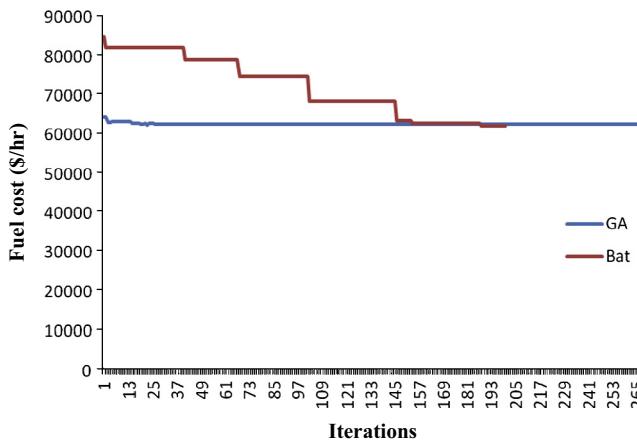
Bus no.	LMP's at all buses (\$/MW h)												
	Without loss model case 1						Concentrated loss model case 2			Distributed loss model case 3			
	LP approach fixed bids		GA approach		Bat linear bids		LP approach fixed bids		GA approach		Bat linear bids		
1	14.09	11.16	11.3	8.65	14.09		11.14	11.46	8.92	14.09	11.14	11.28	8.79
2	14.09	11.16	11.3	8.65	14.09		11.08	11.39	8.82	14.09	11.08	11.22	8.70
3	14.09	11.16	11.3	8.65	14.09		11.22	11.54	8.93	14.09	11.21	11.36	8.81
4	14.09	11.16	11.3	8.65	14.09		11.24	11.57	8.95	14.09	11.24	11.39	8.83
5	14.09	11.16	11.3	8.65	14.09		11.18	11.51	8.89	14.09	11.18	11.33	8.76
6	14.09	11.16	11.3	8.65	14.09		11.16	11.48	8.86	14.09	11.16	11.30	8.73
7	14.09	11.16	11.3	8.65	14.09		11.24	11.56	8.95	14.09	11.24	11.38	8.82
8	14.09	11.16	11.3	8.65	14.09		11.26	11.58	8.97	14.09	11.26	11.40	8.85
9	14.09	11.16	11.3	8.65	14.09		11.21	11.53	8.98	14.09	11.21	11.35	8.85
10	14.09	11.16	11.3	8.65	14.09		11.07	11.39	8.76	14.09	11.06	11.21	8.62
11	14.09	11.16	11.3	8.65	14.09		11.1	11.42	8.80	14.09	11.09	11.24	8.66
12	14.09	11.16	11.3	8.65	14.09		11.1	11.42	8.80	14.09	11.1	11.24	8.66
13	14.09	11.16	11.3	8.65	14.09		11.1	11.42	8.80	14.09	11.1	11.24	8.66
14	14.09	11.16	11.3	8.65	14.09		11.18	11.5	8.87	14.09	11.17	11.32	8.75
15	14.09	11.16	11.3	8.65	14.09		11.23	11.55	8.90	14.09	11.22	11.36	8.79
16	14.09	11.16	11.3	8.65	14.09		11.19	11.51	8.84	14.09	11.17	11.32	8.74
17	14.09	11.16	11.3	8.65	14.09		11.21	11.53	8.88	14.09	11.19	11.34	8.78
18	14.09	11.16	11.3	8.65	14.09		11.23	11.55	8.91	14.09	11.22	11.36	8.81
19	14.09	11.16	11.3	8.65	14.09		10.96	11.27	8.59	14.09	10.99	11.13	8.52
20	14.09	11.16	11.3	8.65	14.09		10.96	11.27	8.59	14.09	10.99	11.13	8.52
21	14.09	11.16	11.3	8.65	14.09		11.13	11.45	8.75	14.09	11.09	11.23	8.66
22	14.09	11.16	11.3	8.65	14.09		10.98	11.30	8.57	14.09	10.92	11.06	8.49
23	14.09	11.16	11.3	8.65	14.09		11.00	11.32	8.60	14.09	10.95	11.09	8.48
24	14.09	11.16	11.3	8.65	14.09		11.19	11.52	8.84	14.09	11.17	11.32	8.74
25	14.09	11.16	11.3	8.65	14.09		11.04	11.36	8.75	14.09	11.03	11.17	8.68
26	14.09	11.16	11.3	8.65	14.09		11.11	11.43	8.77	14.09	11.08	11.23	8.69
27	14.09	11.16	11.3	8.65	14.09		11.20	11.53	8.87	14.09	11.18	11.33	8.78
28	14.09	11.16	11.3	8.65	14.09		10.96	11.28	8.56	14.09	10.92	11.06	8.50
29	14.09	11.16	11.3	8.65	14.09		10.85	11.16	8.43	14.09	10.8	10.94	8.37
30	14.09	11.16	11.3	8.65	14.09		11.08	11.39	8.82	14.09	11.08	11.22	8.70
31	11.16	11.16	11.3	11.55	11.16		11.16	11.48	11.76	11.16	11.16	11.3	11.63
32	14.09	11.16	11.3	8.65	14.09		11.07	11.39	8.76	14.09	11.06	11.21	8.62
33	11.24	14.06	14.2	8.65	11.24		13.86	14.17	8.59	11.24	13.89	14.03	8.52
34	14.09	11.16	11.3	8.65	14.09		10.96	11.27	8.59	14.09	10.99	11.13	8.52
35	14.09	11.16	11.3	8.65	14.09		10.98	11.3	8.57	14.09	10.92	11.06	8.49
36	14.09	11.16	11.3	8.65	14.09		11.00	11.32	8.6	14.09	10.95	11.09	8.48
37	14.09	11.16	11.3	8.65	14.09		11.04	11.36	8.75	14.09	11.03	11.17	8.68
38	14.09	11.16	11.3	8.65	14.09		10.85	11.16	8.43	14.09	10.8	10.94	8.37
39	14.09	11.16	11.3	8.65	14.09		11.17	11.49	8.98	14.09	11.17	11.32	8.85



