



Strategic bidding using fuzzy adaptive gravitational search algorithm in a pool based electricity market

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ABSTRACT

A novel stochastic optimization approach to solve optimal bidding strategy problem in a pool based electricity market using fuzzy adaptive gravitational search algorithm (FAGSA) is presented. Generating companies (suppliers) participate in the bidding process in order to maximize their profits in an electricity market. Each supplier will bid strategically for choosing the bidding coefficients to counter the competitors bidding strategy. The gravitational search algorithm (GSA) is tedious to solve the optimal bidding strategy problem because, the optimum selection of gravitational constant (G). To overcome this problem, FAGSA is applied for the first time to tune the gravitational constant using fuzzy “IF/THEN” rules. The fuzzy rule-based systems are natural candidates to design gravitational constant, because they provide a way to develop decision mechanism based on specific nature of search regions, transitions between their boundaries and completely dependent on the problem. The proposed method is tested on IEEE 30-bus system and 75-bus Indian practical system and compared with GSA, particle swarm optimization (PSO) and genetic algorithm (GA). The results show that, fuzzification of the gravitational constant, improve search behavior, solution quality and reduced computational time compared against standard constant parameter algorithms.

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1. Introduction

Restructuring of the power industry mainly aims at abolishing the monopoly in the generation and trading sectors, thereby, introducing competition at various levels wherever it is possible. But the sudden changes in the electricity markets have a variety of new issues such as oligopolistic nature of the market, supplier's strategic bidding, market power misuse, price-demand elasticity and so on. Theoretically, in a perfectly competitive market, supplier should bid at their marginal production cost to maximize payoff. However, practically the electricity markets are oligopolistic nature, and power suppliers may seek to increase their profit by bidding a price higher than marginal production cost. Knowing their own costs, technical constraints and their expectation of rival and market behavior, suppliers face the problem of constructing the best optimal bid. This is known as a strategic bidding problem [1].

In general, there are three basic approaches to model the strategic bidding problem, viz. (i) based on the estimation of market clearing price, (ii) estimation of rival's bidding behavior and (iii) on game theory. David [2] developed a conceptual optimal bidding

model for the first time in which a dynamic programming (DP) based approach has been used. Gross and Finlay adopted a Lagrangian relaxation-based approach for strategic bidding in England–Wales pool type electricity market [3]. Wang et al. [4] used evolutionary game approach to analyzing bidding strategies by considering elastic demand. Ebrahim and Galiana developed Nash equilibrium based bidding strategy in electricity markets [5]. David and Wen [6] proposed to develop an overall bidding strategy using two different bidding schemes for a day-ahead market using genetic algorithm (GA). The same methodology has been extended for spinning reserve market coordinated with energy market by David and Wen [7]. Chanwit et al. proposed an optimal risky bidding strategy for a generating company (GenCo) by self-organizing hierarchical particle swarm optimization with time-varying acceleration coefficients (SPSO-TVAC) [8]. To construct linear bid curves in the Nord-pool market stochastic programming model has been used by Fleten et al. [9]. The opponents' bidding behaviors are represented as a discrete probability distribution function solved using Monte Carlo method by David and Wen [10].

A new approach based on fuzzy cognitive map (FCM) is introduced to model and simulate GENCO's behavior in the electricity market with respect to profit maximization [11]. pay-as-bid (PAB) has been proposed to replace the market clearing price (MCP) in deregulated electricity markets, with the expectation that it would lower market prices

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and reduce price volatility [12]. Mahvi et al. presented a new method for determination of the optimal bidding strategies among generating companies (GenCo) in the electricity markets using agent-based approach and numerical sensitivity analysis (NSA). While agent-based approach provides for decision making, NSA can help with identifying the critical control points that lead to proper decisions to be taken by GenCos [13]. Hosseini et al. illustrates how a generator profit may be affected by the pricing method of an oligopoly market model. Through utilizing a bi-level optimization technique and game theory concepts, supply function equilibria (SFE) of pay-as-bid pricing (PABP) and marginal pricing (MP) mechanisms are derived [14].

Azadeh et al. formed optimal bidding problem for a day-ahead market as a multi objective problem and solved using GA [15]. Jain and Srivastava considered risk constrained bidding strategy and solved using GA [16]. Ahmet et al. used PSO to determine bid prices and quantities under the rules of a competitive power market [17]. Kanakasabhapathy and Swarup [18] developed strategic bidding for pumped-storage hydroelectric plant using evolutionary tristate PSO. Bajpai and Singh developed blocked bid model bidding strategy in a uniform price spot market using fuzzy adaptive particle swarm optimization (FAPSO) [19]. Venkaiah and Vinod Kumar used fuzzy adaptive bacterial foraging algorithm (FABFA) for optimal rescheduling of active power of generators [20]. The combination of PSO and simulated annealing (SA) is used to predict the bidding strategy of generation companies [21]. Fevrier et al. developed a new hybrid approach by combining the advantages of PSO and GA using fuzzy logic [22].

In general, strategic bidding is an optimization problem that can be solved by various conventional and non-conventional (heuristic) methods. Depending on the bidding models, non-differentiable optimization is well established area of the mathematical optimization field with well known conventional, non-heuristic methods. Heuristic methods such as GA, simulated annealing (SA) and evolutionary programming (EP), and particle swarm optimization (PSO) have main limitations of their sensitivity to the choice of parameters, such as the crossover and mutation probabilities in GA, temperature in SA, scaling factor in EP, inertia weight, learning factors in PSO.

In gravitational search algorithm (GSA) [23] the agent direction is calculated based on the overall force obtained by all other agents. Therefore, agents with a higher performance have a greater gravitational mass. As a result, the agents tend to move toward the best agent, but the main drawback of the GSA is the difficulty for the appropriate selection of gravitational constant (G), which controls the search accuracy and may not give a global solution all the time. Hence, to overcome this drawback, the gravitational constant (G) has been fuzzified.

The main contribution of this paper is, the gravitational constant (G) has been fuzzified for the first time using “IF/THEN” rules to overcome the limitations of GSA and thereafter maximization of the profit of the suppliers using FAGSA. The paper is organized as follows. Section 2 presents the mathematical formulation of optimal bidding problem. Section 3 contains a brief overview of the proposed GSA method and application of GSA for solving the optimal bidding problem. Section 4 presents the proposed FAGSA and application of FAGSA to the optimal bidding problem. Section 5 reports the case studies solving optimal bidding problem using FAGSA for IEEE 30-bus system and 75-bus Indian practical system and Section 6 summed up the final outcome of the paper as conclusions.

