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International Journal of Systems Science

ISSN: 0020-7721 (Print) 1464-5319 (Online) Journal homepage: www.tandfonline.com/journals/tsys20

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To cite this article: D. Dutta & Pavan Kumar (2015) Application of fuzzy goal programming approach to multi-objective linear fractional inventory model, International Journal of Systems Science, 46:12, 2269-2278, DOI: 10.1080/00207721.2013.860639

To link to this article: https://doi.org/10.1080/00207721.2013.860639

Published online: 20 Nov 2013.

International Journal of Systems Science, 2015
Vol. 46, No. 12, 2269–2278, http://dx.doi.org/10.1080/00207721.2013.860639

Application of fuzzy goal programming approach to multi-objective linear fractional inventory model

D. Dutta and Pavan Kumar*

Department of Mathematics, National Institute of Technology, Warangal, Andhra Pradesh, India

(*Received 22 April 2013; accepted 1 October 2013*)

In this paper, we propose a model and solution approach for a multi-item inventory problem without shortages. The proposed model is formulated as a fractional multi-objective optimisation problem along with three constraints: budget constraint, space constraint and budgetary constraint on ordering cost of each item. The proposed inventory model becomes a multiple criteria decision-making (MCDM) problem in fuzzy environment. This model is solved by multi-objective fuzzy goal programming (MOFGP) approach. A numerical example is given to illustrate the proposed model.

Keywords: inventory model; membership function; goal programming; multi-objective optimisation; fractional programming

1. Introduction

Charnes and Cooper (1962) published their classical paper in which they show that a linear fractional programme with one ratio can be reduced to a linear programme using a nonlinear variable transformation. Brief definitions of Multi-Objective Linear Fractional Programming (MOLFP) problem and Fuzzy Linear Programming (FLP) are presented in Appendix. Zimmermann (1976) first extended the FLP approach to a conventional multi-objective linear programming (MOLP) problem. For each of the objective functions in this problem, the DM was assumed to have a fuzzy goal, such as ‘the objective function should be substantially less than or equal to some value’. Later, Zimmermann (1978) described the fuzzy programming and linear programming with several objective functions. The study of fractional programmes with only one ratio has largely dominated the literature in this field until about 1980. Kornbluth and Steuer (1981) proposed the goal programming with linear fractional criteria. Luhandjula (1984) tried to apply the variable transformation to solve the MOLFP problem. Since then two other monographs solely devoted to fractional programming appeared, one authored by Craven (1988). Membership functions, such as linear, piecewise linear, exponential and hyperbolic functions, were used in different analysis. In general, the non-increasing and non-decreasing linear membership functions are frequently applied for the inequalities with less than or equal to and greater than or equal to relationships, respectively.

Dutta, Tiwari, and Rao (1992) developed a set theoretic approach to solve a multiple objective linear fractional programming. Furthermore, Dutta, Tiwari, and Rao (1993a) commented over the fuzzy approaches for multiple criteria linear fractional optimisation. Subsequently, Dutta, Tiwari, and Rao (1993b) studied the effect of tolerance in fuzzy linear fractional programming. A Goal Programming Method for Solving Fractional Programming Problems via Dynamic Programming was proposed by Pal and Basu (1995). At the same time, Barros and Frenk (1995) studied the generalised fractional programming and cutting plane algorithms. Hariri and Abou-El-Ata (1997) proposed a multi-item production lot-size inventory model with varying order cost under a restriction. Geometric programming approach was applied to solve this inventory model.

Some researchers considered multi-objective approach to supply chain management. Sabri and Beamon (2000) proposed a multi-objective approach to simultaneous strategic and operational planning in supply chain design. Later, Maulik, Pal, and Moitra (2003) developed a goal programming procedure for fuzzy MOLFP problem. At the same time, Fung, Tang, and Wang (2003) published a paper on multi-product aggregate production planning with fuzzy demands and fuzzy capacities.

Rafiei, Mohammadi, and Torabi (2013) presented a multi-segment multi-product multi-period supply chain network design model, which minimises the expected cost. They proposed an algorithm based on genetic algorithm (GA)-priority to solve this model. Moreover, Ramana, Rao, and Kumar (2013) developed a generic hierarchy model for decision-makers who can prioritise the supply chain metrics under performance dimensions of leagile supply chain.

Different models arise naturally in decision-making when several rates are to be optimised simultaneously and

*Corresponding author. Email: pavankmaths@gmail.com

a compromise is sought which optimises a weighted sum of these rates. In light of the applications of single-ratio fractional programming numerators and denominators may be representing output, input, profit, cost, capital, risk or time, for example. A multitude of applications of the sum-of-ratios problem can be found in this way. Included is the case where some of the ratios are not proper quotients. This describes situations where a compromise is sought between absolute and relative terms like profit and return on investment (profit/capital) or return and return/risk, for example. A fuzzy approach to solve a multi-objective linear fractional inventory model was used by Sadjadi, Aryanezhad, and Sarfaraj (2005). They considered two goals as fractional under two constraints: space capacity constraint and budget constraint. Their aim was simultaneously to maximise the profit ratio to holding cost and to minimise the back orders ratio to total ordered quantities. Chen (2005) used fractional programming approach to stochastic inventory problems. Some applications of MOLFP in inventory were proposed by Toksarı (2008). An application of fuzzy goal programming approach with different importance and priorities to aggregate production planning was introduced by Belmokaddem, Mekidiche, and Sahed (2009). They made an attempt to minimise total production and work force costs, inventory carrying costs and rates of changes in work force.

