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Equal embedded algorithm for economic load dispatch problem with transmission losses

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A B S T R A C T

This paper presents equal embedded algorithm (EEA) to solve the economic dispatch (ED) problem with quadratic and cubic fuel cost functions and transmission losses. The proposed algorithm involves selection of lambda values, then the expressions of output powers of generators are derived in terms of lambda by interpolation and finally optimal value of lambda is evaluated from the power balance equation by Muller method. The proposed method is implemented and tested by considering 3, 15 and 26 generators to solve the ED problem. Simulation results such as quality of solution, convergence characteristic and computation time of the proposed method are compared with some existing methods like genetic algorithm (GA), particle swarm optimization (PSO) and Lambda iterative method. It is observed from different case studies that the proposed EEA algorithm provides the qualitative solution with less computational time irrespective of the size of the system.

1. Introduction

Main objective of the economic dispatch (ED) problem is to determine the allocation of output powers of generators so as to meet the power demand at minimum operating cost under various system and operating constraints [1–3]. The fuel cost function of each generator is represented by a quadratic function [4]. The minimization of cost of power generation depends on the efficiency of generator, fuel cost and minimization of transmission loss [5]. It is necessary to consider the incremental transmission losses for the ED problem.

Earlier, conventional optimization techniques such as Lambda iteration method, Lambda projection method and gradient methods [6] have been employed to solve the ED problems. In these methods, computational time increases when the size of the system increases and therefore more time is needed to get the optimal solution. In real time power system operation, the incremental fuel cost may not always monotonically increase. To overcome the above difficulty, dynamic programming (DP) [8,9,25] was used for solving the ED problem with monotonically increased and decreased fuel cost functions as it will not impose any restrictions on the nature of the cost curve. However, the DP suffers from problem of increase of computational time with increased dimensionality. Thus, this method is not suitable for online application of the ED problem.

In order to get the qualitative solution for the ED problem, artificial neural network techniques such as back propagation (BP) algorithm based neural network [10] and Hopfield neural network (HNN) [11,12] have been successfully applied for thermal generators with piecewise quadratic function and prohibited zone constraints [13]. The BP algorithm takes more iterations due to improper selection of learning and momentum rates. Similarly, the Hopfield model suffers from excessive iterations due to an unsuitable sigmoid function [14]. Therefore, it takes more time to give optimal solution at required power demand. In the past decade, global optimization technique like genetic algorithm (GA) has been used to solve the ED problem with quadratic, piece wise quadratic fuel cost function and valve point loading [15]. It is a parallel search technique, which imitates natural genetic operation. Due to its high potential for global optimization, the GA has received great attention in solving the ED problems with a quadratic and piecewise quadratic cost function and valve point loading including transmission losses, ramp rates and prohibited zones [16]. But recent research identified some deficiencies in the GA performance as the cross over and mutation operations cannot ensure the better fitness of offspring because the chromosomes in the population have similar structures and their average fitness is high towards the end of the evolutionary process [17]. Recently, meta-heuristic techniques such as evolutionary programming (EP) [19], particle swarm optimization (PSO) [20], ant colony searching

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algorithm (ACSA) [21] and the tabu search algorithm (TSA) [22] have been used to solve various kinds of ED problems. In these methods, the quality of solution depends on user-defined factors. The improper selection of these factors may increase the computational time for getting optimal solution.

The main aim of the present paper is to develop a new method by employing equal embedded algorithm for solving the ED problem with generator constraints and transmission losses. The proposed method is formulated based on numerical methods such as interpolation and Muller method. It is observed from the literature available for solution of ED problems that most of the existing methods have failed to provide optimal solution with reduced computational burden. In the present approach, the qualitative solution can be achieved with less computational time. Further, the proposed method provides qualitative solution than the existing algorithms including the evolutionary programming [18], the Hopfield neural network [23], the genetic algorithm [15], the lambda iterative method and the particle swarm optimization method [20].

Organization of the paper is given here. In Section 2, formulation of the ED problem is introduced. Section 3 addresses description of equal embedded algorithm to solve the ED problem. Section 4 gives the details of implementation of the equal embedded algorithm for solving the ED problem. Case studies with various number of generators are presented in Section 5. Conclusions are finally presented in the last section.

2. Formulation of the economic dispatch problem

The ED problem is a non-linear programming optimization problem. Objective of the ED problem is to minimize the total fuel cost of the generators subjected to the operating constraints of power system.