2. Problem formulation for optimal bidding strategy

Consider a system consisting of ' m ' suppliers participating in a pool-based single-buyer electricity market in which the sealed

auction with a uniform market clearing price (MCP) is employed. Assume that each supplier is required to bid a linear supply function to the pool. The j th supplier bid with linear supply curve denoted by $G_j(P_j) = a_j + b_j P_j$ for $j = 1, 2, \dots, m$, where P_j is the active power output, a_j and b_j are non-negative bidding coefficients of the j th supplier. After receiving bids from suppliers, the pool determines a set of generation outputs that meets the load demand and minimizes the total purchasing cost. It is clear that generation dispatching should satisfy the following Eqs. (1)–(3).

$$a_j + b_j P_j = R, \quad j = 1, 2, \dots, m \quad (1)$$

$$\sum_{j=1}^m P_j = Q(R) \quad (2)$$

$$P_{\min,j} \leq P_j \leq P_{\max,j}, \quad j = 1, 2, \dots, m \quad (3)$$

where R is the market clearing price (MCP) of electricity to be determined, $Q(R)$ is the aggregate pool load forecast as follows:

$$Q(R) = Q_0 - KR \quad (4)$$

where Q_0 is a constant number and K is a non-negative constant used to represent the load price elasticity. When the inequality constraint Eq. (3) is ignored, the solution to Eqs. (1) and (2) are

$$R = \frac{Q_0 + \sum_{j=1}^m (a_j/b_j)}{K + \sum_{j=1}^m (1/b_j)} \quad (5)$$

$$P_j = \frac{R - a_j}{b_j}, \quad j = 1, 2, \dots, m \quad (6)$$

$P_{\min,j}$ and $P_{\max,j}$ are the generation output limits of the j th supplier. If the solution of Eq. (3) exceeds the maximum limit $P_{\max,j}$, P_j is set to $P_{\max,j}$. When P_j is less than $P_{\min,j}$, P_j is set to $P_{\min,j}$. The j th supplier has the cost function denoted by $C_j(P_j) = e_j P_j + f_j P_j^2$, where e_j and f_j are the cost coefficients of the j th supplier. The profit maximization objective of supplier j ($j = 1, 2, \dots, m$) in a unit time for building bidding strategy can be described as:

$$\begin{aligned} \text{Maximize : } & F(a_j, b_j) = RP_j - C_j(P_j) \\ \text{Subject to : } & \text{Eqs. (5) and (6)} \end{aligned} \quad (7)$$

The objective is to determine bidding coefficients a_j and b_j so as to maximize $F(a_j, b_j)$ subject to the constraints Eqs. (5) and (6) [10]. It is clear that market participants can set MCP at the level that returns the maximum profit to them if they know bidding strategy of other firms. But in sealed bid auction based electricity market, information for the next day bidding period is confidential in which suppliers cannot solve optimization problem given in Eq. (7) directly. However, bidding information of previous day will be disclosed after independent system operator (ISO) decide MCP and everyone can make use of this information to strategically bid for the next hour of the present day transaction between suppliers. An immediate problem for each supplier is how to estimate the bidding coefficients of rivals. It is assumed that the suppliers have the freedom to price away from their marginal production costs, and they bid linear supply functions and the market is cleared at a uniform price.

The bidding coefficients (a_j, b_j) are interdependent; therefore one of the coefficients make as a constant and other is randomly varied using probability density function (pdf). The probability density function of a continuous random variable is a function which can be integrated to obtain the probability that the random variable takes a value in a given interval. Let, from the i th supplier's point

of view, rival's j th ($j \neq i$) bidding coefficients (a_j, b_j) obey a joint normal distribution with pdf given by:

$$pdf_i(a_j, b_j) = \frac{1}{2\pi\sigma_j^{(a)}\sigma_j^{(b)}\sqrt{1-\rho_j^2}} \times \exp \left\{ -\frac{1}{2(1-\rho_j^2)} \left[\left(\frac{a_j - \mu_j^{(a)}}{\sigma_j^{(a)}} \right)^2 - \frac{2\rho_j(a_j - \mu_j^{(a)})(b_j - \mu_j^{(b)})}{\sigma_j^{(a)}\sigma_j^{(b)}} + \left(\frac{b_j - \mu_j^{(b)}}{\sigma_j^{(b)}} \right)^2 \right] \right\} \quad (8)$$

where ρ_j is the correlation coefficient between a_j and b_j , $\mu_j^{(a)}, \mu_j^{(b)}$, $\sigma_j^{(a)}$ and $\sigma_j^{(b)}$ are the parameter of the joint distribution. The marginal distributions of a_j and b_j are both normal with mean values $\mu_j^{(a)}$ and $\mu_j^{(b)}$, and standard deviations $\sigma_j^{(a)}$ and $\sigma_j^{(b)}$ respectively. Based on historical bidding data these distributions can be determined [10]. The probability density function Eq. (8) represents the joint distributions between a_j and b_j , the task of optimally coordinating the bidding strategies for a supplier with objective function Eq. (7), and constraints (5) and (6), becomes stochastic optimization problem. The proposed Fuzzy Adaptive gravitational search algorithm (FAGSA) is applied to solve the above stochastic optimization problem.

3. Gravitational search algorithm (GSA)

In this section, a brief review of GSA is introduced. In GSA, agents are considered as objects and their performance is measured by their masses. All these objects attract each other by a gravity force, and this force causes a movement of all objects globally toward the objects with heavier masses. The heavy masses correspond to good solutions of the problem [23]. In the Newton gravitational law, each particle attracts every other particle with a “gravitational force”. The gravitational force between two bodies is directly proportional to the product of their masses and inversely proportional to the square of their distance.

In GSA, each mass (agent) has four specifications: its position, its inertial mass, its active gravitational mass, and its passive gravitational mass. The position of the mass corresponds to a solution of the problem, and its gravitational and inertial masses are determined using a fitness function. In other words, each mass presents a solution, and the algorithm is navigated by properly adjusting the gravitational and inertia masses. By lapse of time, we expect that masses be attracted by the heaviest mass. This mass will present an optimum solution in the search space. The GSA could be considered as an isolated system of masses. It is like a small artificial world of masses obeying the Newtonian laws of gravitation and motion. More precisely, masses obey the following laws.

Law of gravity. Each particle attracts every other particle and the gravitational force between two particles is directly proportional to the product of their masses and inversely proportional to the distance 'R' between them.

Law of motion. The current velocity of any mass is equal to the sum of the fraction of its previous velocity of mass and the variation in the velocity. Variation in the velocity or acceleration of any mass is equal to the force acted on the system divided by mass of inertia.

Now, consider a system with N agents (masses), the position of the i th agent is defined by:

$$X_i = (x_i^1, \dots, x_i^d, \dots, x_i^n) \quad \text{for } i = 1, 2, \dots, N \quad (9)$$

where x_i^d presents the position with N agents (masses), the position of the i th agent in the d th dimension and n is the space dimension.

