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Soret and dufour effects on MHD mixed convection heat and mass transfer in a micropolar fluid

Research Article

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Abstract: This paper analyzes the flow, heat and mass transfer characteristics of the mixed convection on a vertical plate in a micropolar fluid in the presence of Soret and Dufour effects. A uniform magnetic field of magnitude is applied normal to the plate. The governing nonlinear partial differential equations are transformed into a system of coupled nonlinear ordinary differential equations using similarity transformations and then solved numerically using the Keller-box method. The numerical results are compared and found to be in good agreement with previously published results as special cases of the present investigation. The rate of heat and mass transfer at the plate are presented graphically for various values of coupling number, magnetic parameter, Prandtl number, Schmidt number, Dufour and Soret numbers. In addition, the skin-friction coefficient, the wall couple stress are shown in a tabular form.

Keywords: Similarity Solutions • Mixed convection • Micropolar Fluid • MHD • Soret and Dufour effects • Heat and mass transfer

1. Introduction

Mixed convection flows are of great interest because of their various engineering, scientific, and industrial applications in heat and mass transfer. Mixed convection of heat and mass transfer occurs simultaneously in the fields of design of chemical processing equipment, formation and dispersion of fog, distributions of temperature, moisture over agricultural fields, groves of fruit trees, damage of crops due to freezing and pollution of the environment. Extensive studies of mixed convection heat and mass transfer of a non-isothermal vertical surface under boundary layer approximation have been undertaken by several authors. The majority of these studies dealt with the traditional Newtonian fluids. It is well known that most fluids which are encountered in chemical and allied processing applications do not satisfy the classical Newton's law and are accordingly known as non-Newtonian fluids. A number of mathematical models were proposed to explain the rheological behavior of non-Newtonian fluids. Among these, the fluid model introduced by Eringen [1] exhibits some microscopic effects arising from the local structure and micro-motion of the fluid elements. Further, they can sustain couple

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stresses and include classical Newtonian fluid as a special case. The model of micropolar fluid represents fluids consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium where the deformation of the particles is ignored. Micropolar fluids have been shown to accurately simulate the flow characteristics of polymeric additives, geomorphologic sediments, colloidal suspensions, haematological suspensions, liquid crystals, lubricants etc. The mathematical theory of equations of micropolar fluids and applications of these fluids in the theory of lubrication and in the theory of porous media are presented by Lukaszewicz [2]. The heat and mass transfer in micropolar fluids is also important in the context of chemical engineering, aerospace engineering and also industrial manufacturing processes. The problem of mixed convection heat and mass transfer in the boundary layer flow along a vertical surface submerged in a micropolar fluid has been studied by a number of investigators.

In recent years, several simple boundary layer flow problems have received new attention within the more general context of magnetohydrodynamics (MHD). Several investigators have extended many of the available boundary layer solutions to include the effects of magnetic fields for those cases when the fluid is electrically conducting. The study of magneto-hydrodynamic flow for an electrically conducting fluid past a heated surface has important applications in many engineering problems such as plasma studies, petroleum industries, MHD power generators, cooling of nuclear reactors, the boundary layer control in aerodynamics, and crystal growth. In addition, there has been a renewed interest in studying MHD flow and heat transfer in porous media due to the effect of magnetic fields on flow control and on the performance of many systems using electrically conducting fluids. The problem of MHD mixed convection heat and mass transfer in the boundary layer flow along a vertical surface submerged in a micropolar fluid has been studied by several investigators. Kim and Fedorov [3] considered the case of mixed convection flow of a micropolar fluid past a semi-infinite moving vertical porous plate with varying suction velocity normal to the plate in the presence of radiation. Seddeek [4] investigated the analytical solution for the effect of radiation on flow of a magneto-micropolar fluid past a continuously moving plate with suction and blowing. Mahmoud [5] analyzed the effects of slip and heat generation/absorption on MHD mixed convection flow of a micropolar fluid over a heated stretching surface. Hayat [6] studied the effects of heat and mass transfer on the mixed convection flow of a MHD micropolar fluid bounded by a stretching surface using Homotopy analysis method. Das [7] considered

