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FREE CONVECTION IN MHD MICROPOLAR FLUID WITH RADIATION AND CHEMICAL REACTION EFFECTS

Article Highlights

- Flow of a MHD micropolar fluid with radiation and chemical reaction effects studied numerically
- Increase in radiation parameter leads to increase in the temperature, velocity and microrotation
- A higher value of radiation parameter implies higher Nusselt number and Sherwood number
- Velocity, microrotation and concentration decreased as chemical reaction parameter increase
- Increase in chemical reaction parameter causes rise in Nusselt number and fall in Sherwood number

Abstract

In this paper, the effects of radiation and first order chemical reaction on free convection heat and mass transfer in a micropolar fluid is considered. A uniform magnetic field is applied normal to the plate. The plate is maintained with variable surface heat and mass fluxes. The governing nonlinear partial differential equations are transformed into a system of coupled nonlinear ordinary differential equations using similarity transformations and then solved numerically using the Keller-box method. The numerical results are compared and found to be in good agreement with previously published results as special cases of the present investigation. The dimensionless velocity, microrotation, temperature, concentration and heat and mass transfer rates are presented graphically for various values of the coupling number, magnetic parameter, radiation parameter, chemical reaction parameter. The numerical values of the skin friction and wall couple stress for different values of governing parameters are also tabulated.

Keywords: free convection, micropolar fluid, MHD, chemical reaction, radiation, variable heat and mass flux.

Free convection flows are of great interest because of their various engineering, scientific and industrial applications in heat and mass transfer. Free convection of heat and mass transfer occurs simultaneously in the fields of design of chemical processing equipment, formation and dispersion of fog, distributions of temperature, moisture over agricultural fields, groves of fruit trees, damage of crops due to freezing and pollution of the environment. Extensive studies of free convection heat and mass transfer of a non-isothermal vertical surface under boundary layer approximation for Newtonian fluids have been undertaken by several authors. In recent years, several simple boundary layer flow problems have received new attention within the more general context of magnetohydrodynamics (MHD). Several investigators have extended many of the available boundary layer solutions to include the effects of magnetic fields for those cases when the fluid is electrically conducting. The study of magnetohydrodynamic flow for an electrically conducting fluid past a heated surface has important applications in many engineering problems such as plasma studies, petroleum industries, MHD power generators, cooling of nuclear reactors, the

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boundary layer control in aerodynamics, and crystal growth. Alam *et al.* [1] studied the problem of free convection heat and mass transfer flow past an inclined semi-infinite heated surface of an electrically conducting viscous incompressible fluid with magnetic field and heat generation. The effects of heat and mass transfer on MHD free convection along a moving permeable vertical surface have been analyzed by Abdelkhalek [2].

The study of non-Newtonian fluid flows has gained much attention from the researchers because of their applications in biology, physiology, technology and industry. In addition, the effects of heat and mass transfer in non-Newtonian fluids also have great importance in engineering applications; for instance, the thermal design of industrial equipment dealing with molten plastics, polymeric liquids, foodstuffs, or slurries. Several investigators have extended many of the available convection heat and mass transfer problems to include the non-Newtonian effects. The fluid model introduced by Eringen [3] exhibits some microscopic effects arising from the local structure and micro motion of the fluid elements. Further, they can sustain couple stresses and include classical Newtonian fluid as a special case. The model of micropolar fluid represents fluids consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium where the deformation of the particles is ignored. Micropolar fluids have been shown to accurately simulate the flow characteristics of polymeric additives, geomorphological sediments, colloidal suspensions, haematological suspensions, liquid crystals, lubricants, etc. The mathematical theory of equations of micropolar fluids and applications of these fluids in the theory of lubrication and in the theory of porous media are presented by Lukaszewicz [4]. The heat and mass transfer in micropolar fluids is also important in the context of chemical engineering, aerospace engineering and also industrial manufacturing processes.

The problem of free convection heat and mass transfer in the boundary layer flow along a vertical surface submerged in a micropolar fluid has been studied by a number of investigators. Ahmadi [5] analyzed the boundary layer flow of a micropolar fluid over a semi-infinite plate. Takhar and Soundelgekar [6] considered the heat transfer on a semi-infinite plate of micropolar fluid. Agarwal and Dhanapal [7] examined the flow and heat transfer in a micropolar fluid past a flat plate with suction and heat sources. Gorla *et al.* [8] studied the non-similar problem of natural convection boundary layer flow of a micropolar fluid over a vertical plate with uniform heat flux boundary condition. El-Hakien *et al.* [9] discussed the effects of the viscous and Joule heating on MHD-free convection flows with variable plate temperatures in a micropolar fluid. El-Amin [10] considered MHD free-convection and mass transfer flow in a micropolar fluid over a stationary vertical plate with a constant suction. Sibanda and Awad [11] studied the flow of a micropolar fluid in channel with heat and mass transfer. Recently, Srinivasacharya and Ramreddy [12] analyzed the flow, heat and mass transfer characteristics of the free convection on a vertical plate with uniform and constant heat and mass fluxes in a doubly stratified micropolar fluid.

Radiation effects on convective heat transfer and MHD flow problems have assumed an increasing importance in electrical power generation, astrophysical flows, solar power technology, space vehicle re-entry and other industrial areas. Since the solution for convection and radiation equation is quite complicated, there are few studies about simultaneous effects of convection and radiation for internal flows. Ghaly [13] discussed the effect of radiation on heat and mass transfer over a stretching sheet in the presence of a magnetic field. Raptis *et al.* [14] studied the effect of radiation on two-dimensional steady MHD optically thin gray gas flow along an infinite vertical plate taking into account the induced magnetic field. Seddeek *et al.* [15] obtained an analytical solution for the effect of radiation on flow of a magneto- micropolar fluid past a continuously moving plate with suction and blowing. Mohamed *et al.* [16] considered the thermal radiation and MHD effects on free convective flow of a polar fluid through a porous medium in the presence of internal heat generation and chemical reaction.

Chemical reaction effects on heat and mass transfer are of considerable importance in hydrometallurgical industries and chemical technology. Research on combined heat and mass transfer with chemical reaction and thermophoresis effect can help to design for chemical processing equipment, chemically-reactive vapor deposition boundary layers in optical materials processing, catalytic combustion boundary layers, chemical diffusion in disk electrode modeling and carbon monoxide reactions in metallurgical mass transfer and kinetics. Several investigators have examined the effect of chemical reaction on the flow, heat and mass transfer past a vertical plate. Deka *et al.* [17] examined the effect of homogeneous first-order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Chamkha [18] analyzed the MHD flow of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. Magyari and Chamkha [19] obtained analytical solution to the steady combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface. Das [20] analyzed the effect of first order chemical reaction and thermal radiation on hydromagnetic free convection heat and mass transfer flow of a micropolar fluid *via* a porous medium bounded by a semi-infinite porous plate with a constant heat source in a rotating frame of reference. Bakr [21] considered the steady and unsteady MHD micropolar flow and mass transfers flow with a constant heat source in a rotating frame of reference in the presence of chemical reaction of the first-order, taking an oscillatory plate velocity and a constant suction velocity at the plate. Recently, Chaudhary and Jha [22] considered the effects of chemical reactions on MHD micropolar fluid flow past a vertical plate in slip-flow regime.