Very recently, Dutta and Kumar (2012) proposed a goal programming approach using trapezoidal membership function for fuzzy multi-objective linear fractional optimisation. Multi-objective optimisation models were proposed by several researches in fuzzy environment as well as in stochastic environment. Most of researchers assume that the product quality is perfect. However, practically, it can often be observed that the product quality is not always perfect.

Gani and Maheswari (2013) presented a model on inspection cost and imperfect quality items with multiple imprecise goals in supply chains in an uncertain environment. Chakraborty and Chatterjee (2013) considered the material selection problem as multi-criteria decision-making (MCDM). They solved five material selection problems by using different MCDM techniques. Finally, they observed that choices of the best suited materials solely depend on the criterion having the maximum priority value.

Chakrabortty and Hasin (2013) attempted to solve an aggregate production planning problem by fuzzy-based genetic algorithm (FBGA) approach. Sadi-Nezhad and Shahnazari-Shahrezaei (2013) investigated the ranking fuzzy numbers using the concept of preference ration. They introduced the weakness of this method. Then, they proposed a new approach based on the concept of utility function which takes the opinion of decision-maker (DM) for ranking fuzzy numbers into account.

This paper presents an application of fuzzy goal programming approach to solve the multi-objective fractional inventory model. Many realistic inventory problems deal with more than one objective function which may be in conflict with each other. Our model refers to a multi-item inventory problem, with limited capacity of warehouse, constraints on investment in inventory and budgetary constraint on ordering cost of each item. Our aim is to simultaneously maximise the profit ratio to backordered quantity and to minimise the holding cost ratio to total ordered quantities.

The paper is outlined as follows. The notations and assumptions are defined in Section 2. The proposed inventory model is presented in Section 3. In Section 4, we describe the solution procedure. A numerical example is solved in Section 5. In Section 6, we analyse the results. Finally, we conclude in Section 7.

2. Notations and assumptions

2.1. Notations

In our proposed inventory model, we adopt the following notations:

$n =$ Number of items
$k =$ Fixed cost per order
$B =$ Maximum available budget for all items
$F =$ Maximum available space for all items

For ith item: ($i = 1, 2, \ldots n$), we define the following:

$Q_i =$ Ordering quantity of item i (a decision variable)
$h_i =$ Holding cost per item per unit time for ith item
$P_i =$ Purchasing price of ith item
$S_i =$ Selling price of ith item
$D_i =$ Demand quantity per unit time of ith item
$f_i =$ Space required per unit for ith item
$OC_i =$ Ordering cost of ith item.

2.2. Assumptions

The following assumptions are being made in developing the mathematical model:

(1) A multi-item inventory model is considered.
(2) Time horizon is infinite, and there is only one period in the cycle time.
(3) Demand rate is constant over time for each item.
(4) Lead time is zero.
(5) Holding cost is known and constant for each item.
(6) Purchase price of the item is constant for each item, i.e. no discount is available.
(7) Shortages are not allowed.
(8) No deterioration allowed.

3. Proposed inventory model

A multi-item inventory system under resources constraints is introduced as a linear fraction programme. This model refers to a multi-item inventory problem, with limited capacity of warehouse and constraints on investment in inventories. For each item, we also imposed the constraint on ordering cost. Demand for each item is known and constant and it must be met over an infinite horizon without shortages or backlogging. Replenishments are instantaneous and we assume a zero lead time. In real life, we observe inventory problems deal with more than one objective function. These objectives may be in conflict with each other, or may not be. In such type of inventory models, the decision-maker is interested to maximise or minimise two or more objectives simultaneously over a given set of decision variables. We call this model inventory as linear fractional inventory model.

We propose a multi-item inventory system with fractional objective functions. Without loss of generality, we assume there is only one period in the cycle time. Then

$$\text{Profit} = \sum_{i=1}^{n} (S_i - P_i)\, Q_i \tag{1}$$

$$\text{Holding cost} = \sum_{i=1}^{n} \frac{h_i Q_i}{2} \tag{2}$$

$$\text{Ordering cost} = \sum_{i=1}^{n} \frac{kD_n}{Q_n} \tag{3}$$

$$\text{Back ordered quantity} = \sum_{i=1}^{n} (D_i - Q_i) \tag{4}$$

$$\text{Total ordering quantity} = \sum_{i=1}^{n} Q_i. \tag{5}$$

Constraints: We consider the following constraints in this modelling formulation.

Upper limit of the total amount investment:

$$\sum_{i=1}^{n} P_i Q_i \le B. \tag{6}$$

Limitation on the available warehouse floor space in the store:

$$\sum_{i=1}^{n} f_i Q_i \le F. \tag{7}$$

Budgetary constraints on ordering cost as follows.