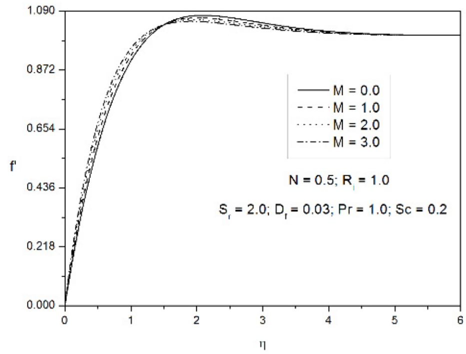

Figure 1. Velocity profile for various values of Magnetic parameter M

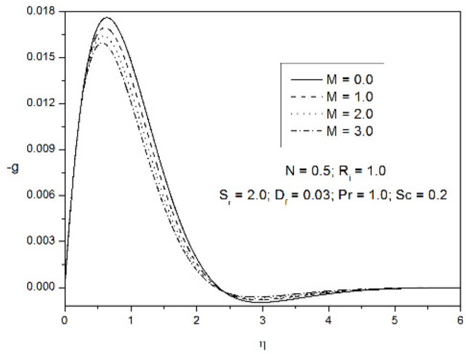

Figure 2. Microrotation profile for various values of Magnetic parameter M

the effects of partial slip on steady boundary layer stagnation point flow of an electrically conducting micropolar fluid impinging normally towards a shrinking sheet in the presence of a uniform transverse magnetic field. Kafoussias [8] analyzed the action of a localized magnetic field on forced and free convective boundary layer flow of a magnetic fluid over a semi-infinite vertical plate. Pal and Talukdar [9] investigated the influence of thermal radiation and first-order chemical reaction on unsteady MHD convective flow, heat and mass transfer of a viscous incompressible electrically conducting fluid past a semi-infinite vertical flat plate in the presence of transverse magnetic field under oscillatory suction and heat source in slip-flow regime.

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of a more intricate nature. It has been observed that an energy flux can be generated not only

