Applied Mathematics and Computation 268 (2015) 991–1000

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A spectral relaxation method for linear and non-linear stratification effects on mixed convection in a porous medium

Ch. RamReddy [a], P.A. Lakshmi Narayana [b,*], S.S. Motsa [c]

[a] Department of Mathematics, National Institute of Technology Warangal 506004, Telangana State, India
[b] Department of Mathematics, Indian Institute of Technology Hyderabad 502205, Telangana State, India
[c] School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Pietermaritzburg, South Africa

ARTICLE INFO

Keywords:
Mixed convection
Porous medium
Thermo-solutal convection
Double stratification
Spectral relaxation method

ABSTRACT

In this composition, we use a new spectral relaxation method (SRM) to investigate the effects of linear and non-linear stratification on mixed convective transport along a vertical surface embedded in a porous medium and it is viewed for the first time in both aiding and opposing buoyancy cases. The governing partial differential equations are transformed into ordinary differential equations using similarity transformation and then the resulting differential equations are solved numerically using SRM. A comparison is also made about the accuracy of SRM results in relation to the results obtained using the shooting method. We show that the proposed technique is an efficient numerical algorithm with assured convergence that serves as an alternative to common numerical methods for solving nonlinear boundary value problems. A parametric study of the physical parameters involved in the problem is conducted and a representative set of numerical results is illustrated, with accent on the comparison between linear and non-linear stratification. It is significant to notice that the separation of flow is found to be more in the absence of stratification whereas it is less in the presence of stratification. Finally, thermal and solutal stratifications significantly affect the heat and mass transfer rates, besides delay the boundary layer separation.

1. Introduction

The analysis of mixed convection boundary layer flow on a vertical surface embedded in porous media has received considerable theoretical and realistic contribution. The phenomenon of mixed convection occurs in many technical and industrial problems such as electronic devices cooled by fans, nuclear reactors cooled during an emergency shutdown, a heat exchanger placed in a low-velocity environment, solar collectors and thus along. Various authors have examined the problem of mixed convection about different surface geometries and various models were offered to explain mathematical and physical aspects related with the boundary layer flow and convective heat transport in porous media. Among these, the Darcy law gained a good deal of tending. Boundary layer assumptions were successfully employed to these theoretical accounts and much work has been performed on them for different body geometries in the last three decades. A detailed literature concerning these procedures can be establish in recent volumes by Vafai [1], Pop and Ingham [2] and Nield and Bejan [3]. In recent years, several investigators have studied the heat and mass transport problems. Aldoss et al. [4] have considered the magnetohydrodynamic mixed convection from a vertical plate embedded in a porous medium. Bourhan [5] analyzed the effects of magnetic and buoyancy on melting

* Corresponding author. Tel.: +91 4023017050.
E-mail address: ananth@iith.ac.in, puranam.ananth@gmail.com (P.A. Lakshmi Narayana).

http://dx.doi.org/10.1016/j.amc.2015.07.007

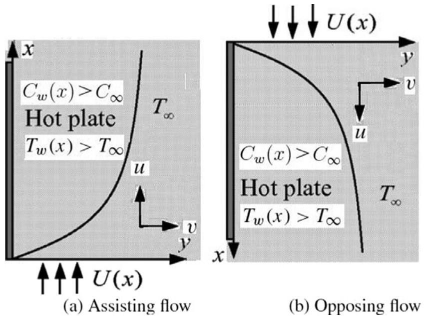

Fig. 1. Physical model and coordinate system.

from a vertical plate embedded in a saturated porous medium. The unsteady MHD combined convection over a moving vertical sheet in a fluid saturated porous medium with uniform surface heat flux was studied by El-Kabeira et al. [6].

Since the combined free and forced convection flows arise in a stratified environment in various industrial and technology problems. The input of thermal energy in enclosed fluid regions, due to the emission of hot fluid or heat removal from heated bodies, frequently contributes to the propagation of a stable thermal stratification. Stratification of fluid goes up due to temperature fluctuations, concentration differences or the bearing of dissimilar fluids. It is important to examine the temperature stratification and concentration differences of hydrogen and oxygen in lakes and ponds as they may directly affect the growth rate of all cultured species. Also, the analysis of thermal stratification is important for solar engineering because higher energy efficiency can be achieved with better stratification. It has been shown by scientists that thermal stratification in energy storage may considerably increase system performance. Prandtl [8] first shown that the outcome of the stratification on the mean field is the constitution of a neighborhood with a temperature deficit (i.e. a negative temperature) and flow reversal in the outer portion of the boundary layer of an infinite wall and later on by Jaluria and Himasekhar [9] for semi-infinite walls. Further, the comprehensive survey given by Gebhart et al. [7] have shown that stratification enhances the local heat transfer rate and reduces the velocity and buoyancy levels. In the recent literature, the researchers reported that the temperature and concentration became negative in the boundary layer depending on the relative intensity of the thermal and solutal stratifications (e.g., readers can see the articles given by Rathish Kumar and Shalini [10], Lakshmi Narayana and Murthy [11], Murthy et al. [12] and Srinivasacharya and RamReddy ([16,17])). The mixed convection boundary layer flow through a stable stratified porous medium bounded by a vertical surface is investigated by Ishak et al. [18].

In order to explore theoretical and experimental insight, Jaluria and Gebhart [13] analyzed the stability of the flow adjacent to a vertical plate dissipating a uniform heat flux into a stratified medium. In this study, authors have reported that a theoretical similarity solution exists when the ambient stratification varies like $x^{1/5}$, where x is the downstream coordinate. Unlike the case of linear stratification, the flow reversal and temperature deficit in this case [Gebhart [7]], where the variation of the ambient temperature is relatively weak, are extremely small. But there is no literature which focused on the liner and non-linear stratification.

A novel iteration scheme called the spectral relaxation method (SRM) (see Motsa and Makukula [19], Motsa [20]) is an iterative algorithm for the solution of nonlinear boundary layer problems which are characterized by flow properties that decay exponentially to constant levels far from the boundary surface. The main advantages of the method are the decoupling of the governing nonlinear systems into a sequence of smaller sub-systems which are then discretized using spectral collocation methods. The method is very efficient in solving boundary layer equations of the type under investigation in this study. The current results were validated by comparison with the results obtained using the shooting method. From the literature survey, it appears that the problem of mixed convection from vertical surface in a porous medium in the presence of linear and non-linear stratification under the different wall properties has not been investigated so far and is a fundamental problem of considerable interest. Therefore, this study aims to analyze the effects of linear and non-linear stratification on mixed convection along a vertical surface in a doubly stratified fluid embedded in a porous medium.