The formulation of ED problem is given below.

$$F_i(P_i) = a_i + b_iP_i + c_iP_i^2/\$\,\mathrm{h} \tag{1}$$

The objective function is

$$\text{Minimize} \quad F_T = \sum_{i=1}^{ng} F_i(P_i) \tag{2}$$

subjected to

(a) Equality constraint given by the power balance equation

$$\sum_{i=1}^{n_g} P_i = P_D + P_L, \quad i = 1, 2, 3 \ldots\ldots n_g. \tag{3}$$

where the total transmission loss is assumed as a quadratic function of the generator power outputs [24] and given by

$$P_L = \sum_{i=1}^{n_g}\sum_{j=1}^{n_g} P_iB_{ij}P_j + B_{i0}P_j + B_{00} \tag{4}$$

(b) Inequality constraints given by minimum and maximum of output power of each generating unit

$$P_i^{\min} \leqslant P_i \leqslant P_i^{\max} \tag{5}$$

From the Eqs. (2)–(5), the formulation of Lagrange function for the ED problem is given by

$$\chi = F_T + \lambda \times \left(P_D + P_L - \sum_{i=1}^{ng} P_i\right) \tag{6}$$

The expressions of lambda and output power are

$$\lambda_i = \frac{b_i + (2 \times c_i \times P_i)}{1 - \left(2 \times \sum_{i=1}^{ng} B_{ij}P_j + B_{i0}\right)} \tag{7}$$

$$P_i = \frac{\lambda_i \times \left(1 - B_{i0} - 2 \times \sum_{i=1}^{ng} B_{ij}P_j\right) - b_i}{2 \times (c_i + \lambda_iB_{ii})} \tag{8}$$

From (8), it is observed that numerator and denominator of the output power of the generator are in linear relation with lambda. Therefore, the output power of the generator can be viewed as

$$P_{i\mathrm{Num}} = \lambda_i \times \left(1 - B_{i0} - 2 \times \sum_{i=1}^{ng} B_{ij}P_j\right) - b_i \tag{9}$$

$$P_{i\mathrm{Den}} = 2 \times (c_i + \lambda_iB_{ii}) \tag{10}$$

where

a_i, b_i, c_i	Fuel cost co-efficients of *i*th generator
$F_i(P_i)$	Fuel cost ($) of generator of *i*th generator
λ_i	Incremental fuel cost ($/MW) of *i*th generator
F_t	Total fuel cost ($)
P_i	Output power of *i*th generator
P_D	Power demand (MW)
P_L	Transmission losses (MW)
Ng	Number of generators.
$P_i^{\min}$	Minimum output power of *i*th generator
$P_i^{\max}$	Maximum output power of *i*th generator
B_{ij}, B_{0i}, B_{00}	Coefficients of B_{Loss} matrix
$P_{i\mathrm{Num}}$	Numerator of generator of *i*th generator
$P_{i\mathrm{Den}}$	Denominator of generator of *i*th generator

3. Equal embedded algorithm for solving economic dispatch problem

In this section, a new algorithm has been proposed based on well established numerical methods such as interpolation and Muller method.

The power balance Eq. (3) is written as

$$\sum_{i=1}^{ng} P_i - (P_D + P_L) = 0 \tag{11}$$

From (11), the power balance equation is written as

$$f(\lambda, P_D) = \sum_{i=1}^{ng} P_i - (P_D + P_L) \tag{12}$$

3.1. Mathematical formulation of equal embedded algorithm

3.1.1. Selection of lambda values

Two values of lambda are selected such that

$$f(\lambda_1, P_D) < 0 \quad \text{and} \quad f(\lambda_2, P_D) > 0 \tag{13}$$

Suitable values of lambda are selected from RMPPD table, which is explained in Section 3.2.1.

It is observed from Eqs. (9) and (10) that the numerator and denominator of output power of generator are linear relation with lambda. The expressions of numerator and denominator of output powers are derived in terms of lambda for all generators by interpolation.

3.1.2. Interpolation

It is a process used to estimate an unknown value between two known values by utilizing a common mathematical relation. It is

Table 1
Interpolation table.

λ	$P_{i\text{Num}}$	$P_{i\text{Den}}$	$P_i = \frac{P_{i\text{Num}}}{P_{i\text{Den}}}$
λ_1	A_{i1}	B_{i1}	$C_i = \frac{A_{i1}}{B_{i1}}$
λ_2	E_{i1}	F_{i1}	$G_i = \frac{E_{i1}}{F_{i1}}$

most often used in situations where a table of values has missing data. A polynomial can be estimated from the known input and the known output data by an interpolation [7,25,26]. At desired input, the unknown output value is evaluated from the polynomial.

In the ED problem with transmission losses, the output power of generating unit is a non-linear relation with lambda whereas numerator and denominator expressions of output power of generators are linear relation with lambda. Here, the lambda is assumed as an input and the numerator and denominator of the output power are taken as two outputs. A simple interpolation table for the problem is shown in Table 1.

The expressions of numerator and denominator of output power are derived in terms of lambda by linear interpolation from Table 1. The corresponding equations are given below.

$$P_{i\text{Num}} = A_{i1} + \left(\frac{E_{i1} - A_{i1}}{\lambda_2 - \lambda_1} \times (\lambda - \lambda_1)\right) \quad (14)$$

$$P_{i\text{Den}} = A_{i1} + \left(\frac{F_{i1} - B_{i1}}{\lambda_2 - \lambda_1} \times (\lambda - \lambda_1)\right) \quad (15)$$

$$P_i = \frac{P_{i\text{Num}}}{P_{i\text{Den}}} \quad (16)$$

where the expression of the output power is in terms of λ. Since, P_D is fixed at a particular power demand in the ED problems. Therefore Eq. (12) can be written as,

$$f(\lambda) = 0 \quad (17)$$

where $f(\lambda) = 0$ is non-linear relation in λ. This equation has one variable. The non-linear equation with one variable can be solved with root finding techniques available in numerical methods. Here, Muller method is used. Description of the Muller method is given below.

