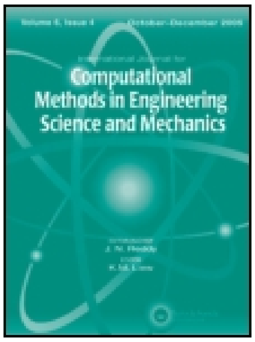

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Mixed Convection Flow of Chemically Reacting Couple Stress Fluid in a Vertical Channel with Soret and Dufour Effects

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The heat and mass transfer characteristics of mixed convection flow of a chemically reacting couple stress fluid between vertical parallel plates in the presence of Dufour and Soret effects are studied. The governing nonlinear partial differential equations are transformed into a system of ordinary differential equations using similarity transformations. The resulting equations are solved using Homotopy Analysis Method (HAM). Profiles of dimensionless velocity, temperature, and concentration are shown graphically for various values of Dufour number, Soret number, Couple stress parameter, and chemical reaction parameter.

Keywords Couple stress fluid, Soret and Dufour effect, Heat and mass transfer, Chemical reaction, HAM

1. INTRODUCTION

Convective flow in channels has been of special interest over the past few years due to vast applications, such as solar collectors, electronic equipment, transistors, and nuclear reactors. Heat exchanger technology involves convective flows in vertical channels. Several researchers have studied analytically and mostly numerically the problem of mixed convection heat transfer and fluid flow between vertical parallel plates. Aung and Worku [1] presented an exact solution for fully developed mixed convection in a parallel-plate vertical channel. Hamadah and Wirtz [2] extended this study by examining different thermal boundary conditions and obtained the velocity and temperature profiles as well as expressions for the Nusselt numbers. Barletta et al. [3] have investigated the dual mixed convection flows in a vertical channel. Ameni et al. [4] investigated numerically the mixed convection in a vertical heated channel. Makinde and Olanrewaju [5] presented the buoyancy effects on thermal boundary layer over a vertical plate with a convective surface boundary condition.

Chemical reaction effects on heat and mass transfer are of considerable importance in hydrometallurgical industries and chemical technology. Chemical reaction can be described as either heterogeneous or homogeneous processes, which depends on whether it occurs at an interface or as a single-phase volume reaction. In many materials, processing systems, chemical reaction effects may exert a significant role. Research on combined heat and mass transfer with chemical reaction and thermophoresis effect can help to design for chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields as well as groves of fruit trees, damage of crops due to freezing, food processing, and cooling towers. Cooling towers are the cheapest way to cool large quantities of water. In particular, the study of heat and mass transfer with chemical reaction is of considerable importance in chemical and hydrometallurgical industries. Das et al. [6] have studied the effect of a homogeneous first-order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer using Laplace transform technique. The effects of chemical reaction, heat, and mass transfer on boundary layer flow over a porous wedge with heat radiation in the presence of suction or injection was studied by Kandasamy et al. [7]. Muthucumaraswamy [8] studied chemical reaction effects on the vertical oscillating plate with variable temperature. Recently, Makinde and Olanrewaju [9] presented the Soret and Dufour effects in an unsteady mixed convection flow past a porous plate moving through a binary mixture of chemically reacting fluid.

The energy flux caused by a concentration gradient is termed the diffusion-thermo (Dufour) effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the thermal-diffusion (Soret) effect. In most of the studies related to heat and mass transfer process, Soret and Dufour effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and

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Fick's laws. But these effects are considered as second-order phenomena and may become significant in areas such as hydrology, petrology, geosciences, etc. The Dufour effect was recently found to be of an order of considerable magnitude such that it cannot be neglected [10]. Kafoussias [11] presented the local similarity solution for combined free-forced convective and mass transfer flow past a semi-infinite vertical plate. Dursunkaya and Worek [12] studied diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface, whereas Kafoussias and Williams [13] presented the same effects on mixed convective and mass transfer transfer steady laminar boundary layer flow over a vertical flat plate with temperature–dependent viscosity. The effect of Soret and Dufour parameters on free convection heat and mass transfers from a vertical surface in a doubly stratified Darcian porous medium has been reported by Lakshmi Narayana and Murthy [14]. Awad and Sibanda [15] studied the Dufour and Soret effects on heat and mass transfer in a micropolar fluid in a horizontal channel. Free convection heat and mass transfer flow in a vertical channel with the Dufour effect studied by Ajibade and Jha [16]. Makinde [17] studied the Soret and Dufour effects on MHD mixed convection flow past a vertical plate embedded in a porous medium. Later, Olanrewaju and Makinde [18] extended the work to a porous plate moving through a binary mixture of chemically reacting fluid. Srinivasacharya and Kaladhar [19] presented the mixed convective flow of couple stress fluid with Soret and Dufour effects.

The growing importance of non-Newtonian fluids in modern technology has attracted researchers for the consideration of such fluids; this is because the traditional Newtonian fluids cannot precisely describe the characteristics of the real fluids. Different models have been proposed to explain the behavior of non-Newtonian fluids. Among these, couple stress fluids introduced by Stokes [20] have distinct features, such as the presence of couple stresses, body couples, and non-symmetric stress tensor. The main feature of couple stresses is to introduce a size–dependent effect. Classical continuum mechanics neglects the size effect of material particles within the continua. This is consistent with ignoring the rotational interaction among particles, which results in symmetry of the force-stress tensor. The study of couple-stress fluids has applications in a number of processes that occur in industry, such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, and colloidal solutions, etc. A review of couple stress (polar) fluid dynamics was reported by Stokes [21]. Recently, analytical solution for free convective flow of couple stress fluid in an annulus with Hall and ion-slip effects presented by Srinivasacharya and Kaladhar [22].