**Fig. 15.** LMP's comparison for case 1 of New England 39 bus system.

its fitness will be measured by taking the reciprocal of the fuel cost function, or else it needs to be penalized. Using the exterior penalty function approach, the violated operating constraints are incorporated as penalties in objective function.

Calculate the fitness function,  $FF = 100/(1 + J + \text{penalties})$ . The penalties are calculated for (10) or (14), (11) and slack bus power if they are violated as follows:



**Fig. 16.** Convergence characteristics comparison of GA and BA for case 3 of New England 39 bus system.

Penalty function for line flows:

$$\text{pcost\_f} = \text{lambda\_f}(k) * \text{df} * (|\text{pflow}(k)| - \text{limit})^2.$$

Penalty function for power balance:

pcost\_error= lambda\_error \* (error)<sup>2</sup>.

Penalty function for slack bus power:

$$\text{pcost\_s} = \text{lambda\_s} * \text{ds} * (\text{pgen}(\text{nsslack}) - \text{s\_limit})^2.$$

where  $\lambda_f(k)$ ,  $df$ ,  $\lambda_{\text{error}}$ ,  $\lambda_s$ ,  $ds$  are all constant values and are maintained same value for all the three loss cases in each test system. This constraint handling procedure is same in GA and BA.

### *Pseudo code*

Initialize the bat population  $x_i$  ( $i = 1, 2, \dots, n$ ) and  $v_i$   
 Initialize frequencies  $f_i$ , pulse rates  $r_i$  and the loudness  $A_i$

**while** ( $t < \text{Max number of iterations}$ )

Generate new solutions by adjusting frequency,  
and updating velocities and locations/solutions (23)–(25)

Select a solution among the best solutions

**if** (rand >  $r_i$ )

Generate a local solution around the selected best solution

*end if*

## Evaluate new solutions

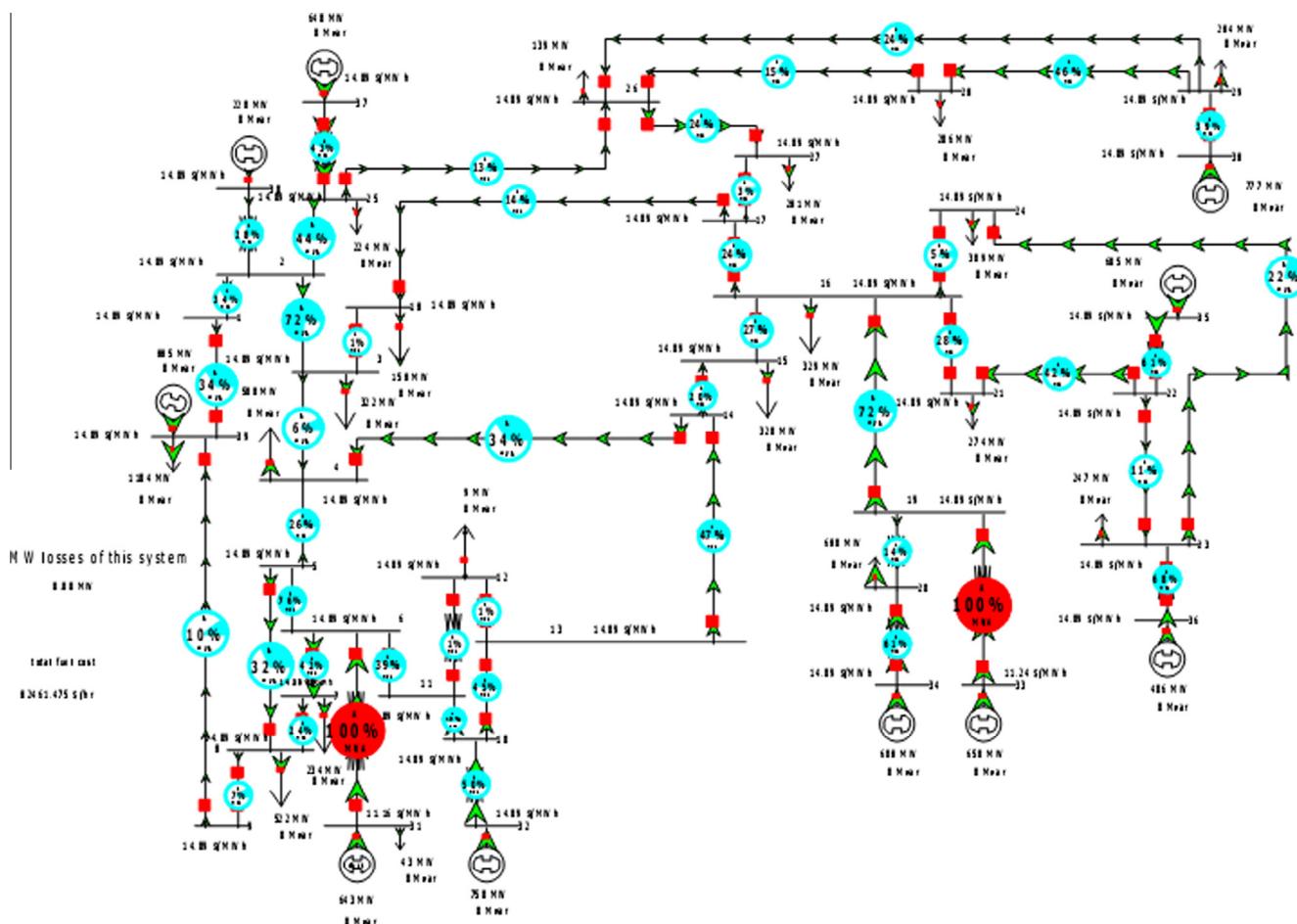
**if** (rand <  $A_i$  &  $f(x_i) < f(x^*)$ )