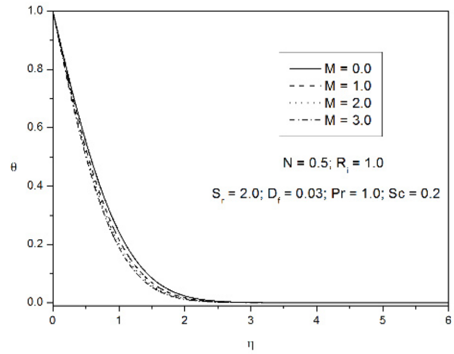

Figure 3. Temperature profile for various values of Magnetic parameter M

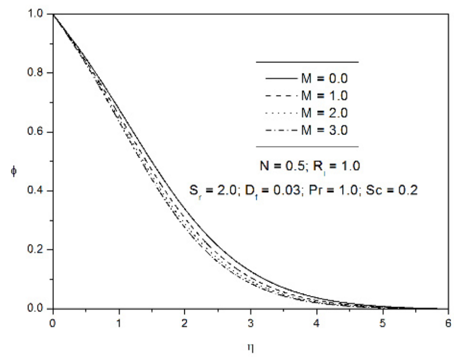

Figure 4. Concentration profile for various values of Magnetic parameter M

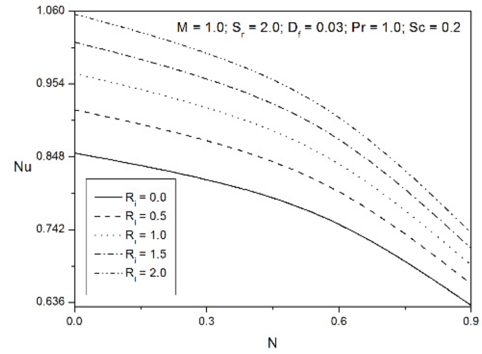

Figure 5. Effect of mixed convection parameter R_i on Heat Transfer Rate

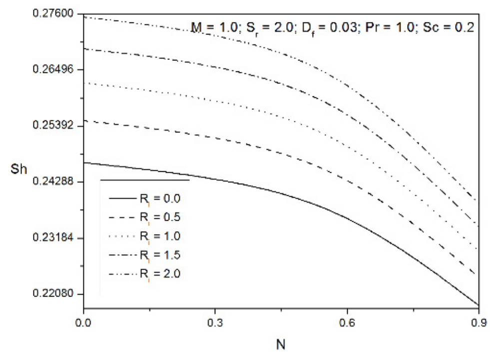

Figure 6. Effect of mixed convection parameter R_i on Mass Transfer Rate

by temperature gradients but also by concentration gradients. The energy flux caused by a concentration gradient is termed the diffusion-thermo (Dufour) effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the thermal-diffusion (Soret) effect. In most of the studies related to heat and mass transfer process, Soret and Dufour effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fouriers and Ficks laws. But these effects are considered as second order phenomena and may become significant in areas such as hydrology, petrology, geosciences, etc. The Soret effect, for instance, has been utilized for isotope separation and in mixture between gases with very light molecular weight and of medium molecular weight. The Dufour effect was recently found to be of order of considerable magnitude so that it cannot be neglected Eckert and Drake [10]. Dursunkaya and Worek [11] studied diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface, whereas Kafoussias and Williams [12] presented the same effects on mixed convective and mass transfer steady laminar boundary layer flow over a vertical flat plate with temperature dependent viscosity. Postelnicu [13] studied numerically the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Abreu et al. [14] considered both free and forced convection boundary layer flows with Soret and Dufour effects . Alam and Rahman [15] investigated the Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction. Chamkha [16] focused on the numerical modeling of the effects of Dufour and Sorer and radiation on heat and mass transfer by MHD mixed convection from a semi-infinite, isothermal, vertical and permeable surface immersed in a uniform porous medium. Rawat and Bhargava [17] presented a mathematical model for the steady thermal convection heat and mass transfer in a micropolar fluid saturated Darcian porous medium in the presence of significant Dufour and Soret effects and viscous heating. Srinivasacharya and RamReddy [18] studied Soret and Dufour effects on the mixed convection from a semi-infinite vertical plate embedded in a stable micropolar fluid with uniform and constant heat and mass flux conditions. Makinde [19] considered the mixed convection flow of an incompressible Boussinesq fluid under the simultaneous action of buoyancy and transverse magnetic field with Soret and Dufour effects over a vertical porous plate with constant heat flux embedded in a porous medium. Olanrewaju and Makinde [20] analyzed the effects of thermal-diffusion and diffusion-thermo on chemically reacting MHD boundary layer flow of heat and mass transfer past a moving vertical plate with suction/injection. Makinde *et. al.* [21] presented numerical study of chemically-reacting hydromagnetic boundary layer flow with soret/dufour effects and a convective surface boundary condition.

Motivated by the investigations mentioned above, the aim of this investigation is to consider the effects of transverse magnetic field, Soret and Dufour effects on the mixed convection heat and mass transfer along a vertical plate embedded in a micropolar fluid. It is established that similarity solutions are possible only when the variation in the temperature of the plate and the difference in the concentration is a linear function of the distance from the leading edge measured along the plate. Using these similarity transformations, the governing system of partial differential equations is transformed into a system of non-linear ordinary differential equations. This system of nonlinear ordinary differential equations is solved numerically using Keller-box method given in Cebeci and Bradshaw [22]. The effects of various parameters on the heat and mass transfer rates are presented graphically.

2. Mathematical formulation

Consider a steady, laminar, incompressible, two-dimensional mixed convective heat and mass transfer along a semi infinite vertical plate embedded in a free stream of electrically conducting micropolar fluid with velocity $U(x)$, temperature T_∞ and concentration C_∞. Choose the co-ordinate system such that x - axis is along the vertical plate and y - axis normal to the plate. The plate is maintained at variable temperature $T_w(x)$ and concentration $C_w(x)$. These values are assumed to be greater than the ambient temperature T_∞ and concentration C_∞. A uniform magnetic field of magnitude B_0 is applied normal to the plate. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in comparison with the applied magnetic field. The physical model and coordinate system are shown in Fig.(1). In addition, the Soret and Dufour effects are considered.

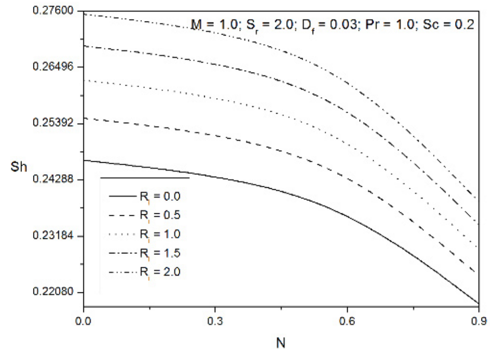

Figure 7. Effect of mixed convection parameter R_i on Mass Transfer Rate

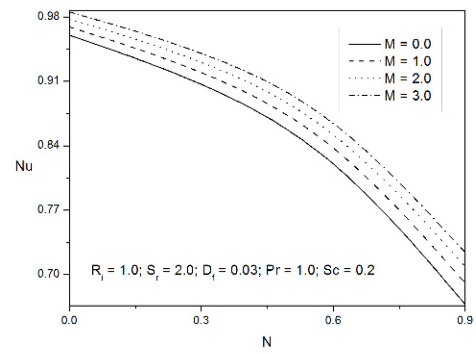

Figure 8. Effect of Magnetic parameter M on Heat Transfer Rate

Using the boussinesq and boundary layer approximations, the governing equations for the micropolar fluid [6, 9] are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U(x)\frac{dU(x)}{dx}+\left(\frac{\mu+\kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{\kappa}{\rho}\frac{\partial \omega}{\partial y}+ g^*(\beta_T(T-T_\infty)+\beta_c(C-C_\infty))+\frac{\sigma B_0^2}{\rho}(U(x)-u) \tag{2}$$

$$u\frac{\partial \omega}{\partial x}+v\frac{\partial \omega}{\partial y}=\frac{\gamma}{\rho j}\frac{\partial^2 \omega}{\partial y^2}-\frac{\kappa}{\rho j}\left(2\omega+\frac{\partial u}{\partial y}\right) \tag{3}$$