From literature survey, it seems that the similarity solutions for the effects of transverse magnetic field, radiation and first order chemical reaction on the free convection heat and mass transfer along a vertical plate embedded in a micropolar fluid have not been reported to date. In view of this, the authors, in the present investigations, aim to study the free convection on a vertical plate with variable heat and mass fluxes embedded in a stable micropolar fluid in the presence of magnetic, radiation and first order chemical reaction effects. The novelty of this paper is the use of similarity transformations to find the solution of the problem. Most of the similar studies reported in the literature used local similarity transformations to solve the problems. The governing system of partial differential equations is transformed into a system of non-linear ordinary differential equations using similarity transformations. This system of nonlinear ordinary differential equations is solved numerically using Keller-box method given by Cebeci and Bradshaw [23]. The effects of various parameters on the velocity, microrotation, temperature, concentration, skin friction coefficient, wall couple stress, heat and mass transfer rates are presented graphically.

MATHEMATICAL FORMULATION

Consider a steady, laminar, incompressible, two-dimensional free convective heat and mass transfer along a semi-infinite vertical plate embedded in a free stream of electrically conducting micropolar fluid with temperature T_∞ and concentration C_∞. Choose the coordinate system such that x-axis is along the vertical plate and y-axis normal to the plate. The physical model and coordinate system are shown in Figure 1. The plate is taken with variable surface heat flux $q_w(x)$ and mass fluxes $q_m(x)$. A uniform magnetic field of magnitude B_0 is applied normal to the plate. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in comparison with the applied magnetic field. The fluid is considered to be a gray, absorbing emitting radiation but non-scattering medium and the Rosseland approximation [24] is used to describe the radiative heat flux in the energy equation. The radiative heat flux in the x-direction is considered negligible in comparison to the y-direction. Also, it is assumed that there exists a homogenous chemical reaction of first-order with rate constant R^* between the diffusing species and the fluid.

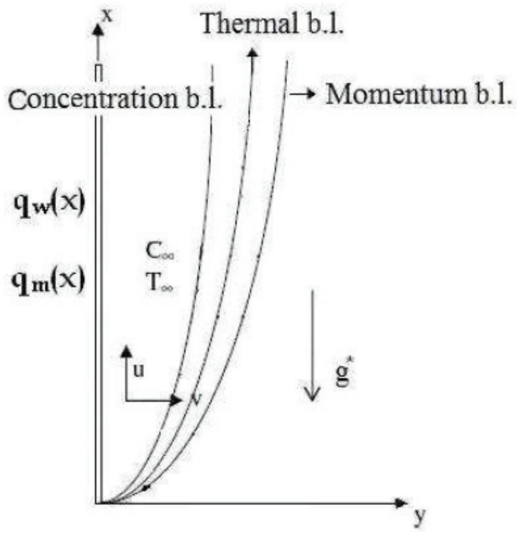


Figure 1. Physical model and coordinate system.

Using the boussinesq and boundary layer approximations, the governing equations for the MHD micropolar fluid are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=(\mu+\kappa)\frac{\partial^2 u}{\partial y^2}+\kappa\frac{\partial \omega}{\partial y}+ +\rho g^*\left(\beta_T(T-T_\infty)+\beta_C(C-C_\infty)\right)-\sigma B_0^2 u \tag{2}$$

$$\rho j\left(u\frac{\partial \omega}{\partial x}+v\frac{\partial \omega}{\partial y}\right)=\gamma\frac{\partial^2 \omega}{\partial y^2}-\kappa\left(2\omega+\frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-R^*(C-C_\infty) \quad (5)$$

where u and v are the components of velocity along x and y directions respectively, ω is the component of microrotation whose direction of rotation lies normal to the xy-plane, g^* is the gravitational acceleration, T is the temperature, C is the concentration, β_T is the coefficient of thermal expansion, β_C is the coefficient of solutal expansion, C_p is the specific heat capacity, B_0 is the coefficient of the magnetic field, μ is the dynamic coefficient of viscosity of the fluid, q_r is the radiative heat flux, ρ is the density, κ is the vortex viscosity, j is the micro-inertia density, γ is the spin-gradient viscosity, σ is the magnetic permeability of the fluid, α is the thermal diffusivity and D is the molecular diffusivity.

The boundary conditions are:

$$u=0\,,\ v=0\,,\ \omega=0\,,\ -k\frac{\partial T}{\partial y}=q_w(x)\,,$$

$$-D\frac{\partial C}{\partial y}=q_w(x)\ \text{at}\ y=0 \quad (6a)$$

$$u\to 0,\ v\to 0,\ \omega\to 0,\ T\to T_\infty,\ C\to C_\infty\ \text{as}\ y\to\infty \quad (6b)$$

where the subscripts w and ∞ indicates the conditions at wall and at the outer edge of the boundary layer, respectively.

The radiative heat flux q_r is described by the Rosseland approximation such that:

$$q_r=-\frac{4\sigma^*}{3k_1}\frac{\partial T^4}{\partial y} \quad (7)$$

where σ^* and k_1 are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. We assume that the differences of the temperature within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding in a Taylor series about T_∞ and neglecting higher-order terms, thus:

$$T^4\cong 4T_\infty^3 T-3T_\infty^4 \quad (8)$$

Using Eqs. (7) and (8) in the last term of Eq. (4), we obtain:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^* T_\infty^3}{3\rho C_p k_1}\frac{\partial^2 T}{\partial y^2} \quad (9)$$

The continuity equation (1) is satisfied by introducing the stream function ψ such that:

$$u=\frac{\partial\psi}{\partial y}\,,\ v=-\frac{\partial\psi}{\partial x} \quad (10)$$

In order to explore the possibility for the existence of similarity, we assume:

$$\psi=A\,x^a f(\eta),\ \eta=B\,x^b\,y,\ \omega=E\,x^c\,g(\eta)$$

$$T=T_\infty+\frac{q_w(x)}{k}\theta(\eta),\ \ \frac{q_w(x)}{k}=M_1Bx^l,$$

$$C=C_\infty+\frac{q_m(x)}{D}\varphi(\eta),\ \ \frac{q_m(x)}{D}=N_1Bx^m \quad (11)$$

where A, B, E, M_1, N_1, a, b, c, l, and m are constants. Substituting (10) and (11) in (2), (3), (4) and (9), it is found that similarity exists only *if* $a = 1$, $b = 0$, $c = 1$, $l = m = 1$. Hence, appropriate similarity transformations are:

$$\psi=A\,x\,f(\eta),\ \eta=B\,y,\ \omega=E\,xg(\eta),$$

$$T=T_\infty+\frac{q_w(x)}{k}\theta(\eta),\ \ \frac{q_w(x)}{k}=M_1Bx,$$

$$C=C_\infty+\frac{q_m(x)}{D}\varphi(\eta),\ \ \frac{q_m(x)}{D}=N_1Bx \quad (12)$$

Making use of the dimensional analysis, the constants A, B, E, M_1 and N_1 have, respectively, the dimensions of velocity, reciprocal of length, the reciprocal of the product of length and time, the ratio of (temperature/length) and of the ratio (concentration/length).