2. Mathematical formulation

Consider the two dimensional, laminar and steady mixed convection flow over a heated semi-infinite vertical flat plate in a doubly stratified fluid saturated porous medium. The geometry and the coordinate system are schematically shown in Fig. (1). The x-axis is taken along the plate and y-axis normal to it. The plate is maintained at variable temperature and concentration $T_w(x) = T_{\infty,0} + ax^m$ and $C_w(x) = C_{\infty,0} + bx^m$. The external velocity $U(x)$, the surface temperature $T_\infty(x)$ and the surface concentration $C_\infty(x)$ are assumed to vary as x^m, where x is measured from the leading edge of the vertical surface and m is a constant. The values $T_w(x)$ and $C_w(x)$ are assumed to be greater than the ambient temperature $T_\infty(x)$ and concentration $C_\infty(x)$ respectively at any arbitrary reference point in the medium (inside the boundary layer).

Subjecting to the Boussinesq and boundary layer approximations, the governing equations for the stratified fluid in a porous medium are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial y} = \pm \frac{Kg}{\nu}\left[\beta_T \frac{\partial T}{\partial y} + \beta_C \frac{\partial C}{\partial y}\right] \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

where u and v are the Darcian velocity components along x and y directions, T is the temperature, C is the concentration, K is the permeability constant, g is the acceleration due to gravity, ν is the kinematic viscosity, ρ is the density, α is the thermal diffusivity and D is the solutal diffusivity of the medium, β_T is the coefficient of thermal expansion, β_C is the coefficient of solutal expansion. The subscript ∞ indicate the condition at the outer edge of the boundary layer.

The boundary conditions are

$$v = 0, \quad T = T_w(x), \quad C = C_w(x) \quad at \quad y = 0 \tag{5a}$$

$$u = U(x), \quad T = T_\infty(x), \quad C = C_\infty(x) \quad as \quad y \to \infty \tag{5b}$$

In order that (1)–(4) subject to the boundary conditions (5) admit similarity solutions, we assume that

$$\begin{aligned} T_w(x) &= T_{\infty,0} + ax^m, C_w(x) = C_{\infty,0} + bx^m, \\ U(x) &= cx^m, T_\infty(x) = T_{\infty,0} + dx^m, C_\infty(x) = C_{\infty,0} + ex^m \end{aligned} \tag{6}$$

where where a, b, c, d and e are positive constants, m is a parameter.

In view of the continuity Eq. (1), we introduce the stream function ψ by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{7}$$

Substituting (7) in (2)–(4) and then using the following similarity transformation

$$\left.\begin{aligned} \eta &= \frac{y}{x}\left(\frac{Pe_x}{2}\right)^{1/2}, \quad f(\eta) = \frac{Pe_x^{-1/2}}{\sqrt{2}\alpha}\psi, \\ \theta(\eta) &= \frac{(T - T_\infty(x))}{(T_w - T_{\infty,0})}, \quad \phi(\eta) = \frac{(C - C_\infty(x))}{(C_w - C_{\infty,0})}, \end{aligned}\right\} \tag{8}$$

where $Pe_x = \frac{U(x)x}{\alpha}$ is the local Peclet number, we get the following system of ordinary differential equations

$$f'' \mp \lambda(\theta' + N\phi') = 0 \tag{9}$$

$$\theta'' + (m+1) f \theta' - 2m\varepsilon_1 f' - 2m f' \theta = 0 \tag{10}$$

$$\phi'' + (m+1) Le f \phi' - 2m Le \varepsilon_2 f' - 2m Le f' \phi = 0 \tag{11}$$

where the primes indicate differentiation with respect to η. Further, $Ra_x = \frac{Kg\beta_T(T_w - T_{\infty,0})x}{\alpha\nu}$ is the local Darcy–Reyleigh number and $\lambda = \frac{Ra_x}{Pe_x}$ is the mixed convection parameter. In usual notations, $Le = \frac{\alpha}{D}$, $N = \frac{\beta_C(C_w - C_{\infty,0})}{\beta_T(T_w - T_{\infty,0})}$ are Lewis number and buoyancy ratio. Finally, $\varepsilon_1 = \frac{d}{c}$ and $\varepsilon_2 = \frac{e}{c}$ are the constant thermal and solutal stratification parameters.

The boundary conditions (5) in terms of f, θ and ϕ become

$$f(0) = 0, \quad \theta(0) = 1 - \varepsilon_1, \quad \phi(0) = 1 - \varepsilon_2 \tag{12a}$$

$$f'(\infty) = 1, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0. \tag{12b}$$

The heat and mass fluxes from the plate are given by

$$q_w = -k\left[\frac{\partial T}{\partial y}\right]_{y=0}, \quad q_m = -D\left[\frac{\partial C}{\partial y}\right]_{y=0} \tag{13}$$

The local Nusselt number $Nu_x = \frac{q_w x}{k(T_w - T_{\infty,0})}$ and local Sherwood number $Sh_x = \frac{q_m x}{D(C_w - C_{\infty,0})}$ are readily obtained in the forms

$$\frac{Nu_x}{(Pe_x/2)^{1/2}} = -\theta'(0), \quad \frac{Sh_x}{(Pe_x/2)^{1/2}} = -\phi'(0) \tag{14}$$

Table 1
Comparison of two numerical schemes for fixed values of $\lambda = 1,\ m = 1$.

				$-\theta'(0)$		$-\phi'(0)$	
N	ε_1	ε_2	Le	Shooting method	SRM	Shooting method	SRM
−0.5	0.0	0.5	0.0	−2.09658	−2.09658	0	0
−0.5	0.0	0.5	0.4	−2.19434	−2.19434	−2.2687	−2.2687
−0.5	0.5	0.5	0.0	−0.92106	−0.92106	0	0
−0.5	0.5	0.5	0.4	−0.97425	−0.97425	−1.97098	−1.97098
1.0	0.0	0.5	0.0	−2.58949	−2.58949	0	0
1.0	0.0	0.5	0.4	−2.42205	−2.42205	−2.58484	−2.58484
1.0	0.5	0.5	0.0	−1.19118	−1.19118	0	0
1.0	0.5	0.5	0.4	−1.10255	−1.10255	−2.32744	−2.32744
−0.5	0.3	0.0	0.0	−1.22583	−1.22583	0	0
−0.5	0.3	0.0	0.4	−1.37154	−1.37154	−3.93676	−3.93676
−0.5	0.3	0.5	0.0	−1.36652	−1.36652	0	0
−0.5	0.3	0.5	0.4	−1.43539	−1.43539	−2.09499	−2.09499
1.0	0.3	0.0	0.0	−1.9345	−1.9345	0	0
1.0	0.3	0.0	0.4	−1.70715	−1.70715	−5.26834	−5.26834
1.0	0.3	0.5	0.0	−1.72834	−1.72834	0	0
1.0	0.3	0.5	0.4	−1.60605	−1.60605	−2.43356	−2.43356