3.1.3. Muller method

It is a root finding algorithm for solving equation of the form of $f(x) = 0$, where $f(x)$ is a non-linear function of x. it was first presented by Muller in 1956. It is based on the secant method and is used to find the root of the $f(x) = 0$, when no information about the derivative exists. In this method, three points are used to find an interpolating quadratic polynomial.

In Muller method, higher order polynomial is approximated by a quadratic curve in the vicinity of a root. The roots of quadratic equation are then assumed to be approximately equal to be the roots of the equation $f(x) = 0$. This method is iterative and converges almost quadratically [7,25,26].

Let x_{i-2}, x_{i-1}, x_i are three distinct approximations to a root of $f(x) = 0$ and y_{i-2}, y_{i-1} and y_i are the corresponding values of $y = f(x)$. The relation between y and x can be represented by

$$y = A(x - x_i)^2 + B(x - x_i) + y_i \quad (18)$$

where

$$A = \frac{(x_{i-2} - x_{i-1})(y_{i-1} - y_i) - (x_{i-1} - x_i)(y_{i-2} - y_i)}{(x_{i-1} - x_{i-2})(x_{i-1} - x_i)(x_{i-2} - x_i)} \quad (19)$$

$$B = \frac{(x_{i-2} - x_i)^2(y_{i-1} - y_i) - (x_{i-1} - x_i)^2(y_{i-2} - y_i)}{(x_{i-1} - x_{i-2})(x_{i-1} - x_i)(x_{i-2} - x_i)} \quad (20)$$

$$x_{i-1}^{(1)} = x_{i-1}^{(0)} - \frac{2 \cdot y_i}{B \pm \sqrt{B^2 - 4Ay_i}} \quad (21)$$

The sign in the denominator should be chosen properly so as to make the denominator largest in magnitude. With this choice, Eq. (21) gives the next approximation to the root.

The advantage of this method is that it converges quadratically to find the root of the polynomial, which in the present case is the value of lambda.

The lambda values of all generators are varying from minimum to maximum lambda for different power demands. At required power demand, lambda values of all generators are embedded at one value to provide an optimal solution and it is equal for all generators. Hence the proposed algorithm is named as "equal embedded algorithm".

3.2. Equal embedded algorithm for solving the economic dispatch with transmission losses

The proposed equal embedded algorithm for solving the ED problem is given below.

3.2.1. Selection of lambda values

From the Eq. (7), lambda values are evaluated at the minimum and maximum output powers for all generators.

$$\lambda_i = \frac{b_i + (2 \times c_i \times P_i)}{1 - (2 \times \sum_{i=1}^{ng} B_{ij}P_j + B_{i0})} \quad \text{at } P_i = P_i^{\min}, P_i^{\max} \quad (22)$$

where $\lambda_i = \lambda_i^{\min}$ at $P_i = P_i^{\min}$ and $\lambda_i = \lambda_i^{\max}$ at $P_i = P_i^{\max}$

All the lambda values are arranged in ascending order.

3.2.2. Formation of PPD, MPPD and RPPD tables

Purpose of PPD, MPPD and RPPD table is to find the suitable range of the lambda values at desired power demand. The procedure of formulation of these tables is given below

I. *Pre-prepared power demand (PPD)*: Table output powers and transmission losses are computed for all values of lambda by using (8). All lambda values, output powers, transmission loss, sum of output power minus transmission loss (SOP) are formulated as a table. This table is called PPD table.
II. *Modified pre-prepared power demand (MPPD)*: Table numerator and denominators of the output powers, output power, transmission loss and SOP (sum of output powers at lambda minus transmission loss) are calculated and arranged in a table for all lambda values is known as modified pre-prepared power demand (MPPD) table.
III. *Reduced modified pre-prepared power demand (RMPPD)*: Table at required power demand, the upper and lower rows of the MPPD table are selected such that the power demand lies within the SOP limits and these two rows are formulated in a table is known as reduced MPPD (RMPPD) table.

3.2.3. Interpolation

At required power demand, the expressions of the numerator and denominators of the output powers of the generators are obtained in terms of the lambda from RMPPD table by Newton forward interpolation method. In the ED problem, the application of Newton forward interpolation method to obtain the output power of the generator in terms of a lambda is as follows:

(i) The expression of output power in terms of lambda is given in (8). Also the expressions of numerator and denominator of the output power are given in (9) and (10).
(ii) At desired power demand, λ_j, λ_{j+1}, $P_{i\text{Num}}$, $P_{i\text{Den}}$ and P_i obtained from the MPPD table.

Table 2
RMPPD table.