For an effective inventory management, the control over the ordering cost is also one of the priorities of the decision-maker (DM). The low ordering cost plays an important role to minimise the total inventory cost, and to optimise value of ordering quantity. We are interested to impose the upper limit of ordering cost as a constraint.

Since $OC_1, OC_2, \ldots, OC_n$ are the ordering cost of first item, second item,. . ., nth item, therefore, we can express the concerned constrained as follows:

$$\begin{array}{lll} \text{For first item} & \dfrac{kD_1}{Q_1} \le OC_1 & \\ & \Rightarrow \quad \mathrm{k}D_1 - (OC_1)Q_1 \le 0 & \\ \text{For second item} & \frac{kD_2}{Q_2} \le OC_2 & \\ & \Rightarrow \quad \mathrm{k}D_2 - (OC_2)Q_2 \le 0 & \\ \text{Similarly, for } n\text{th item} & \frac{kD_n}{Q_n} \le OC_n & \\ & \Rightarrow \quad \mathrm{k}D_n - (OC_n)Q_n \le 0. & \end{array} \tag{8}$$

For optimal policy of inventory, the decision-maker desires to generate an optimal policy so that the profit can be maximised, and the holding cost can be minimised.

With the maximisation of profit, if the backordered quantity is minimised, then this policy will be most preferred, and economical. Likewise, with the minimisation of holding cost, the policy is such that the total order quantity is optimised. So, it is practical and wise to consider the objective functions in the form of fractions and automatically trade-offs between the above objectives will be considered. Such objectives are called fractional objectives.

Hence, the multi-objective fractional programming model is:

$$\begin{array}{ll} \text{Maximise} & \dfrac{\text{Profit}}{\text{Back ordered quantity}} \\ \text{Minimise} & \dfrac{\text{Holding cost}}{\text{Ordering quantity}} \\ \text{Subject to} & \text{Total Amount Investment Constraint,} \\ & \text{Warehouse Floor Space Constraint,} \\ \text{and} & \text{Budgetary Constraints on Ordering Cost} \\ & \text{of each item.} \end{array}$$

Mathematically, we can rewrite above optimisation problem as follows:

$$\text{Maximise} \quad Z_1 = \frac{\sum_{i=1}^{n} (S_i - P_i)\, Q_i}{\sum_{i=1}^{n} (D_i - Q_i)} \tag{9}$$

$$\text{Minimise} \quad Z_2 = \frac{\sum_{i=1}^{n} \frac{h_i Q_i}{2}}{\sum_{i=1}^{n} Q_i} \tag{10}$$

$$\text{Subject to} \quad \sum_{i=1}^{n} P_i Q_i \le B, \tag{11}$$

$$\sum_{i=1}^{n} f_i Q_i \le F, \tag{12}$$

$$kD_n - (OC_n)Q_n \le 0 \text{ (for each } n), \tag{13}$$

$$\text{and} \quad \text{all } Q_n \ge 0, \quad OC_n > 0. \tag{14}$$

Obviously, this model is multi-objective linear fractional programming problem (MOLFPP), which can be easily solved by using fuzzy goal programming (FGP) approach.

4. Solution procedure

4.1. *Fuzzy goal programming (FGP) approach*

In our approach, first the objectives are transformed into fuzzy goals by means of assigning an aspiration level to each of them. Then achievement of the highest membership value (unity) to the extent possible of each of the fuzzy goals is considered.

Let be the aspiration level assigned to the *k*th objective (*X*). Then, the fuzzy goals appear as:

$$Z_k(X) \succcurlyeq g_k (\text{for maximising } Z_k(X)), \tag{15}$$

$$\text{and } Z_k(X) \preccurlyeq g_k (\text{for minimising } Z_k(X)), \tag{16}$$

where $\succcurlyeq$, $\preccurlyeq$ indicate the fuzziness of the aspiration levels, and are to be understood as 'essentially more than' and 'essentially less than' in the sense of Zimmermann [2]. Hence, the Fuzzy Linear Fractional Goal Programming (FLFGP) can be stated as follows:

$$\text{Find } X$$

$$\text{Satisfying } Z_k(X) \succcurlyeq g_k, k = 1, 2, \ldots, k_1 \tag{17}$$

$$Z_k(X) \preccurlyeq g_k, k = k_1 + 1, \ldots, K \tag{18}$$

$$\text{Subject to } AX \le, =, \text{or} \ge b,\ X \ge 0. \tag{19}$$

Now, in case of fuzzy programming, the fuzzy goals are characterised by their associated membership functions. For *k*th fuzzy goal $Z_k(X) \succcurlyeq g_k$, the membership function μ_k can be written as:

$$\mu_k(X) = \begin{cases} 1 & \text{if } Z_k \ge g_k \\ \frac{Z_k(X) - l_k}{g_k - l_k} & \text{if } l_k \ge Z_k(X) \le g_k \\ 0 & \text{if } Z_k(X) \le l_k \end{cases}. \tag{20}$$

Also, for *k*th fuzzy goal $Z_k(X)$ W g_k, the membership functionμ_k is:

$$\mu_k(X) = \begin{cases} 1 & \text{if } Z_k(X) \le g_k \\ \frac{u_k - Z_k(X)}{u_k - h_k} & \text{if } h_k \le Z_k(X) \le u_k \\ 0 & \text{if } Z_k(X) \ge u_k \end{cases}, \tag{21}$$

where u_k is the upper tolerance limit, and $l_k \le g_k \le h_k \le u_k$ = real numbers.