The homotopy analysis method [23] was first proposed by Liao in 1992, and is one of the most efficient methods in solving different types of nonlinear equations such as coupled, decoupled, homogeneous, and non-homogeneous. Also, HAM provides us a great freedom to choose different base functions to express solutions of a nonlinear problem [24]. The application of the Homotopy Analysis Method (HAM) in engineering problems is highly considered by scientists because HAM provides us with a convenient way to control the convergence of approximation series, which is a fundamental qualitative difference in analysis between HAM and other methods. Later, Liao [25] presented an optimal Homotopy Analysis approach for strongly nonlinear differential equations. HAM is used to get analytic approximate solutions for heat transfer of a micropolar fluid through a porous medium with radiation by Rashidi et al. [26].

In this paper, we have investigated the Soret and Dufour effects on steady mixed convective flow of couple stress fluid in a vertical channel with chemical reaction. The Homotopy Analysis Method is employed to solve the governing nonlinear equations. Convergence of the derived series solution is analyzed. The behavior of emerging flow parameters on the velocity and temperature is discussed.

2. MATHEMATICAL FORMULATION

Consider a steady laminar mixed convection flow of a couple stress fluid between two vertical plates of a distance of $2d$ apart. Choose the coordinate system such that x-axis be taken along vertically upward direction through the central line of the channel, y, is perpendicular to the plates and the two plates are infinitely extended in the direction of x. The plates of the channel are at $y = d$. The plate $y = -d$ is maintained at a constant temperature T_1 and concentration C_1, while the plate $y = d$ at a constant temperature T_2 and concentration C_2. Since the boundaries in the x direction are of infinite dimensions, without any loss of generality, we assume that the physical quantities depend on y only. The fluid properties are assumed to be constant except for density variations in the buoyancy force term. In addition, the Soret and Dufour effects are considered. The flow is a mixed convection caused by buoyancy forces and uniform pressure gradient in the direction of x. The flow configuration and the coordinates system are shown in Figure 1. The fluid velocity vector $\bar{q} = (u; v)$ is assumed to be parallel to the x-axis, so that only the x component u of the velocity vector does not vanish but the transpiration cross-flow velocity v_0 remains constant, where $v_0 < 0$ is the velocity of suction and $v_0 > 0$ is the velocity of injection.

With the above assumptions and Boussinesq approximations with energy and concentration, the equations governing the steady flow of an incompressible couple stress fluid are

$$v = v_0 = \text{constant} \tag{1}$$

$$\rho v_0 \frac{\partial u}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2} - \eta_1 \frac{\partial^4 u}{\partial y^4} + \rho g \beta_T (T - T_1) + \rho g \beta_c (C - C_1) - \frac{dp}{dx} \tag{2}$$

$$v_0 \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + 2\frac{\mu}{\rho C_p}\left[\frac{\partial u}{\partial y}\right]^2 + \frac{\eta_1}{\rho C_p}\left[\frac{\partial^2 u}{\partial y^2}\right]^2 + \frac{D K_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

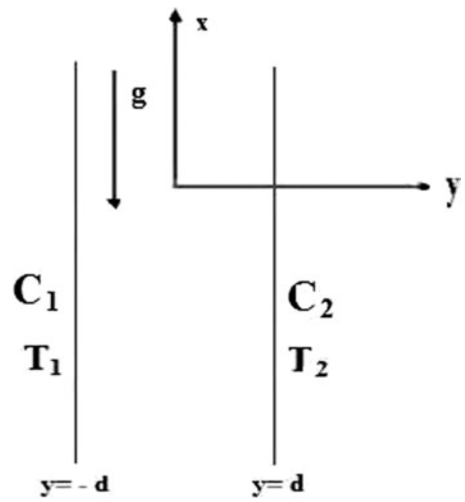

FIG. 1. Physical model and coordinate system.

$$v_0 \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{D K_T}{T_m} \frac{\partial^2 T}{\partial y^2} - k_1 (C - C_1) \tag{4}$$

where u is the velocity component along x direction, ρ is the density, g is the acceleration due to gravity, μ is the coefficient of viscosity, β_T is the coefficient of thermal expansion, β_c is the coefficient of solutal expansion, α is the thermal diffusivity, D is the mass diffusivity, C_p is the specific heat capacity, C_s is the concentration susceptibility, T_m is the mean fluid temperature, K_T is the thermal diffusion ratio, K_f is the coefficient of thermal conductivity, and η_1 is the additional viscosity coefficient which specifies the character of couple-stresses in the fluid.

The boundary conditions are given by

$$u = 0 \quad \text{at} \quad y = \pm d \tag{5a}$$

$$u_{yy} = 0 \quad \text{at} \quad y = \pm d \tag{5b}$$

$$T = T_1 \quad \text{at} \quad y = -d \quad \text{and} \quad T = T_2 \quad \text{at} \quad y = d \tag{5c}$$

The boundary condition (5a) corresponds to the classical no-slip condition from viscous fluid dynamics. The boundary condition (5b) implies that the couple stresses are zero at the plate surfaces.

Introducing the following similarity transformations

$$y = \eta d, \quad u = u_0 f, \quad T - T_1 = (T_2 - T_1)\theta,$$
$$C - C_1 = (C_2 - C_1)\phi, \quad p = \frac{\mu u_0}{d^2} P \tag{6}$$

in Equations (2)–(4), we get the following nonlinear system of differential equations

$$S^2 f^{(iv)} - f'' + R f' - \frac{Gr_T}{Re}\theta - \frac{Gr_C}{Re}\phi + A = 0 \tag{7}$$

$$\theta'' - R\, Pr\, \theta' + 2\, Br\, (f')^2 + S^2\, Br\, (f'')^2 + D_f\, Pr\, \phi'' = 0 \tag{8}$$