At a specific time 't' we define the force acting on mass 'i' from mass 'j' as following:

$$F_{ij}^d(t) = G(t) \frac{M_{pi}(t) \times M_{aj}(t)}{R_{ij}(t) + \varepsilon} (x_j^d(t) - x_i^d(t)) \quad (10)$$

where M_{aj} is the active gravitational mass related to agent j , M_{pi} is the passive gravitational mass related to agent i , $G(t)$ is gravitational

constant at time t , ε is a small constant and $R_{ij}(t)$ is the Euclidian distance between two agents i and j . The total force that acts on agent

i in a dimension d is a randomly weighted sum of d th component of the forces exerted from K_{best} agents:

$$F_i^d(t) = \sum_{j \in K_{best}, j \neq i}^N rand_j F_{ij}^d(t) \quad (11)$$

where $rand_j$ is a random number in the interval [0,1] and K_{best} is the set of first agents with the best fitness value and biggest mass. K_{best} is a function of time, initialized to K_0 at the beginning and decreasing with time. In such a way, at the beginning all agents apply force and as time passes, K_{best} is decreased linearly and at the end, there will be just one agent apply force to others.

By the law of motion, the acceleration of the agent i at time t , and in direction d , $a_i^d(t)$, is given as follows:

$$a_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)}, \quad (12)$$

where M_{ii} is the inertial mass of i th agent. The next velocity of an agent is considered as a fraction of its current velocity added to its acceleration. Therefore, its position and its velocity could be calculated as follows:

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t) \quad (13)$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \quad (14)$$

where $rand_i$ is a uniform random variable in the interval [0,1]. This random number to gives randomized characteristic to the search. The gravitational constant, G , is initialized at the beginning and will be reduced with time to control the search accuracy. Hence, G is a function of the initial value (G_0) and time (t):

$$G(t) = G_0 e^{-\tau} \left(\frac{iter}{iter_{max}} \right) \quad (15)$$

Gravitational and inertia masses are simply calculated by the fitness evaluation. Here G_0 is set to 100. A heavier mass means a more efficient agent. This means that better agents have higher attractions and walk more slowly. Assuming the equality of the gravitational and inertia mass, the value of masses is calculated using the map of fitness. The gravitational and inertial masses are updated by the following equations:

$$M_{ai} = M_{pi} = M_{ii} = M_i; \quad i = 1, 2, \dots, N \quad (16)$$

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \quad (17)$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^N m_j(t)}, \quad (18)$$

where $fit_i(t)$ represent the fitness value of the agent i at time t , and $worst(t)$ and $best(t)$ are defined as follows (for a maximization problem and vice versa for minimization):

$$best(t) = \max_{j \in \{1, \dots, N\}} fit_j(t) \quad (19)$$

$$worst(t) = \min_{j \in \{1, \dots, N\}} fit_j(t) \quad (20)$$

It is obvious that for maximizing the profits of a supplier, bidding coefficients a_j , and b_j cannot be selected independently in other words, a supplier can fix one of these two coefficients and then

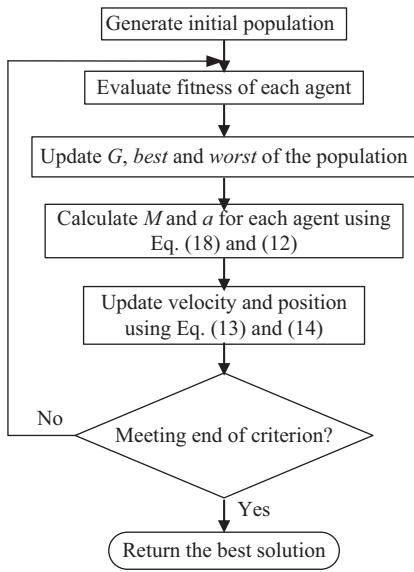


Fig. 1. Flowchart of the GSA.

determine the other by using an optimization procedure. In this regard, GSA is applied to find the optimal bidding coefficients and profit of each supplier. **Fig. 1** illustrates the flowchart of the GSA.

3.1. Gravitational search algorithm (GSA) for finding optimal bidding coefficients (b_j)

Step 1. Initialization of the agents

- (a) Generate random population of b_j solutions (masses), where b_j is the bidding parameter of the j th supplier to be optimized.
- (b) Read input data μ , σ , ρ , a and maximum iterations, where μ is the mean, σ the standard deviation, ρ the correlation coefficient of Eq. (8), and a the cost coefficient.

Step 2. Fitness evaluation and best fitness computation for each agents

$$best(t) = \max_{j \in \{1, \dots, N\}} fit_j(t)$$

$$worst(t) = \min_{j \in \{1, \dots, N\}} fit_j(t)$$

where $fit_j(t)$ represents the fitness of the of j th at iteration t , $best(t)$ and $worst(t)$ represents the best and worst fitness at generation t of Eq. (8)

Step 3. Compute gravitational constant G

Compute gravitational constant G at iteration t using the following equation:

$$G(t) = G_0 e^{-\tau} \left(\frac{iter}{iter_{max}} \right)$$

where G_0 is a constant value and $iter_{max}$ is the maximum number of iterations.

Update $G(t)$, $best(t)$, $worst(t)$ and $M_i(t)$ for $i = 1, 2, \dots, N$

Step 4. Calculate the mass of the each agent at iteration t

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)}$$

Step 5. Calculate accelerations of the agents at iteration t

$$a_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)},$$

where M_{ii} is the inertial mass of i th agent and $F_i^d(t)$ is the total force that acts on i th agent calculated as:

$$F_i^d(t) = \sum_{j \in K_{best1}, j \neq i}^N rand_j F_{ij}^d(t)$$

$F_{ij}^d(t)$ is the force acting on agent i from agent j at d th dimension and t th iteration is computed as:

$$F_{ij}^d(t) = G(t) \frac{M_{pi}(t) \times M_{aj}(t)}{R_{ij}(t) + \varepsilon} (x_j^d(t) - x_i^d(t))$$

where $R_{ij}(t)$ is the Euclidian distance between two agents i and j

Step 6. Update velocity and position of agents

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t)$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1)$$

Step 7. Repeat from steps 2–6 until iteration reaches their maximum limit. Return the best fitness (optimal bid value b_j) computed at final iteration as a global fitness. Using b_j values, calculate MCP from Eq. (5).

3.2. Maximization of profit using GSA

Step 1. Initialization of the agents

- (a) Generate random population of profit F_j solutions (masses) in the search space, where F_j is the profit of the j th supplier.
- (b) Read input data of generators (i.e. cost coefficients, P_{min} , P_{max}), demand (Q_o) and maximum number of iterations.