In fuzzy programming, the highest degree of membership function is 1. Hence, for the above-defined membership functions, the flexible membership goals with the aspiration level 1 can be written as:

$$\frac{Z_k(X) - l_k}{g_k - l_k} + d_k^- - d_k^+ = 1, \tag{22}$$

$$\text{and } \frac{u_k - Z_k(X)}{u_k - h_k} + d_k^- - d_k^+ = 1, \tag{23}$$

where d_k^- (≥ 0) and d_k^+ (≥ 0) with $d_k^- d_k^+ = 0$ are the under-deviation and over-deviation, respectively, from the aspiration levels. We see that the membership goals in Equations (22) and (23) are inherently nonlinear in nature and this may create computational difficulties in the solution process. To avoid such problems, the linearisation of membership goals must be done. So, we express the *k*th membership goal in Equation (22) as follows:

$$L_k Z_k(X) - L_k l_k + d_k^- - d_k^+ = 1, \text{ where } L_k = \frac{1}{g_k - l_k},$$

$$\Rightarrow L_k Z_k(X) + d_k^- - d_k^+ = 1 + L_k l_k,$$

$$\Rightarrow L_k(c_k X + \alpha_k) + d_k^-(d_k X + \beta_k) - d_k^+(d_k X + \beta_k)$$

$$= L_k'(d_k X + \beta_k); \text{ by Equation (1) and letting } L_k' = 1 + L_k l_k,$$

$$\Rightarrow C_k X + d_k^-(d_k X + \beta_k) - d_k^+(d_k X + \beta_k) = G_k, \tag{24}$$

$$\text{where } C_k = L_k c_k - L_k' d_k \text{ and } G_k = L_k'\beta_k - L_k\beta_k. \tag{25}$$

Similar goal expressions for the membership goal in Equation (23) can also be obtained. Now, using the method of variable change, the goal expression in Equation (24) can be linearised as follows.

Taking $D_k^- = d_k^-(d_k X + \beta_k)$ and $D_k^+ = d_k^+ (d_k X + \beta_k)$, the linear form of the expression in Equation (24) is obtained as

$$C_k X + D_k^- - D_k^+ = G_k \tag{26}$$

$$\text{with } D_k^-,\ D_k^+ \ge 0 \text{ and } D_k^- D_k^+ = 0 \text{ since } d_k^-, d_k^+ \ge 0 \text{ and } d_k X + \beta_k > 0. \tag{27}$$

Now, to minimise d_k^- means to minimise $D_k^-/(d_k X + \beta_k)$ which is also a nonlinear expression. Also, when a membership goal is fully achieved, $d_k^- = 0$ and when its achievement is zero, $d_k^- = 1$ are found in the solution. So, to involve $d_k^- \le 1$ in the solution, we need to impose the following constraint to the model of the problem:

$$D_k^-/(d_k X + \beta_k) \le 1$$

$$\Rightarrow -d_k X + D_k^- \le \beta_k. \tag{28}$$

It may be noted that any such constraint corresponding to does not arise in the model formulation. Now, if the simplest version of GP is introduced to formulate the model of the problem under consideration, then the GP model formulation becomes:

$$\left.\begin{array}{ll}\text{Find } X \text{ so as to} & \\ \text{Minimise} & Z = \sum_{k=1}^{K} w_k^- D_k^- \\ \text{Satisfying} & C_k X + D_k^- - D_k^+ = G_k \\ \text{Subject to} & AX \geq b \\ & -d_k X + D_k^- \leq \beta_k, \\ X \geq 0,\ D_k^-, D_k^+ \geq 0 \text{ for } k = 1, 2, \ldots, K & \end{array}\right\}, \tag{29}$$

where Z is the fuzzy achievement function consisting of the weighted under-deviational variables, in which the numerical weights w_k^- (≥ 0), $k = 1, 2, \ldots, K$ are the relative importance of achieving the aspiration levels of the respective fuzzy goals subject to the constraints set.

Now, for the relative importance of the fuzzy goals properly, the weighting scheme can be used to assign the values to w_k^- ($k = 1, 2, \ldots, K$). In our formulation, w_k^- can be determined as

$$w_k^- = \begin{cases} \dfrac{1}{g_k - l_k}, & \text{for maximisation fuzzy goal} \\ \dfrac{1}{u_k - h_k}, & \text{for minimisation fuzzy goal} \end{cases}. \tag{30}$$

The mathematical programming problem in Equation (29) can be solved by using min–sum method. The final goal expression is:

$$C_k X + D_k^- - D_k^+ = G_k, \tag{31}$$

which is actually the linearised form of the original goal expression

$$\begin{aligned} L_k (c_k X + \alpha_k) &+ d_k^- (d_k X + \beta_k) - d_k^+ (d_k X + \beta_k) \\ &= L_k' (d_k X + \beta_k) \\ &\Rightarrow L_k \frac{(c_k X + \alpha_k)}{(d_k X + \beta_k)} + d_k^- - d_k^+ = L_k', \end{aligned} \tag{32}$$

where $L_k = 1/(g_k - l_k)$, $L_k' = 1 + L_k l_k$, and where $(g_k - l_k)$ denotes the admissible violation constant.