Substituting (12) into the Eqs.(2), (3), (5) and (9), we obtain:

$$\left(\frac{1}{1-N}\right)f'''+f\,f''+\left(\frac{N}{1-N}\right)g'-\left(f'\right)^2+$$
$$+\theta+L\phi-M\,f'=0 \quad (13)$$

$$\lambda\,g''-\left(\frac{N}{1-N}\right)\vartheta\,\left(2g+f'''\right)-f'g+fg'=0 \quad (14)$$

$$\frac{1}{3\Pr}(3+4R)\theta''+f\,\theta'-f'\theta=0 \quad (15)$$

$$\frac{1}{Sc}\varphi''-\delta\varphi'+f\,\varphi'-f'\varphi=0 \quad (16)$$

where $\Pr=\nu/\alpha$ is the Prandtl number, $Sc=\nu/D$ is the Schmidt number, $\lambda=\gamma/(j\,\rho\nu)$ is the spin-gradient viscosity, $N=\kappa/(\kappa+\mu)$, $(0 \le N < 1)$ is the Coupling number, $L=\beta_c N_1/(\beta_T M_1)$ is the buoyancy parameter, $M=\sigma B_0^2/(\mu B^2)$ is the magnetic field parameter, $\vartheta=1/(jB^2)$ is the micro-inertia density, $R=4\sigma^* T_\infty^3/(k\,k_1)$ is the radiation parameter and $\delta=R^*/(\nu B^2)$ is the chemical reaction parameter. The primes denote differentiation with respect to the similarity variable η.

The boundary conditions (6) in terms of f, g, θ and ϕ become:

$$f(0)=0,\ f'(0)=0,\ g(0)=0,\quad \theta'(0)=-1,\ \phi'(0)=-1 \tag{17a}$$

$$f'(\infty)\to 0,\quad g(\infty)\to 0,\quad \theta(\infty)\to 0,\ \varphi(\infty)\to 0 \tag{17b}$$

Skin friction and wall couple stress

The wall shear stress and wall couple stress:

$$\tau_w=\left[(\mu+\kappa)\frac{\partial u}{\partial y}+\kappa\omega\right]_{y=0},\quad m_w=\gamma\left[\frac{\partial \omega}{\partial y}\right]_{y=0} \tag{18}$$

The dimensionless wall shear stress:

$$C_f=2\tau_w/(\rho A^2)$$

wall couple stress:

$$M_w=Bm_w/(\rho A^2)$$

where A is the characteristic velocity, are given by:

$$C_f=\left(\frac{2}{1-N}\right)f''(0)\bar{x}\ \text{ and }\ M_w=\left(\frac{\lambda}{\vartheta}\right)g'(0)\bar{x} \tag{19}$$

where $\bar{x}$ = Bx.

Heat and mass transfer rates

The heat and mass transfers from the plate respectively are given by:

$$q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}-\frac{4\sigma^*}{3k_1}\left(\frac{\partial T^4}{\partial y}\right)_{y=0}$$

and

$$q_m=-D\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{20}$$

The non-dimensional rate of heat transfer, called the Nusselt number $Nu=q_w/(Bk(T_w-T_\infty))$ and rate of mass transfer, called the Sherwood number $Sh_x=q_m/BD(C_w-C_\infty)$ are given by:

$$Nu=\frac{1}{\theta(0)}\left(1+\frac{4R}{3}\right)\ \text{ and }\ Sh=\frac{1}{\varphi(0)} \tag{21}$$

RESULTS AND DISCUSSION

The flow Eqs. (13) and (14) which are coupled, together with the energy and concentration Eqs. (15) and (16), constitute non-linear nonhomogeneous differential equations for which closed-form solutions cannot be obtained. Hence, the governing Eqs. (13) to (16) have been solved numerically using the Keller-box implicit method [23]. This method has a second order accuracy, unconditionally stable, and is easy to be programmed, thus making it highly attractive for production use. A uniform grid was adopted, which is concentrated towards the wall. The calculations are repeated until some convergent criterion is satisfied and the calculations are stopped when $\delta f_0''\le 10^{-8}$, $\delta\theta_0'\le 10^{-8}$ and $\delta\varphi_0'\le 10^{-8}$. In the present study, the boundary conditions for η at ∞ are replaced by a sufficiently large value of η where the velocity, temperature and concentration approach zero. In order to see the effects of step size ($\Delta\eta$) we ran the code for our model with three different step sizes as $\Delta\eta$ = 0.001, $\Delta\eta$ = 0.01 and $\Delta\eta$ = 0.05 and in each case we found very good agreement between them on different profiles. After some trials, we imposed a maximal value of η at ∞ as 6 and a grid size of $\Delta\eta$ as 0.01. In order to study the effects of coupling number N, magnetic field parameter M, thermal radiation parameter R, chemical reaction parameter δ, Prandtl number Pr and Schmidt number Sc on the physical quantities of the flow, the remaining parameters are fixed as L = 1, λ = 1 and ϑ = 0.1. The values of micropolar parameters λ and $\mathcal{J}$ are chosen so as to satisfy the thermodynamic restrictions on the material parameters given by Eringen [3].

In the absence of coupling number N, magnetic parameter M, radiation parameter R and chemical reaction parameter δ and buoyancy number L with ϑ = 0, $\lambda\to 0$, Pr = 1.0 and Sc = 0.24 the results have been compared with the exact values [25] and it was found that they are in good agreement, as shown in Table 1.

Table 1. Comparison between skin friction $f''(0)$ and temperature $\theta(0)$ calculated by the present method and that of Merkin [25] for N = M = R = δ = L = ϑ = λ = 0 and Pr = 1.0

$f''(0)$		$\theta(0)$	
Merkin [25] with Pr = 1	Present	Merkin [25] with Pr = 1	Present
1.0097	1.0097	1.5148	1.5147

The variation of the non-dimensional velocity, microrotation, temperature and concentration profiles with η for different values of magnetic parameter is illustrated in Figure 2. It is observed from Figure 2a that velocity decreases as the magnetic parameter (M) increases. This is due to the fact that the introduction of a transverse magnetic field, normal to the flow direction, has a tendency to create the drag known as the Lorentz force, which tends to resist the flow. Hence, the horizontal velocity profiles decrease as the magnetic parameter M increases. From Figure 2b, it is clear that the microrotation component increases near the plate and deceases far away from