3. Numerical solution using the spectral relaxation method(SRM)

The spectral relaxation method (SRM) is a proposed new algorithm for the solution of boundary value problems of the form (9)–(12). The algorithm utilizes the idea of the Gauss–Seidel method to decouple the governing systems of equations. From the decoupled equations an iteration scheme is developed by evaluating linear terms in the current iteration level (denoted by $r+1$) and all other terms (linear and nonlinear) in the previous iteration level (denoted by r). The decoupled equation system is solved using the Chebyshev pseudo - spectral method ([21], [22]). The basic thought behind the spectral collocation method is the first appearance of a differentiation matrix D which is applied to approximate the differential coefficients of the unknown variables, for example, $f(\eta)$ at the collocation points as the matrix vector product

$$\frac{df}{d\eta} = \sum_{k=0}^{\bar{N}} \mathbf{D}_{lk} f(\tau_k) = \mathbf{D}f, \ \ l = 0, 1, \ldots, \bar{N}, \tag{15}$$

where $\bar{N}+1$ is the number of collocation points (grid points), $\mathbf{D} = \frac{2\mathcal{D}}{\eta_\infty}$, and $f = [f(\tau_0), f(\tau_1), \ldots, f(\tau_{\bar{N}})]^T$ is the vector function at the collocation points. Higher order derivatives are obtained as powers of $\mathbf{D}$, that is

$$f^{(p)} = \mathbf{D}^p \mathbf{Z}. \tag{16}$$

where p is the order of the derivatives, η_∞ is a finite length that is chosen to be numerically large enough to approximate the conditions at infinity in the governing problem and τ is a variable used to map the truncated interval $[0, \eta_\infty]$ to the interval $[-1, 1]$ on which the spectral method can be implemented.

With the spectral relaxation method to (9)–(12), we obtain the following iteration scheme:

$$f''_{r+1} = \pm\lambda\,(\theta'_r + N\,\phi'_r) \tag{17}$$

$$\theta''_{r+1} + (m+1)\, f_{r+1}\, \theta'_{r+1} - 2\,m\, f'_{r+1}\, \theta_{r+1} = 2\,m\,\varepsilon_1\, f'_{r+1} \tag{18}$$

$$\phi''_{r+1} + (m+1)\, Le\, f_{r+1}\, \phi'_{r+1} - 2\,m\, Le\, f'_{r+1}\, \phi_{r+1} = 2\,m\, Le\, \varepsilon_2\, f'_{r+1} \tag{19}$$

subject to boundary conditions:

$$f_{r+1}(0) = 0, \quad f'_{r+1}(\infty) = 1 \tag{20}$$

$$\theta_{r+1}(0) = 1 - \varepsilon_1, \quad \theta_{r+1}(\infty) = 0 \tag{21}$$

$$\phi_{r+1}(0) = 1 - \varepsilon_2, \quad \phi_{r+1}(\infty) = 0 \tag{22}$$

Eqs. (17) –(22) are now written in the form:

$$A_1\, \mathbf{f_{r+1}} = B_1, \quad f_{r+1}(\tau_{\bar{N}}) = 0, \quad f'_{r+1}(\tau_0) = 1 \tag{23}$$

$$A_2\, \boldsymbol{\theta}_{\mathbf{r+1}} = B_2, \quad \theta_{r+1}(\tau_{\bar{N}}) = 1 - \varepsilon_1, \quad \theta_{r+1}(\tau_0) = 0 \tag{24}$$

$$A_3\, \boldsymbol{\phi}_{\mathbf{r+1}} = B_3, \quad \phi_{r+1}(\tau_{\bar{N}}) = 1 - \varepsilon_2, \quad \phi_{r+1}(\tau_0) = 0 \tag{25}$$