Its generalised fractional form is given by

$$Z_k(X) + (g_k - l_k) d_k^- - (g_k - l_k) d_k^+ = g_k. \tag{33}$$

Now, the conventional form of a goal in GP is

$$Z_k(X) + n_k - p_k = b_k, \tag{34}$$

where $Z_k(X)$ may be linear/nonlinear including fractional, where n_k and p_k represent the under and over-deviational variables, respectively. Comparing the above two forms of goals, we find that

$$n_k = (g_k - l_k) d_k^-, \text{ which gives } d_k^- = n_k/(g_k - l_k). \tag{35}$$

Now, due to the minimising the weighted goal deviational variables in GP, we observe that the GP formulations for the two forms of goals are equivalent with $d_k^- = n_k/(g_k - l_k)$. Thus, the weights of the goals in the proposed model are reciprocal to the admissible violation constants.

5. Numerical example

We consider a three item inventory problem. All input parameter values are given in Table 1. Using this data, the multi-objective inventory problem can be expressed as:

$$\begin{aligned} \text{Maximise } Z_1 &= \frac{\sum_{i=1}^{3} (S_i - P_i) Q_i}{\sum_{i=1}^{3} (D_i - Q_i)} \\ &= \frac{25Q_1 + 20Q_2 + 10Q_3}{4500 - Q_1 - Q_2 - Q_3} \end{aligned} \tag{36}$$

$$\text{Minimise } Z_2 = \frac{\sum_{i=1}^{3} \frac{h_i Q_i}{2}}{\sum_{i=1}^{3} Q_i} = \frac{6Q_1 + 8Q_2 + 9Q_3}{Q_1 + Q_2 + Q_3} \tag{37}$$

$$\text{Subject to } 625Q_1 + 730Q_2 + 440Q_3 \leq 900000, \tag{38}$$

$$2Q_1 + 4Q_2 + 2Q_3 \leq 13000, \tag{39}$$

$$7(1000) - 320Q_1 \leq 0 \tag{40}$$

$$7(2000) - 350Q_2 \leq 0 \tag{41}$$

Table 1. Input data for three-item inventory problem.

Item	Holding cost per item per year, h_i (Rs.)	Purchasing price, P_i (Rs.)	Selling price, S_i (Rs.)	Demand quantity, D_i (units/year)	Ordering cost per order, QC_i (Rs.)	Space required for item, f_i (sq. metre)	Fixed cost per order, k	Maximum available space, F (sq. metre)	Maximum inventory budget, B (Rs.)
$i = 1$	12	625	650	1000	320	2	7	13,000	900,000 (nine lacs)
$i = 2$	16	730	750	2000	350	4			
$i = 3$	18	440	450	1500	250	2			

$$7(1500) - 250Q_3 \le 0 \quad (42)$$

$$\text{all } Q_i \ge 0, \quad i = 1, 2, 3. \quad (43)$$

We find the individual best solution: $Z_1 \equiv 11.56$ at $(1363.71, 40, 42)$ and $Z_2 \equiv 6.14$ at $(1363.71, 40, 42)$. Let the fuzzy aspiration levels of the two objectives be $g_{Z_1} = 13$ and $g_{Z_2} = 5$, respectively. Then the problem can be designed as:

Find $Q\,(Q_1, Q_2, Q_3)$ so as to satisfy the following two fuzzy goals:

$$Z_1 = \frac{25Q_1 + 20Q_2 + 10Q_3}{4500 - Q_1 - Q_2 - Q_3} \succcurlyeq 13 \quad (44)$$

$$Z_2 = \frac{6Q_1 + 8Q_2 + 9Q_3}{Q_1 + Q_2 + Q_3} \preccurlyeq 5. \quad (45)$$

Subject to constraints (38)–(43).

For the fuzzy goal $Z_1 \succcurlyeq 13$, let the lower tolerance limit be $l_{z_1} = 8$. For the fuzzy goal $Z_2 \preccurlyeq 5$, let the upper tolerance limit be $u_{z_2} = 10$, as shown in Figures 1 and 2.

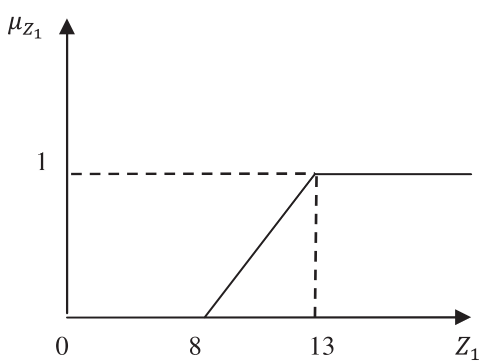

Figure 1. Membership function of Z_1. (Maximisation type).