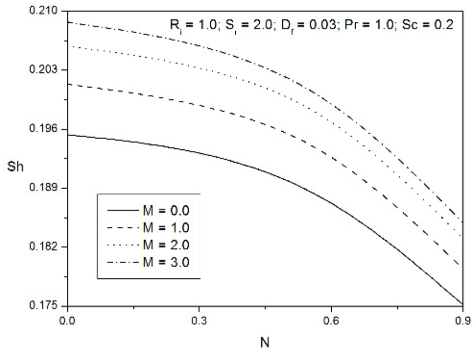

Figure 9. Effect of Magnetic parameter M on Mass Transfer Rate

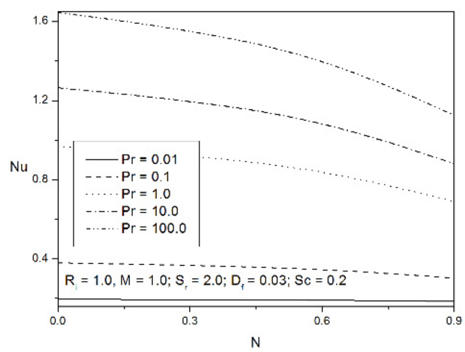

Figure 10. Effect of Prandtl number Pr on Heat Transfer Rate

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{D\,K_T}{C_s\,C_p}\frac{\partial^2 C}{\partial y^2} \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}+\frac{D\,K_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{5}$$

where u and v are the components of velocity along x and y directions respectively, ω is the component of microrotation whose direction of rotation lies in the xy-plane, g^* is the gravitational acceleration, T is the temperature, C is the concentration, β_T is the coefficient of thermal expansions, β_c is the coefficient of solutal expansions, C_p is the specific heat capacity, B_0 is the coefficient of the magnetic field, μ is the dynamic coefficient of viscosity of the fluid, κ is the vortex viscosity, j is the micro-inertia density, γ is the spin-gradient viscosity, σ is the magnetic permeability of the fluid, α is the thermal diffusivity, D is the molecular diffusivity, K_T is the thermal diffusion ratio, C_s is the concentration susceptibility, T_m is the mean fluid temperature.

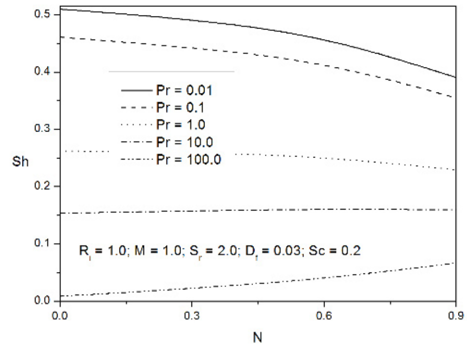

Figure 11. Effect of Prandtl number Pr on Mass Transfer Rate

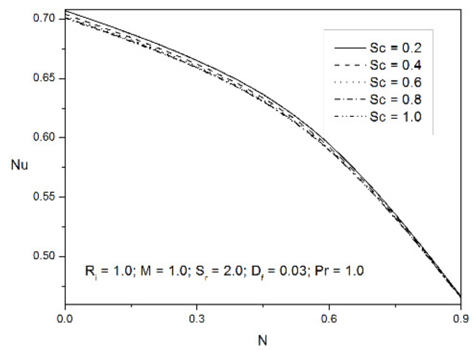

Figure 12. Effect of Schmidt number Sc on Heat Transfer Rate

The boundary conditions are:

$$u=0,\quad v=0,\quad \omega=0,\quad T=T_w(x),\quad C=C_w(x)\quad \text{at}\quad y=0 \tag{6a}$$

$$u\to U(x),\quad \omega\to 0,\quad T\to T_\infty,\quad C\to C_\infty\quad \text{as}\quad y\to\infty \tag{6b}$$

where the subscripts w and ∞ indicates the conditions at wall and at the outer edge of the boundary layer, respectively.