λ	P_{iNum}	P_{iDen}	$P_i = \frac{P_{iNum}}{P_{iDen}}$	P_{LOSS}	$SOP = \sum_{i=1}^{k=ng} P_i - P_{Loss}$
λ_j	A_{i1}	B_{i1}	$C_i = \frac{A_{i1}}{B_{i1}}$	d	$\sum_{i=1}^{ng} C_i - d$
λ_{j+1}	E_{i1}	F_{i1}	$G_i = \frac{E_{i1}}{F_{i1}}$	h	$\sum_{i=1}^{ng} G_i - h$

A simple model of MPPD table is given in Table 2. The expressions of output power numerator and denominator are derived in terms of lambda from the Table 2 by using Newton forward interpolation method and the corresponding equations are

$$P_{iNum} = A_{i1} + \frac{(E_{i1} - A_{i1})}{(\lambda_{j+1} - \lambda_j)}(\lambda - \lambda_j) \tag{23}$$

$$P_{iDen} = B_{i1} + \frac{(F_{i1} - B_{i2})}{(\lambda_{j+1} - \lambda_j)}(\lambda - \lambda_j) \tag{24}$$

$$P_i = \frac{P_{iNum}}{P_{iDen}} \tag{25}$$

3.2.4. Muller method

The application of Muller method to find the lambda value from the power balance equation at required power demand in the ED problem is as follows:

At specified power demand,

$$f(\lambda) = \sum_{i=1}^{ng} P_i(\lambda) - (P_D + P_L(\lambda)) \tag{26}$$

The above equation is non-linear and the solution is obtained by Muller method.

At the required power demand,

$$x_{i-2} = \lambda_j \quad \text{and} \quad y_{i-2} = SOP_j \tag{27}$$

$$x_i = \lambda_{j+1} \quad \text{and} \quad y_i = SOP_{j+1} \tag{28}$$

$$x_{i-1} = (\lambda_j + \lambda_{j+1})/2 \tag{29}$$

From (21), the lambda value can be evaluated by an iterative approach.

4. Implementation of equal embedded algorithm for solving the ED problem with transmission loss

Step-1: The input data.

- Fuel cost data of generators.
- Co-efficient of B_{Loss}.
- Power demand.

Step-2: Calculate lambda values by (7) for all generators at their maximum and minimum output powers and arrange in ascending order.

Step-3: Compute output powers and transmission loss for all values of lambda by using (8). All the lambda values, output powers and transmission loss are formulated as a table is called as a PPD table. At any lambda value in PPD table, if the power demand is matched with SOP then it is the optimal solution. Otherwise go to step-4

Step-4: Obtain themodified pre-prepared power demand (MPPD) table.

Step-5: At required power demand, obtain reduced MPPD (RMPPD) table.

Step-6: Find expressions of numerator and denominator of output power in terms of lambda by Newton forward interpolation from the RMPPD table.

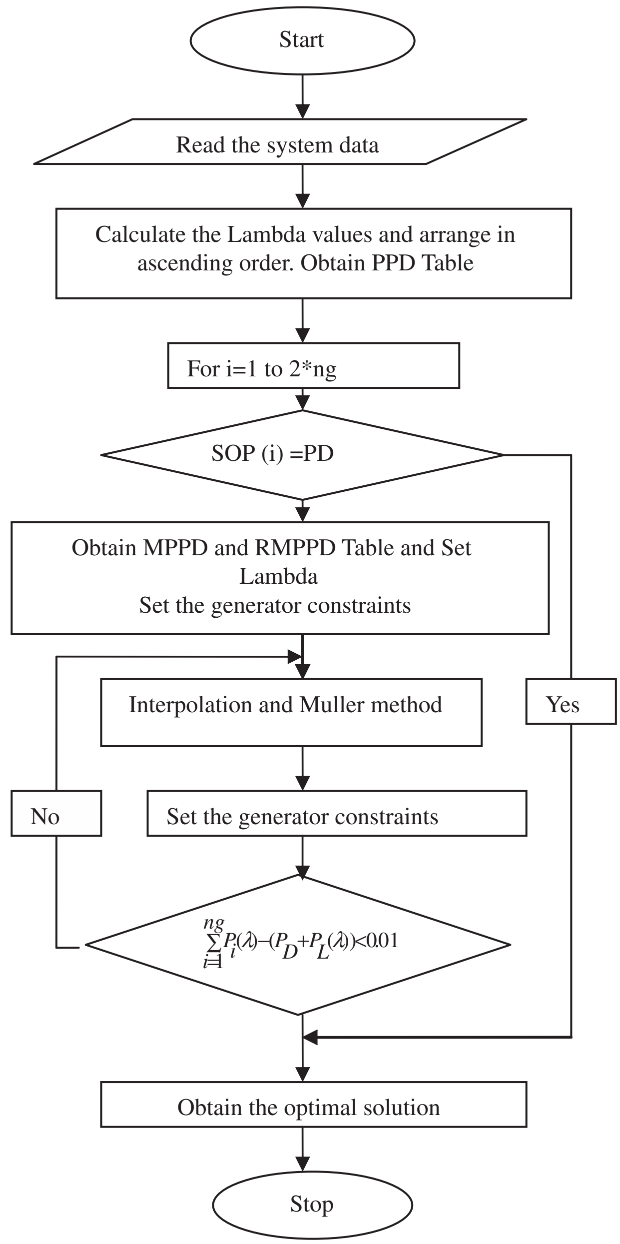

Fig. 1. Flow chart of the proposed method.