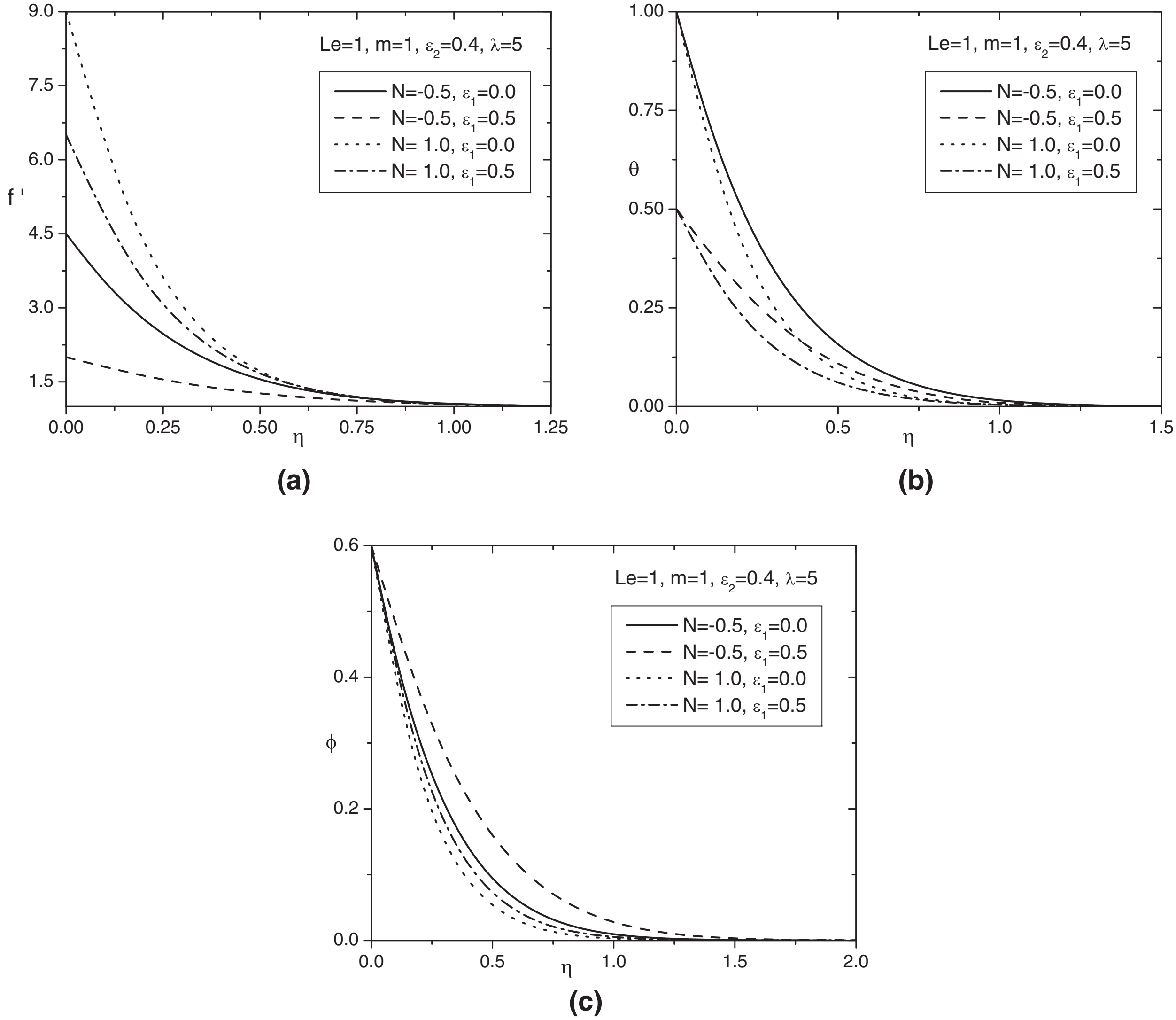

Fig. 2. (a) Velocity, (b) temperature, and (c) concentration profiles for various values of ε_1 for aiding and opposing buoyancy cases.

where

$$A_1 = \mathbf{D}^2, \quad B_1 = \pm\lambda\,(\theta_r' + N \quad \phi_r') \tag{26}$$

$$A_2 = \mathbf{D}^2 + (m+1)\,\mathrm{diag}[f_{r+1}]\,\mathbf{D} - 2\,m\,\mathrm{diag}[f_{r+1}']\,I, \quad B_2 = 2\,m\,\varepsilon_1\,f_{r+1}' \tag{27}$$

$$A_3 = \mathbf{D}^2 + (m+1)\,Le\,\mathrm{diag}[f_{r+1}]\,\mathbf{D} - 2\,m\,Le\,\mathrm{diag}[f_{r+1}']\,I, \quad B_3 = 2\,m\,Le\,\varepsilon_2\,f_{r+1}' \tag{28}$$

In Eqs. (26)–(28), **I** is an identity matrix, **D** is the differentiation matrix and diag[ ] is a diagonal matrix, all of size $(\overline{N}+1)\times(\overline{N}+1)$, where $\overline{N}$ is the number of grid points, **f**, $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$ are the values of the functions f, θ and ϕ when evaluated at the grid points. The subscript r denotes the iteration number.

The initial guesses to start the SRM scheme for Eqs. (17)–(19) are chosen as functions that satisfy the boundary conditions. From physical considerations, the velocity and temperature profiles for the boundary layer problem discussed in this work decay exponentially at $\eta = \infty$. For this reason, it is convenient to choose the following exponential functions as initial guesses:

$$f_0(\eta) = 1 - e^{-\eta} + \eta,\ \theta_0 = (1-\varepsilon_1)\,e^{-\eta},\ \phi_0 = (1-\varepsilon_2)\,e^{-\eta}$$

