Optimal Placement of PMU's Considering Sensitivity Analysis

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Abstract—**This paper presents various aspects of optimal Phasor measurement unit (PMU) placement problem with load sensitivity analysis. Binary integer linear programming based methodology for optimal placement of PMU in a given power network for full observability of that network is presented in this paper. First, complete observability of the given network is formulated conventionally and then, zero injection bus constraints are added in conventional formulation. Load sensitivity analysis is done using Newton-Rapson Load flow and most sensitive buses based on load sensitivity analysis, are sorted out. Minimum number of PMU's, less than the optimal number (without considering load sensitivity) are placed such that it covers most sensitive buses and also most of the buses are observed. In this paper optimal PMU placement problem considering sensitivity analysis is presented for IEEE-14 bus and IEEE-30 bus systems.**

Keywords-*Phasor Measurement Units (PMUs); Sensitivity Analysis; Optimization Algorithm*;

I. INTRODUCTION

Phasor Measurement Units provide time synchronized Phasor measurements in power system. PMUs were first introduced in 1980's and since then have become a mature technology with many applications. Assisted by global positioning systems (GPS), Phasor measurement units (PMUs) acquire time-synchronized Phasor measurement data for power system operations at high speed and high accuracy [1], [2]. They have been proved capable of significantly improving the performance of power system monitoring, protection, and control [3], [4]. As a result, PMUs are considered necessary components in smart grid.

Throughout this paper, it is presumed that a PMU placed on a bus measures the following parameters:

1) Voltage magnitude and phase angle of the bus;
2) Branch current Phasor of all branches connected to that bus.

PMU placement at all substations allows direct measurement of the state of the network. However, PMU placement on each bus of a system is difficult to achieve either due to cost factor or due to non-existence of communication facilities in some substations. Moreover, as a consequence of Ohm's Law, when a PMU is placed at a bus, neighboring busses also become observable [5], [6]. This implies that a system can be made observable with a lesser number of PMUs than the number of busses.

In [7], the author has proposed Recursive Tabu Search method to solve optimal PMU placement (OPP) problem and used numerical method to check the network observability. Author in [8], introduced a new method for OPP problem, in which problem is formulated as a quadratic minimization problem subject to nonlinear observability constraints. The optimal solution is obtained by an unconstrained nonlinear weighted least squares (WLS) approach. The OPP problem is solved based on Probabilistic cost/benefit analysis in [9] where the reduction of system rick cost is recognized as benefit with use of minimum number of PMU's. In [10], author considered power system controlled islanding so that power system remains observable in controlled islanding operation as well as normal operating conditions. In [11], author has proposed a new algorithm, Memetic Algorithm (MA) for OPP problem, which combines the global optimization power of genetic algorithms with local solution tuning using the hill-climbing method.

In this paper, OPP problem is solved using Integer Linear Programming considering sensitivity analysis. As the cost of one PMU is very high, so it is required to place minimum number of PMU's without neglecting the safety and security of the system. So the most sensitive buses in given power system based on load changes, are found and PMU's are optimally placed such that all the sensitive buses are observed. Thus the number of PMU's are further reduced than the proposed number in [8], [9], [10], [11] and [12], reducing the system cost.

II. PROBLEM FORMULATION

PMU placed at a given bus is capable of measuring the voltage Phasor of the bus as well as the Phasor currents for all lines incident to that bus. Thus, the entire system can be made observable by placing PMUs at strategic buses in the system. Moreover, when zero injection buses, namely, buses with no loads or generations connected, are present in a power system, the number of PMUs needed to achieve full observability can be further reduced. In this section, two formulations of the OPP problem are stated. The first formulation does not consider zero injections, whereas the second one takes them into account.

A. Without considering zero injection buses

For an n-bus system, the PMU placement problem can be formulated as follows:

$$\min \sum_{i}^{n} w_i \, . x_i \quad (1)$$

$$s.t \; f(X) \geq \hat{1}$$

Where

X is a binary decision variable vector, whose entries are defined as:

$$x_i = \begin{cases} 1 & if\ a\ PMU\ is\ installed\ at\ bus\ i \\ & . \\ 0 & Otherwise \end{cases}$$

w_i, is the cost of the PMU installed at bus i;

$f(X)$ is a vector function, whose entries are non-zero if the corresponding bus voltage is solvable using the given measurement set and zero otherwise.