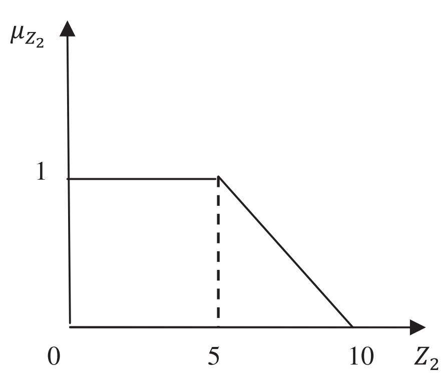

Figure 2. Membership function of Z_1. (Minimisation type).

Analytically, the linear membership functions are defined as follows:

For $Z_1 \preccurlyeq 13$,

$$\mu_{Z_1} = \frac{Z_1(Q) - l_{Z_1}}{g_{Z_1} - l_{Z_1}}, \quad \text{when } l_{Z_1} \le Z_1(Q) \le g_{Z_1}$$

$$= \frac{\frac{25Q_1+20Q_2+10Q_3}{4500-Q_1-Q_2-Q_3} - 8}{13 - 8}, \quad \text{when } 8 \le Z_1(Q) \le 13$$

$$= \frac{33Q_1 + 28Q_2 + 18Q_3 - 36000}{5(4500 - Q_1 - Q_2 - Q_3)}, \quad (46)$$

and for $Z_2 \preccurlyeq 5$,

$$\mu_{Z_2} = \frac{u_{Z_2} - Z_2(Q)}{u_{Z_2} - g_{Z_2}}, \quad \text{when } g_{Z_2} \le Z_2(Q) \le u_{Z_2}$$

$$= \frac{10 - \frac{6Q_1+8Q_2+9Q_3}{Q_1+Q_2+Q_3}}{10 - 5}, \text{ when } 5 \le Z_2(Q) \le 10$$

$$= \frac{4Q_1 + 2Q_2 + Q_3}{5(Q_1 + Q_2 + Q_3)}. \quad (47)$$

Then the membership goals can be expressed as

$$\frac{33Q_1 + 28Q_2 + 18Q_3 - 36000}{5(4500 - Q_1 - Q_2 - Q_3)} + d_1^- - d_1^+ = 1 \quad (48)$$

$$\frac{4Q_1 + 2Q_2 + Q_3}{5(Q_1 + Q_2 + Q_3)} + d_2^- - d_2^+ = 1, \quad (49)$$

where $d_i^-, d_i^+ \ge 0$, $d_1^- \le 1$, $d_2^- \le 1$ and $d_i^- d_i^+ = 0$ for all $i = 1, 2$. (50)

From Equations (48) and (49), we obtain

$$38Q_1 + 33Q_2 + 23Q_3 + D_1^- - D_1^+ = 58500 \quad (51)$$

$$-Q_1 - 3Q_2 - 4Q_3 + D_2^- - D_2^+ = 0 \quad (52)$$

$$\text{where } D_1^- = 5d_1^-(4500 - Q_1 - Q_2 - Q_3), \quad (53)$$

$$D_1^+ = 5d_1^+(4500 - Q_1 - Q_2 - Q_3), \quad (54)$$

$$D_2^- = 5d_2^-(Q_1 + Q_2 + Q_3), \quad (55)$$

$$\text{and } D_2^+ = 5d_2^+(Q_1 + Q_2 + Q_3). \quad (56)$$

Now, from the restrictions $d_1^- \le 1$ and $d_2^- \le 1$, we obtain

$$\frac{D_1^-}{5(4500 - Q_1 - Q_2 - Q_3)} \le 1, \text{ and}$$

$$\frac{D_2^-}{5(Q_1+Q_2+Q_3)} \le 1$$

$$\Rightarrow D_1^- + 5Q_1 + 5Q_2 + 5Q_3 \le 22500, \text{ and}$$

$$D_2^- - 5Q_1 - 5Q_2 - 5Q_3 \le 0. \quad (57)$$

Thus the equivalent GP formulation is obtained as: Find $Q\,(Q_1, Q_2, Q_3)$ so as to

$$\text{Minimise } \frac{1}{5}D_1^- + \frac{1}{5}D_2^- \qquad (58)$$

$$\left.\begin{aligned}
\text{Subject to } & 38Q_1 + 33Q_2 + 23Q_3 + D_1^- - D_1^+ = 58500,\\
& -Q_1 - 3Q_2 - 4Q_3 + D_2^- - D_2^+ = 0,\\
& D_1^- + 5Q_1 + 5Q_2 + 5Q_3 \le 22500,\\
& D_2^- - 5Q_1 - 5Q_2 - 5Q_3 \le 0,\\
& 625Q_1 + 730Q_2 + 440Q_3 \le 900000,\\
& 2Q_1 + 4Q_2 + 2Q_3 \le 13000,\\
& 7(1000) - 320Q_1 \le 0,\\
& 7(2000) - 350Q_2 \le 0,\\
& 7(1500) - 250Q_3 \le 0,\\
& \text{and } Q_1, Q_2, Q_3 \ge 0,\ D_i^-, D_i^+ \ge 0\\
& \text{for all } i = 1, 2.
\end{aligned}\right\} \qquad (59)$$

The LINGO computer software package is operated to run the linear programming model. The optimal solution is obtained as: $Q_1 = 1363.712$, $Q_2 = 40$, $Q_3 = 42$. The achieved objective function values are $Z_1 = 11.5617$, $Z_2 = 6.1424$. The resulting membership values are $\mu_{z_1} = 0.7123$, $\mu_{z_2} = 0.7715$. The optimal results obtained for the different values of demand quantity are shown in Table 2–4.