Step 2. Calculate generator output each supplier using Eq. (6)

- (a) If generation violates lower limit set as a lower limit If generation violates upper limit set as an upper limit

(b) Add all generations

- (c) Error = total system generation – total system demand

Step 3. Fitness evaluation and best fitness computation for each agents

$$best(t) = \max_{j \in \{1, \dots, N\}} fit_j(t)$$

$$worst(t) = \min_{j \in \{1, \dots, N\}} fit_j(t)$$

where $fit_j(t)$ represents the fitness of the of j th at iteration t , $best(t)$ and $worst(t)$ represents the best and worst fitness at generation t of Eq. (7)

Step 4. Compute gravitational constant G

Compute gravitational constant G at iteration t using the following equation:

$$G(t) = G_0 e^{-\tau} \left(\frac{iter}{iter_{max}} \right)$$

where G_0 is a constant value and $iter_{max}$ is the maximum number of iterations.

Update $G(t)$, $best(t)$, $worst(t)$ and $Mi(t)$ for $i = 1, 2, \dots, N$
Step 5. Calculate the mass of the each agent at iteration t

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)}$$

Step 6. Calculate accelerations of the agents at iteration t

$$a_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)},$$

where M_{ii} is the inertial mass of i th agent and $F_i^d(t)$ is the total force that acts on i th agent calculated as:

$$F_i^d(t) = \sum_{j \in Kbest1, j \neq i}^N rand_j F_{ij}^d(t)$$

$F_{ij}^d(t)$ is the force acting on agent i from agent j at d th dimension and t th iteration is computed as:

$$F_{ij}^d(t) = G(t) \frac{M_{pi}(t) \times M_{aj}(t)}{R_{ij}(t) + \varepsilon} (x_j^d(t) - x_i^d(t))$$

where $R_{ij}(t)$ is the Euclidian distance between two agents i and j .
Step 7. Update velocity and position of agents

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t)$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1)$$

Step 8. Repeat from steps 3–7 until iteration reaches their maximum limit. Return the best fitness (maximum profit) computed at final iteration as a global fitness.

Step 9. Print c.p.u time and plot number of iterations versus %error.

$$\%Error = \frac{Generation - demand}{Generation} \times 100$$

4. Proposed fuzzy adaptive gravitational search algorithm (FAGSA)

Metaheuristic which include the GSA method, are approximate algorithms designed to be applied to engineering problems. It is clearly desirable that these algorithms be applicable to real optimization problems without the need for highly skilled labor. However, till date, their application has required significant time and labor for tuning the parameters, and hence, from engineering perspective, it is desirable to add robustness and adaptability to these algorithms. The latter adaptability property is especially important from the viewpoint of practical applications.

Two significant relationships must be understood in order to add adaptability to an optimization algorithm. One is the analysis of the qualitative and quantitative relationship between parameters and the behavior of the algorithm. The other is the analysis of the qualitative and quantitative relation between the behavior of the algorithm and success, or failure, of the search. The modification of the algorithm due to the results of these analyses should be carefully weighed so that an ideal algorithm behavior may be determined relative to the success of the search, so that an adaptive algorithm which feeds back the conditions of the search in order to maintain this behavior may be understood.

In Eq. (10) of GSA, force acting on the masses is related to the value of gravitational constant (G). Hence, the acceleration of the agent varies by varying the value of gravitational constant from Eq. (12). Therefore, the gravitational constant determines the influence of agent's previous velocity in the next iteration and also the search ability of GSA is reduced when the scale of the problem becomes large, because the search finishes before the phase of searching shifts from diversification to intensification. Suitable selection of the gravitational constant (G) provides a balance between global exploration, local exploration and exploitation, which results in less number of iterations on average to find a sufficiently optimal solution. Although the GSA algorithms can converge very quickly toward the nearest optimal solution for many optimization problems, it has been observed that GSA experiences difficulties in reaching the global optimal solution.

The gravitational constant (G) characterizes the behavior of agents, and experience shows that the success or failure of the search is heavily dependent on the value of the gravitational constant. The main causes of the search failures are given by the following:

- The velocity of the agents (masses) increase rapidly, and agents go out of the search space.
- The velocity of the agents (masses) decrease rapidly, and agents become immobile.
- Agents (masses) cannot escape local optimal solutions.

In order to avoid these undesirable situations, it is important to analyze the relationship between the parameters and the behavior of agents (masses), with special regard to divergence and convergence of agents (masses). Therefore, the fuzzy adaptive GSA is proposed, to design a fuzzy adaptive dynamic gravitational constant using fuzzy "IF/THEN" rules for solving the optimal bidding problem. In FAGSA concept, the velocity and position update equations are same as in the case of GSA. But the gravitational constant is dynamically adjusted, as iteration grows, using fuzzy "IF/THEN" rules. The fuzzy inference system maps crisp set of input variables into a fuzzy set using membership functions. According to the pre-defined logic, the output is assigned based on these fuzzy input sets. The variables selected as input to the fuzzy inference system are the current best performance evaluation (normalized fitness value) and current gravitational constant; whereas output variable is change in the gravitational constant as shown in Fig. 2.

The fuzzy system consists of four principle components: fuzzification, fuzzy rules, fuzzy reasoning and defuzzification which are described in the following subsections [24].

4.1. Fuzzification

To obtain a better gravitational constant value under the fuzzy environment, two inputs are considered: (i) normalized fitness value (NFV); (ii) current gravitational constant (G) and output is the correction of the gravitational constant (ΔG). The triangular membership functions considered for the fuzzification of the input variables are presented in three linguistic values, S (small), M (medium) and L (large), whereas the output variable (ΔG) is presented in three fuzzy sets of linguistic values; NE (negative), ZE (zero) and PE (positive) with associated triangular membership functions, as shown in Fig. 3.

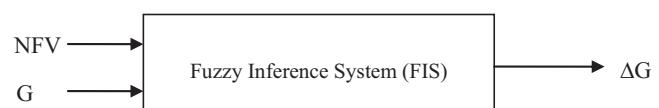


Fig. 2. Inputs and outputs of fuzzy inference system for the proposed FAGSA.