$\hat{1}$ is a vector whose entries are all ones.

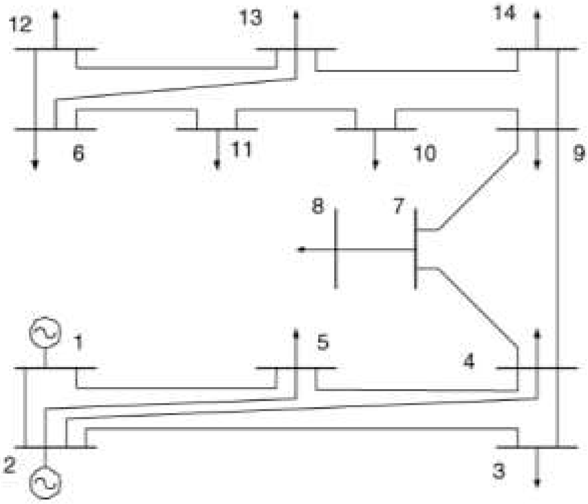

Fig-1: IEEE 14 Bus Test System (Seventh bus is zero injection bus)

Consider the IEEE-14 bus system shown in Fig. 1. The constraints are written as [12]:

$Bus-1: x_1 + x_2 + x_5 \geq 1$ (2)

$Bus-2: x_1 + x_2 + x_3 + x_4 + x_5 \geq 1$ (3)

$Bus-3: x_2 + x_3 + x_4 \geq 1$ (4)

$Bus-4: x_2 + x_3 + x_4 + x_5 + x_7 + x_9 \geq 1$ (5)

$Bus-5: x_1 + x_2 + x_4 + x_5 \geq 1$ (6)

$Bus-6: x_6 + x_{11} + x_{12} + x_{13} \geq 1$ (7)

$Bus-7: x_4 + x_7 + x_8 + x_9 \geq 1$ (8)

$Bus-8: x_7 + x_8 \geq 1$ (9)

$Bus-9: x_4 + x_7 + x_9 + x_{10} + x_{14} \geq 1$ (10)

$Bus-10: x_9 + x_{10} + x_{11} \geq 1$ (11)

$Bus-11: x_6 + x_{10} + x_{11} \geq 1$ (12)

$Bus-12: x_6 + x_{12} + x_{13} \geq 1$ (13)

$Bus-13: x_6 + x_{12} + x_{13} + x_{14}$ (14)

$Bus-14: x_9 + x_{13} + x_{14} \geq 1$ (15)

Objective (1) gives the minimum number of PMU's required and equations (2) – (15) are the constraints. The above equations can be written in form of $AX = B$ and solved using binary integer linear programming in MATLAB. The matrix A [12] is formed using above constraints and matrix B consists of all one's. For above IEEE-14 bus system, the optimal locations are given as buses 2, 6, 7 and 9. Similarly results for IEEE-30 bus and IEEE-57 bus are given in Table-I. These are optimal locations when zero injection buses are not considered. When zero bus injections are considered the number of PMU's required becomes less than we obtained now.

B. Considering zero injection buses

When zero injection buses are considered, the number of PMU's required for full system observability can be reduced. Zero injection buses are buses with no load and no injection i.e., during planning of power system, it was observed that in long transmission lines there is a voltage drop at receiving end. Therefore, to maintain voltage profile, at a bus only voltage improvement devices are installed. These buses are zero injection buses. Thus at zero injection buses no current is injected into system. This concept is used to reduce the number of PMU's. To illustrate this, consider a small system shown in Fig-2, suppose that Bus 3 is a zero injection bus. There is only one PMU needed to be placed at Bus 1 for full observability. When a PMU is installed at Bus 1, the voltage Phasor at Buses 2 and 3, namely, V_2andV_3, together with the current phasor between Buses 1 and 3, $I_{1,3}$ are known. By Ohm's Law, the current phasor between Buses 2 and 3,$I_{2,3}$is solvable. Since Bus 3 is a zero injection bus, applying Kirchhoff's Current Law (KCL), we have:

$$I_{1,3} + I_{2,3} + I_{3,4} = 0 \quad (16)$$

Where all currents are leaving from bus 3

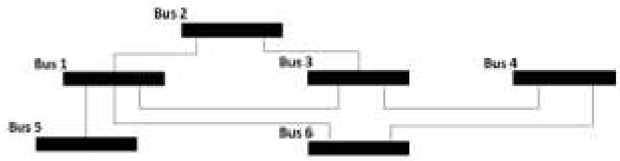

Fig-2: Zero injection bus modeling

Thus from (16), $I_{3,4}$ is solvable which makes Bus 4 observable. Earlier, without considering bus 3 as zero injection bus, bus 4 was not observable when PMU placed at bus 2 but now considering zero injection bus, bus 4 also observable. Now to solve for optimal number of PMU's, the effect of zero injection buses is to be included in matrix A. for that follow two rules:

1. If buses connected to observable zero injection bus, are observable except one then that bus is also observable.
2. When buses connected to an unobservable zero injection bus are all observable, the zero injection bus also be identified as observable by applying the KCL at zero injection bus.

The equations at buses, which are connected to zero injection bus are modified using above rules and solved using linear integer programming and the equation at zero injection bus is neglected, which can be represented in matrix A by making zero injection bus row and columns as zero and corresponding element in matrix B also. For IEEE-14 bus system equation at bus 8 changes as:

$$Bus-8: x_4+x_8+x_9 \geq 1 \qquad (17)$$

For IEEE-14 bus system with above changes, the optimal number of PMU's required is reduced to 3,which are placed at buses 2, 6 and 9.Hence with considering zero injection buses number of PMU's required are reduced. Similarly for IEEE-30 and IEEE-57 bus systems results are given in Table-II.

TABLE I. OPTIMAL PMU PLACEMENT WITHOUT CONSIDERING ZERO INJECTION BUSES

IEEE bus system	No. of PMU's required	PMU location
IEEE-14 bus system	4	2,6,8,9
IEEE-30 bus system	10	1,7,9,10,12,18,24,25,27,28
IEEE-57 bus system	17	1,4,6,10,20,23,25,27,29,32,36,39,41,45,46, 49,54

TABLE II. OPTIMAL PMU PLACEMNET CONSIDERING ZERO INJECTION BUSES

IEEE bus system	No. of zero injection buses	Zero injection buses	No. of PMU's required	PMU location
IEEE-14 bus system	1	7	3	2,6,9
IEEE-30 bus system	8	6,9,11.13,22,25,27,28	7	3,7,10,12,15,18, 25
IEEE-57 bus system	15	4,7,11,21,22,24,26,34, 37, 39,40,45,46, 48	12	1,9,10,15,18, 22,28,30,32,49, 53, 56

III. PMU PLACEMENT ACCORDING TO SENSITIVITY ANALYSIS

The cost of a single PMU is very expensive so to make system work in economical way, it is required to reduce the number of PMU's in system without neglecting safety and protection the system. So, now PMU's less than actual number are placed based sensitivity analysis, which is carried out on given power system. With sensitivity analysis most sensitive are found. Here, the sensitive bus means system is more sensitive with change of load at that bus. The sensitivity of a particular bus is found by using the load flow analysis [13]. For a change in load at a bus, the changes in line flows are calculated. After calculating changes in line flows for change in load at all buses, the bus which has got maximum changes in flows in considered most sensitive bus. The PMU's are placed such that most sensitive buses are observed with reduced number of PMU's than obtained in [8], [11] and [12].

Sensitivity of a bus based on load is decided by calculating the sum of changes in line flows with the change in load at that bus. Steps to find load sensitivity are:

Step 1: Run the basic NR load flow [13], [14].

Step 2: Calculate and save the base line flows in a vector $base\_flow$

Step 3: Change load at 1st bus by 10% and run rerun NR Load flow and save the new line flows asnew_flow.

Step 4: Calculate change in line flows

$$diff = new\_flow - base\_flow$$

Step 5: Calculate the sum of changes in line flows

$$sum\_dif = \sum_{k=1}^{nline} diff(i)$$

Step 6: Repeat from step 3 to step 5 for all buses for different loading conditions

Step 7: The bus with high sum_dif value is the most sensitive bus.