$$\phi'' - R\, Sc \phi' + S_r\, Sc\, \theta'' - K\, Sc\, \phi = 0 \tag{9}$$

$Re = \frac{\rho u_0 d}{\mu}$ is the Reynolds number, $R = \frac{\rho v_0 d}{\mu}$ is the suction/injuction parameter, $Pr = \frac{\mu C_p}{K_T}$ is the Prandtl number, u_0 is the entrance velocity, $Gr_T = \frac{\rho^2 g \beta_T d^3}{\mu^2}(T_2 - T_1)$ is the temperature Grashof number, $Gr_C = \frac{\rho^2 g \beta_C d^3}{\mu^2}(C_2 - C_1)$ is the mass Grashof number, $Br = \frac{\mu u_0^2}{K_T(T_2 - T_1)}$ is the Brinkman number, $S_r = \frac{D K_T (T_2 - T_1)}{v T_m (C_2 - C_1)}$ is the Soret number, $D_f = \frac{D K_T (C_2 - C_1)}{v C_s C_p (T_2 - T_1)}$ is the Dufour number, $K = \frac{k_1 d^2}{v}$ is the chemical reaction parameter, $A = \frac{dP}{dx}$ is the constant pressure gradient, $S = \frac{1}{d}\sqrt{\frac{\eta_1}{\mu}}$ is the couple stress parameter; the effects of couple-stress are significant for large values of $S(= l/d)$, where $l = \sqrt{\frac{\eta_1}{\mu}}$ is the material constant. If l is a function of the molecular dimensions of the liquid, it will vary greatly for different liquids. For example, the length of a polymer chain may be a million times the diameter of a water molecule [20]. Therefore, there are many reasons to expect that couple-stresses appear in noticeable magnitudes in liquids with large molecules. The last terms on the right-hand side of the energy equation (8) and concentration equation (9) signify the Dufour (diffusion-thermo) effect and the Soret (thermal-diffusion) effect, respectively.

Boundary conditions (5) in terms of f, θ, ϕ become

$$f = 0, \quad f'' = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{at} \quad \eta = -1$$
$$f = 0, \quad f'' = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 1 \tag{10}$$

The shear stress [20] is $\tau = \mu \left(\frac{\partial u}{\partial y}\right)_{y=\pm d}$ and friction factors at the plates are

$$C_{f_1} Re = 2 f'(-1) \quad \text{and} \quad C_{f_2} Re = 2 f'(1) \tag{11}$$

The heat and mass transfer rates on the plates are

$$Nu = -\left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=\pm 1} \quad \text{and} \quad Sh = -\left.\frac{\partial \phi}{\partial \eta}\right|_{\eta=\pm 1} \tag{12}$$

3. THE HAM SOLUTION OF THE PROBLEM

For HAM solutions, we choose the initial approximations of $U(\eta)$, $\theta(\eta)$, and $\phi(\eta)$ as follows:

$$U_0(\eta) = 0, \quad \theta_0(\eta) = \frac{1+\eta}{2}, \quad \phi_0(\eta) = \frac{1+\eta}{2} \tag{13}$$

and choose the auxiliary linear operators:

$$L_1 = \frac{\partial^4}{\partial \eta^4}, \quad L_2 = \frac{\partial^2}{\partial \eta^2} \tag{14}$$

such that

$$L_1(c_1 + c_2\eta + c_3\eta^2 + c_4\eta^3) = 0, \quad L_2(c_5 + c_6\eta) = 0. \tag{15}$$

where $c_i (i = 1, 2, \ldots, 6)$ are constants. Introducing non-zero auxiliary parameters h_1, h_2 and h_3, we develop the zeroth-order deformation problems as follow:

$$(1-p)L_1[f(\eta;p)-f_0(\eta)] = ph_1N_1[f(\eta;p)] \quad (16)$$

$$(1-p)L_2[\theta(\eta;p)-\theta_0(\eta)] = ph_2N_2[\theta(\eta;p)] \quad (17)$$

$$(1-p)L_2[\phi(\eta;p)-\phi_0(\eta)] = ph_3N_3[\phi(\eta;p)] \quad (18)$$

subject to the boundary conditions

$$f(-1;p)=0,\ f(1;p)=0,\ f''(-1;p)=0,\ f''(1;p)=0$$
$$\theta(-1;p)=0,\ \theta(1;p)=1,\ \phi(-1;p)=0,\ \phi(1;p)=1 \quad (19)$$

where $p \in [0,1]$ is the embedding parameter and the nonlinear operators N_1, N_2 and N_3 are defined as:

$$N_1[f(\eta,p),\theta(\eta,p),\phi(\eta,p)] = S^2 f^{(iv)} - f'' + Rf' - \frac{Gr_T}{Re}\theta - \frac{Gr_C}{Re}\phi + A \quad (20)$$

$$N_2[f(\eta,p),\theta(\eta,p),\phi(\eta,p)] = \theta'' - R\,Pr\,\theta' + 2\,Br\,(f')^2 + S^2\,Br\,(f'')^2 + D_f\,Pr\,\phi'' \quad (21)$$

$$N_3[f(\eta,p),\theta(\eta,p),\phi(\eta,p)] = \phi'' - R\,Sc\phi' + S_r\,Sc\,\theta'' - K\,Sc\,\phi \quad (22)$$

For $p=0$ we have the initial guess approximations

$$f(\eta;0)=f_0(\eta),\ \theta(\eta;0)=\theta_0(\eta),\ \phi(\eta;0)=\phi_0(\eta) \quad (23)$$

When $p=1$, Equations (16)–(18) are the same as (7)–(9), respectively, therefore at $p=1$ we get the final solutions

$$f(\eta;1)=f(\eta),\ \theta(\eta;1)=\theta(\eta),\ \phi(\eta;1)=\phi(\eta) \quad (24)$$