Accept the new solution

## Increasing

**end if**

Rank the



**Fig. 17.** Power World simulation of New England 39 bus system for 650 case 2 with LP-FB.

Once the convergence is arrived at:

- (i) Calculate the energy price of the reference bus either with fixed bids or with linear bids and then calculate LMP's at all buses using (17) and the decomposition of LMP using (18)–(20).
- (ii) Calculate ISO payments to generators ( $\sum_i$  power generation at bus  $i * LMP$  at bus  $i$ ) and then calculate generator profit (ISO payment to generators-fuel cost).
- (iii) Calculate load payment to ISO ( $\sum_i$  load at bus  $i * LMP$  at bus  $i$ ), and ISO profit (load payment to ISO–ISO payment to generators) for all the cases.
- (iv) Finally calculate the Social Welfare or Social surplus (Generator profit + ISO profit) for all loss cases.

#### BA flow chart

Fig. 3 represents flow chart of all the steps involved in Bat algorithm.

#### Results and discussion

The developed BA based DCOPF for cases 1–3 for LMP estimation are applied on IEEE 14 bus system [27], New England 39 bus system [24] and 75 bus Indian power system [34]. GA parameters used are Population size: 40, number of bits for each generator in the chromosome: 12, Elitism probability: 0.15, Crossover probability:

0.85, Mutation probability: 0.01, Tolerance: 0.0001. BA parameters used are Population size: 25 (10–25), Loudness: 0.25 (0–1), Pulse rate: 0.5 (0–1),  $f_{\min} = 0$ ,  $f_{\max} = 0.02$ ,  $\alpha = 0.9$ ,  $\gamma = 0.9$ . IEEE 14 bus system has 2 generators, 39 bus system has 10 generators and 75 bus system has 15 generators. The solution reported is the best solution over 20 different runs. The proposed BA-LB is compared with GA-LB, GA-FB and LP-FB for all the three cases. After the spot prices have been obtained using the above approaches, ISO payments to generators, Load payments to ISO, generator profit, ISO profit and Social surplus (Social welfare) have been computed.

#### Case study 1: IEEE 14 bus test system

IEEE 14 bus system [27] has 2 generators, 17 lines and 3 transformers. Results for all the cases are presented in Tables 1 and 2. For the base case loading, i.e., 259 MW only 9th line connecting 4–9 buses is congested with all approaches whose shadow price is 91.747 \$/MW h for case 1 and 109.253 \$/MW h for cases 2 and 3. The corresponding generator power outputs are listed in Table 1. Spot prices at all buses are presented in Table 2. From Table 1, it can be observed that BA-LB gives the most optimal fuel cost for cases 1, 2 and 3 and it can also be illustrated that slack bus power in distributed loss model is reduced than in concentrated loss model and burden on the slack generator is removed. As this study of spot pricing is a real time study; the CPU time of the algorithm is also an important parameter which should also be taken care of by

Table 5

Comparison of results with different methods for three loss models of 75 bus Indian power system.