6. Analysis of results

From above Table 2–4, we observe the following points:

(1) The optimal order quantity is $Q_1 = 1363.712$ units, $Q_2 = 40$, $Q_3 = 42$ and achieved objective function values are $Z_1 = 11.5617$, $Z_2 = 6.1424$. The resulting membership values are: $\mu_{z_1} = 0.7123$, $\mu_{z_2} = 0.7715$. This means the resulting achievement degrees for the two fuzzy goals are achieved up to the level of 71.23%, and 77.15% respectively, all of which satisfy the requirements of decision-makers.

(2) The maximum achievement degrees for the two fuzzy goals are 0.7145 and 0.7732 corresponding to the demand values (900, 1900, 1400), as displayed in Table 2.

Table 2. Optimal solution for different values of demand quantity.

S. no.	Demand quantity (D_1, D_2, D_3)	Optimal solution Q_1	Q_2	Q_3	Z_1	Z_2	Resulting achievement degrees for two fuzzy goals μ_{z_1}	μ_{z_2}
1	(600, 1600, 1100)	1082.516	32.0	30.8	8.3497	6.1365	0.0699	0.7727
2	(700, 1700, 1200)	1181.716	34.0	33.60	9.4007	6.1351	0.2801	0.7729
3	(800, 1800, 1300)	1280.916	36.0	36.40	10.5212	6.1338	0.5042	0.7732
4	(900, 1900, 1400)	1368.019	38.0	39.20	11.5728	6.1339	0.7145	0.7732
5	(1000, 2000, 1500)	1363.712	40.0	42.0	11.5617	6.1424	0.7123	0.7715
6	(1100, 2100, 1600)	1359.405	42.0	44.80	11.5506	6.1510	0.7101	0.7698
7	(1200, 2200, 1700)	1355.098	44.0	47.60	11.5395	6.1595	0.7079	0.7681
8	(1300, 2300, 1800)	1350.790	46.0	50.40	11.5283	6.1680	0.7056	0.7664
9	(1400, 2400, 1900)	1346.483	48.0	53.2	11.5172	6.1765	0.7034	0.7647
10	(1500, 2500, 2000)	No feasible solution found						

Table 3. Optimal solution for different values of purchasing price.

S. no.	Purchasing price (P_1, P_2, P_3)	Optimal solution Q_1	Q_2	Q_3	Z_1	Z_2	Resulting achievement degrees for two fuzzy goals μ_{z_1}	μ_{z_2}
1	(600, 200, 200)	21.8750	81.8399	42.0	13.0001	7.9831	1.000	0.4033
	↑		↑					
2	(600, 705, 415)	859.7460	40.0	42.0	13.000	6.2187	1.000	0.7562
3	(605, 710, 420)	940.9310	40.0	42.0	13.000	6.2013	1.000	0.7597
4	(610, 715, 425)	1037.434	40.0	42.0	13.000	6.1840	1.000	0.7632
5	(615, 720, 430)	1154.042	40.0	42.0	13.000	6.1666	1.000	0.7666
6	(620, 725, 435)	1297.767	40.0	42.0	13.000	6.1493	0.9999	0.7701
7	(625, 730, 440)	1363.712	40.0	42.0	11.5617	6.1424	0.7123	0.7715
8	(630, 735, 445)	1352.238	40.0	42.0	9.0857	6.1436	0.2171	0.7712
9	(635, 740, 450)			No feasible solution found				

Table 4. Optimal solution obtained for different values of selling price.

S. no.	Selling price (S_1, S_2, S_3)	Optimal solution Q_1	Q_2	Q_3	Z_1	Z_2	Resulting achievement degrees for two fuzzy goals μ_{z_1}	μ_{z_2}
1	(640, 740, 440)	No feasible solution found						
2	(645, 745, 445)	1363.712	40.0	42.0	9.1950	6.1424	0.2390	0.7715
3	(650, 750, 450)	1363.712	40.0	42.0	11.5617	6.1424	0.7723	0.7715
4	(655, 755, 455)	1297.767	40.0	42.0	13.000	6.1493	1.000	0.7701
5	(660, 760, 460)	1154.042	40.0	42.0	13.000	6.1666	1.000	0.7666
6	(665, 765, 465)	1037.434	40.0	42.0	13.000	6.1840	1.000	0.7632
7	(670, 770, 470)	940.9310	40.0	42.0	13.000	6.2046	1.000	0.7590
8	(675, 775, 475)	859.7460	40.0	42.0	13.000	6.2187	1.000	0.7562
9	(680, 780, 480)	790.5000	40.0	42.0	13.000	6.2361	1.000	0.7527
10	(685, 785, 485)	730.7397	40.0	42.0	13.000	6.2534	1.000	0.7493
	↑			↑				
11	(1085, 1185, 885)	43.4334	40.0	42.0	12.6326	7.6423	0.9265	0.4715
12	(1185, 1285, 985)	21.8750	40.0	42.0	12.8592	7.9831	0.9718	0.4033

(3) In Table 3, the maximum achievement degrees for the two fuzzy goals are 1.0 and 0.7715, which corresponding to different set of purchasing price values.