The vector sum_dif is sorted in descending order and the bus with maximum sum_dif value is called most sensitive bus. Now PMU's less than actual number are placed such that they covers most sensitive buses and makes system almost observable. Results for 10% and 20% change in load for IEEE-14 bus and IEEE-30 bus system are shown in Table-III and Table-IV respectively and PMU's location based on load sensitivity for IEEE-14 and IEEE-30 bus system are given in Table-V.

PMU's are placed based on topological observation of the power network. First, most sensitive bus is considered and possible places to place PMU are searched so that the bus is observed. Among these possibilities, best one is chosen such that placement of PMU at that bus will also observe next sensitive buses and covers many buses.

IV. RESULTS

Load sensitivity analysis is done on IEEE-14 and IEEE-30 bus systems and the results are shown in Table-III and Table-IV respectively. From Table-III, it is observed that, for IEEE-30 bus system, buses 5, 8, 21, 30 and 7 are most sensitive buses. Any small change in the load at these buses, creating more change in line flows so it is important to observe these buses for the safety and security of the system. Considering zero injection buses, number of PMU's required were 7 but considering load sensitivity analysis, the number of PMU's used are only 6 unlike in [15], it is used 9 PMU's for IEEE-30 bus system. These PMU's are placed on topological observation of the system. With PMU's placed at locations given in Table-IV, only two buses (3 and 26) are not observable and are neglected as they are of not sensitive buses. For IEEE-14 bus system, the most sensitive buses are 3, 9, 4 and 14. Least sensitive buses are 6, 11. 2 and 5, therefore they need not to be observed and hence used only 2 PMU's rather than 3 PMU's.

TABLE III. SENSITIVITY OF BUSES FOR A 10% AND 20% CHANGE IN LOAD FOR IEEE-14 BUS SYSTEM

Bus number	sum_dif for 10% change in load	sum_dif for 20% change in load
3	0.2785	0.5592
9	0.1413	0.2828
4	0.1365	0.2734
14	0.0803	0.1609
13	0.0705	0.1408
10	0.0478	0.0955
6	0.0476	0.0949
12	0.0336	0.0670
2	0.0303	0.0605
5	0.0186	0.0369
11	0.0176	0.0351
1	0	0
7	0	0
8	0	0

TABLE IV. SENSITIVITY OF BUSES FOR A 10% AND 20% CHANGE IN LOAD FOR IEEE-30 BUS SYSTEM

Bus number	sum_dif for 10% change in load	Bus number	sum_dif for 20% change in load
5	0.3431	5	0.6896
21	0.1402	21	0.2708
8	0.1343	8	0.2692
30	0.1208	30	0.1986
7	0.0925	7	0.1857
19	0.0731	19	0.1570
24	0.0686	24	0.1481
12	0.0670	17	0.1399
17	0.0648	12	0.1335
15	0.0584	15	0.1166
14	0.0452	14	0.0901
10	0.0367	10	0.0738
2	0.0344	26	0.0692
26	0.0311	2	0.0684
16	0.0268	16	0.0537
23	0.0255	23	0.0506
18	0.0247	18	0.0492
4	0.0234	4	0.0463
29	0.0206	29	0.0408
20	0.0161	20	0.0324
3	0.0080	3	0.0154
1	0	1	0
6	0	6	0
9	0	9	0
11	0	11	0
13	0	13	0
22	0	22	0
25	0	25	0
27	0	27	0
28	0	28	0

TABLE V. PMU PLACEMENT BASED ON SENSITIVITY ANALYSIS FOR IEEE-14 AND IEEE-30 BUS SYSTEMS

IEEE bus system	No. of PMU's placed	PMU location	Non observable buses
IEEE-14 bus system	2	2,9	6,11,12,13
IEEE-30 bus system	6	2, 10, 12,18,24,27	3,26

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V. CONCLUSION AND FUTURE WORK

In this paper, optimal placement of PMU's using linear integer programming is presented first. Since the cost of PMU is very high, therefore sensitivity analysis is used to reduce the number of PMU's further. With sensitivity analysis some buses are unobservable therefore, there needs action to do with those non-observable buses which can be taken for future work

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