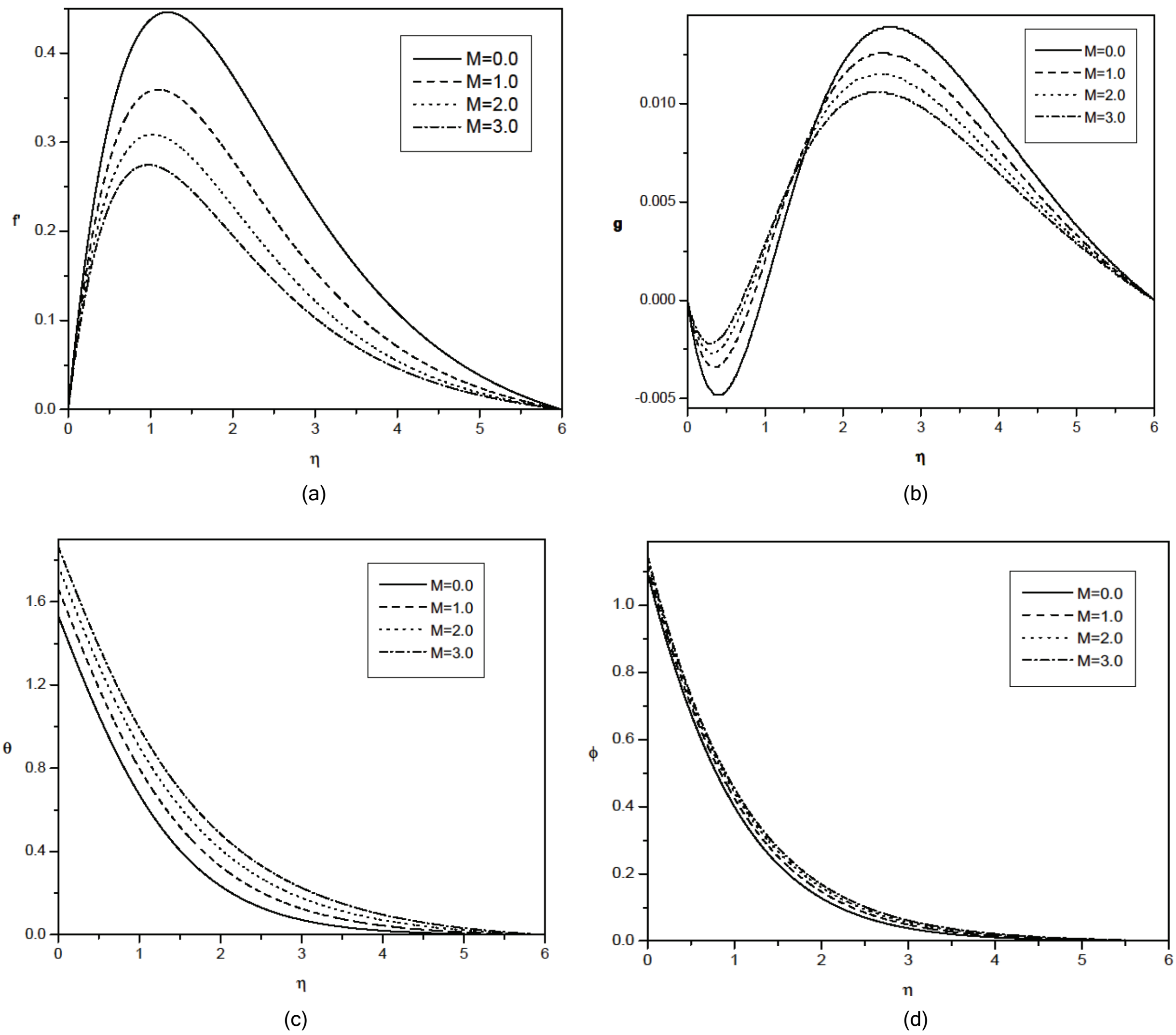


Figure 2. a) Velocity, b) microrotation, c) temperature and d) concentration profiles for various values of magnetic parameter M.

the plate for increasing values of M. It is noticed from Figure 2c that the temperature increases with increasing values of magnetic parameter. It is clear from Figure 2d that the non-dimensional concentration increases with increasing values of M. As explained above, the transverse magnetic field gives rise to a resistive force known as the Lorentz force of an electrically conducting fluid. This force makes the fluid experience a resistance by increasing the friction between its layers and thus increases its temperature and concentration.

Figure 3 depicts the variation of coupling number (N) on the velocity, microrotation, temperature and concentration. The coupling number N characterizes the coupling of linear and rotational motion arising from the micromotion of the fluid molecules. Hence, N signifies the coupling between the Newtonian and rotational viscosities. As $N \rightarrow 1$, the effect of microstructure becomes significant, whereas with a small value of N the individuality of the substructure is much less pronounced. As $\kappa \rightarrow 0$, *i.e.*, $N \rightarrow 0$, the micropolarity is lost and the fluid behaves as nonpolar fluid. Hence, $N \rightarrow 0$ corresponds to viscous fluid. It is observed from Figure 3a that the velocity decreases with the increase of N. The maximum of velocity decreases in amplitude and the location of the maximum velocity moves farther away from the wall with an increase of N. The velocity in case of micropolar fluid is less than that in the viscous fluid case ($N \rightarrow 0$ corresponds to viscous fluid). It is seen from Figure 3b that the microrotation component decreases near the vertical plate and increases far away from the plate with increasing coupling number N. The microrotation tends to zero as $N \rightarrow 0$ as is expected that in the limit $\kappa \rightarrow 0$, *i.e.*, $N \rightarrow 0$ the Eqs. (1) and (2) are uncoupled with Eq. (3) and they reduce to viscous fluid flow equations. It is noticed from Figure 3c that the temperature increases with increasing values of