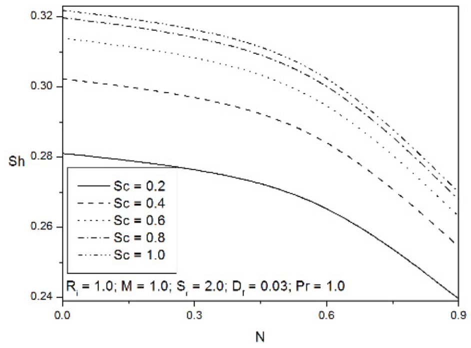

Figure 13. Effect of Schmidt number Sc on Mass Transfer Rate

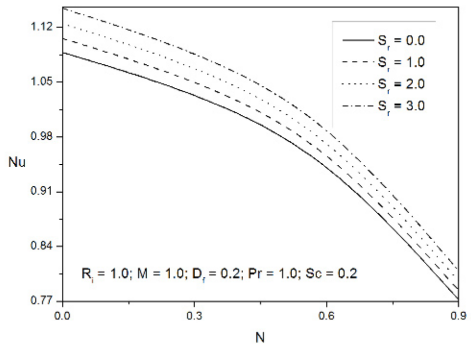

Figure 14. Effect of Soret number S_r on Heat Transfer Rate

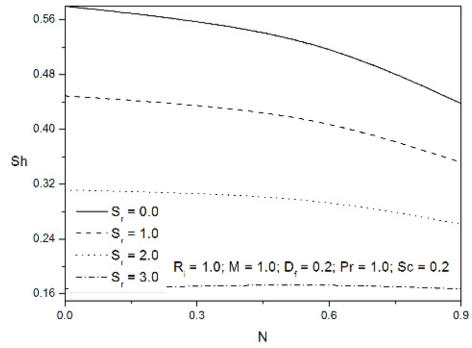

Figure 15. Effect of Soret number S_r on Mass Transfer Rate

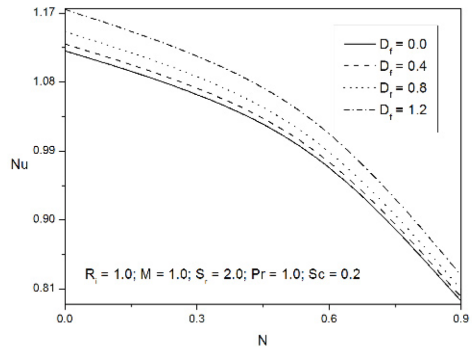

Figure 16. Effect of Dufour number D_f on Heat Transfer Rate

3. Method of solution

The continuity equation (1) is satisfied by introducing the stream function ψ such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{6}$$

In order to explore the possibility for the existence of similarity, we assume

$$\left.\begin{aligned} &\psi = Ax^a f(\eta), \quad \eta = Byx^b, \quad \omega = E\,x^c\,g(\eta), \\ &\frac{T - T_\infty}{T_w(x) - T_\infty} = \theta(\eta), \quad T_w(x) - T_\infty = M_1\,x^l, \\ &\frac{C - C_\infty}{C_w(x) - C_\infty} = \phi(\eta), \quad C_w(x) - C_\infty = N_1\,x^m \end{aligned}\right\} \tag{7}$$

where $A, B, E, M_1, N_1, a, b, c, l,$ and m are constants. We define $U(x) = A\,B\,x^d$. Substituting (6) and (7) in (2), (3), (4) and (5), it is found that similarity exists only if $a = 1, b = 0, c = d = l = m = 1$. Hence, appropriate similarity transformations are

$$\left.\begin{aligned} &\psi = A\,x\,f(\eta), \quad \eta = B\,y, \quad \omega = E\,x\,g(\eta), \\ &\frac{T - T_\infty}{T_w(x) - T_\infty} = \theta(\eta), \quad T_w(x) - T_\infty = M_1\,x, \\ &\frac{C - C_\infty}{C_w(x) - C_\infty} = \phi(\eta), \quad C_w(x) - C_\infty = N_1\,x \end{aligned}\right\} \tag{8}$$

Making use of the dimensional analysis, the constants A, B, E, M_1 and N_1 have, respectively, the dimensions of velocity, reciprocal of length, the reciprocal of the product of length and time, the ratio of (temperature/length) and of the ratio (concentration/length).

Substituting (8) in (2), (3), (4) and (5), we obtain

$$f'^2 - f f'' = 1 + \left[\frac{1}{1-N}\right] f''' + \left[\frac{N}{1-N}\right] g' + R_i[\theta + L\,\phi] + M[1 - f'] \quad (9)$$

$$f' g - f g' = \lambda g'' - \left(\frac{N}{1-N}\right) \mathcal{J}\,\left(2g + f''\right) \quad (10)$$

$$f'\theta - f\theta' = \frac{1}{Pr}\theta'' + D_f\,\phi'' \quad (11)$$

$$f'\phi - f\phi' = \frac{1}{Sc}\phi'' + S_r\,\theta'' \quad (12)$$

where the ratio of buoyancy forces to the inertia forces in terms of the mixed convection parameter is defined as $R_i = \frac{g^*\beta_T M_1}{A^2 B^2}$ and is used to delineate the free, forced and mixed convection regimes. $R_i \leq 1$ corresponds to pure forced convection, whereas $R_i \geq 1$ corresponds to pure free convection, $Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $Sc = \frac{\nu}{D}$ is the Schmidth number, $\mathcal{J} = 1/(jB^2)$ is the micro-inertia density, $\lambda = \frac{\gamma}{j\rho\nu}$ is the spin-gradient viscosity, $N = \frac{\kappa}{\mu+\kappa}$ $(0 \leq N < 1)$is the Coupling number, $L = \frac{\beta_c}{\beta_T}\frac{N_1}{M_1}$ is the buoyancy parameter, $M = \frac{\sigma B_0^2}{\mu B^2}$ is the magnetic field parameter, $D_f = \frac{D\,K_T\,N_1}{C_s\,C_p\,\nu M_1}$ is the Dufour number, and $S_r = \frac{D\,K_T\,M_1}{T_m\,\nu N_1}$ is the Soret number.