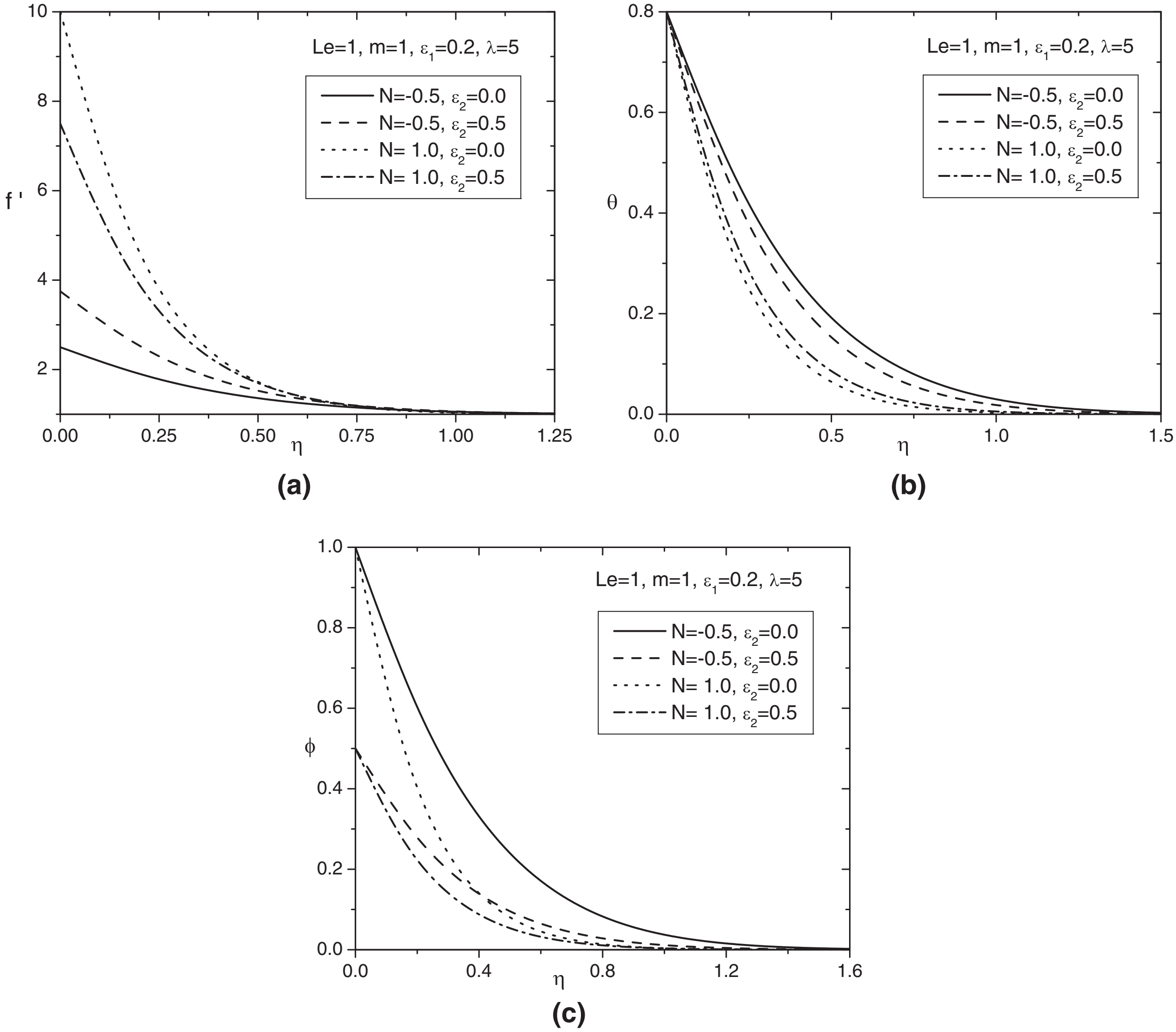

Fig. 3. (a) Velocity, (b) temperature, and (c) concentration profiles for various values of ε_2 for aiding and opposing buoyancy cases.

4. Results and discussion

It is worth mentioning that Eqs. (9)–(11) reduce to those derived by Aly et al. [14] when $\varepsilon_1 = 0$, $\varepsilon_2 = 0$, while for $\varepsilon_1 = 0$, $\varepsilon_2 = 0$ and $m = 0$ these equations reduce to those of Merkin [15]. A comparison is also made of the accuracy of the SRM results in relation to the results obtained using the Shooting method (Nag Routine: D02HAF) solver and they are given in Table 1.

4.1. Boundary-layer distributions of velocity, temperature and concentration

(a) **With varying thermal and solutal stratification parameters**

The first and second set of figures, (Figs. 2 and 3), are for $Le = 1.0$, $m = 1$ and $\lambda = 5.0$, and refer to the variation of the non-dimensional velocity f', temperature θ and concentration ϕ across the boundary layer. Fig. 2(a)–(c) are for the effect of thermal stratification, while Fig. 3(a)–(c) are for the effect of solutal stratification with both aiding and opposing buoyancy cases. It can be noticed that the speed of the fluid decreases with the gain of the thermal stratification parameter. This is because of thermal stratification reduces the effective convective potential between the heated plate and the ambient fluid in the medium. Hence, the thermal stratification effect reduces the velocity in the boundary layer. As the solutal stratification increases, the velocity of the fluid is less in the case of opposing buoyancy in comparison with the aiding buoyancy. The outcome of varying thermal and solutal stratification is seen to be qualitatively the same for the thermal boundary layer in the comparison between these two aiding and opposing positions.

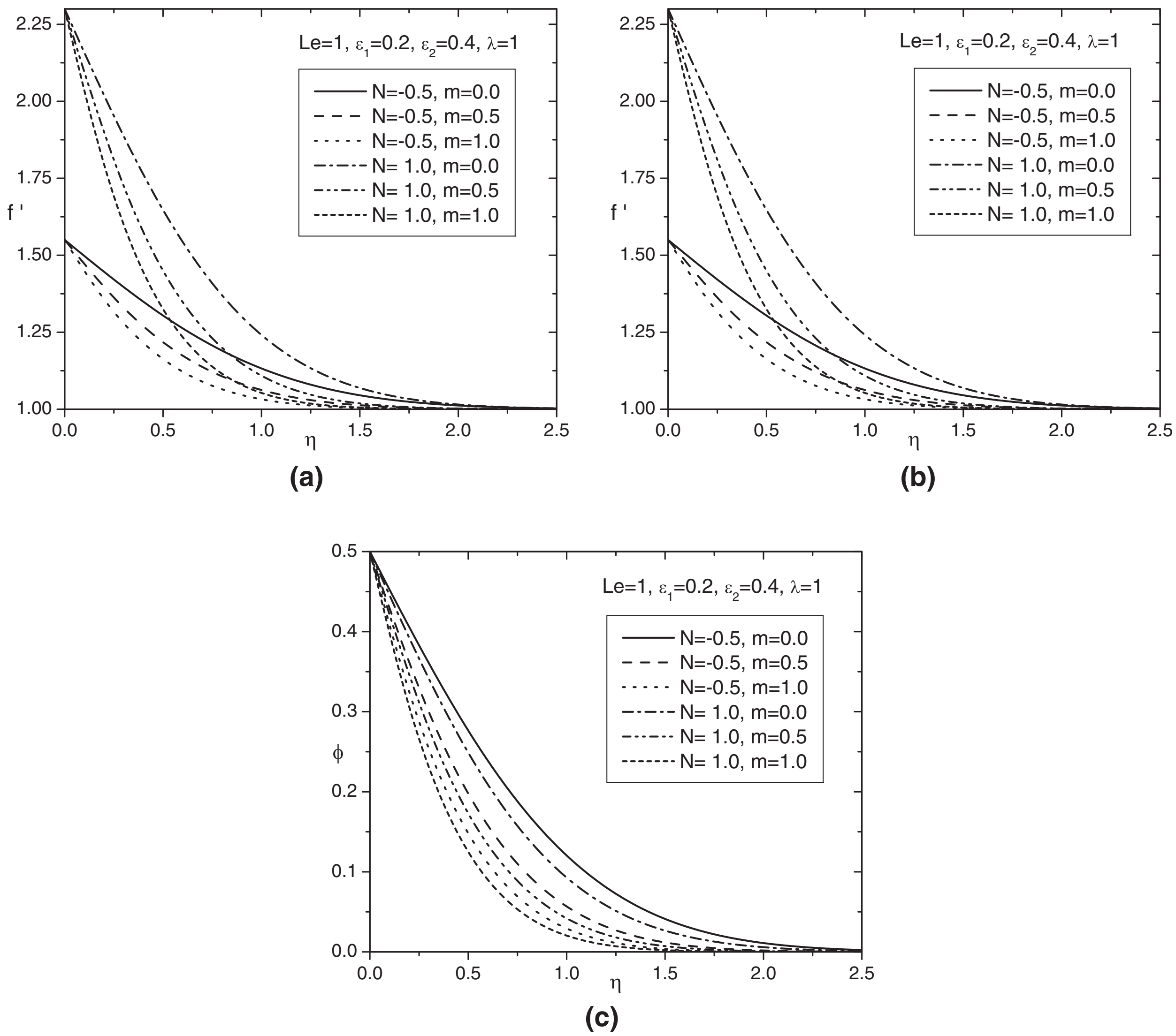

Fig. 4. (a) Velocity, (b) temperature, and (c) concentration profiles for various values of *m* for aiding and opposing buoyancy cases.