Generator bus no.	Power generations in MW without loss case 1				Power generations in MW with concentrated loss case 2				Power generations in MW with distributed loss case 3			
	LP approach fixed bids	GA approach		Bat linear bids	LP approach fixed bids	GA approach		Bat linear bids	LP approach fixed bids	GA approach		Bat linear bids
		Fixed bids	Linear bids			Fixed bids	Linear bids			Fixed bids	Linear bids	
1 (Slack bus)	684.2	730.05	730.05	814.56	795.367	793.64	793.64	902.698 (Limit violated)	786.682	785.98	785.98	798.79
2	360	282.52	282.52	353.64	360	282.52	282.52	353.64	360	270.41	270.41	225.73
3	280	279.93	279.93	260.10	280	279.93	279.93	260.10	280	279.93	279.93	278.28
4	185	189.87	189.87	198.09	185	189.87	189.87	198.09	193.683	199.02	199.02	199.72
5	25	25.0	25.0	25.00	25	25.0	25.0	25.00	25	75.95	75.95	32.68
6	220	219.95	219.95	199.70	220	219.95	219.95	199.70	220	219.95	219.95	189.25
7	160	159.96	159.96	159.24	160	159.96	159.96	159.24	160	159.96	159.96	159.13
8	180	179.96	179.96	179.00	180	179.96	179.96	179.00	180	179.96	179.96	179.30
9	505.92	201.59	201.59	358.29	525	201.59	201.59	358.29	525	305.5	305.5	447.66
10	180	179.96	179.96	179.40	180	179.96	179.96	179.40	180	179.96	179.96	179.77
11	209	208.95	208.95	208.98	209	208.95	208.95	208.98	209	208.95	208.95	208.50
12	775	1106.8	1106.8	1045.55	775	1106.8	1106.8	1045.55	775	962.5	962.5	1090.54
13	1000	999.76	999.76	931.97	1000	999.76	999.76	931.97	1000	999.76	999.76	997.32
14	250	249.94	249.94	249.03	250	249.94	249.94	249.03	250	249.94	249.94	249.93
15	554	553.87	553.87	405.49	554	553.87	553.87	405.49	554	553.87	553.87	438.56
System loss (MW)					130.247	63.59	63.59	88.135	130.247	63.58	63.58	106.516
Fuel cost (\$/hr)	56097.0356	55981.2	48528.21	48141.71	57347.664	56696.62	49237.91	49175.81	58070.319	57376.59	49569.53	49557.2
ISO payment to generator (\$/hr)	60239.80	63277.41	61718.34	64529.94	63427.678	65089.39	65660.11	70079.86	63395.79	65166.35	65469.86	66910.11
Generator profit (\$/hr)	4142.7644	7296.2	13190.12	16388.22	6080.014	8392.77	16422.2	20904.05	5325.471	7789.75	15900.32	17352.83
Load payment to ISO (\$/hr)	60859.55	62641.35	61093.03	63878.68	62641.35	65243.39	65814.16	70428.97	62641.35	65218.56	65519.71	67138.81
ISO profit (\$/hr)	619.75	−636.06	−625.3	−651.26	−786.328	153.99	153.94	349.11	−754.44	52.21	49.84	228.692
Social surplus (\$/hr)	4762.51	6660.14	12564.82	15736.96	5293.686	8546.76	16576.14	21253.16	4571.031	7841.96	15950.17	17581.52
Iterations			396	125					396	125		
CPU time (sec)			897.928	236.978					672.491	114.817	312	60
											703.208	118.389

the system operator. The CPU time of convergence with BA is much less than GA and moreover Social surplus or social welfare is also improved with BA as compared to other methods in all the loss cases. If the difference in the fuel cost is small, then system operator chooses a method which consumes less CPU time. So in this connection BA gains more priority than GA and hence BA can be considered as better algorithm for spot pricing problem. Figs. 4–6 present a comparison of fuel cost for all the three loss cases. Figs. 7–9 show the spot price comparison for cases 1, 2 and 3. From these figures it can be observed that due to congestion in the line 4–9, LMP at 4th bus is minimum and LMP at 9th bus is maximum, thus a large variation in LMPs can be observed. Fig. 10 shows the convergence characteristics of GA and BA. Fig. 11 shows the Power World simulation of case 1 with LP-FB.

Fig. 10 shows that GA takes 68 iterations for convergence where as BA converges only in 10 iterations; and CPU time per iteration is also very less for BA than for GA. So, total convergence time with BA is drastically reduced.

The simulation diagrams of LP-FB with Power World simulator for cases 2 & 3 are similar to Fig. 11; and therefore they are not shown here. In Fig. 11 for the congested line (1 line is overloaded i.e., 4–9 line), loading is indicated in red color.

#### Case study 2: New England 39 bus test system

This system is a 345 kV transmission system in New England having 10 generators, 34 lines and 12 transformers. For this system also congestion occurred for base case loading (6150.5 MW) only. The generation dispatch for New England 39 bus system [24] is presented in Table 3. For all the three cases of loss in LP-FB approach (Power World simulation) lines 37 and 39 connecting

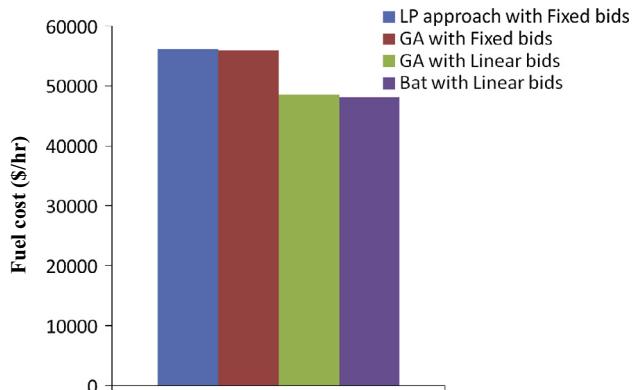


Fig. 18. Fuel 676 cost comparison for case 1 of 75 bus Indian power system.

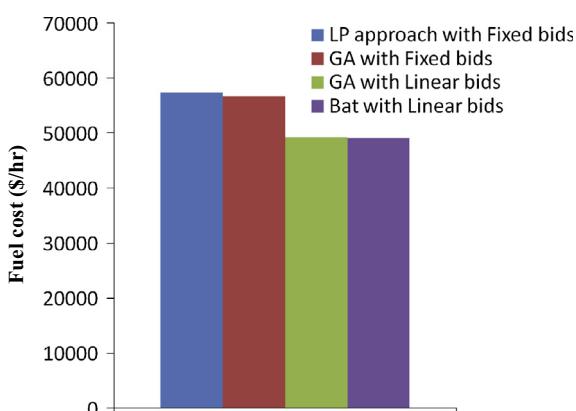


Fig. 19. Fuel cost comparison for case 2 of 75 bus Indian power system.