The boundary conditions (6) in terms of f, g, θ and ϕ becomes

$$\eta = 0: \quad f'(0) = 0, f(0) = 0, \quad g(0) = 0, \quad \theta(0) = 1, \quad \phi(0) = 1 \quad (14a)$$

$$as \quad \eta \to \infty: f'(\infty) = 1, \quad g(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0 \quad (14b)$$

The wall shear stress and the wall couple stress are

$$\tau_w = \left[(\mu+\kappa)\frac{\partial u}{\partial y} + \kappa\,\omega\right]_{y=0} \quad \text{and} \quad m_w = \gamma\left[\frac{\partial \omega}{\partial y}\right]_{y=0} \quad (13)$$

The non-dimensional skin friction $C_f = \frac{2\tau_w}{\rho A^2}$ and wall couple stress $M_w = \frac{B}{\rho A^2} m_w$, where A is the characteristic velocity, are given by

$$C_f = \left(\frac{2}{1-N}\right) f''(0)\overline{x} \quad \text{and} \quad M_w = \frac{\lambda}{\mathcal{J}}\,g'(0)\overline{x} \quad (14)$$

where $\overline{x} = B\,x$.
The heat and mass transfers from the plate respectively are given by

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad \text{and} \quad q_m = -D\left[\frac{\partial C}{\partial y}\right]_{y=0} \quad (15)$$

The non dimensional rate of heat-transfer, called the Nusselt number $Nu = \frac{q_w}{B\,k\,(T_w - T_\infty)}$ and rate of mass transfer, called the Sherwood number $Sh = \frac{q_m}{D\,B[C_w - C_\infty]}$ are given by

$$Nu = -\theta'(0) \quad \text{and} \quad Sh = -\phi'(0) \quad (16)$$

4. Numerical procedure and validation

The flow Eqs. (9) and (10) which are coupled, together with the energy and concentration Eqs.(11) and (12), constitute non-linear nonhomogeneous differential equations for which closed-form solutions cannot be obtained. Hence the governing Eqs.(9) to (12) have been solved numerically using the Keller-box implicit method [22]. This method has four main steps. The first step is converting the equations (9) to (12) into a system of first-order equations. The second step is replacing partial derivatives by central finite difference approximation. The third step is linearizing the nonlinear algebraic equations by Newton's method and then casting as the matrix vector form. The last step is solving linearized system of equations using the block-tridiagonal-elimination technique. Here, the initial values for velocity, temperature and concentration are arbitrarily chosen so that they satisfy the boundary conditions. The calculations are repeated until some convergent criterion is satisfied and the calculations are stopped when $\delta f''(0) \leq 10^{-8}$, $\delta\theta'(0) \leq 10^{-8}$ and $\delta\phi'(0) \leq 10^{-8}$. In the present study, the boundary conditions for η at ∞ are replaced by a sufficiently large value of η where the velocity approach unity and temperature and concentration approach zero. The independence of the step size $(\Delta\eta)$ is tested by taking three different step sizes $\Delta\eta = 0.001$,

$\Delta\eta = 0.01$ and $\Delta\eta = 0.05$. In each case we found very good agreement between them on different profiles. This method has been proven to be adequate and gave accurate results for boundary layer equations. This method has a second order accuracy and unconditionally stable. After some trials we imposed a maximal value of η at ∞ as 6 and a grid size of $\Delta\eta$ as 0.01. In order to study the effects of the coupling number N, magnetic field parameter M, Prandtl number Pr, Schmidt number Sc, Dufour number D_f and Soret number S_r on the physical quantities of the flow, the remaining parameters are fixed as $R_i = 1.0$, $L = 1$, $\lambda = 1$ and $\mathcal{J} = 0.1$. The values of micropolar parameters λ and $\mathcal{J}$ are chosen so as to satisfy the thermodynamic restrictions on the material parameters given by Eringen [1]. In the present analysis the values of Soret number S_r and Dufour number D_f are chosen in such a way that their product is constant according to their definition provided that the mean temperature T_m is kept constant [12].

In the absence of coupling number N, Magnetic parameter M , Soret number S_r, Dufour number D_f and buoyancy parameter L with $R_i = 1.0$, $\lambda = \mathcal{J} = 0$, and $Sc = 0.24$ the results of $f''(0)$ and $-\theta(0)$ have been compared with the values of Ramachandran et al.[23] and found that they are in good agreement, as shown in table 1.