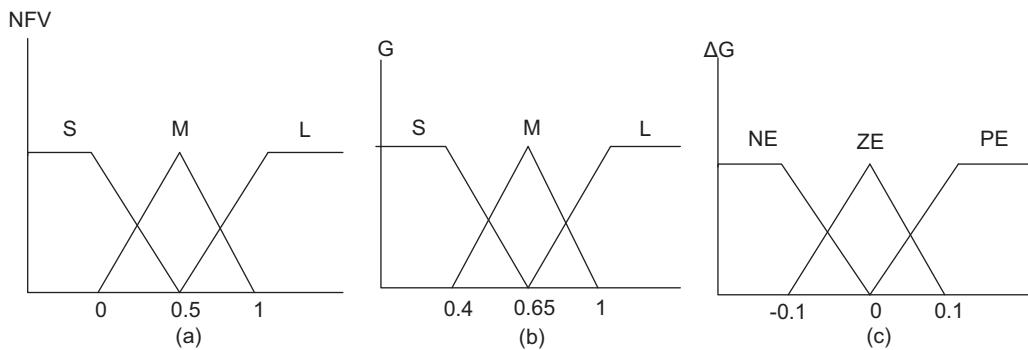


Fig. 3. Membership functions of input variables (a) normalized fitness value (NFV), (b) G, and output variable (c) ΔG .

The membership function of a fuzzy set, usually expressed as $\mu_A(x)$ define how the grade of the membership function (x) associated with the fuzzy set (A), that depends not only on the concept to be represented, but also on the context which it is used. The number of membership functions is problem dependent. Greater resolution is achieved by using more membership functions at the price of greater computational complexity. In the manuscript submitted, triangular membership functions are considered for simplicity as fuzzy set is not too sensitive to variation in shape.

4.2. Fuzzy rules

The Mamdani-type fuzzy rules are used to formulate the conditional statements that comprise fuzzy logic. For example:

IF (NFV is S) AND (G is M) THEN change in gravitational constant (ΔG) is NE

In this paper, the rule base is designed based on the ideal velocity characteristics of the agents (masses) with respect to the generations shown in Fig. 4. The fuzzy rules are designed to determine the change in gravitational constant (ΔG). From the characteristics it can be observed that if the NFV is smaller than the G, then NFV is to be increased to meet G which can be achieved by increasing the gravitational constant. If NFV is greater than G, then NFV is to be decreased to meet G which can be achieved by decreasing the gravitational constant. To incorporate these, three linguistic variables 'Negative', 'Zero' and 'Positive' (NE, ZE, PE) are considered. Therefore, nine ($3 \times 3 = 9$) fuzzy rules can be designed from Table 1. These nine fuzzy rules are sufficient for the fuzzification of change in gravitational constant. As the number of fuzzy rules increases, complexity of the problem increases.

4.3. Fuzzy reasoning

The fuzzy control strategy is used to map the inputs to the output. The AND operator is typically used to combine the membership values for each fired rule to generate the membership values for the fuzzy sets of output variables in the consequent part of the rule. Since there may be other rules fired in the rule sets, for some fuzzy

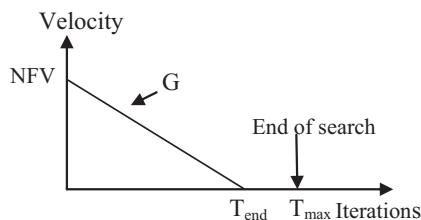


Fig. 4. NFV and G of agents (masses) for the proposed FAGSA.

sets of the output variables there may be different membership values obtained from different fired rules.

To obtain a better gravitational constant under the fuzzy environment, the variables selected as input to the fuzzy system are the current best performance evaluation (NFV) and current gravitational constant (G); whereas the output variable is the change in the gravitational constant (ΔG). The NFV is defined as:

$$\text{NFV} = \frac{\text{FV} - \text{FV}_{\min}}{\text{FV}_{\max} - \text{FV}_{\min}} \quad (21)$$

The fitness value (FV) calculated from Eq. (7) at the first iteration may be used as FV_{\min} for the next iterations, whereas FV_{\max} is a very large value and is greater than any acceptable feasible solution. The gravitational constant (G) is one of the most popular strategies for tuning the parameters of FAGSA. The value of the parameter ' G ' is large at the beginning of the search process and gradually it becomes small as the iterations are increasing. Hence on the universe of discourse the range is selected between 1 (maximum) and 0.4 (minimum). In this paper ($-0.1, +0.1$) has been considered on the universe of discourse as the change in gravitational constant (ΔG) is small and requires both positive and negative corrections.

$$G^{t+1} = G^t + \Delta G \quad (22)$$

4.4. Defuzzification

For defuzzification of every input and output, the method of centroid (center-of-sums) is used for the membership functions shown in Fig. 3. The flow chart of the proposed FAGSA is shown in Fig. 5. The pseudo-code for the proposed FAGSA is given in Table 2.

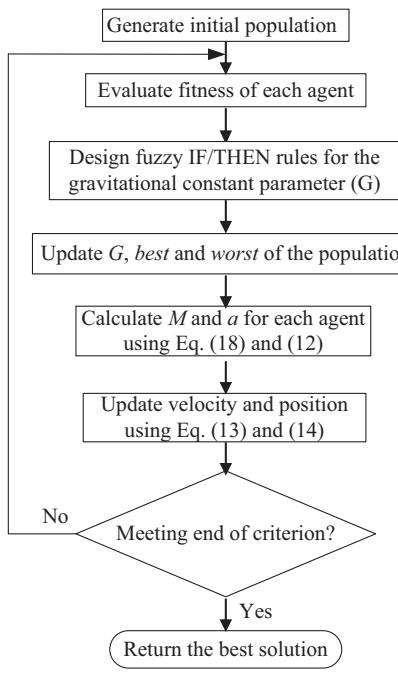
4.5. Implementation of fuzzy adaptive gravitational search algorithm (FAGSA) for optimal bidding problem

4.5.1. FAGSA for computing optimal bidding coefficients (b_j)

Step 1. Initialization of the agents

Table 1
Fuzzy rules for variation of the gravitational constant (G).

Rule No	Antecedent		Consequent
	NFV	G	
1	S	S	ZE
2	S	M	NE
3	S	L	NE
4	M	S	PE
5	M	M	ZE
6	M	L	NE
7	L	S	PE
8	L	M	ZE
9	L	L	NE

**Fig. 5.** Flowchart of the proposed FAGSA.

(a) Generate random population of b_j solutions (masses), where b_j is the bidding parameter of the j th supplier to be optimized.
 (b) Read input data μ , σ , ρ , a and maximum iterations, where μ is the mean, σ the standard deviation, ρ the correlation coefficient of Eq. (8), a is the cost coefficient.

Step 2. Fitness evaluation and best fitness computation for each agents

$$best(t) = \max_{j \in \{1, \dots, N\}} fit_j(t)$$

$$worst(t) = \min_{j \in \{1, \dots, N\}} fit_j(t)$$

where $fit_j(t)$ represents the fitness of the j th at iteration t , $best(t)$ and $worst(t)$ represents the best and worst fitness at generation t of Eq. (8).

Step 3. Compute gravitational constant parameter (G) using fuzzy "IF/THEN" rules.