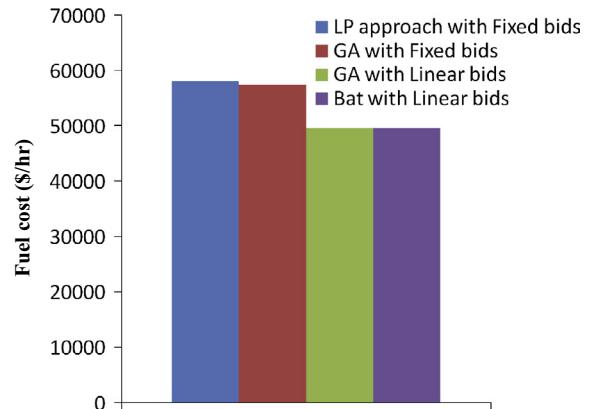


Fig. 20. Fuel cost comparison for case 3 of 75 bus Indian power system.

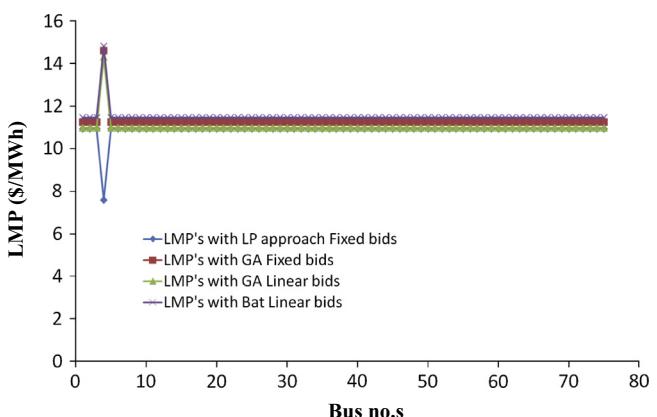


Fig. 21. LMP's comparison 686 for case 1 of 75 bus Indian power system.

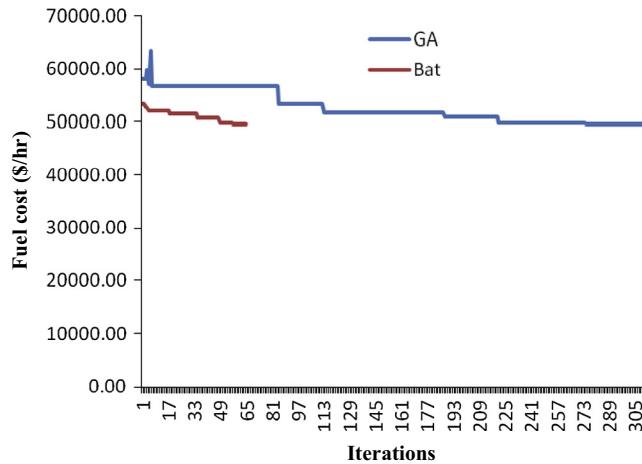
buses (6–31), (19–33) respectively are congested, whereas in GA-FB and in GA-LB methods, line 39 is only congested and in BA-LB, line 37 is congested for all the three loss cases. Figs. 12–14 show the comparison of fuel cost. In this system also BA-LB gives minimum fuel cost and convergence time is also very less with BA as compared with GA for without loss, concentrated loss and distributed loss models. Prices are also calculated at all buses and are presented in Table 4. Fig. 15 shows the nodal prices comparison of case 1. Due to congestion in the system, LMP's at buses 31 and 33 are different, indicating large variation in prices. Fig. 16 shows the convergence characteristics comparison of GA and BA. Fig. 17 shows the Power World simulation of case 2 with LP-FB. From Table 3 it is observed that slack bus generation in distributed loss model is reduced than in concentrated loss model and therefore burden on slack generator is removed. The shadow price for both the congested lines is 2.9 \$/MW h for cases 1, 2 and 3.

Bus spot price comparison graphs of cases 2 & 3 are as similar as Fig. 15, for which data is provided in Table 4 and hence they are not shown here. In Fig. 15 in all the methods, the LMP's for the buses except at the congested buses are same due to the absentia of loss price in the LMP components.

GA takes 188 iterations to converge the OPF problem whereas BA converges only in 50 iterations as is shown by Fig. 16. Therefore the net execution time of BA reduces, and this makes the Independent System Operator (ISO) to calculate the prices of the buses in less time than with GA.

In Fig. 17 for the congested lines (2 lines congested) loading is indicated in red color.<sup>1</sup> Fig. 17 illustrates that the total system loss

<sup>1</sup> For interpretation of color in Fig. 17, the reader is referred to the web version of this article.



**Fig. 22.** Convergence characteristics comparison of GA and BA for case 3 of 75 bus Indian power system.

is added to the slack bus 31 as an extra load to consider the effect of concentrated loss in the simulation. Similarly cases 1 & 3 are also simulated with Power World Simulators.