5. Results and Discussion

The variation of the non-dimensional velocity, microrotation, temperature and concentration profiles with η for different values of magnetic parameter M is illustrated in Figs. from (1) to (4). It is observed from Fig. (1) that the velocity increase near the plate and then decreases far away from the plate as the magnetic parameter (M) increases. When $U(x) > u$ (i.e., imposed pressure terms $\left[\left(\sigma B_0^2/\rho\right) U(x)\right]$) dominates Lorentz force imposed by a transverse magnetic field normal to the flow direction $\left[\left(\sigma B_0^2/\rho\right) u\right]$ in Eq. (2), the effect of the magnetic interaction parameter is to increase the velocity. Similarly, for $U(x) < u$, the effect of the magnetic interaction parameter is to decrease the velocity. From Fig. (2), it is observed that the microrotation component increases near the plate and deceases far away from the plate for increasing values of M. It is noticed from Fig. (3) that the temperature decreases with increasing values of magnetic parameter. It is clear from Fig. (4) that the non-dimensional concentration decreases with increasing values of M. As explained above, the magnetic field gives rise to a motive force to an electrically conducting fluid, this force makes the fluid experience an acceleration by decreasing the friction between its layers and thus decreases its temperature and concentration.

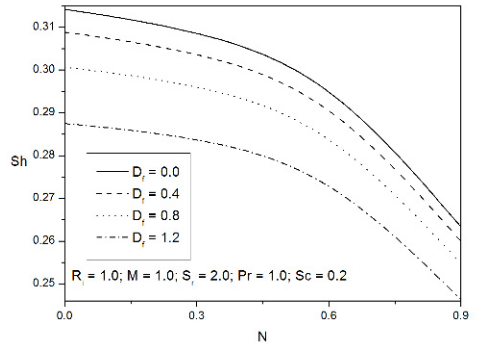

Figure 17. Effect of Dufour number D_f on Mass Transfer Rate

Figures from 5 - 7 show the effect of various values of the ratio of buoyancy forces to the inertia forces in terms of mixed convection parameter R_i on the non-dimensional heat and mass transfer rates. Figs. 5 and 7 explains that the dimensionless heat transfer rate (Nusselt number Nu) and mass transfer rate (Sherwood number Sh) rises as mixed convection parameter R_i increases. The higher value of R_i leads to the greater buoyancy effect in mixed convection flow, hence it accelerate the flow. This causes the increasing effect of convection cooling as a result of the great buoyancy effect for large R_i. As R_i accelerate the fluid velocity, then the hot fluid and high concentration of the fluid nearby the vertical plate are replaced by cooled fluid chunks, hence the heat and mass transfer rates increase. The Nusselt number and Sherwood number of the fluid increase from pure forced convection (as $R_i \to 0$) to pure free convection ($R_i > 1$).

Figures (8) and (9) depict the variation of heat and mass transfer rates (Nusselt number Nu and Sherwood number Sh) with coupling number N for different values of magnetic parameter M. The coupling number N characterizes the coupling of linear and rotational motion arising from the micromotion of the fluid molecules. Hence, N signifies the coupling between the Newtonian and rotational viscosities. As $N \to 1$, the effect of microstructure becomes significant, whereas with a small value of N the individuality of the substructure is much less pronounced. As $\kappa \to 0$ i.e. $N \to 0$, the micropolarity is lost and the fluid behaves as nonpolar fluid. Hence, $N \to 0$ corresponds to viscous fluid. It is observed from Figs. (8) and (9) that the both Nusselt number and Sherwood number decrease

Table 1. : Comparison of $f''(0)$ and $\theta'(0)$ for various values of Pr calculated by the present method and that of Ramachandran et al. [23] for $N = M = S_r = D_f = L = 0$, $\mathcal{J} = \lambda = 0$ $R_i = 1.0$ and $Sc = 0.24$

	$f''(0)$		$-\theta'(0)$	
Pr	Ramachandran et al. [23]	Present	Ramachandran et al. [23]	Present
0.7	1.7063	1.70633618	0.7641	0.76406928
7.0	1.51790	1.51792246	1.7224	1.72247984
20	1.4485	1.44849210	2.4576	2.45790153
40	1.4101	1.41006659	3.1011	3.10174561
60	1.3903	1.39028343	3.5514	3.55240591
80	1.3774	1.37740137	3.9095	3.91083193
100	1.3680	1.36804285	4.2116	4.21334959

Table 2. : Effect of skin friction and wall couple stress for various values of N, Pr, Sc, M, D_f and S_r