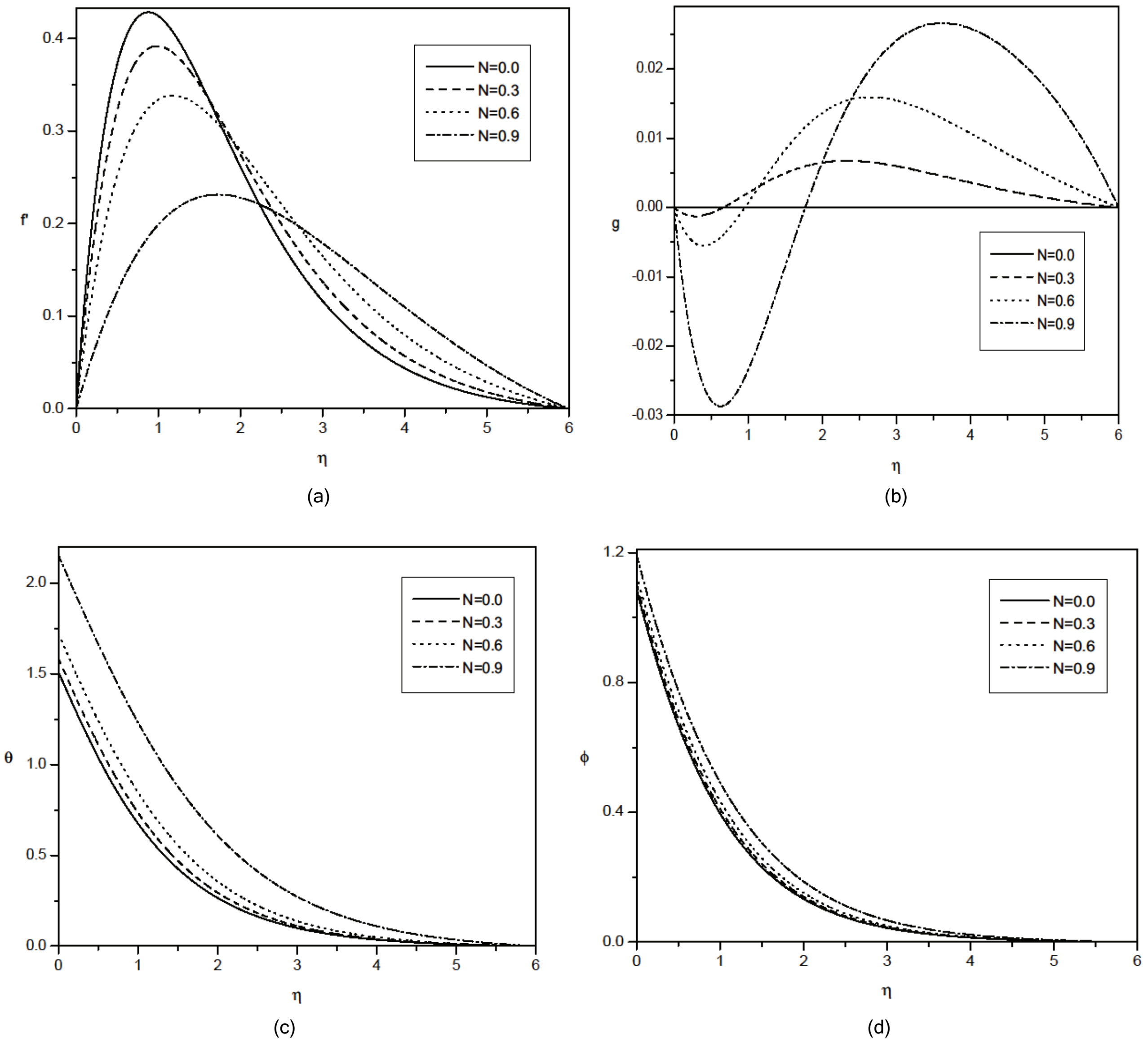


Figure 3. a) Velocity, b) microrotation, c) temperature and d) concentration profiles for various values of coupling number N.

coupling number. It is clear from Figure 3d that the non-dimensional concentration increases with increasing values of *N*.

The effect of radiation parameter on the velocity, microrotation, temperature and concentration is shown in Figure 4. It is observed from Figure 4a that the velocity increases with the increase of R. From Figure 4b, it is clear that the microrotation component increases with R. It is noticed from Figure 4c that the temperature increases with increasing values of radiation parameter. Higher values of radiation parameter *R* imply higher values of wall temperature. Consequently, the temperature gradient increases. It is clear from Figure 4d that the non-dimensional concentration decreases with increasing values of *R*.

The influence of chemical reaction parameter on the velocity, microrotation, temperature and concentration is shown in Figure 5. It is observed from Figure 5a that the velocity decreases with the increase of δ. From Figure 5b, it is clear that the microrotation component decreases with δ. It is noticed from Figure 5c that the temperature increases with increasing values of chemical reaction parameter. It is clear from Figure 5d that the non-dimensional concentration decreases with increasing values of δ. Increase in the chemical reaction parameter produces a decrease in the species concentration. This causes the concentration buoyancy effects to decrease as δ increases. Consequently less flow is induced along the plate, resulting in decrease in the fluid velocity in the boundary layer. In addition the concentration boundary layer

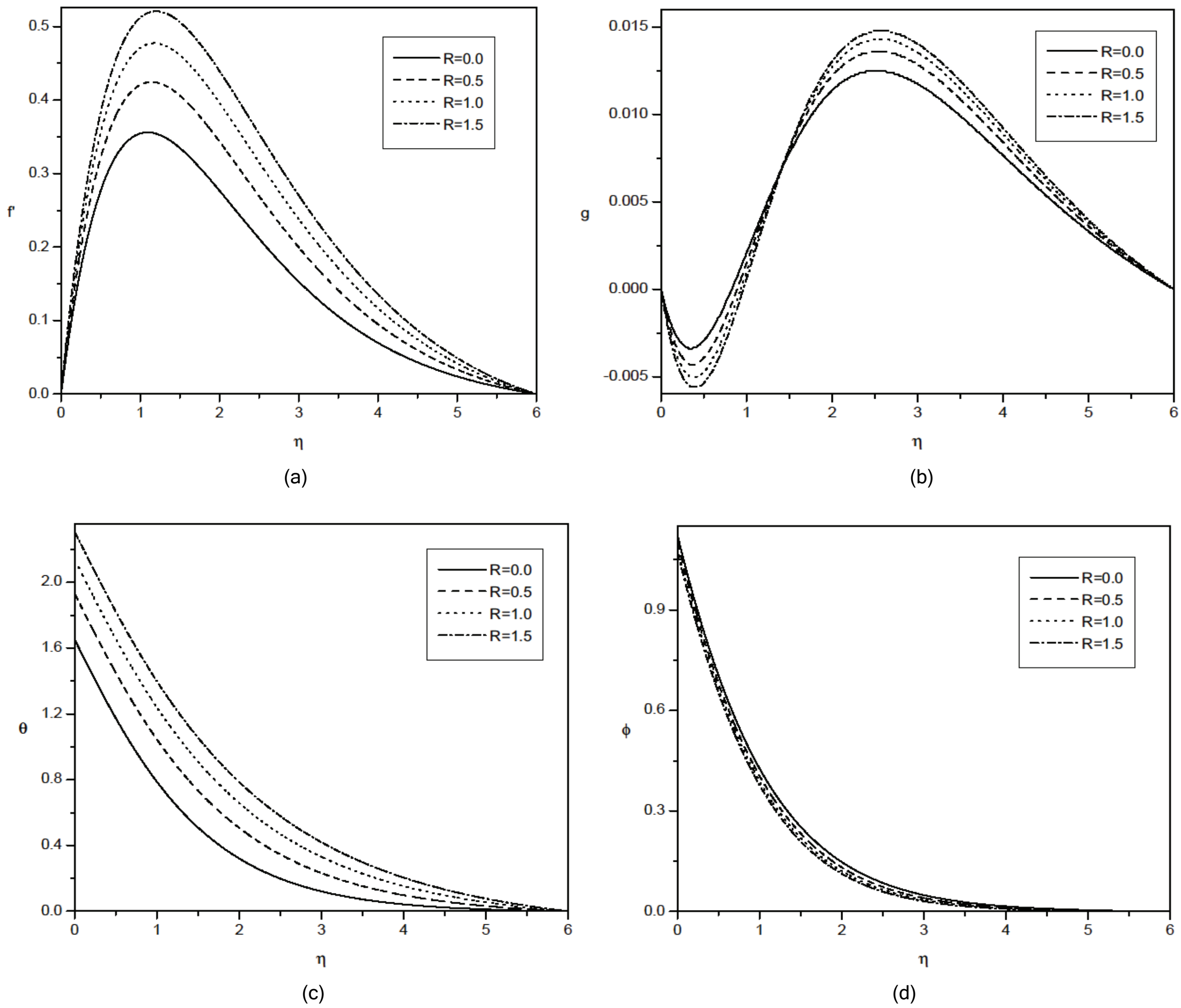


Figure 4. a) Velocity, b) microrotation, c) temperature and d) concentration profiles for various values of radiation parameter R.

thickness decreases as δ increases. On the other hand, increasing the chemical reaction parameter produces an increase in temperature.