Hence the process of giving an increment to p from 0 to 1 is the process of $f(\eta;p)$ varying continuously from the initial guess $f_0(\eta)$ to the final solution $f(\eta)$ (similar for $\theta(\eta,p)$ and $\phi(\eta,p)$). This kind of continuous variation is called deformation in topology so that we call system Eqs. (16) – (19) the zero-order deformation equation. Next, the m^{th}-order deformation Equations follow as

$$L_1[f_m(\eta)-\chi_m f_{m-1}(\eta)] = h_1R_m^f(\eta) \quad (25)$$

$$L_2[\theta_m(\eta)-\chi_m\theta_{m-1}(\eta)] = h_2R_m^\theta(\eta) \quad (26)$$

$$L_2[\phi_m(\eta)-\chi_m\phi_{m-1}(\eta)] = h_3R_m^\phi(\eta) \quad (27)$$

with the boundary conditions

$$f_m(-1)=0,\ f_m(1)=0,\ f_m''(-1)=0,\ f_m''(1)=0$$
$$\theta_m(-1)=0,\ \theta_m(1)=0,\ \phi_m(-1)=0,\ \phi_m(1)=0 \quad (28)$$

where

$$R_m^f(\eta) = S^2 f^{(iv)} - f'' + Rf' - \frac{Gr_T}{Re}\theta - \frac{Gr_C}{Re}\phi + A(1-\chi_m) \quad (29)$$

$$R_m^\theta(\eta) = \theta'' - RPr\theta' + 2Br\sum_{n=0}^{m-1} f'_{m-1-n}f'_n + S^2Br\sum_{n=0}^{m-1} f''_{m-1-n}f''_n + D_fPr\phi'' \quad (30)$$

$$R_m^\phi(\eta) = \phi'' - R\,Sc\phi' + S_r\,Sc\,\theta'' - K\,Sc\,\phi \quad (31)$$

for m being integer

$$\chi_m = 0 \quad \text{for} \quad m \le 1 \quad = 1 \quad \text{for} \quad m > 1 \quad (32)$$

The initial guess approximations $f_0(\eta)$, $\theta_0(\eta)$ and $\phi_0(\eta)$, the linear operators L_1, L_2, and the auxiliary parameters h_1, h_2 and h_3 are assumed to be selected such that Equations (16) - (19) have a solution at each point $p \in [0,1]$ and also with the help of Taylor's series, and due to Eq. (23); $f(\eta;p)$, $\theta(\eta;p)$ and $\phi(\eta;p)$ can be expressed as

$$f(\eta;p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)p^m \quad (33)$$

$$\theta(\eta;p) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta)p^m \quad (34)$$

$$\phi(\eta;p) = \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta)p^m \quad (35)$$

in which h_1, h_2 and h_3 are chosen in such a way that the series (33) – (35) is convergent [25] at $p=1$. Therefore we have from (24) that

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) \quad (36)$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) \quad (37)$$

$$\phi(\eta) = \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta) \quad (38)$$

for which we presume that the initial guesses to f, θ and ϕ the auxiliary linear operators L and the non-zero auxiliary parameters h_1, h_2 and h_3 are so properly selected that the deformations $f(\eta,p)$, $\theta(\eta,p)$ and $\phi(\eta,p)$ are smooth enough and their mth-order derivatives with respect to p in Equations (36)–(38) exist and are given respectively by $f_m(\eta) = \frac{1}{m!}\frac{\partial^m f(\eta;p)}{\partial p^m}\Big|_{p=0}$, $\theta_m(\eta) = \frac{1}{m!}\frac{\partial^m \theta(\eta;p)}{\partial p^m}\Big|_{p=0}$, $\phi_m(\eta) = \frac{1}{m!}\frac{\partial^m \phi(\eta;p)}{\partial p^m}\Big|_{p=0}$. It is clear that the convergence of Taylor series at $p=1$ is a prior assumption, whose justification is provided via a theorem [27], so that the system in (36)–(38) holds true. The formulae in (36)–(38) provide us with a direct relationship between the initial guesses and the exact solutions. All the effects of interaction of the chemical reaction as well as of the heat and mass transfer, Soret and Dufour effects and couple stress flow field can be studied

TABLE 1
Convergence of HAM solutions for different order of approximations

Order	$U(0)$	$\theta(0)$	$\phi(0)$
5	0.05463457394417	0.6014718469641	0.7501884356211
10	0.0544547839349	0.6009835960752	0.7492017782657
15	0.05445002207132	0.6009834479197	0.7491938981286
20	0.05445002143533	0.6009834478693	0.7491938973199
30	0.05445002143508	0.6009834478036	0.7491938973098
40	0.05445002143482	0.6009834478035	0.7491938973088
50	0.05445002143482	0.6009834478035	0.7491938973088

from the exact formulas (36)–(38). Moreover, a special emphasis should be placed here that the m^{th}-order deformation system (25)–(28) is a linear differential equation system with the auxiliary linear operators L, whose fundamental solution is known.

4. CONVERGENCE OF THE HAM SOLUTION

The expressions for f, θ and ϕ contain the auxiliary parameters h_1, h_2 and h_3. As pointed out by Liao [23], the convergence and the rate of approximation for the HAM solution strongly depend on the values of auxiliary parameter h. For this purpose, h-curves are plotted by choosing h_1, h_2 and h_3 in such a manner that the solutions (33)–(35) ensure convergence [23]. Here to see the admissible values of h_1, h_2 and h_3, the h-curves are plotted for the fifteenth-order of approximation in Figures (2)–(4) by taking the values of the parameters $Pr = 0.71$, $Sc = 0.22$, $Br = 0.5$, $Re = 2$, $R = 2$, $Gr_T/Re = Gr_C/Re = 2$, $K = 0.1$, $A = 1$, $S = 0.5$, $D_f = 0.03$ and $S_r = 2.0$. It is clearly noted from Fig. 2 that the range for the admissible values of h_1 is $-1.25 < h_1 < -0.3$. From