#### Case study 3: 75 bus Indian power system

The developed algorithms are also tested on 75 bus Indian power system (Uttar Pradesh State Electricity Board data i.e., 400 kV, 220 kV and 132 kV grid data) [34]. This system has 15 generators, 72 lines and 24 transformers. For base case loading of this system 9th line connecting 4–28 buses shows congestion for all the

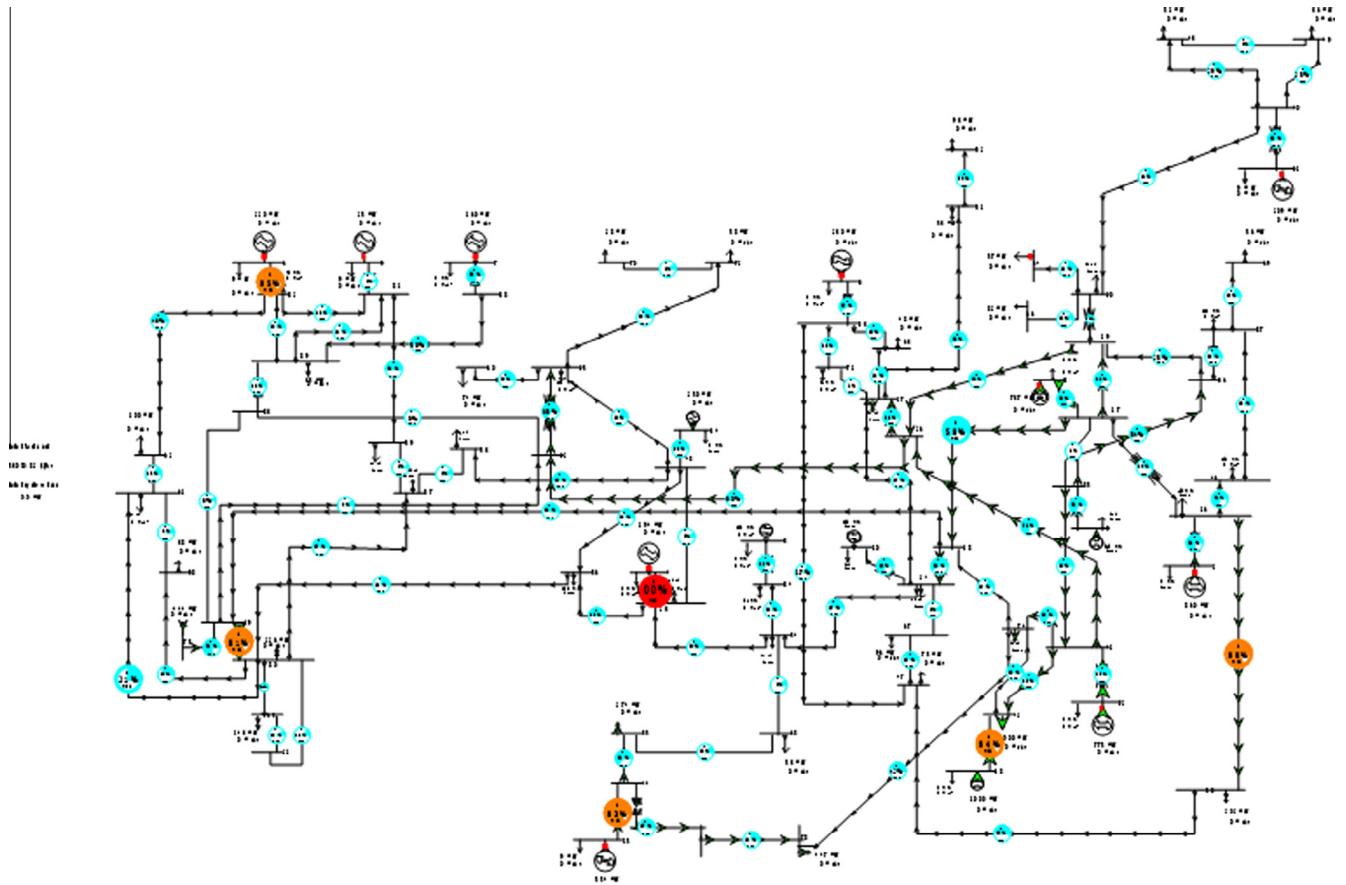
three cases of losses with all methods. The corresponding results are listed in Table 5. The spot prices at all buses are also calculated but not shown here. The fuel cost comparison graphs for cases 1, 2 and 3 are shown in Figs. 18–20 respectively. Fig. 21 shows the spot prices comparison for case 1 and this graph implies that due to congestion in the line 4–28, LMP at 4th bus undergoes a large variation. Convergence characteristics of GA and BA is represented in Fig. 22. Fig. 23 shows Power World simulation of case 3 with LP-FB. This case study also, highlighted that BA approach with linear bids of generators minimizes the total fuel cost of the system and improves the Social surplus of the system. Slack generator power in distributed loss model is comparatively reduced than in concentrated loss model. The shadow price of congested line is 3.35 \$/MW h for case 1, 3.67 \$/MW h for case 2 and 3.7 \$/MW h for case 3.

From the above Table 5 it can be observed that in case 2, bat linear bids, slack bus generation violated the maximum limit which is the major drawback of case 2 pointed out in this paper. In case 3, bat linear bids, slack bus generation sets within its limits.

Bus spot price comparison graphs of cases 2 & 3 are also similar as Fig. 21 and are not shown here. In Fig. 21 in all the methods, the LMP's for all the buses except at the 4th bus are same due to the absence of loss price in the LMP components.

From Fig. 22 it is revealed that GA converges in 312 iterations but BA converges only in 60 iterations. Thus it can be concluded that convergence time of OPF with BA is significantly reduced. This helps the system operator to analyze the situation satisfactorily and quickly.

In Fig. 23 for the congested line (100% loaded line i.e., 4–28 line), loading is indicated in red color. In Fig. 23 the system loss is distributed to all the generators and the circuit is modeled as an equivalent model of distributed loss system in Power World



**Fig. 23.** Power World simulation of 75 bus Indian power system for case 3 with LP-FB.

Simulator and then the simulation is carried out. The simulations for cases 1 & 2 are also simulated in a similar manner using Power World Simulator.