Step-7: Evaluate lambda using Muller method from the power balance equation at required power demand.

Step-8: Obtain the solution

Flow chart of the proposed EEA algorithm is shown in Fig. 1.

5. Case studies and simulation results

In this section, various test cases are considered to test the performance of the proposed method. Coding of the algorithm is developed in MATLAB-7.0 and executed on Pentium Dual core, 2.7 GHz personal computer with 1 GB RAM to solve the ED problem of a power system having 3, 15 and 26 generators with transmission loss. The results obtained from the proposed method are compared in terms of the solution quality, convergence characteristics and computation efficiency with various methods such as GA, PSO and Lambda iterative method. During the execution of GA and PSO, the following parameters have been chosen.

Binary coded GA method	PSO method
• Population size = 100	• Population size = 100
• Iterations = 100	• Iterations = 100
• Cross over rate = 0.75	• $w_{max} = 0.7$ and $w_{min} = 0.2$
• Elitism rate = 0.1	• $V^{max} = 0.1(\lambda^{max} - \lambda^{min})$
• Mutation rate = 0.01	• $c_1 = 2$ and $c_2 = 2$

Similarly, the lambda value is selected such that the lowest lambda value among all lambda values of the generators at their minimum and maximum output powers for the execution of conventional lambda iterative method. In all cases, the lambda is taken as a control parameter.

5.1. Case studies

In this section, different cases are considered to test the applicability of equal embedded algorithm for solving the ED problems.

5.1.1. Case 1

In this case, three generators are considered. The fuel cost data of three generators is taken from [25] and is given in Table 3.

Table 3
Fuel cost data of three generators system.

Unit	a_i ($)	b_i ($/MW)	c_i ($/MW^2)	P_{imin} (MW)	P_{imax} (MW)
1	213.1	11.660	0.00533	050.0	200.0
2	200.0	10.330	0.00889	037.5	150.0
3	240.0	10.833	0.00741	045.0	180.0

Coefficients of B matrix are

$$B_{11} = 0.01^* \begin{bmatrix} 0.06760 & 0.00953 & -0.00507 \\ 0.00953 & 0.05210 & 0.00901 \\ -0.00507 & 0.00901 & 0.0294 \end{bmatrix}$$

$$B_{10} = [-0.0766 \quad -0.00342 \quad 0.01890], \quad B_{00} = 4.0357$$

The power demand in this case is 210 MW.

This case shows the results of the proposed method when all the constraints including generator constraints and system transmission loss are involved. The PPD table of three generating system is shown in Table 4. The dimension of PPD table is 6 × 7. Similarly, the MPPD table is also formulated and is shown in Table 5. Dimension of the MPPD table is 6 × 13. At required power demand, the RMPPD is obtained from the MPPD table for three generators. It is shown in Table 6. Numerator and denominator of output powers of all generators are derived in terms of lambda by linear interpolation. Finally, lambda is evaluated by the Muller method. The optimal solution satisfies the system constraints such as the generator constraints and transmission losses. Table 7 listed the statistical

Table 7
The optimum solution of each unit by various methods.

Unit power output (MW)	Iterative method	GA	PSO	Proposed method
$P1$	73.5275	73.8324	73.8124	73.9723
$P2$	69.5074	69.9595	69.9460	69.6656
$P3$	75.7826	75.0201	74.9962	75.1645
Power loss (MW)	8.8165	8.827	8.813	8.8024
Fuel cost ($/h)	3163.9	3163.63	3162.938	3163.694
Iterations	11	13	29	3
CPU time (s)	0.015	0.7	0.16	0.015

Table 4
PPD table of three generators system.

Lambda ($/MW)	$P1$ (MW)	$P2$ (MW)	$P3$ (MW)	$\sum_{i=1}^{3} P_i$	P_{Loss}	SOP $= \sum_{i=1}^{3} P_i - P_{Loss}$
11.57	27.952	39.309	21.705	88.966	3.9438	85.022
12.091	47.795	52.526	44.409	144.73	5.2793	139.45
12.138	49.522	53.679	46.41	149.61	5.4507	144.16
15.679	160.97	128.95	184.74	474.66	36.203	438.46
16.421	180.29	142.13	210.7	533.13	45.701	487.43
17.495	206.31	159.93	246.65	612.89	60.569	552.32

Table 5
MPPD Table of three generators system.

Lambda	Numerator terms			Denominator terms			Output powers			$\sum_{i=1}^{3} P_i$	P_{Loss}	SOP $= \sum_{i=1}^{3} P_i - P_{Loss}$
	$P11$	$P12$	$P13$	$P21$	$P22$	$P23$	$P1$ (MW)	$P2$ (MW)	$P3$ (MW)			
11.57	0.73522	1.1728	0.46933	0.026303	0.029836	0.021623	50	39.309	45	134.31	4.4774	129.83
12.091	1.2908	1.5957	0.97386	0.027007	0.030379	0.02193	50	52.526	45	147.53	5.2977	142.23
12.138	1.3406	1.6333	1.019	0.02707	0.030428	0.021957	50	53.679	46.41	150.09	5.4489	144.64
15.679	5.1281	4.3994	4.441	0.031858	0.034117	0.024039	160.97	128.95	180	469.92	35.572	434.35
16.421	5.9245	4.9591	5.157	0.032861	0.03489	0.024475	180.29	142.13	180	502.43	41.37	461.06
17.495	7.0789	5.7589	6.1927	0.034313	0.036009	0.025107	200	150	180	530	46.826	483.17

Table 6
RMPPD table of three generators system.