The effect of varying thermal and solutal stratification is seen to be qualitatively the difference for the solutal boundary layer in the comparison between these two aiding and opposing situations. In the case of aiding buoyancy, the concentration is increasing but it is showing an opposite trend in the case of opposing buoyancy situation. It can be observed from Fig. (3) a rapid stabilization of the temperature profiles in the downstream direction whereas the concentration profiles in the upstream direction with the increasing value of thermal stratification parameter. It shows a relative rapid stabilization for stratified fluids of the temperature and concentration profiles immediately downward of the leading edge, which means that similarity solutions are quickly achieved in the flow development. This an interesting aspect, put into evidence by the present numerical analysis. It is remarked that the strong influence of thermal and solutal stratification on the temperature and concentration profiles in the boundary layer. The results are in tune with the observation made in references [Gebhart et al. [7]; Prandtl[8]; Jaluria and Himasekhar [9]; Lakshmi Narayana and Murthy [11]; Murthy et al. [12]; Srinivasacharya and RamReddy ([16, 17])].

(b) **With varying values of *m***

The third set of figures, (Fig. 4), is plotted for $Le = 1.0$, $\lambda = 1.0$, $\varepsilon_1 = 0.2$ and $\varepsilon_2 = 0.5$ and refer to the variation of the non-dimensional velocity f', temperature θ and concentration ϕ across the boundary layer. The dimensionless velocity component for different values of m with fixed values of the other parameters, is depicted in Fig. 4(a). In both aiding and opposing buoyancy cases, it is observed that the velocity of the fluid decreases with the increase of m. Fig. 4(b) shows that the thermal boundary layer thickness decreases in the downstream direction, for both aiding and opposing buoyancy cases. Fig. 4(c) depicts the effect of solutal stratification in both the cases of aiding and opposing buoyancies. Fig. 4(c) demonstrates the similar behavior of the thermal and concentration boundary layers, in comparison with what described above for the thermal stratification parameter.

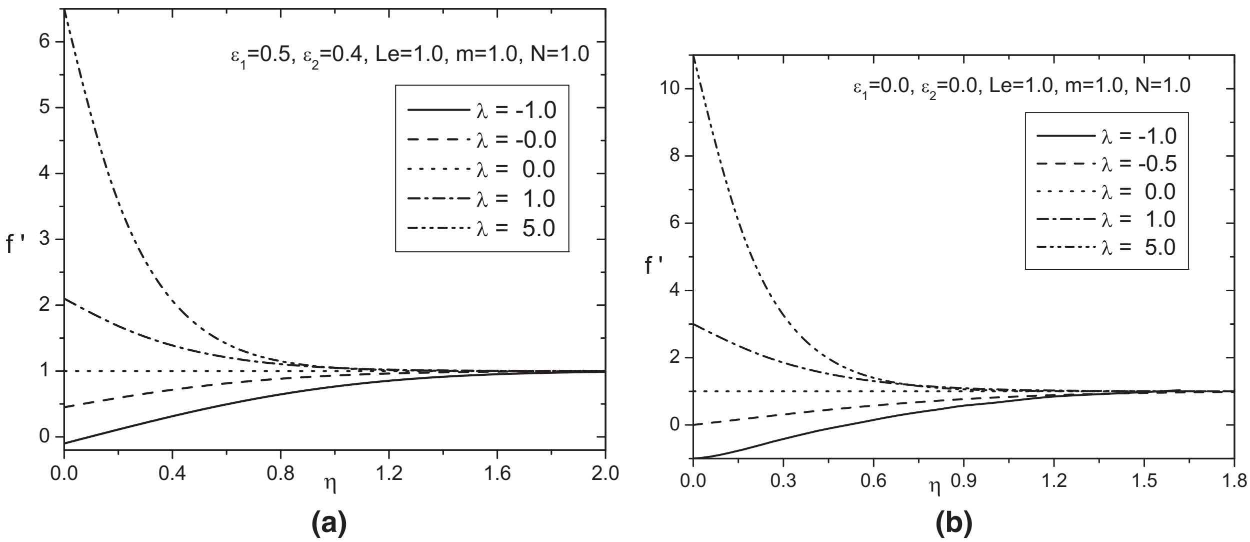

Fig. 5. Velocity profile for various values of λ in (a) the presence, and (b) the absence of stratification.

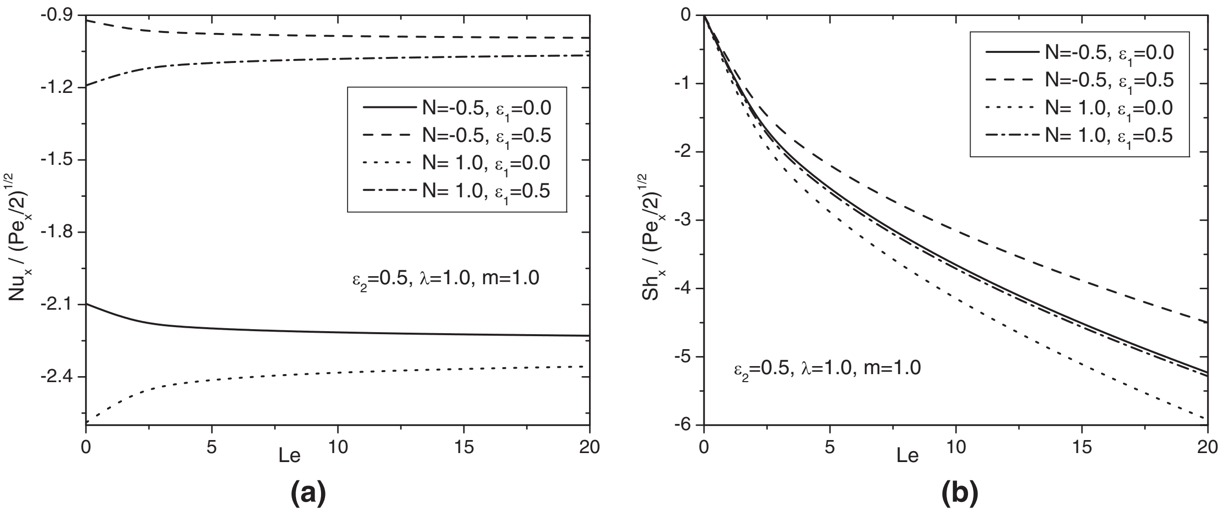

Fig. 6. Effect of ε_1 on (a) heat transfer rate (b) mass transfer rates versus *Le* for aiding and opposing buoyancy cases.