Update $G(t)$, $best(t)$, $worst(t)$ and $M_i(t)$ for $i = 1, 2, \dots, N$

Table 2
Pseudo-code for the proposed FAGSA.

```

Begin;
Initialize the number of agents,  $N$ 
Initialize the positions of a system with  $N$  masses
Compute gravitational constant using fuzzy logic
For each individual  $i \in N$ : calculate fitness ( $i$ ) and the gravitational and inertial
  masses.
Compute the force acting on mass  $i$  from mass  $j$  at time  $t$ 
Compute the total force that acts on object  $i$  in dimension  $d$ 
Find the acceleration of object  $i$  in  $d$ th dimension
Compute the velocity and the position of the object for  $t = t + 1$ 
If the norm of two consecutive best values of  $x_i$  is smaller than a specified
  tolerance value, or the best values do not change for a specified number of
  iterations stop, otherwise readjust the gravitational constant
End;
  
```

Step 4. Calculate the mass of the each agent at iteration t

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)}$$

Step 5. Calculate accelerations of the agents at iteration t

$$a_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)},$$

where M_{ii} is the inertial mass of i th agent and $F_i^d(t)$ is the total force that acts on i th agent calculated as:

$$F_i^d(t) = \sum_{j \in Kbest, j \neq i}^N rand_j F_{ij}^d(t)$$

$F_{ij}^d(t)$ is the force acting on agent i from agent j at d th dimension and t th iteration is computed as:

$$F_{ij}^d(t) = G(t) \frac{M_{pi}(t) \times M_{qj}(t)}{R_{ij}(t) + \varepsilon} (x_j^d(t) - x_i^d(t))$$

where $R_{ij}(t)$ is the Euclidian distance between two agents i and j .
 Step 6. Update velocity and position of agents

$$v_i^d(t + 1) = rand_i \times v_i^d(t) + a_i^d(t)$$

$$x_i^d(t + 1) = x_i^d(t) + v_i^d(t + 1)$$

Step 7. Repeat from steps 2–6 until iteration reaches their maximum limit. Return the best fitness (optimal bid value b_j) computed at final iteration as a global fitness. Using b_j values, calculate MCP from Eq. (5).

4.5.2. FAGSA for profit maximization

Step 1. Initialization of the agents

(a) Generate random population of profit F_j solutions (masses) in the search space, where F_j is the profit of the j th supplier.

(b) Read input data of Generators (i.e. cost coefficients, P_{\min} , P_{\max}), demand (Q_o) and maximum number of iterations.

Step 2. Calculate generator output each supplier using Eq. (6)

(a) If generation violates lower limit set as a lower limitIf generation violates upper limit set as an upper limit

(b) Add all generations

(c) Error = total system generation – total system demand

Step 3. Fitness evaluation and best fitness computation for each agents

$$best(t) = \max_{j \in \{1, \dots, N\}} fit_j(t)$$

$$worst(t) = \min_{j \in \{1, \dots, N\}} fit_j(t)$$

where $fit_j(t)$ represents the fitness of the j th at iteration t , $best(t)$ and $worst(t)$ represents the best and worst fitness at generation t of Eq. (7)

Step 4. Compute gravitational constant parameter (G) using fuzzy logic

Update $G(t)$, $best(t)$, $worst(t)$ and $Mi(t)$ for $i = 1, 2, \dots, N$
 Step 5. Calculate the mass of the each agent at iteration t

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)}$$

Step 6. Calculate accelerations of the agents at iteration t

$$a_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)},$$

where M_{ii} is the inertial mass of i th agent and $F_i^d(t)$ is the total force that acts on i th agent calculated as:

$$F_i^d(t) = \sum_{j \in Kbest1, j \neq i}^N rand_j F_{ij}^d(t)$$

$F_{ij}^d(t)$ is the force acting on agent i from agent j at d th dimension and t th iteration is computed as:

$$F_{ij}^d(t) = G(t) \frac{M_{pi}(t) \times M_{aj}(t)}{R_{ij}(t) + \varepsilon} (x_j^d(t) - x_i^d(t))$$

where $R_{ij}(t)$ is the Euclidian distance between two agents i and j .
 Step 7. Update velocity and position of agents

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t)$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1)$$

Step 8. Repeat from steps 3–7 until iteration reaches their maximum limit. Return the best fitness (maximum profit) computed at final iteration as a global fitness.

Step 9. Print c.p.u time and plot number of iterations versus %Error.

$$\%Error = \frac{\text{Generation} - \text{demand}}{\text{Generation}} \times 100$$

5. Results and discussions

In order to evaluate the performance of proposed FAGSA for solving optimal bidding problem, IEEE 30-bus system and 75-bus Indian practical system are considered. The performance of the proposed FAGSA has been compared with GSA [23], PSO [17], GA [15] and a traditional optimization method called golden section search (GSS) method [6]. In this work, the parameters used for GSA, PSO and GA (binary coded) are given in Table 3, where N : population size; G : gravitational constant for GSA; $c1$ and $c2$: learning factors, w : inertia weight for PSO; l : chromosome length; P_e : Elitism probability; P_c : crossover probability, P_m : mutation probability for GA;

Table 3

Parameters for different approaches for IEEE 30-bus system and 75-bus Indian system.

GSA	PSO	GA
$N = 50$; Max. iterations = 1000; $G = 100$	No. of particles = 50; Max. iterations = 1000; $c1 = c2 = 2.0$; $w = 0.9-0.4$	Population size = 50; Generations = 1000; $l = 12$ $P_e = 0.15$; $P_c = 0.85$; $P_m = 0.005$

Table 4
 Generator data for IEEE 30-bus system.

Generator	e	f	P_{\min} (MW)	P_{\max} (MW)
1	2.0	0.00375	20	160
2	1.75	0.0175	15	150
3	1.0	0.0625	10	120
4	3.25	0.00834	10	100
5	3.0	0.025	10	130
6	3.0	0.025	10	130

Table 5
 Optimal bidding strategies of generators for IEEE 30-bus system.

Generator	FAGSA	GSA [23]	PSO [17]	GA[15]	Traditional GSS [6]
	b_i	b_j	b_j	b_j	b_j
1	0.021437	0.021004	0.001092	0.001045	0.15800
2	0.121878	0.090472	0.050953	0.048786	0.04745
3	0.337380	0.263450	0.181976	0.174234	0.13099
4	0.023806	0.054320	0.024283	0.023250	0.02458
5	0.100457	0.108594	0.072791	0.069694	0.05614
6	0.063465	0.108594	0.072791	0.069694	0.56140

simulations are carried on 2.66 GHz, PIV processor, 3GB RAM and MATLAB 7.8 version is used.