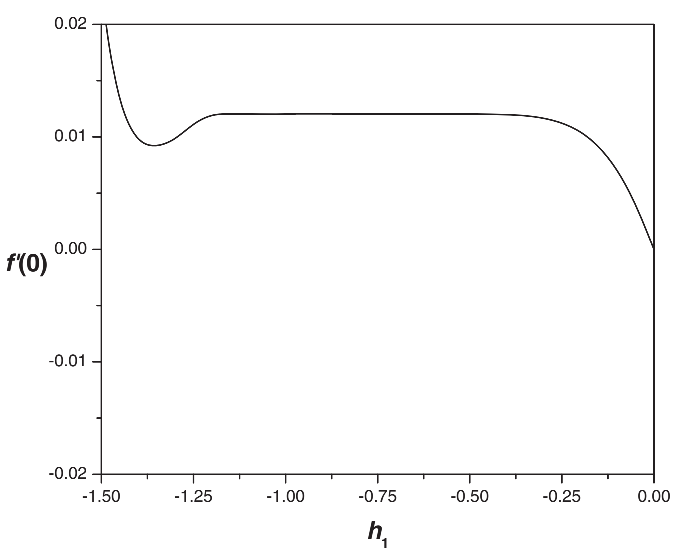

FIG. 2. The h curve of $f(\eta)$ when $D_f = 0.03$, $S_r = 2.0$, $S = 1.0$, $K = 0.1$.

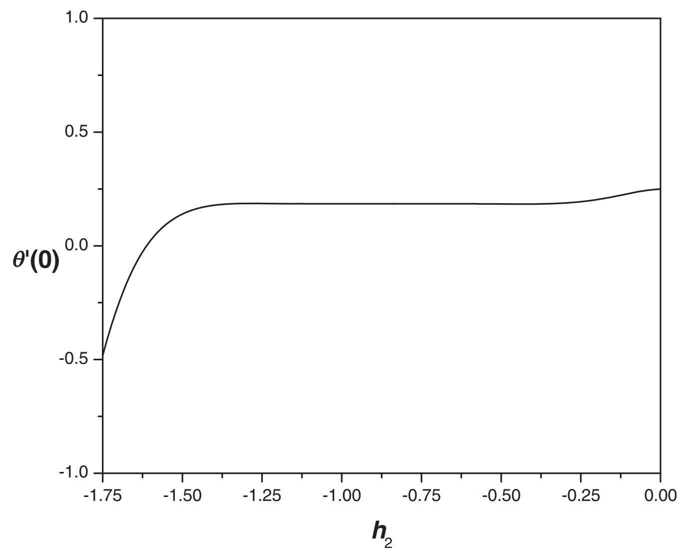

FIG. 3. The h curve of $\theta(\eta)$ when $D_f = 0.03$, $S_r = 2.0$, $S = 1.0$, $K = 0.1$.

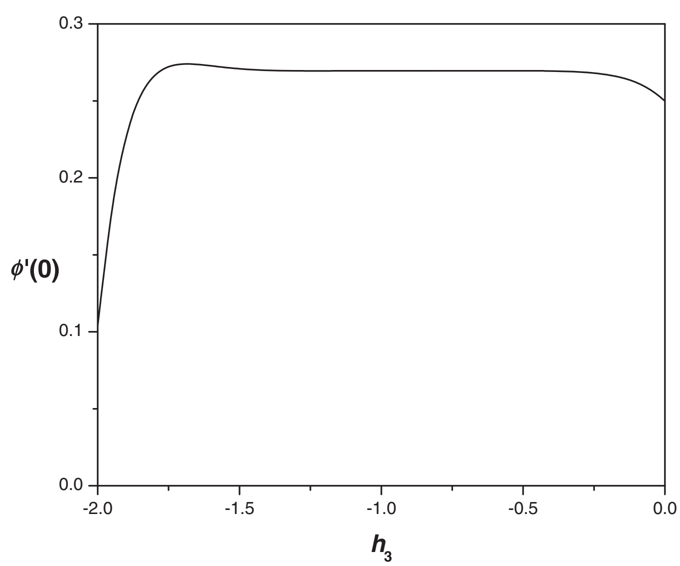

FIG. 4. The h curve of $\phi(\eta)$ when $D_f = 0.03$, $S_r = 2.0$, $S = 1.0$, $K = 0.1$.

Fig. 3, it can be seen that the h-curve has a parallel line segment that corresponds to a region $-1.4 < h_2 < -0.3$. Fig. 4 depicts that the admissible value of h_3 is $-1.5 < h_3 < -0.4$. A wide valid zone is evident in these figures, ensuring convergence of the series. To choose the optimal value of the auxiliary parameter, the average residual errors (see Ref. [25] for more details) are defined as

$$E_{f,m} = \frac{1}{2K} \sum_{i=-K}^{K} \left(N_1 \left[\sum_{j=0}^{m} f_j(i\triangle t) \right] \right)^2 \quad (39)$$

$$E_{\theta,m} = \frac{1}{2K} \sum_{i=-K}^{K} \left(N_2 \left[\sum_{j=0}^{m} \theta_j(i\triangle t) \right] \right)^2 \quad (40)$$

$$E_{\phi,m} = \frac{1}{2K} \sum_{i=-K}^{K} \left(N_3 \left[\sum_{j=0}^{m} \phi_j(i\triangle t) \right] \right)^2 \quad (41)$$

where $\triangle t = 1/K$ and K = 5. These average residual errors are calculated at different order of approximations (m) and they are minimum at $h_1 = -0.8,\ h_2 = -0.85,\ h_3 = -0.7$, respectively. Therefore, the optimum values of convergence control parameters are taken as $h_1 = -0.8,\ h_2 = -0.85,\ h_3 = -0.7$.