Lambda	Numerator terms of output powers			Denominator terms of output powers			Output powers			$\sum_{i=1}^{3} P_i$	P_{Loss}	SOP $= \sum_{i=1}^{3} P_i - P_{Loss}$
	$P11$	$P12$	$P13$	$P21$	$P22$	$P23$	$P1$ (MW)	$P2$ (MW)	$P3$ (MW)			
12.138	1.3406	1.6322	1.0196	0.02707	0.030428	0.021957	50	53.643	46.437	150.08	5.4477	144.63
15.679	5.1206	4.4128	4.441	0.031858	0.034117	0.024039	160.73	129.34	180	470.07	35.613	434.46

results that involved fuel cost, convergence characteristics and computational time. The convergence characteristic of the proposed method is shown in Fig. 2.

The proposed method provides optimal solution with minimum fuel cost. It is observed during the execution of the algorithm that the proposed method gives the optimal solution within three iterations at a specified power demand. Also, it is observed that the proposed method provide optimal solution within 3–4 iterations for any power demand.

5.1.2. Case 2

In this case, the system contains 15 generators [9]. The fuel cost data is given in Table 8. Power demand is 2630 MW.

It is clear from the Table 11 that the proposed equal embedded algorithm provides solution in two iterations for different power demands.

5.2. Comparison of methods

5.2.1. Solution quality

Various tables in the different cases demonstrate the effectiveness of the proposed method for getting the qualitative solution. The proposed method yields better solution with low generation cost with considerable computational time. Quality of the solution depends on the error tolerance (the difference between the generated power and power demand includes transmission loss) In GA, PSO and lambda iterative method. Fixation of the tolerance is absolute value of 0.01 for fast convergence. But, the optimal value of lambda is solution of the power balance equation in the proposed method. It indicates that the proposed method provides qualitative

$$B_{ij} = 10^{-3} \cdot \begin{bmatrix} 1.4 & 1.2 & 0.7 & -0.1 & -0.3 & -0.1 & -0.1 & -0.1 & -0.3 & -0.5 & -0.3 & -0.2 & 0.4 & 0.3 & -0.1 \\ 1.2 & 1.5 & 1.3 & 0 & -0.5 & -0.2 & 0 & 0.1 & -0.2 & -0.4 & -0.4 & 0 & 0.4 & 1 & -0.2 \\ 0.7 & 1.3 & 7.6 & -0.1 & -1.3 & -0.9 & -0.1 & 0 & -0.8 & -1.2 & -1.7 & 0 & -2.6 & 11.1 & -2.8 \\ -0.1 & 0 & -0.1 & 3.4 & -0.7 & -0.4 & 1.1 & 5 & 2.9 & 3.2 & -1.1 & 0 & 0.1 & 0.1 & -2.6 \\ -0.3 & -0.5 & -1.3 & -0.7 & 9 & 1.4 & -0.3 & -1.2 & -1 & -1.3 & 0.7 & -0.2 & -0.2 & -2.4 & -0.3 \\ -0.1 & -0.2 & -0.9 & -0.4 & 1.4 & 1.6 & 0 & -0.6 & -0.5 & -0.8 & 1.1 & -0.1 & -0.2 & -1.7 & 0.3 \\ -0.1 & 0 & -0.1 & 1.1 & -0.3 & 0 & 1.5 & 1.7 & 1.5 & 0.9 & -0.5 & 0.7 & 0 & -0.2 & -0.8 \\ -0.1 & 0.1 & 0 & 5 & -1.2 & -0.6 & 1.7 & 16.8 & 8.2 & 7.9 & -2.3 & -3.6 & 0.1 & 0.5 & -7.8 \\ -0.3 & -0.2 & -0.8 & 2.9 & -1 & -0.5 & 1.5 & 8.2 & 12.9 & 11.6 & -2.1 & -2.5 & 0.7 & -1.2 & -7.2 \\ -0.5 & -0.4 & -1.2 & 3.2 & -1.3 & -0.8 & 0.9 & 7.9 & 11.6 & 20 & -2.7 & -3.4 & 0.9 & -1.1 & -8.8 \\ -0.3 & -0.4 & -1.7 & -1.1 & 0.7 & 1.1 & -0.5 & -2.3 & -2.1 & -2.7 & 14 & 0.1 & 0.4 & -3.8 & 16.8 \\ -0.2 & 0 & 0 & 0 & -0.2 & -0.1 & 0.7 & -3.6 & -2.5 & -3.4 & 0.1 & 5.4 & -0.1 & -0.4 & 2.8 \\ 0.4 & 0.4 & -2.6 & 0.1 & -0.2 & -0.2 & 0 & 0.1 & 0.7 & 0.9 & 0.4 & -0.1 & 10.3 & -10.1 & 2.8 \\ 0.3 & 1 & 11.1 & 0.1 & -2.4 & -1.7 & -0.2 & 0.5 & -1.2 & -1.1 & -3.8 & -0.4 & -10.1 & 57.8 & -9.4 \\ -0.1 & -0.2 & -2.8 & -2.6 & -0.3 & 0.3 & -0.8 & -7.8 & -7.2 & -8.8 & 16.8 & 2.8 & 2.8 & -9.4 & 128.3 \end{bmatrix}$$