5.1. IEEE 30-bus system

The IEEE 30-bus system consists of six suppliers, who supply electricity to aggregate load. The generator data is shown in Table 4. Q_o is 500 with inelastic load ($K=0$), considered for aggregated demand. Bidding strategies are shown in Table 5. The optimal bid prices and profits are shown in Table 6. From Tables 5 and 6 it is observed that, the proposed FAGSA giving maximum power outputs and higher profits. Therefore, the bidding parameters obtained by FAGSA are optimum compared to GSA, PSO, GA and GSS method. Fig. 6 shows the convergence characteristics of proposed FAGSA, GSA, PSO and GA. From Fig. 6 it is observed that, the time taken for the convergence of the proposed method is drastically reduced because of the fuzzification of gravitational constant (G). The gravitational constant adjusts the accuracy of the search, so it decreases with the time, which leads to a fast convergence rate compared to reported methods. In GSA the optimum selection of gravitational constant (G) is tedious and improper selection of gravitational constant results the velocity of the agents (masses) decreases rapidly, and agents become immobile. The performance of the PSO greatly dependent on the inertia weight, therefore, improper selection of the inertia weight may lead to premature convergence of the

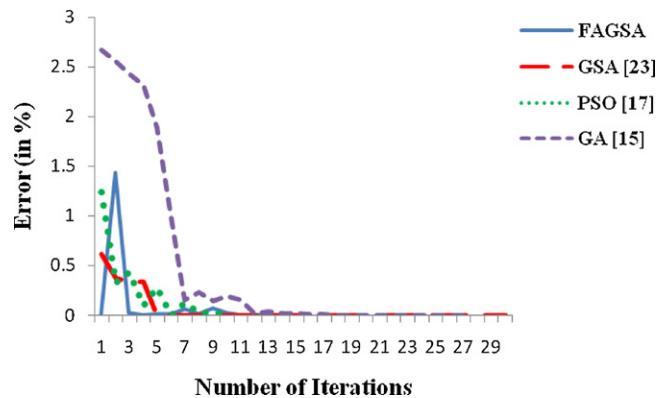


Fig. 6. Convergence characteristics of FAGSA, GSA, PSO and GA for IEEE 30-bus system.

Table 6

Market clearing price (MCP) (\$/MWh) and profit (\$) of generators for IEEE 30-bus system.

Generator	FAGSA		GSA [23]		PSO [17]		GA [15]		Traditional GSS [6]	
	P (MW)	Profit (\$)	P (MW)	Profit (\$)						
1	160	1034.9	160	959.38	160.00	772.41	160	741.45	160.00	557
2	60.04	376.38	99.67	417.85	100.83	340.10	101.2	321.32	91.3	249
3	58.91	157.22	38.83	167.06	32.35	125.06	32.68	119.33	38.8	103
4	100	498.47	98.42	441.38	100.00	280.36	100	261.01	100.00	200
5	60.41	275.38	51.53	221.99	53.40	136.32	53	125.56	54.90	94
6	60.41	275.38	51.53	221.99	53.40	136.32	53	125.56	54.90	94
MCP	9.06		8.59		6.88		6.69		6.08	
Total profit	2617.73		2429.65		1790.57		1694.23		1297	

particles. GA has limitation of sensitivity of the choice of the parameters such as crossover and mutation probabilities.

5.2. 75-bus Indian practical system

The 75-bus Indian practical system [25] consists of 15 suppliers, who supply electricity to aggregate load. Q_0 is 5000 with inelastic load ($K=0$) considered for aggregated load. Bidding coefficients, generator output, MCP and profit of suppliers are calculated using FAGSA, shown in Tables 7 and 8. It can be evident from Tables 7 and 8 that, proposed FAGSA method producing higher profits compared with other reported algorithms. Therefore, the bidding parameters obtained by FAGSA are optimum compared to GSA, PSO, GA and GSS method. Fig. 7 shows the convergence

characteristics of different methods. From Fig. 7 it is observed that the proposed FAGSA converges fast compared to GSA, PSO and GA because it has the advantages of agents with higher performance have a greater gravitational mass, as a result, the agents tend to move toward the best agent, which avoids premature convergence and also a bigger inertia causes a higher attraction of agents, this permits faster convergence. Even if the size of the system increases still proposed method converges fast. This shows the robustness of the FAGSA.

The superiority of the FAGSA approach is demonstrated through comparison of simulation results with GSA, PSO and GA. Due to the randomness of the evolutionary algorithms, their performance cannot be judged by the result of a single run. Many trials with different initializations should be made to reach a valid conclusion about

Table 7

Optimal bidding strategies of generators for 75-bus Indian practical system.

Generator	FAGSA		GSA [23]		PSO [17]		GA [15]		Traditional GSS [6]	
	b_j	b_j	b_j	b_j	b_j	b_j	b_j	b_j	b_j	b_j
1	0.002223		0.002924		0.002200		0.002944		0.001993	
2	0.004708		0.004509		0.003232		0.004774		0.003233	
3	0.010916		0.002369		0.006976		0.003918		0.002653	
4	0.008912		0.006296		0.006762		0.002844		0.001926	
5	0.203729		0.334051		0.134368		0.196915		0.133342	
6	0.008007		0.011553		0.011002		0.005410		0.003664	
7	0.024431		0.019105		0.006162		0.012133		0.008216	
8	0.013519		0.004908		0.009800		0.004973		0.003367	
9	0.010950		0.010374		0.008323		0.004177		0.002828	
10	0.006197		0.006258		0.003392		0.003222		0.002182	
11	0.005578		0.005537		0.002952		0.003222		0.002182	
12	0.008748		0.007409		0.005573		0.003063		0.002074	
13	0.008645		0.004727		0.002035		0.002964		0.002007	
14	0.005838		0.002403		0.005630		0.002665		0.001805	
15	0.002253		0.006110		0.002529		0.003640		0.002465	

Table 8

Market clearing price (MCP) (Rs/MWh) and profit of generators for 75-bus Indian practical system.