From Tables 1, 3 and 5 it was observed that generator profits, ISO profits and Social surplus are negative in some cases. This is because, the objective function of this problem is fuel cost minimization and in the process of minimizing the fuel cost alone, parameters like generator profit, ISO profit and Social surplus are found to be negative. These negative values can be removed, when consumers are modeled as elastic consumers while modifying the objective function as social welfare maximization for LMP calculation. From the results of all the test systems it was also observed that BA-LB gives the most optimal fuel cost, improved Social Welfare compared to others for all the three cases. The advantage of using linear bid over fixed bid is, it recovers true marginal cost of slack generator or energy price for determining spot price. As mentioned in case 3 of 'Spot price calculation with different loss cases', in concentrated loss model it is not feasible to add total system loss to slack generator which may create a burden on slack generator or slack generator may violate its limit. So distributed loss model is also proposed in this paper, in which total system loss is distributed among all the generators, and hence burden on slack generator is removed. It is observed that, slack bus generation in distributed loss model is reduced than in concentrated loss model for all the test systems studied in this paper.

## Conclusion

This paper proposed Bat algorithm based DCOPF for evaluating the spot prices in pool based electricity market. It presented a transmission pricing scheme to recover congestion cost, loss cost and some percentage of existing system cost using LP-FB, GA-FB, GA-LB and BA-LB methods with different loss cases during congestion in the system in a pool type electricity market. Nodal prices have been estimated for all loss cases of the system. The product of difference in LMP's of two ends of a line with line flow recovers the sum of congestion price and loss price of that particular line and sum of this amount on all the lines in the system forms merchandising surplus or transmission opportunity cost. This amount is kept with ISO as ISO profit, and utilized for future transmission expansion. The generator profit and Social surplus have also been evaluated. Bat algorithm with linear generator bids evolves as a better optimization algorithm for LMP estimation in the power systems. This study also explored the effect of type of generator bids and various loss models on spot prices. Distributing loss to all generators (distributed loss model), rather than concentrating at slack generator removes the burden on slack generator. Further, considering generator linear bids leads to true fuel cost of generators. Spot prices with linear bids are calculated to avoid the nonsmooth nature of bid curve in fixed bids. Finally this paper concludes that, Bat algorithm based on DCOPF with generator linear bids exhibits reliable convergence with reduced fuel cost and improved Social Welfare of the system in most of the loss cases for all test systems studied and can assist the system operator in obtaining correct economic signals during congestion in the transmission system.

## Appendix A

$$\left( \sum_{i=1}^N DF_i \times P_i \right) + P_{loss}$$

where  $P_i = G_i - D_i$

$P_i$  = net power injection at bus 'i'.

$G_i$  = power generation at bus 'i'.

$D_i$  = power demand at bus 'i'.

$$\begin{aligned}
 &= \left( \sum_{i=1}^N (1 - LF_i) \times P_i \right) + P_{loss} \\
 &= \left( \sum_{i=1}^N \left( 1 - \frac{\partial P_{loss}}{\partial P_i} \right) \times P_i \right) + P_{loss} \\
 &= \left( \sum_{i=1}^N P_i - \sum_{i=1}^N \frac{\partial P_{loss}}{\partial P_i} \times P_i \right) + P_{loss} \\
 &= \left( \sum_{i=1}^N P_i - \sum_{i=1}^N \left( \left( \sum_{k=1}^M 2R_k \times F_k \times GSF_{k-i} \right) \times P_i \right) \right) + P_{loss} \\
 &= \left( \sum_{i=1}^N P_i - \sum_{k=1}^M \left( 2R_k \times F_k \times \sum_{i=1}^N (GSF_{k-i} \times P_i) \right) \right) + P_{loss} \\
 &= \left( \sum_{i=1}^N P_i - \sum_{k=1}^M (2R_k \times F_k \times F_k) \right) + P_{loss} \\
 &= \left( \sum_{i=1}^N P_i - 2 \cdot \sum_{k=1}^M (R_k \times F_k^2) \right) + P_{loss} \\
 &= (P_{loss}^{schd} - 2P_{loss}^{act}) + P_{loss} \\
 &= -P_{loss} + P_{loss} = 0
 \end{aligned}$$