$$B_{10} = [-1 \quad -2 \quad 28 \quad -1 \quad 1 \quad -3 \quad -2 \quad 6 \quad 39 \quad -17 \quad -00 \quad -32 \quad 67 \quad -64]$$

$$B_{00} = 0.0055$$

PSO, GA, lambda iterative method and proposed method are executed for 15 generators with generator constraints and transmission losses. The main aim here is to enlighten the effectiveness of the proposed method to solve large scale ED problem with mixed generators. The output powers are given in Table 9.

It has been observed from the Table 9 that the qualitative solution is nearly same for all methods. But, huge computational time and more iterations are taken by the other methods compared to the proposed method.

5.1.3. Case 3

In this case, 26 generators system is considered. The fuel cost data of 26 generators system is given in Table 10. Here, higher order (cubic) cost functions are considered in the ED problem without transmission loss to prove the applicability of the proposed method. Here, the power expressions in terms of lambda are obtained directly from the PPD table using quadratic interpolation and then Muller method is applied to get the optimal solution. The simulation results for various power demands are shown in Table 11.

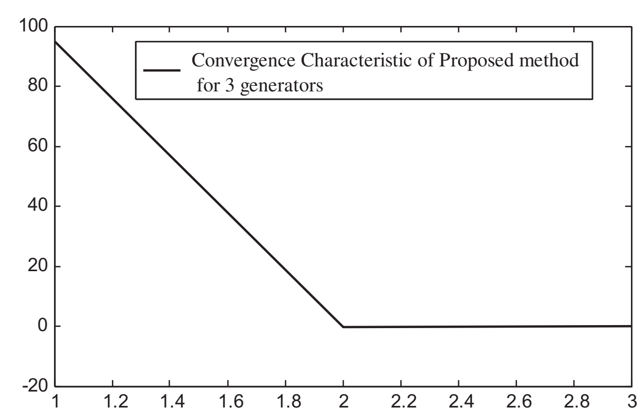

Fig. 2. Convergence characteristic of the proposed method of three generators system.

Table 8
Fuel cost data of fifteen generators.

Unit	a_i ($)	b_i ($/MW)	c_i ($/MW^2)	$P_{i\min}$ (MW)	$P_{i\max}$ (MW)
1	671	10.1	0.000299	150	455
2	574	10.2	0.000183	150	455
3	374	8.8	0.001126	20	130
4	374	8.8	0.001126	20	130
5	461	10.4	0.000205	150	470
6	630	10.1	0.000301	135	460
7	548	9.8	0.000364	135	465
8	227	11.2	0.000338	60	300
9	173	11.2	0.000807	25	162
10	175	10.7	0.001203	25	160
11	186	10.2	0.003586	20	80
12	230	9.9	0.005513	20	80
13	225	13.1	0.000371	25	85
14	309	12.1	0.001929	15	55
15	323	12.4	0.004447	15	55

Table 9
Optimal solution of 15 generating unit system by various methods.

Output powers (MW)	Iterative method	GA	PSO	Proposed method
*P*1	455	455	455	455
*P*2	455	455	455	455
*P*3	130	130	130	130
*P*4	130	130	130	130
*P*5	172.640590480130	172.591	172.691	172.63
*P*6	460	460	460	460
*P*7	465	465	465	465
*P*8	60	60	60	60
*P*9	25	25	25	25
*P*10	119.403390991819	119.51	119.41	119.42
*P*11	80	80	80	80
*P*12	50.6174846428085	50.62	50.62	50.61
*P*13	25	25	25	25
*P*14	15	15	15	15
*P*15	15	15	15	15
Total power (MW)	2657.67	2658.224	2657.67	2657.6722
Power loss (MW)	27.67	28.224	27.67	27.6722
Fuel cost ($/h)	32594.436	32588.54	32589.16	32588.80
Iterations	64	13	24	3
CPU time (s)	0.04	2.56	0.4	0.039

solution than the other methods, which are mentioned in the case studies.

5.2.2. Convergence characteristics

The proposed method provides the qualitative solution within few iterations. It is clear from the different tables that the proposed method converges within 3–6 iterations for different case studies. Irrespective of the system complexity, the proposed method converges within few iterations at any power demand. Convergence characteristics of the GA depend on various factors such as the selection of chromosomes, the cross over probability, elitism rate and mutation rate. Depending upon the system complexity, these factors are selected by the system operator judiciously. Similarly, PSO takes more iterations due to the velocity modification by random process and identification of best particle. Usually number of iterations is varying with the increasing of system size in the GA and the PSO method. But, the convergence characteristics of the proposed method will not depends on the size of the system.