Figure 6 depicts the variation of heat and mass transfer rates (Nusselt number *Nu* and Sherwood number *Sh*) with coupling number *N* for different values of magnetic parameter *M*. It is observed from Figures 6a and 6b that both the Nusselt number and Sherwood number decrease as coupling number increases. It is noticed that the heat and mass transfer rates are more in case of viscous fluids. Therefore, the presence of microscopic effects arising from the local structure and micromotion of the fluid elements reduce the heat and mass transfer rates. Further, it is seen that both the Nusselt number and Sherwood number are decreasing as the magnetic parameter is increasing. This is due to the Lorentz force created by traverse magnetic field, which tends to resist the flow.

The effect of radiation parameter on heat and mass transfer coefficient is displayed in Figures 7a and 7b. It is noticed from these figures that both the Nusselt number and Sherwood number increase with the increase in the radiation parameter. Higher values of radiation parameter *R* imply higher values of wall temperature. Consequently, the temperature gradient and hence the Nusselt number and Sherwood number increase.

The variation of heat and mass transfer coefficients with coupling number for different values of chemical reaction parameter δ is depicted in Figures 8a and 8b. It is clear from these figures that an increase in the chemical reaction parameter δ, leads to a decrease in the Nusselt number and an increase in the Sherwood number. Increase in the values of δ implies more interaction of species concentration with the momentum boundary layer and less interaction

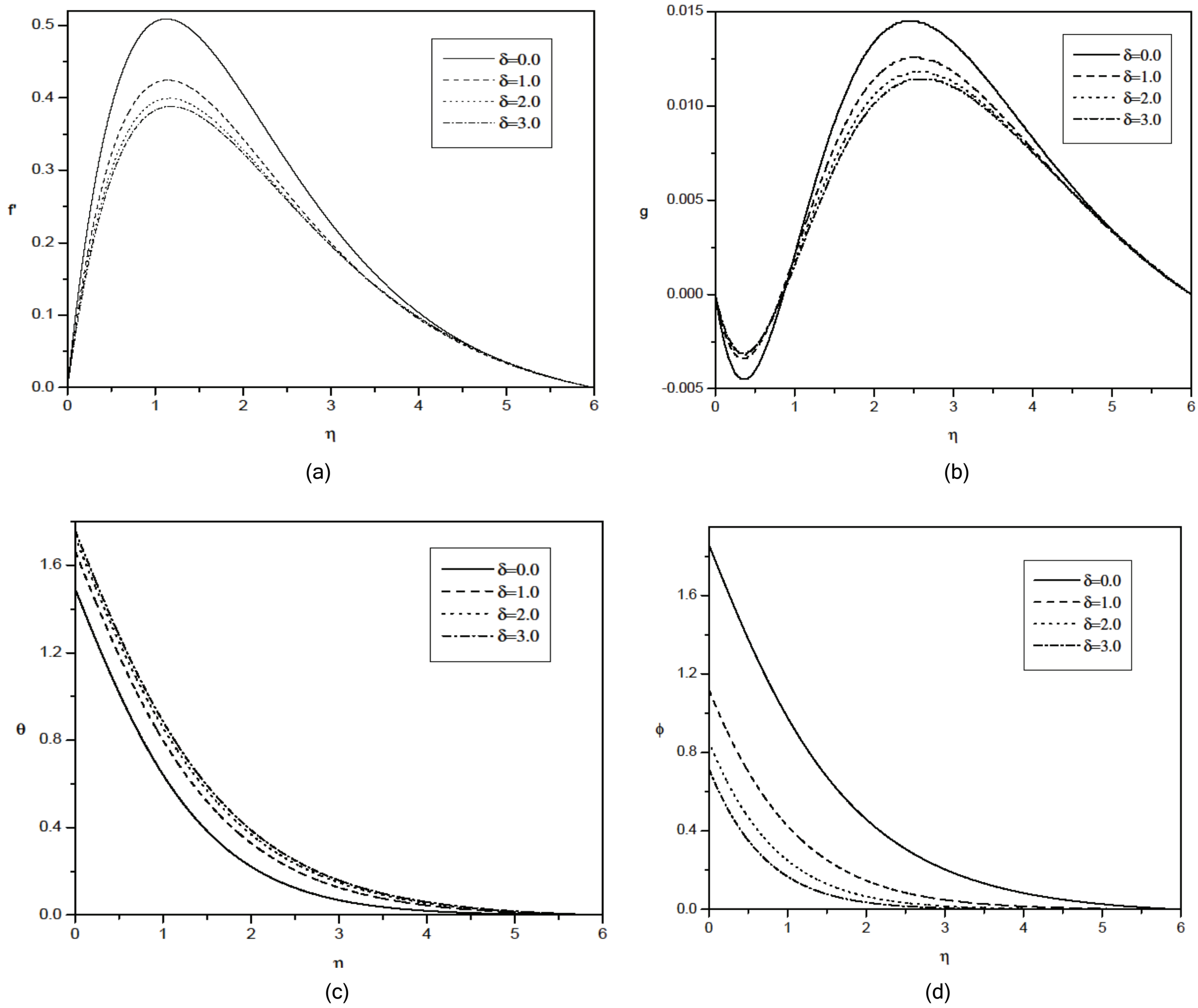


Figure 5. a) Velocity, b) microrotation, c) temperature and d) concentration profiles for various values of chemical reaction parameter δ.