Generator	FAGSA		GSA [23]		PSO [17]		GA [15]		Traditional GSS [6]	
	P (MW)	Profit (Rs)	P (MW)	Profit (Rs)						
1	95.2	786.72	467.24	315.1	471.3	160.02	571.7	119.7	502.39	86.16
2	173.44	305.69	198.71	83.2	175.4	25.6	56.6	22.9	209.62	7.04
3	64.92	207.66	180	82.4	174.1	27.5	162.6	23.0	141.29	5.68
4	60.52	405.43	100	242.0	92.6	158.8	94	99.8	94.7	72.61
5	70.15	16.19	175.13	6.2	170.3	3.7	163.4	3.5	164.08	2.79
6	375.19	250.58	80.16	56.7	54.2	24.3	52.7	22.5	50.82	7.67
7	77.44	383.38	52.17	262.8	60	225.7	40	149.8	42	128.09
8	103.46	218.37	80	114.4	77.2	43.5	73.5	40.2	68.17	19.68
9	156.58	237.32	294.09	73.2	227.7	37.6	274.3	34.5	291.71	13.543
10	725.38	299.08	80	152.0	80	92.7	76	57.1	74	32.69
11	150	356.25	109	185.4	109	116.7	94.5	73.3	92.5	44.91
12	270	613.98	554.31	322.2	252.5	257.1	225.4	233.1	267.36	186.45
13	95.2	539.15	329.22	350.8	603.1	198.1	602.6	182.2	531.48	135.23
14	173.44	629.541	149.5	425.2	150	343.0	132.5	189.2	145	155.20
15	64.92	398.51	149.93	96.2	302.1	37.6	379.1	32.39	318.65	10.66
MCP	8.83		8.01		7.68		7.56		7.38	
Total Profit	5647.91		2768.57		1752.6		1283.89		908.40	

Table 9

Performance comparison of different approaches for IEEE 30-bus system.

		FAGSA	GSA [23]	PSO [17]	GA [15]
Total profit (\$)	Best (\$)	2617.73	2429.65	1790.57	1694.23
	Worst (\$)	2591.18	2392.34	1574.85	1464.27
	Average (\$)	2604.45	2410.99	1682.71	1579.25
	PD (%)	0.010	0.015	0.120	0.135
Average execution time (s)		1.97	1.97	2.06	6.24

Table 10

Performance comparison of different approaches for 75-bus Indian practical system.

		FAGSA	GSA [23]	PSO [17]	GA [15]
Total profit (Rs)	Best (Rs)	5647.91	2768.57	1752.6	1283.89
	Worst (Rs)	5492.18	2643.91	1627.34	1092.06
	Average (Rs)	5570.04	2706.24	1689.97	1187.97
	PD (%)	0.027	0.045	0.071	0.149
Average execution time (s)		48.84	63.46	78.37	93.56

Table 11The statistical results of the *t*-test for 20 independent runs.

Test system	FAGSA		GSA		<i>t</i> -Value between FAGSA and GSA
	Mean	Standard deviation	Mean	Standard deviation	
IEEE 30-bus system	2600.17	304.18	2409.67	190.42	38.30
75-bus Indian practical system	5557.00	2809.61	2699.35	2134.5	42.93

the performance of the algorithms. An algorithm is robust, if it can guarantee an acceptable performance level under different conditions. Since FAGSA, GSA, PSO and GA random in nature therefore, the bidding data was executed for 20 runs for all the approaches. The best, worst, average values, total profit and PD for the given data are tabulated in [Tables 9 and 10](#) for IEEE 30-bus and practical 75 bus Indian system respectively. The percentage deviation (PD) is computed as follows:

$$PD \ (\%) = \frac{(Best - Worst)}{Best} \times 100$$

From the result it is observed that the PD is minimum for the proposed FAGSA compared to GSA, PSO and GA, for the given test system and optimal bidding strategies obtained by FAGSA producing higher profits compared to GSA, PSO and GA. In addition to that, FAGSA shows good consistency by keeping small variation between the best and worst solution. In other words, the simulation results show that the FAGSA converges to global solution has a shorter execution time and small percentage deviation because, it has the advantage of agents with higher performance have greater gravitational mass. As a result, the agents tend to move toward the best

agent, which avoids premature convergence and also bigger inertia mass cause's higher attraction of agents, this permits a faster convergence.

In order to prove further the number of the runs is reasonable to evaluate the convergence and robustness of FAGSA, a statistical analysis is carried out by applying *t*-test to compare the means of the best results produced by the FAGSA and GSA. The statistical results of the *t*-test for FAGSA and GSA on IEEE 30-bus and 75-bus Indian practical system are listed in [Table 11](#). The *t*-values between the FAGSA and GSA for both the test systems are 38.30 and 42.93 respectively. From [Table 11](#) it is observed that, when the *t*-value is higher than 2.02 (for degrees of freedom = 38 and probability = 0.05), there is a significant difference between the two algorithms with a 95% confidence level. Therefore, the performance of the FAGSA is statistically significantly better than that of other optimization methods with a 95% confidence level. From these results it is understood that selecting the number of runs to be 20 is reasonable to evaluate the convergence and robustness of FAGSA in comparison with reported methods.

6. Conclusion

In this paper, a new optimization algorithm called fuzzy adaptive gravitational search algorithm (FAGSA) has been proposed to achieve a better balance between global and local searching abilities of the agents (masses). The result of gravitational search algorithm (GSA) greatly depends on gravitational constant (*G*) and the method often suffers from the problem of being trapped in local optima. To overcome this drawback, gravitational constant has been adjusted dynamically and nonlinearly by using fuzzy "IF/THEN" rules in order to reach the global solution.

The performance of the proposed FAGSA is tested on IEEE 30-bus system and 75-bus Indian practical system. The test results of proposed method are compared with the well-known heuristic search methods reported in literature. From the test results, it is observed that, the proposed FAGSA converge to global best solution due to fuzzification of gravitational constant. Proper selection of gravitational constant makes a great intensity of attraction as a

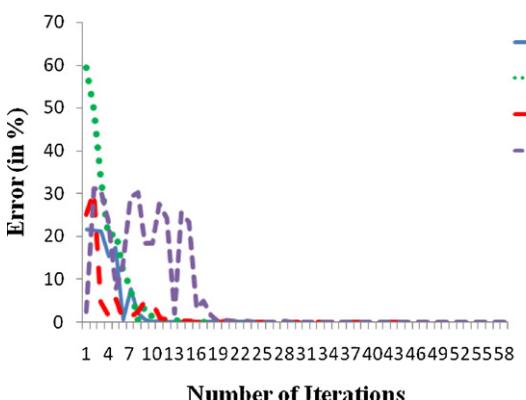


Fig. 7. Convergence characteristics of FAGSA, GSA, PSO and GA for 75-bus Indian system.

result the agents tend to move toward the best agent compared to gravitational search algorithm (GSA), particle swarm optimization (PSO) and genetic algorithm (GA). The proposed FAGSA takes minimum execution time due to the gravitational constant has been dynamically adjusted using simple "IF/THEN" rules and also FAGSA outperformed the reported algorithms in a statistically meaningful way. Therefore, in conclusion, the proposed FAGSA outperform the GSA, PSO and GA reported in literature in terms of global best solution, standard deviation and computation time. Thus, the proposed FAGSA is more effective for the optimal bidding strategy in giving the best optimal solution in comparison to the GSA, PSO and GA with respect to total profit and computation time. Future research should focus on extending the optimal bidding problem for a day-ahead market including inter-temporal constraint of the generating units.

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