5.2.3. Computational time

Due to the less iterations, the proposed method has better computation performance than the GA, PSO and lambda iterative methods. The evaluation process involved in the proposed method is that the power expressions are derived in terms of lambda by interpolation and the lambda is evaluated from the power balance equation by the Muller method. So, the computational time is less. But, the evolution process involved in the GA is the decoding of

Table 10
Fuel cost data of 26 generators system with cubic fuel cost function.

Unit	a_i ($)	b_i ($/MW)	c_i ($/MW^2)	d_i ($/MW^3)	$P_{i\min}$ (MW)	$P_{i\max}$ (MW)
1	24.38	25.54	0.025	5.08E−09	2.40E+00	12
2	24.41	25.67	0.026	−1.01E−08	2.40E+00	12
3	24.63	25.8	0.028	1.01E−08	2.40E+00	12
4	24.76	25.93	0.028	−5.08E−09	2.40E+00	12
5	24.88	26.06	0.028	−5.72E−16	2.40E+00	12
6	117.75	37.55	0.011	8.31E−08	4.00E+00	20
7	118.1	37.66	0.012	8.56E−08	4.00E+00	20
8	118.45	37.77	0.013	8.15E−08	4.00E+00	20
9	118.82	37.88	0.014	8.29E−08	4.00E+00	20
10	81.13	13.32	0.008	−5.80E−10	1.52E+01	76
11	81.29	13.35	0.008	−5.47E−10	1.52E+01	76
12	81.46	13.38	0.009	−5.49E−10	1.52E+01	76
13	81.62	13.4	0.009	−5.50E−10	1.52E+01	76
14	217.89	18	0.006	1.25E−18	2.50E+01	100
15	218.33	18.09	0.006	−1.19E−18	2.50E+01	100
16	218.77	18.2	0.005	2.44E−18	2.50E+01	100
17	142.73	10.69	0.004	1.11E−10	5.43E+01	155
18	143.02	10.71	0.004	1.03E−10	5.43E+01	155
19	143.31	10.73	0.004	1.03E−10	5.43E+01	155
20	143.59	10.75	0.004	1.03E−10	5.43E+01	155
21	259.3	23	0.002	1.07E−10	6.90E+01	197
22	259.64	23.1	0.002	1.04E−10	6.90E+01	197
23	260.17	23.2	0.002	1.00E−10	6.90E+01	197
24	177.05	10.86	0.001	−4.42E−19	1.40E+02	350
25	310	7.49	0.001	−1.10E−19	1.00E+02	400
26	311.91	7.5	0.001	−3.55E−20	1.00E+02	400

Table 11
Simulation results of 26 generators by proposed method for different power demands.

Output power of generator (MW)	Power demand (MW)		
	2400	2600	2900
1	2.4	2.4	2.4
2	2.4	2.4	2.4
3	2.4	2.4	2.4
4	2.4	2.4	2.4
5	2.4	2.4	2.4
6	4	4	4
7	4	4	4
8	4	4	4
9	4	4	4
10	76	76	76
11	76	76	76
12	76	76	76
13	76	76	76
14	36.76	99.50	100
15	29.38	92.14	100
16	25.00	99.49	100
17	155	155	155
18	155	155	155
19	155	155	155
20	155	155	155
21	68.95	68.95	190.99
22	68.95	68.95	166
23	68.95	68.95	141.00
24	350	350	350
25	400	400	400
26	400	400	400
Incremental fuel cost ($/MW)	18.4419	19.195	23.764
Fuel cost ($)	32642.41	36406.3	43436.5
Computational time (s)	0.015	0.01	0.01
No. of iterations	2	2	2

chromosomes, evaluation of fitness function and then genetic operators like parent selection, reproduction, elitism and mutation. Therefore, it takes more computational time to give the optimal solution. Similarly, the evolution process involved in the PSO is the fitness function evaluation, velocity modification by random process and identification of best particle.

6. Conclusion

This paper suggested equal embedded algorithm for solving the ED problem of a power system having 3, 15 and 26 generators with the generator constraints and transmission loss. A salient feature of the proposed method is that it gives qualitative solution with fast convergence characteristics, which are mentioned in the case studies. Due to the fast convergence, the computational time is less for getting the optimal solution. The additional advantages of the proposed method are mentioned below.

(i) It will not depend on user defined parameters.
(ii) System constraints such as ramp rate limits and prohibited zones can be incorporated easily.
(iii) Irrespective of the system complexity, the proposed method provides qualitative solution within few iterations.
(iv) The proposed method can easily implemented for the ED problems with higher order fuel cost function such as cubic cost function.

In real time operation of power system, mixed generators in large scale are usually involved in the ED problem. Irrespective of the size of the system, the proposed EEA algorithm provides the solution in less iteration at different power demands. Therefore, the algorithm can be used in real time environment for solving the ED problems effectively.

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