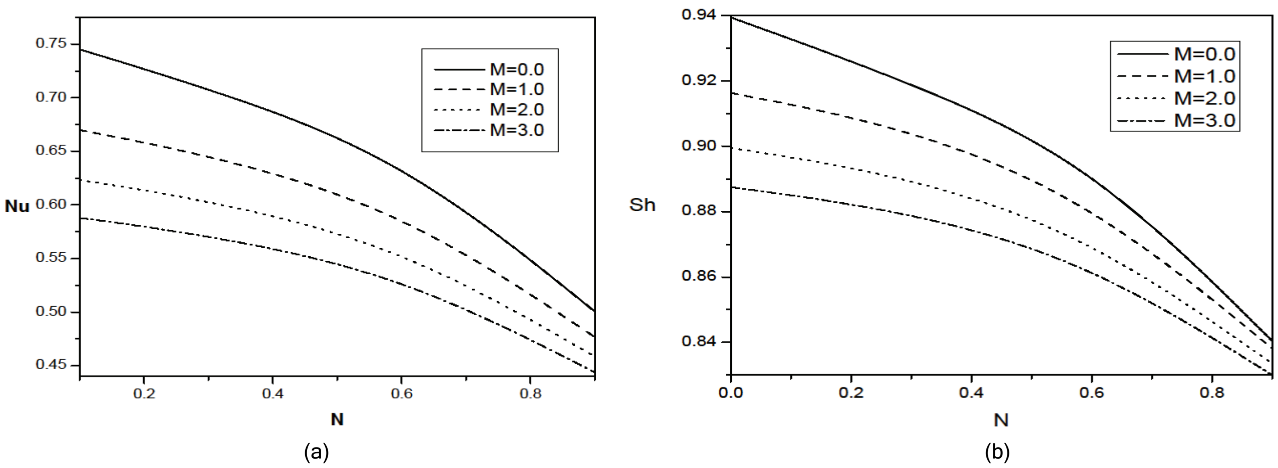
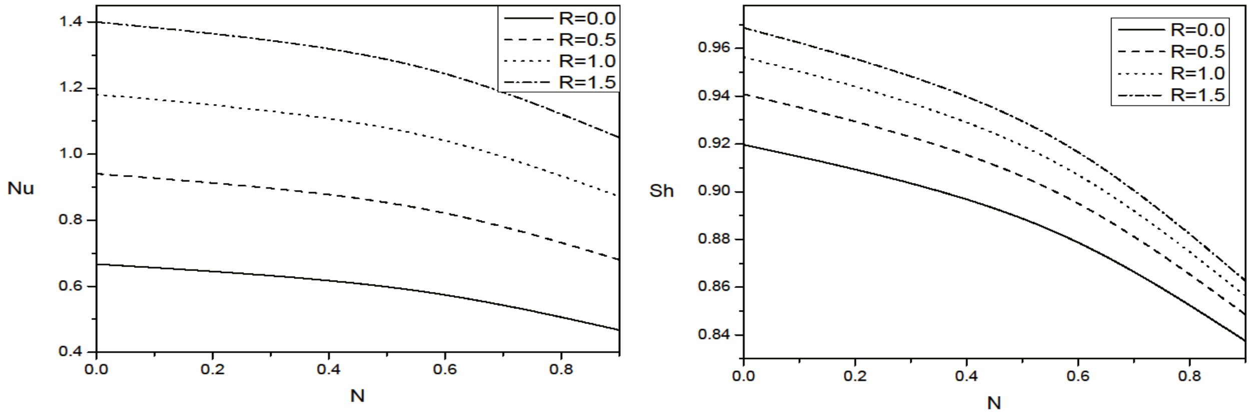


Figure 6. Effect of magnetic parameter M on a) heat transfer rate b) mass transfer rate.

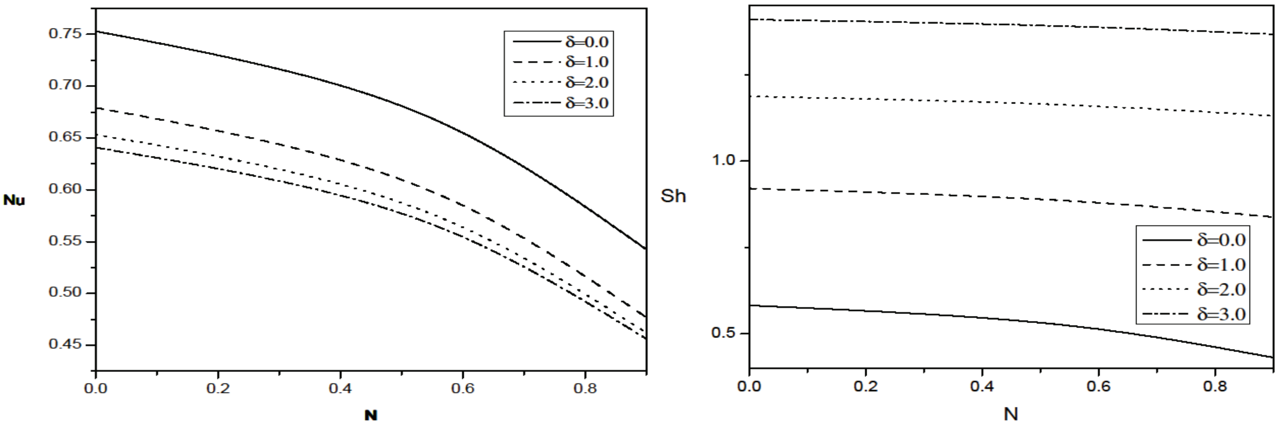
with the thermal boundary layer. Hence, chemical reaction parameter has more significant effect on the Sherwood number than it does on the Nusselt number.

Table 2 shows the effects of the coupling number *N*, Prandtl number *Pr*, Schmidt number *Sc*, radiation parameter *R*, chemical reaction parameter δ and the magnetic parameter M on the skin friction C_f



(a) (b)

Figure 7. Effect of radiation parameter R on a) heat transfer rate b) mass transfer rate.



(a) (b)

Figure 8. Effect of chemical reaction parameter δ on a) heat transfer rate b) mass transfer rate.

Table 2. Effect of skin friction and wall couple stress for various values of N, Pr, Sc, M, R and δ

N	Pr	Sc	M	R	δ	$f''(0)$	$-g'(0)$
0.0	1.0	0.6	1.0	0.02	1.0	1.2691	0.0000
0.3	1.0	0.6	1.0	0.02	1.0	1.0332	0.0097
0.6	1.0	0.6	1.0	0.02	1.0	0.7440	0.0318
0.9	1.0	0.6	1.0	0.02	1.0	0.3112	0.1108
0.5	0.01	0.6	1.0	0.02	1.0	2.3371	0.0737
0.5	0.1	0.6	1.0	0.02	1.0	1.5493	0.0480
0.5	1.0	0.6	1.0	0.02	1.0	0.8490	0.0219
0.5	10.0	0.6	1.0	0.02	1.0	0.5191	0.0120
0.5	100	0.6	1.0	0.02	1.0	0.4077	0.0104
0.5	1.0	0.2	1.0	0.02	1.0	1.0697	0.0296
0.5	1.0	0.4	1.0	0.02	1.0	0.9149	0.0240
0.5	1.0	0.6	1.0	0.02	1.0	0.8490	0.0219
0.5	1.0	0.8	1.0	0.02	1.0	0.8110	0.0209
0.5	1.0	1.0	1.0	0.02	1.0	0.7857	0.0203
0.5	1.0	0.6	0.0	0.02	1.0	0.9339	0.0274
0.5	1.0	0.6	1.0	0.02	1.0	0.8490	0.0219
0.5	1.0	0.6	2.0	0.02	1.0	0.7951	0.0187
0.5	1.0	0.6	3.0	0.02	1.0	0.7560	0.0166