## References

- [1] Schwepp FC, Caramanis MC, Tabors RD, Bohn RE. Spot pricing of electricity. Boston (MA): Kluwer; 1988.
- [2] Rivier M, Perez-Arriaga IJ, Luengo G. JUANAC: a model for computation of spot prices in interconnected power systems. In: Proc 10th power systems computation conf (PSCC). Graz, Austria; August 1990. p. 254–1.
- [3] Hogan WW. Contract networks for electric s transmission: technical reference. Cambridge, Massachusetts: John F. Kennedy School of Government, Harvard University; February 1992.
- [4] Ring Bj. Dispatch based pricing in decentralised power systems, PhD dissertation. Christchurch, New Zealand: Univ. Canterbury; 1995.
- [5] Ring Bj, Grant Read E. A dispatch based pricing model for the New Zealand electricity market. In: Einhorn M, Siddiqi R, editors. Electricity transmission pricing and technology. Boston: Kluwer Academic Publishers; 1996. p. 183–206.
- [6] Clayton RE, Mukerji R. System planning tools for the competitive markets. IEEE Comput Appl Power 1996;9(3):50–5.
- [7] Baughman ML, Siddiqi SN, Zarnikau JW. Advanced pricing in electrical systems: Part I: Theory. IEEE Trans Power Syst 1997;12:489–95.
- [8] Baughman ML, Siddiqi SN, Zarnikau JW. Advanced pricing in electrical systems: Part II: Implications. IEEE Trans Power Syst 1997;12:496–502.
- [9] Alvey T, Goodwin D, Ma X, Streiffert D, Sun D. A security constrained bid-clearing system for the New Zealand wholesale electricity market. IEEE Trans Power Syst May 1998;13:340–6.
- [10] Singh H, Hao S, Papalexopoulos A. Transmission congestion management in competitive electricity markets. IEEE Trans Power Syst 1998;13:672–80.
- [11] NYISO Transmission & Dispatch Operations Manual, NYISO; 1999.
- [12] PJM Training Materials-LMP 101, PJM.
- [13] FERC electric rate schedule. ISO-New England; 2001.
- [14] Chen Luonan, Suzuki Hideki, Wachi Tsunehisa, Shimura Yukihiko. Components of nodal prices for electric power systems. IEEE Trans Power Syst 2002;17(1).
- [15] Shahidehpour M, Yamin H, Li Z. Market operations in electric power systems. New York: Wiley; 2002.
- [16] Ramos JLM, Exposito AG, Moron FJC, Bercerra SN. On the use of loss penalty factors for generation scheduling. In: Proc IEEE power eng soc annu meeting, vol. 2; 2003. p. 926–31.
- [17] U.S. Federal Energy Regulatory Commission. Notice of white paper; April 28, 2003.
- [18] Yang J, Li F, Freeman LAA. A market simulation program for the standard market design and generation/transmission planning. In: Proc IEEE power eng soc general meeting; 2003. p. 442–6.
- [19] Kirschen DS, Strbac G. Fundamentals of power system economics. Wiley; 2004.
- [20] Overbye Thomas J, Cheng Xu, Sun Yan. A comparison of the AC and DC power flow models for LMP calculations. In: IEEE proceedings of the 37th Hawaii international conference on system sciences; 2004.
- [21] Litvinov E, Zheng T, Rosenwald G, Shamsollahi P. Marginal loss modeling in LMP calculation. IEEE Trans Power Syst 2004;19(2):880–8.
- [22] Li F, Pan J, Chao H. Marginal loss calculation in competitive spot market. In: Proc IEEE int conf deregulation, restructuring power technol (DRPT), vol. 1; 2004. p. 205–9.
- [23] Purchala K, Meeus L, Dommelen DV, Belmans R. Usefulness of dc power flow for active power flow analysis. In: Proc IEEE power eng soc annu meeting; 2005. p. 454–9.

- [24] <http://sys.elec.kitami-it.ac.jp/ueda/demo/WebPF/39-New-England.pdf>.
- [25] Nabav SMH, Javid Shahram, Masoum MAS, Kazemi A. Congestion management in nodal pricing with genetic algorithm. In: International conference on power electronics, drives and energy systems, PEDES; 2006.
- [26] Fu Yong, Li Zuyi. Different models and properties on LMP calculations. In: IEEE Power engineering society general meeting; 2006.
- [27] Iyambo PK, Tzonneva R. Transient stability analysis of the IEEE 14-bus electric power system. In: IEEE conference, Africon; 2007.
- [28] Li Fangxing, Bo Rui. DCOPF-based LMP simulation: algorithm, comparison with ACOPF, and sensitivity. *IEEE Trans Power Syst* 2007;22(4).
- [29] Rahimi Nezhad A, Mokhtari G, Davari M, Roghani Araghi A, Hosseinian SH, Gharehpetian GB. A new high accuracy method for calculation of LMP as a random variable. In: International conference on electric power and energy conversion systems, EPECS; 2009.
- [30] Litvinov E. Design and operation of the locational marginal prices-based electricity markets. *IET Gener Transm Distrib* 2010;4(2):315–23.
- [31] Hu Zechun, Cheng Haohong, Li Furon. An iterative LMP calculation method considering loss distributions. *IEEE Trans Power Syst* 2010;25(3).
- [32] Mokhtari G, Behnood A, Ebrahimi J, Gharehpetian GB. LMP calculation, considering network topology uncertainty. In: The 4th international power engineering and optimization conf, Malaysia; 2010.
- [33] Ramachandran P, Senthil R. Locational marginal pricing approach to minimize congestion in restructured power market. *J Electr Electron Eng Res* 2010;2(6): 143–53.
- [34] 75 bus Indian Power System data (Uttar Pradesh State Grid data) obtained from Uttar Pradesh State (a state of INDIA) Electricity Board.
- [35] Yang Xin-She. *Nature-inspired metaheuristic algorithms*. 2nd ed. United Kingdom: Luniver Press; 2010.
- [36] Li Fangxing. Fully reference-independent LMP decomposition using reference-independent loss factors. *Electr Power Syst Res* 2011;81:1995–2004.
- [37] Dong Lei, Wang Wenbo, Lin Shan. Probabilistic calculation of locational marginal price taking account of correlated loads. In: IEEE Asia pacific power and energy engineering conference (APPEEC); March 2012.
- [38] Murali M, Sailaja Kumari M, Sydulu M. Genetic algorithm based security constrained economic dispatch approach for LMP calculation. *Int J Energy Sci* 2013;3(2):116–26.