N	Pr	Sc	M	D_f	S_r	$f''(0)$	$-g'(0)$
0.0	1.0	0.2	1.0	0.03	2.0	2.47550	0.00000
0.3	1.0	0.2	1.0	0.03	2.0	2.03405	0.03115
0.6	1.0	0.2	1.0	0.03	2.0	1.48498	0.09424
0.9	1.0	0.2	1.0	0.03	2.0	0.63986	0.29902
0.5	0.01	0.2	1.0	0.03	2.0	1.79578	0.07188
0.5	0.1	0.2	1.0	0.03	2.0	1.76056	0.07006
0.5	1.0	0.2	1.0	0.03	2.0	1.68544	0.06693
0.5	10	0.2	1.0	0.03	2.0	1.62136	0.06537
0.5	1.0	0.2	1.0	0.03	2.0	1.68544	0.06693
0.5	1.0	0.4	1.0	0.03	2.0	1.67416	0.06620
0.5	1.0	0.6	1.0	0.03	2.0	1.66973	0.06587
0.5	1.0	0.8	1.0	0.03	2.0	1.66782	0.06569
0.5	1.0	1.0	1.0	0.03	2.0	1.66710	0.06558
0.5	1.0	0.2	0.0	0.03	2.0	1.52642	0.06556
0.5	1.0	0.2	1.0	0.03	2.0	1.68544	0.06693
0.5	1.0	0.2	2.0	0.03	2.0	1.83064	0.06810
0.5	1.0	0.2	3.0	0.03	2.0	1.96496	0.06912
0.5	1.0	0.2	1.0	0.0	2.0	1.68531	0.06692
0.5	1.0	0.2	1.0	0.4	2.0	1.68695	0.06704
0.5	1.0	0.2	1.0	0.8	2.0	1.68845	0.06715
0.5	1.0	0.2	1.0	1.2	2.0	1.68971	0.06726
0.5	1.0	0.2	1.0	0.2	0.0	1.65224	0.06567
0.5	1.0	0.2	1.0	0.2	1.0	1.66900	0.06632
0.5	1.0	0.2	1.0	0.2	2.0	1.68614	0.06698
0.5	1.0	0.2	1.0	0.2	3.0	1.70370	0.06764

as coupling number increases. It is noticed that the heat and mass transfer rates are more in case of viscous fluids. Therefore, the presence of microscopic effects arising from the local structure and micromotion of the fluid elements reduce the heat and mass transfer rates. Further, it is seen that both the Nusselt number and Sherwood number are increasing as the magnetic parameter is increasing. This is due to the motive force created by magnetic field applied normal to the fluid flow, which tends to accelerate the flow.

Figures (10) and (11) show the variation of heat and mass transfer coefficients with Prandtl number Pr. It is interesting to note that the Nusselt number is increasing whereas the Sherwood number is decreasing as Prandtl number increases. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of Pr. Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

The effect of Schmidt number on the heat and mass transfer coefficients is plotted in Figs. (12) and (13). It is clear that the Nusselt number is decreasing while the Sherwood numer is increasing with increasing values of Sc. The Schmidt number embodies the ratio of the momentum to the mass diffusivity. The Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. Its effect on the species concentration has similarities to the Prandtl number effect on the temperature. That is increase in the values of Sc cause the velocity and species concentration and its boundary layer thickness to decrease significantly.

The effect of Soret number S_r on heat and mass transfer coefficient is displayed in Figs. (14) and (15). It can be notice that the Nusselt number increases with the increase of Soret number S_r while the Sherwood number decrease.

The effect of Dufour number D_f on heat and mass transfer coefficient is displayed in Figs. (16) and (17). It is interesting to notice that the Nusselt number increases with the increase of Dufour number D_f whereas the Sherwood number decrease with the increase in the Dufour number D_f.

Table 2 shows that the effects of the coupling number N, Prandtl number Pr, Schmidt number Sc, Soret number Sr and Dufour number Df and the magnetic parameter M on the skin friction Cf and the dimensionless wall couple stress Mw. It is seen from this table that both the skin friction and the wall couple stress decrease with the increasing coupling number N. The skin friction coefficient decreases and the wall couple stress increases with increasing Prandtl number and Schmidt number. Also, the effect of magnetic parameter is to increase the skin friction coefficient and decrease the wall couple stress. Further, it is observed that the skin friction coefficient is increasing and that of wall couple stress is decreasing with the increasing of Dufour number Df and increasing of Soret number Sr is showing the same effect that of Dufour number on skin-friction and wall couple stress.

6. Conclusions

In this paper, a boundary layer analysis for mixed convection heat and mass transfer in an electrically conducting micropolar fluid over a vertical plate with variable temperature and concentration wall conditions in the presence of a uniform magnetic field of magnitude B_0, Soret and Dufour effects is considered. Using the similarity variables, the governing equations are transformed into a set of non-linear ordinary differential equations and numerical solution for these equations has been presented for different values of physical parameters. It is observed that the both Nusselt number and Sherwood number i) decrease as coupling number increases, ii) increase as the magnetic parameter increases. The skin friction coefficient in the micropolar fluid is less compared to that of the Newtonian fluid. The present analysis has also shown that the flow field is appreciably influenced by the Magnetic parameter (M), Soret number (Sr) and Dufour number (Df).

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