It can be observed from Fig. (4) the temperature and concentration profiles are more in the absence of stratification and less in the presence of stratification. Particularly, the temperature and concentration profiles are decreased when the fluid changes from a non-linear stratified fluid to linearly stratified fluid.

(b) **With varying values of λ**

The fourth set of figures, (Fig. 5), is plotted for $m = 1.0$, $Le = 1.0$ and $N = 1.0$ and refer to the variation of the non-dimensional velocity f' across the boundary layer in the presence and/or in the absence of stratification. The dimensionless velocity components for different values of λ with fixed values of the other parameters in the presence and/or in the absence of stratification, are presented in Fig. 5. It is found that the velocity of the fluid loss in the case of opposing flow when compared to the case of aiding flow. It is important to note that the separation of flow is observed in this study. Also it is found that flow separation is more in the absence of stratification but it is less in the presence of stratification. Finally, thermal and solutal stratifications significantly affect the heat and mass transfer rates, besides delay the boundary layer separation.

4.2. Nusselt and Sherwood numbers

The streamwise variations of the rate of heat and mass transfers versus *Le* at the wall are shown in Figs. 6–8, when $\lambda = 1$. Both these quantities behave similarly; both decrease with the increasing value of thermal stratification parameter. Both these quantities behave differently in the case of solutal stratification: decrease with the increasing value of solutal stratification parameter in the aiding buoyancy case but shows the opposite trend in the case of opposing buoyancy. Heat transfer rate decreases

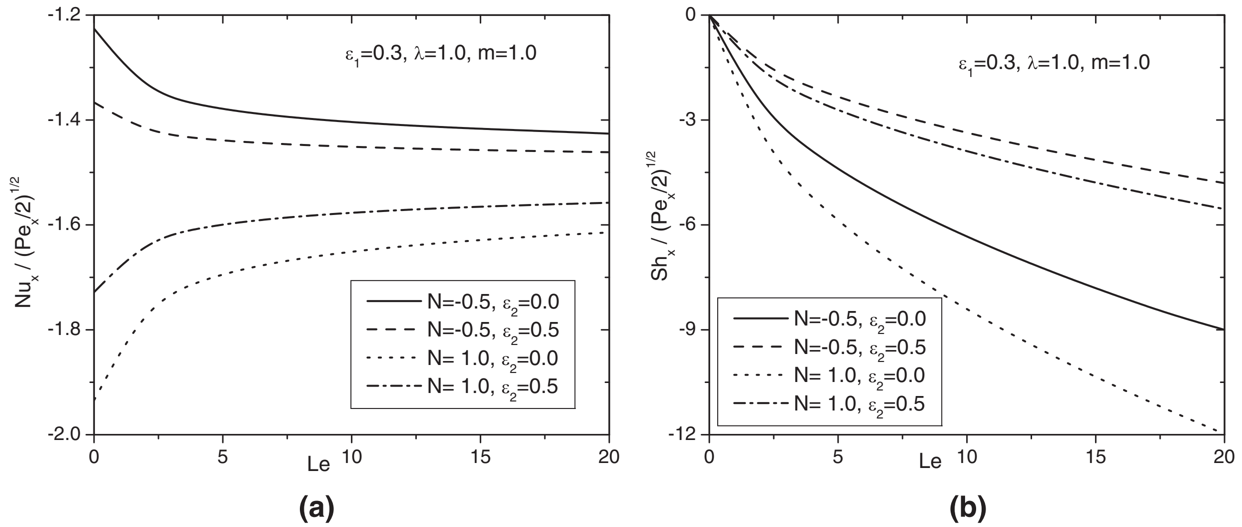

Fig. 7. Effect of ε_2 on (a) heat transfer rate (b) mass transfer rates versus *Le* for aiding and opposing buoyancy cases.

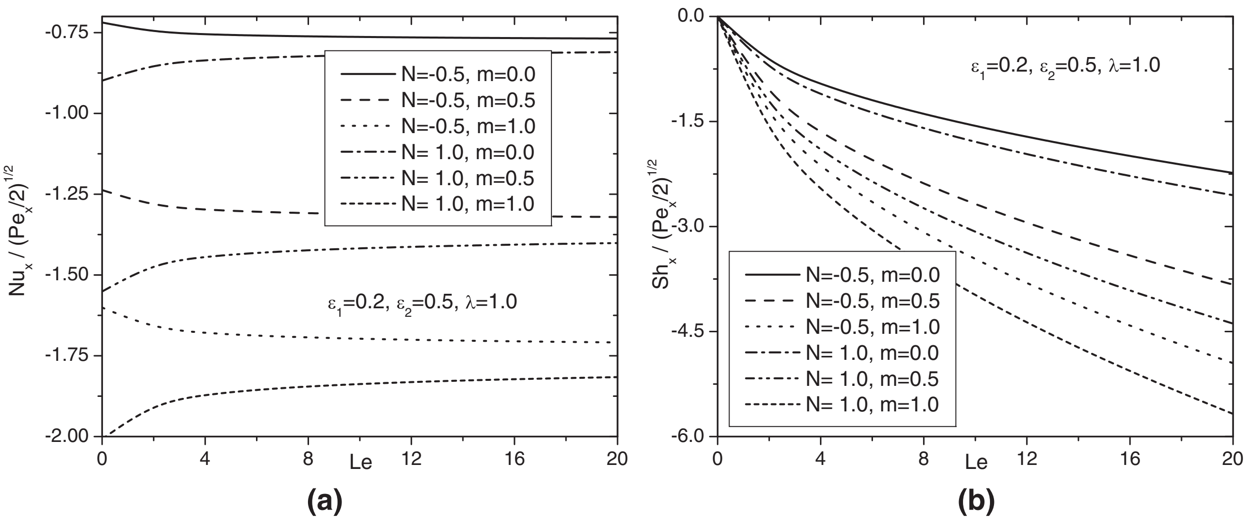

Fig. 8. Effect of *m* on (a) heat transfer rate (b) mass transfer rates versus *Le* for aiding and opposing buoyancy cases.

in the aiding buoyancy case, yet increase in the case of opposing buoyancy whenever the fluid slowly changing from fluid without stratification to stratified fluid. Another important fact demonstrated by these two figures is that higher heat and mass transfer rates possible without stratified fluid in both the cases of aiding and opposing buoyancy.