(4) In Table 4, the maximum achievement degrees for the two fuzzy goals are 1.0 and 0.7715, which corresponding to different set of selling price values.

(5) Using fuzzy goal programming approach to simultaneously optimise two objectives, yields optimal order quantity over a wide range of demand values. The demand variation, up to a finite lower and upper limits, gives the global optimal and feasible results. Beyond these limits, the solution becomes non-optimal and infeasible. This is because of the model assumptions. In both examples, the demand quantity values vary but the selling price and purchasing price do not vary. If we impose another assumption: dependency of demand quantity values with selling price and the purchasing price, then the global optimal results will be obtained over a wide range of variation in demand quantity values. Then, the model may be more realistic.

The proposed model is solved by using LINGO computer package and getting optimal order quantity plan. The proposed model gives an efficient compromise solution and the overall levels of decision-making (DM) satisfaction with the multiple fuzzy goal values.

7. Conclusion

In this paper, we proposed an application of a fuzzy goal programming approach with trapezoidal fuzzy number developed by us (Dutta & Kumar, 2012) to multi-objective linear fractional inventory problem in a simple way. The proposed approach attempts simultaneously to maximise the profit ratio to backordered quantity and to minimise the holding cost ratio to total ordered quantities, so that in the end, the proposed model is solved by using LINGO programme to get optimal order quantities. This model gives optimal results over a wide range of demand values. However, the major limitations of the proposed model concern the assumptions made in determining each of the decision parameters, with reference to forecasted demand, selling price and purchasing.

Acknowledgements

The authors are thankful to the anonymous referees, and the associate editor, for their valuable comments and suggestions, which surprisingly improved the clarity of the proposed work.

Notes on contributors

Pavan Kumar is currently a PhD candidate and SRF-UGC in the Department of Mathematics at the National Institute of Technology, Warangal, India, under the supervision of Prof Debashis Dutta. From CCS University, Meerut, India, he received his MSc degree in Mathematics and BSc degree in Mathematics. His current research interests include inventory models, multi-objective optimisation, fuzzy optimisation and interval optimisation.

Debashis Dutta is currently a professor in the Department of Mathematics at the National Institute of Technology, Warangal in India. He has done PhD in Operations Research. He teaches Optimisation Techniques and Statistics. His prime research interest is in the field of mathematical programming, operations research, uncertainty and atmospheric modelling.

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Appendix

Lets us first define MOLFP and FLP:

Definition 1: The general form of an MOLFP problem is

$$\text{Optimise}: Z_k(X) = \frac{c_k X + \alpha_k}{d_k X + \beta_k}, k = 1, 2, \ldots, K. \quad \text{(A1)}$$

Subject to:

$$X \in S = \left\{X \in R^n | AX \leq, =, \text{or} \geq b,\ X \geq 0,\ b \in R^m\right\}, \quad \text{(A2)}$$

$$\text{where } c_k, d_k \in R^n; \alpha_k,\ \beta_k \text{ are constants, and } S \neq \Phi. \quad \text{(A3)}$$

Without loss of generality, we set $d_k X + \beta_k > 0$, for all $X \in S$. In MOLFPP, if we introduce a fuzzy aspiration level to each of the objectives, then these fuzzy objectives are called as fuzzy goals.

In the conventional approach, value of the parameters of linear programming models must be well-defined and precise. However, in real world environment, this is not a realistic assumption. In the real-life problems, there may exist uncertainty about the parameters. In such a situation the parameters of linear programming problems may be represented as fuzzy numbers.

Definition 2: An FLP problem with *m* fuzzy equality constraints and *n* fuzzy variables may be formulated as follows:

$$\text{Max (or Min)}\ (\bar{C}^T \otimes \bar{X}), \quad \text{(A4)}$$

$$\text{Subject to}\ \bar{A} \otimes \bar{X} = \tilde{b}, \quad \text{(A5)}$$

$$\bar{X} \text{ is a non} - \text{negative fuzzy number}, \quad \text{(A6)}$$

where $\tilde{C}^T = [\bar{c}_j]_{1\times n}$, $\tilde{X} = [\bar{X}_j]_{n\times 1}$, $\tilde{A} = \overline{[a}_{ij}]_{m\times n}$, $\bar{b} = \bar{b}_i]_{m\times 1}$

$$\text{and } \bar{a}_{ij}, \bar{c}_j, \bar{x}_j, \bar{b}_i \in F(R) \quad \text{(A7)}$$

where $F(R)$ is the set of real fuzzy numbers. In various applications of nonlinear programming, a ratio of two functions is to be maximised or minimised under certain number of constraints. In other applications, the objective function involves more than one ratio of functions. The problem of optimising one or several ratios of functions is called a fractional programme. Generally, most of the MOFP problems are first converted into single objective FP problems and then solved. The decision-maker (DM) must simultaneously optimise these conflicting goals in a framework of fuzzy aspiration levels.