To see the accuracy of the solutions, the residual errors are defined for the system as

$$RE_f = S^2 f_n^{(iv)} - f_n'' + R f_n' - \frac{Gr_T}{Re}\theta_n - \frac{Gr_C}{Re}\phi_n + A \quad (42)$$

$$RE_\theta = \theta_n'' - R\, Pr\, \theta_n' + 2\, Br\, (f_n')^2 + S^2\, Br\, (f_n'')^2 + D_f\, Pr\, \phi_n'' \quad (43)$$

$$RE_\phi = \phi_n'' - R\, Sc \phi_n' + S_r\, Sc\, \theta_n'' - K\, Sc\, \phi_n \quad (44)$$

where $f_n(\eta)$, $\theta_n(\eta)$ and $\phi_n(\eta)$ are the HAM solution for $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$. For optimality of the convergence control parameters, residual error [26] are calculated for different values of h in the convergence region and we determined that $h_1 = -0.8,\ h_2 = -0.85,\ h_3 = -0.7$ give a better solution. Table 1 establishes the convergence of the obtained series solution. It is found from the above observations that the series given by (33)-(35) converges in the whole region of η when $h_1 = -0.8,\ h_2 = -0.85,\ h_3 = -0.7$.

5. RESULTS AND DISCUSSION

In order to study the effects of couple stress fluid parameter S, Soret number S_r, Dufour number D_f, and chemical reaction parameter K explicitly, computations were carried out by taking $Pr = 0.71, Sc = 0.22, A = 1, Gr_T/Re = 2.0, Gr_C/Re = 2.0, Br = 0.5, Re = 2, R = 2$ and $h_1 = -0.8, h_2 = -0.85, h_3 = -0.7$. The values of Soret number S_r and Dufour number D_f are chosen in such a way that their product is constant according to their definition provided that the mean temperature T_m is kept constant [13]. These values are used throughout the computations, unless otherwise indicated.

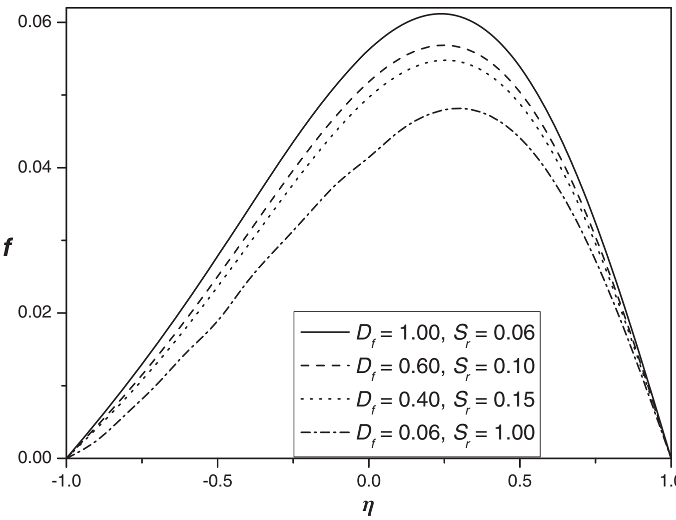

FIG. 5. Velocity profile for different values of D_f, S_r at $S = 1.0$, $K = 0.1$.

Fig. 5 displays the nondimensional velocity for different values of Soret number S_r and Dufour number D_f with $S = 1.0$ and $K = 0.1$. It can be observed from this figure that the velocity of the fluid decreases with the decrease of Dufour number (or increase of Soret number). The dimensionless temperature for different values of Soret number S_r and Dufour number D_f with $S = 1.0$ and $K = 0.1$ is shown in Fig. 6. It is clear that the temperature of the fluid decreases with the decrease of Dufour number (or increase of Soret number). Fig. 7 demonstrates the dimensionless concentration for different values of Soret number S_r and Dufour number D_f with S = 1.0 and K = 0.1. It is seen that the concentration of the fluid decreases with increase of Dufour number (or decrease of Soret number).

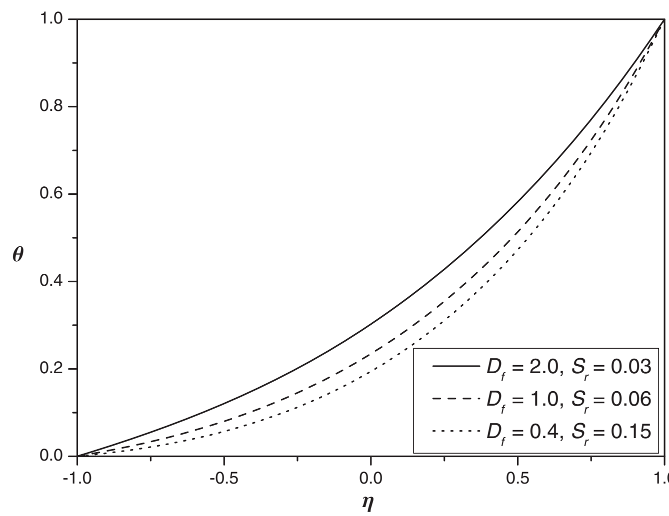

FIG. 6. Temperature profile for different values of D_f, S_r at $S = 1.0$, $K = 0.1$.

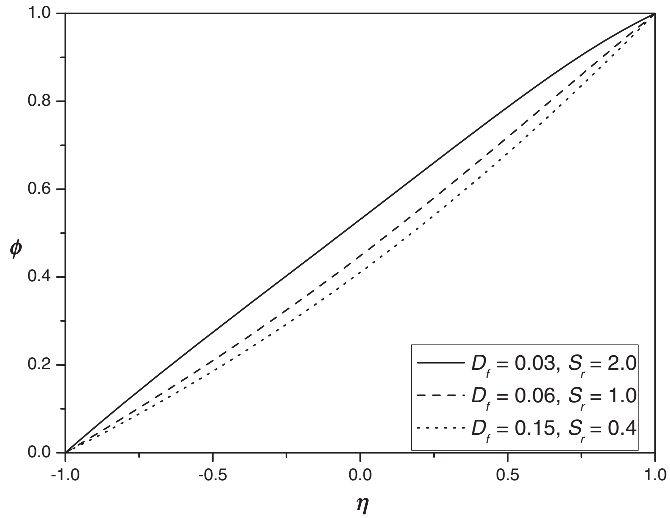

FIG. 7. Concentration profile for different values of D_f, S_r at $S = 1.0$, $K = 0.1$.