Table 2. Continued

N	Pr	Sc	M	R	δ	$f''(0)$	$-g'(0)$
0.5	1.0	0.6	1.0	0.0	1.0	0.8433	0.0217
0.5	1.0	0.6	1.0	0.5	1.0	0.9626	0.0262
0.5	1.0	0.6	1.0	1.0	1.0	1.0522	0.0296
0.5	1.0	0.6	1.0	1.5	1.0	1.1251	0.0324
0.5	1.0	0.6	1.0	0.02	0.0	1.0639	0.0283
0.5	1.0	0.6	1.0	0.02	1.0	0.8490	0.0219
0.5	1.0	0.6	1.0	0.02	2.0	0.7801	0.0203
0.5	1.0	0.6	1.0	0.02	3.0	0.7459	0.0196

and the dimensionless wall couple stress M_w. It is seen from this table that both the skin friction and the wall couple stress decrease with increasing coupling number N. For increasing value of N, the effect of microstructure becomes significant, hence the wall couple stress decreases. The skin friction coefficient decreases and the wall couple stress increases with increasing Prandtl number and Schmidt number. Also, the effect of magnetic parameter is to decrease the skin friction coefficient and increase the wall couple stress. Further, it is observed that the skin friction coefficient is increasing and wall couple stress is decreasing with increasing value of radiation parameter. The increase in chemical reaction parameter decreases the skin friction coefficient and increases the wall couple stress.

CONCLUSIONS

In this paper, a boundary layer analysis for free convection heat and mass transfer in an electrically conducting micropolar fluid over a vertical plate with variable surface heat and mass fluxes conditions in the presence of a first order chemical reaction and radiation is considered. A uniform magnetic field is applied normal to the plate. Earlier, the problems of free convection heat and mass transfer in micropolar fluid were solved using the local similarity transformations. In the present study, the similarity transformations are derived and used to study the effects of magnetic field, radiation and chemical reaction. Using the similarity variables, the governing equations are transformed into a set of similar parabolic equations and numerical solution for these equations has been presented for different values of parameters. From the present study we observe that:

1. The introduction of a transverse magnetic field, normal to the flow direction, has a tendency to create the drag known as the Lorentz force, which tends to resist the flow. Hence, the horizontal velocity profiles, both the Nusselt number and Sherwood number and the skin friction coefficient decrease, microrotation component increases near the plate and decreases far away from the plate, the temperature as well as the concentration and the wall couple stress increase as magnetic parameter M increases.

2. The higher values of the coupling number N (*i.e.*, the effect of microrotation becomes significant) result in lower velocity distribution but higher wall temperature; wall concentration distributions in the boundary layer compared to the Newtonian fluid case.

3. A higher value of radiation parameter R implies higher values of wall temperature. Consequently, the temperature gradient increases, hence, the velocity, microrotation, temperature, Nusselt number and Sherwood number increase and the concentration decreases with the increase in the radiation parameter.

4. Increase in the chemical reaction parameter produces a decrease in the species concentration. This causes the concentration buoyancy effects to decrease as δ increases. Consequently less flow is induced along the plate, resulting in decrease in the fluid velocity, the microrotation component, the Nusselt number and the concentration boundary layer thickness in the boundary layer. On the other hand, increasing the chemical reaction parameter produces an increase in temperature and the Sherwood number.

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Nomenclature

A	Characteristic velocity
B	Reciprocal of length
B_0	Magnetic field coefficient
C	Concentration
c_p	Specific heat at constant pressure
C_∞	Free stream concentration
C_f	Skin-friction coefficient

D	Mass diffusivity
E	Reciprocal of the product of length an time
f	Dimensionless stream function
g	Dimensionless microrotation
g^*	Acceleration due to gravity
j	Dimensional Micro-inertia density
$\mathcal{J}$	Dimensionless Micro-inertia density
k	Thermal conductivity
k_1	Mean absorption coefficient
L	Buoyancy parameter
m_w	Wall couple stress
M	Magnetic parameter
M_1	The ratio (temperature/length)
M_w	Non-dimensional couple stress on the wall
N	Coupling parameter
N_1	Ratio (concentration/length)
N_u	Dimensionless Nusselt number
Pr	Prandtl number
q_r	Radiative heat flux
R	Thermal Radiation parameter
R^*	Rate of chemical reaction
$q_w(x)$	Heat flux
$q_m(x)$	Mass flux
Sc	Schmidt number
Sh	Sherwood number
T	Temperature
T_1	Free stream temperature
u, v	Velocity components in the x- and y- directions respectively
x, y	Cartesian coordinates along the plate and normal to it

Greek symbols

α	Thermal diffusivity
β_T	Coefficient of thermal expansion
β_C	Coefficient of concentration expansion
γ	Spin-gradient viscosity
δ	Chemical reaction parameter
η	Similarity variable
θ	Dimensionless temperature
κ	Thermal conductivity of the fluid
λ	Spin-gradient viscosity
μ	Viscosity of the fluid
ν	Kinematic viscosity
ρ	Density of the fluid
σ	Electrical conductivity
σ^*	Stefan-Boltzmann constant
τ_w	Wall shear stress
ϕ	Dimensionless concentration
Ψ	Stream function
ω	Microrotation component

Subscripts

w	Condition at wall
∞	Condition at infinity

Superscripts

′	Differentiation with respect to η

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NAUČNI RAD

PRIRODNA KONVEKCIJA U MHD MIKROPOLARNOM FLUIDU SA UTICAJEM RADIJACIJE I HEMIJSKE REAKCIJE

U radu su razmotreni uticaji zračenja i hemijske reakcije prvog reda na prenos toplote i mase prirodnom konvekcijom u mikropolarnom fluidu. Uniformno magnetno polje je normalno na ploču. Primenjeni su promenljivi fluksevi toplote i mase na površinu ploče. Bilansne nelinearne parcijalne diferencijalne jednačine su transformisane u sistem spregnutih nelinearnih običnih diferencijalnih jednačina primenom sličnosti transformacija, koji zatim rešen numerički Keller-box metodom. Poređenjem numeričkih rezultata utvrđeno je da se oni dobro slažu sa ranije objavljenim rezultatima specijalnih slučajeva sprovedenog istraživanja. Bezdimenzine veličine: brzina, mikrorotacija, temperature, koncentracija i brzine prenosa toplote i mase su prikazane grafički za različite vrednosti broja kuplovanja, kao magnetnog, radijacionog i hemijskog parametra. Numeričke vrednosti faktora trenja i naponskog sprega zida za različite vrednosti parametara su prikazane tabelarno.

Ključne reči: prirodna konvekcija, mikropolarni fluid, hemijska reakcija, radijacija, fluks toplote i mase.