The variation of non-dimensional heat and mass transfer coefficients along the plate is shown in the Figs. 5–7, when $\lambda = 1$.

- Effects of thermal and soulutal stratification are displayed in Figs. 6–8. It presents a nonlinear decrease/increase of heat transfer coefficient in the aiding/opposing buoyancy case, whereas the nonlinear growth of mass transfer coefficient in both cases of aiding and opposing buoyancy with the increase of *Le*.
- In Fig. 8 illustrated the effect of heat and mass transfer rates against *Le* for different values of *m*. The heat transfer rate increase in both the cases of aiding and opposing buoyancy, but the shape of the curves change from concave to convex. The mass transfer rate increase non-linearly with *Le*. Obviously, the constancy of these coefficients where $m = 0$ signify the occurrence of similarity solutions.

5. Conclusions

In this report, a boundary layer analysis for mixed convection heat and mass transport along a vertical surface in a porous media saturated with doubly stratified fluid in the presence of aiding and opposing buoyancy cases is given. Utilizing a set of suitable similarity variables, the governing equations are translated into a band of ordinary differential equations depending on several non-dimensional parameters. Between these parameters there is also the dimensionless power m, which is varied between 0 and 1 and this approach is a characteristic feature of the present investigation. Consequently, numerical solutions have been presented for a wide range of $(Le, m, N, \lambda, \varepsilon_1, \varepsilon_2)$ for aiding and opposing buoyancy cases.

- Higher values of the thermal stratification parameter, result in lower temperature distribution, heat and mass transfer rates but higher concentration distribution. The same behavior is seen in the case of solutal stratification within the boundary layer.
- Higher values of the stratification parameter, result in lower temperature and concentration distributions and mass transport rate. Further, the heat transfer rate is increasing in the opposing buoyancy and showing reverse nature in the case of aiding buoyancy.
- Increasing the value of the parameter m tends to decrease the temperature and concentration profiles, while to increase the heat and mass transfer coefficients in the medium for both the cases of aiding and opposing buoyancy.
- It is found that flow separation is more in the absence of stratification, but it is to a lesser extent in the presence of stratification. Finally, thermal and solutal stratifications significantly affect the heat and mass transport rates, besides delay the boundary layer separation.

References

[1] K. Vafai, Handbook of Porous Media, Basel, Marcel Dekker, Inc., New York, 2000.
[2] I. Pop, D.B. Ingham, Convective Heat Transfer: Mathematical and Computational Modeling of Viscous Fluids and Porous Media, Oxford, Pergamon, 2001.
[3] D.A. Nield, A. Bejan, Convection in Porous Media, third, Springer-Verlag (2006), New-York, 2006.
[4] T.K. Aldoss, M.A. Al-Nimr, M.A. Jarrah, B.J. Al-Shaer, Magnetohydrodynamic mixed convection from a vertical plate embedded in a porous medium, Numer. Heat Transfer 28 (1995) 635–645.
[5] T. Bourhan, Magnetic and buoyancy effects on melting from a vertical plate embedded in saturated porous media, Energy Convers. Manage. 46 (2005) 2566–2577.
[6] S.M.M. EL-Kabeira, A.M. Rashada, R.S.R. Gorla, Unsteady MHD combined convection over a moving vertical sheet in a fluid saturated porous medium with uniform surface heat flux, Math. Comput. Modell. 46 (2007) 384–397.
[7] B. Gebhart, Y. Jaluria, R. Mahajan, B. Sammakia, Buoyancy Induced Flows and Transport, Hemisphere publishing Corp., 1952.
[8] L. Prandtl, Essentials of Fluid Dynamics, Blackie, London, 1952.
[9] Y. Jaluria, K. Himasekhar, Buoyancy induced two dimensional vertical flows in a thermally stratified environment, Comput. Fluids 11 (1983) 39–49.
[10] B.V. Rathish Kumar, Shalini, Double diffusive natural convection in a doubly stratified wavy porous enclosure, Appl. Math. Comput. 171 (2005) 180–202.
[11] P.A. Lakshmi Narayana, P.V.S.N. Murthy, Free convective heat and mass transfer in a doubly stratified non-Darcy porous medium, J. Heat Transfer 128 (2006) 1204–1212.
[12] P.V.S.N. Murthy, S. Mukherjee, P.V.S.S.S.R. Krishna, D. Srinivasacharya, Mixed convection heat and mass transfer in a doubly stratified non-darcy porous medium, Int. J. Appl. Mechan. Eng. 12 (2007) 109–123.
[13] Y. Jaluria, B. Gebhart, Stability and transition of buoyancy induced flows in a stratified medium, J. Fluid Mechan. 66 (1974) 593–604.
[14] E.H. Aly, L. Elliott, D.B. Ingham, Mixed convection boundary-layer flow over a vertical surface embedded in a porous medium, Eur. J. Mech. B: Fluids 22 (2003) 529–543.
[15] J.H. Merkin, Mixed convection boundary layer flow on a vertical surface in a saturated porous medium, J. Engrg. Math. 14 (1980) 301–313.
[16] D. Srinivasacharya, Ch. RamReddy, Effect of double stratification on free convection in a micropolar fluid, J. Heat Transfer 133/122502 (2011) 1–7.
[17] D. Srinivasacharya, Ch. RamReddy, Mixed convection in a doubly stratified micropolar fluid saturated non-Darcy porous medium, Can. J. Chem. Eng. 90 (2012) 1311–1322.
[18] A. Ishak, R. Nazar, I. Pop, Mixed convection boundary layer flow over a vertical surface embedded in a thermally stratified porous medium, Phys. Lett. A 372 (2008) 2355–2358.
[19] S.S. Motsa, Z.G. Makukula, On spectral relaxation method approach for steady von Karman flow of a Reiner–Rivlin fluid with Joule heating, viscous dissipation and suction/injection, Cent. Eur. J. Phys. 11 (2013) 363–374.
[20] S.S. Motsa, A new spectral relaxation method for similarity variable nonlinear boundary layer flow systems, Chem. Eng. Commun. 201(2) (2014) 241–256.
[21] C. Canuto, M.V. Hussaini, A. Quarteroni, T.A. Zang, Spectral Methods in Fluid Dynamics, Springer, Verlag, 1958.
[22] L.N. Trefethen, Spectral Methods in Matlab, SIAM, 2000.