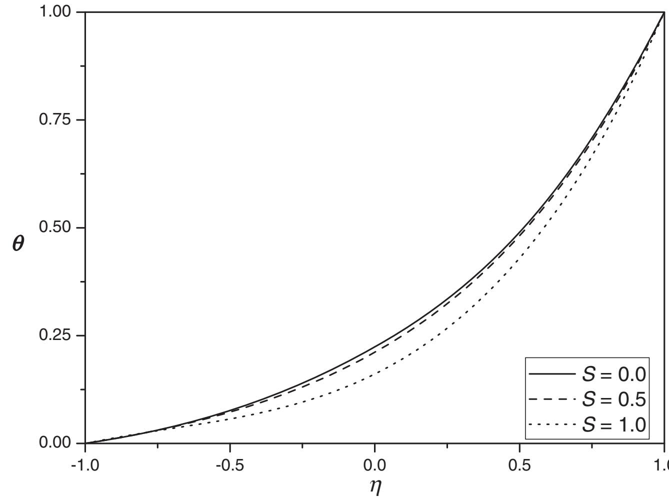

FIG. 9. Temperature profile for different values of S at $D_f = 0.03$, $S_r = 2.0$, $K = 0.1$.

In Figs. 8–10, the effects of the couple stress parameter S on the dimensionless velocity, temperature, and concentration profiles are presented for fixed values of $S_r = 2.0$, $D_f = 0.03$ and $K = 0.1$. As S increases, it can be observed from Fig. 8 that the maximum velocity decreases in amplitude. This happens because of the rotational field of the velocity generated in couple stress fluid. It is clear from Fig. 9 that the temperature decreases with the increase of couple stress fluid parameter S. It can be seen from Fig. 10 that the concentration of the fluid decreases with an increase in couple stress fluid parameter S.

Figures 11 to 13 represent the effect of chemical reaction K on $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$. It can be seen from these figures that the velocity $f(\eta)$ decreases with an increase in the parameter K. The dimensionless temperature decreases as K increases. The concentration $\phi(\eta)$ decrases with an increase in the parameter K. Higher values of K amount to a fall in the chemical molecular diffusivity; i.e., less diffusion. Therefore, they are obtained by species transfer. An increase in K will suppress species concentration. The concentration distribution decreases at all points of the flow field with the increase in the reaction parameter. This shows that heavier diffusing species have greater retarding effect on the concentration distribution of the flow field.

The variations skin-friction coefficient, rate of heat and mass transfers are shown in Table 2 for different values of Soret and Dufour numbers and reaction parameter. From this table, it is observed that the value of f' decreases with the increasing values of chemical reaction parameter. The heat and mass transfer rates decrease with the increasing values of reaction parameters. Finally, for fixed values of K, the effects of Dufour and Soret number on the skin-friction coefficient rate of heat and

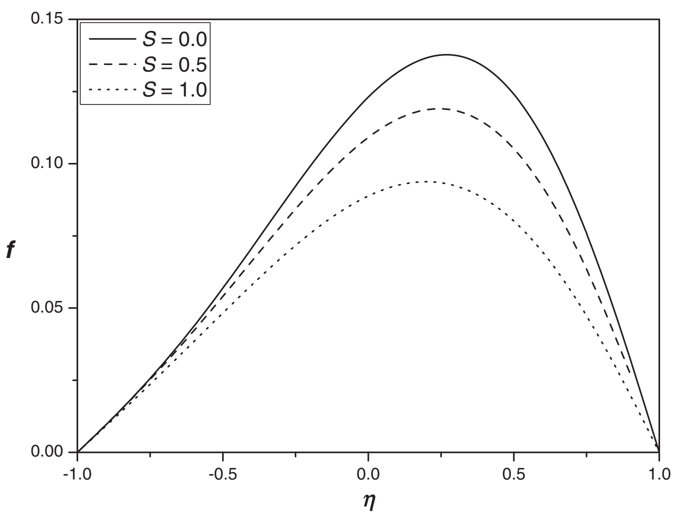

FIG. 8. Velocity profile for different values of S at $D_f = 0.03$, $S_r = 2.0$, $K = 0.1$.

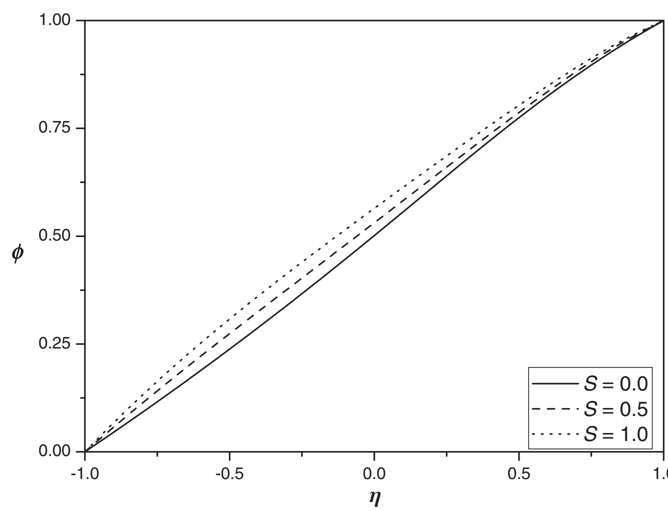

FIG. 10. Concentration profile for different values of S at $D_f = 0.03$, $S_r = 2.0$, $K = 0.1$.

TABLE 2
Effects of skin friction, heat and mass transfer coefficients for varying values of Soret and Dufour numbers and chemical reaction parameters

D_f	S_r	K	C_{f_1}	C_{f_2}	Nu_1	Nu_2	Sh_1	Sh_2
2	0.03	0.1	0.09193	-0.4147	0.24607	0.90448	0.30875	0.76311
1	0.06	0.1	0.04904	-0.3476	0.16837	1.19106	0.31086	0.75612
0.4	0.15	0.1	0.02668	-0.3126	0.12296	1.35825	0.31717	0.73531
0.03	2	0.1	0.08436	-0.4021	0.10229	1.41532	0.44461	0.32861
0.03	2	0.5	0.07139	-0.3822	0.10076	1.42301	0.42313	0.38942
0.03	2	1	0.05640	-0.3591	0.09920	1.43117	0.39835	0.46221

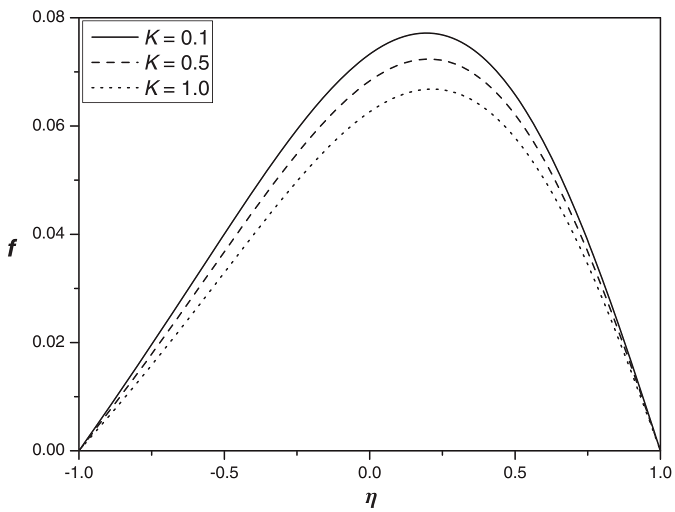

FIG. 11. Velocity profile for different values of K at $D_f = 0.03$, $S_r = 2.0$, $S = 1.0$.

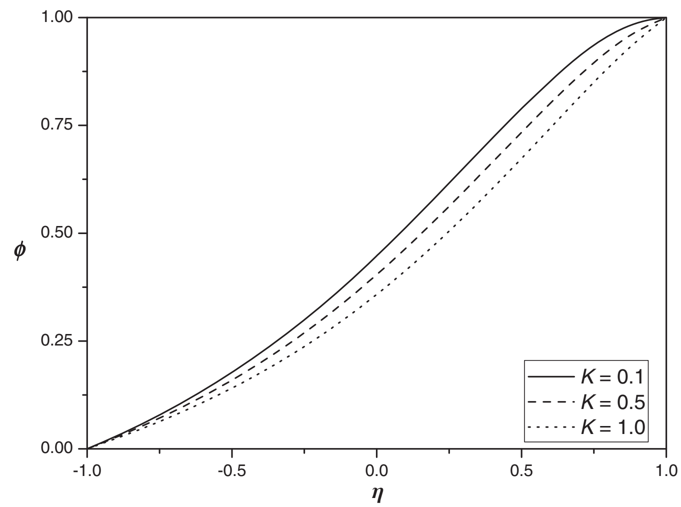

FIG. 13. Concentration profile for different values of K at $D_f = 0.03$, $S_r = 2.0$, $K = 1.0$

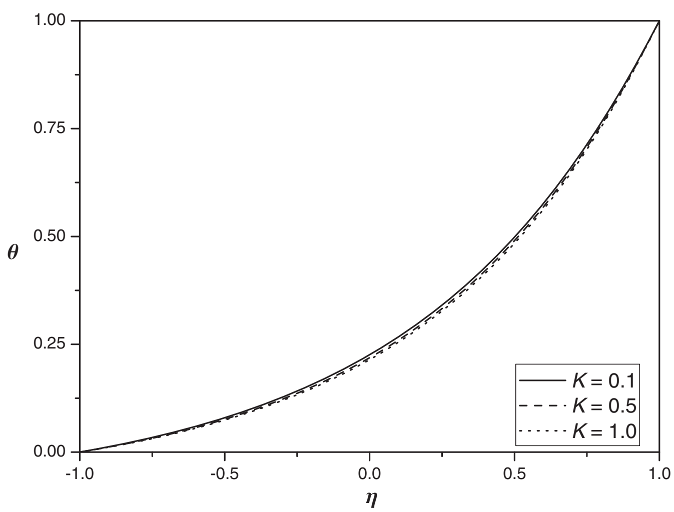

FIG. 12. Temperature profile for different values of K at $D_f = 0.03$, $S_r = 2.0$, $K = 1.0$.

mass transfer are shown in this table. The behavior of these parameters is self-evident from Table 2 and hence is not discussed for briefness.

6. CONCLUSIONS

In this paper, the Dufour and Soret effects on steady mixed convection of a couple stress fluid flowing through a vertical channel with chemical reaction has been studied. Using transformations, the governing equations have been transformed into nonlinear ordinary differential equations. The approximate analytical series solutions are obtained applying homotopy analysis method (HAM). From the present study, we observe that:

1. The velocity, temperature, friction factor, and heat transfer rate of the fluid decreases with the decrease of Dufour number (or increase of Soret number), and with increase of Dufour number (or decrease of Soret number) the concentration and mass transfer rate of the fluid decrease.
2. The presence of couple stresses in the fluid decreases the velocity and temperature and increases the concentration.

3. The velocity, temperature, concentration, skin-friction, and heat mass transfer rates decrease with the increase in the reaction parameter.

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