Hydromagnetic effects on the flow of a micropolar fluid in a diverging channel

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Steady flow of an incompressible and electrically conducting micropolar fluid through a diverging channel is studied. The flow is subjected to a uniform magnetic field perpendicular to the flow direction. Perturbation solutions have been obtained for the velocity and microrotation components in terms of effective Reynolds number. The profiles of velocity and microrotation components are presented for different micropolar fluid parameters and magnetic parameter. The influence of magnetic parameter on the pressure gradient is also studied.

1 Introduction

The flow of a fluid through converging or diverging channels has many industrial and engineering applications, such as flow through nozzles, diffuser and reducers as encountered in polymer processing operations, to simulate flows of dilute polymer solutions through porous media [1], the cold-drawing operation in polymer industry to improve mechanical properties of products such as plastic sheets and rods [2], extrusion of molten polymers through converging dies [3–5] etc. Several authors, Hooper et al. [6], Dennis et al. [7], Drazin [8], to mention but few, have studied the laminar flow of an incompressible viscous fluid in a converging or diverging channels or tubes.

The increasing number of technical applications using magnetohydrodynamic (MHD) effects has made it desirable to extend many of the available hydrodynamic solutions to include the effects of magnetic fields for those cases when the fluid is electrically conducting. The motivations for the studies of the flow of an electrically conducting fluid under the influence of an external magnetic field are industrial processes where electromagnetic processing of materials (EPM) is applied. This is very useful for casting of liquid metals as well as for growing of semiconductor crystals. A vast literature is available regarding the study of MHD flows. A survey of MHD studies in the technological fields can be found in Moreau [9]. The early works concern fully developed, laminar MHD flows in ducts under a uniform magnetic field. Exact solutions have been developed by Hartmann [10] for a one-dimensional problem consisting of rectilinear, laminar MHD flow between two infinite parallel plates. The imposed magnetic field is uniform and normal to the two surfaces. For sufficiently high Hartmann numbers the flow is characterized by a core with uniform velocity and thin boundary layers along the plates. These layers are called Hartmann layers and they are present along walls on which the magnetic field has a normal component. The size of these layers is inversely proportional to the magnetic field strength, which is measured by the dimensionless Hartmann number. Hartmann and Lazarus [11] took measurements of the pressure loss for laminar and turbulent Hartmann flows. In the past decade, considerable advances have been made towards a better theoretical understanding of the problem of transition to turbulence in Hartmann flow. Morecso and Alboussiere [12] investigated experimentally the transition to turbulence in the Hartmann layers that arise in MHD flows through measurements of the friction factor in the electromagnetic flow in ducts. Krasnov et al. [13] investigated the instability and transition to turbulence in the flow of an electrically conducting incompressible fluid between two parallel unbounded insulating walls affected by a wall-normal magnetic field. Beck et al. [14] analyzed the mean flow properties of turbulent MHD channel flow with electrically insulating channel walls using high-resolution direct numerical simulations.

Expansions and contractions are important geometric elements in liquid metal devices and the study of the flow in such geometries is a key issue for applications in fusion reactor blankets where the flow is distributed from small pipes to large boxes. Another application of the study of the flow of an electrically conducting fluid through converging or diverging channels under the influence of an external magnetic field can be found in the field of industrial metal casting i.e. the control

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of molten metal flows and may be of interest is in the motion of liquid metals or alloys in the cooling systems of advanced nuclear reactors. In recent times, many medical diagnostic devices especially those used in diagnosing cardiovascular diseases make use of the interaction of magnetic fields with tissue fluids. In physiological fluid flow, Barnothy has reported experiments [15] where the heart rate decreased by exposing biological systems to an external magnetic field. In this line of application is magnetic resonance imaging, a technique for obtaining high resolution images of various organs within the human body in the presence of a magnetic field. Walker et al. [16] analyzed MHD flows in smoothly expanding insulating channels using inertia less approximation. Rao and Deshikachar [17] have considered the MHD Oscillatory flow of blood through channels of variable cross section. Verma et al. [18] analyzed the Magnetic fluid flow in a two dimensional diverging channel. Mhone and Makinde [19] have studied the unsteady MHD flow with heat transfer in a diverging channel.

It is known that many of the industrially and technologically important fluids are electrically conducting fluids and behave like a non-Newtonian fluid. Balmer and Kauzlarich [20] obtained similarity solutions for the steady flow of non-Newtonian elastic power law fluids in a converging or diverging channel with wall suction or injection. Sinha and Nayak [21] studied the steady two dimensional incompressible laminar visco-elastic flow in a converging or diverging channel with suction and injection. Rajagopal et al. [22] studied the slow flow of an incompressible third grade fluid in a converging/diverging channel. Ozturk et al. [23] considered the slow flow of the Reiner–Rivlin fluid in a converging or diverging channel with suction and injection. Baris [24] has considered the flow of a second-grade viscoelastic fluid in a porous converging channel. Hayat et al. [25], studied the Magnetohydrodynamic flow of an Oldroyd 6-constant fluid.

The theory of micro polar fluids initiated by Eringen [26] exhibits some microscopic effects arising from the local structure and micro motion of the fluid elements. Further, they can sustain couple stresses and include classical Newtonian fluid as a special case. The model of micro polar fluid represents fluids consisting of rigid randomly oriented (or spherical) particles suspended in a viscous medium where the deformation of the particles is ignored. The fluids containing certain additives, some polymeric fluids and animal blood are examples of micro polar fluids. Lukaszewicz [27] has presented the mathematical theory of equations of micro polar fluids and applications of these fluids in the theory of lubrication and in the theory of porous media.

Although many MHD devices involve flow in converging or diverging ducts, no theoretical attempts have been made to analyze them by considering the mechanical behavior of a fluid which possesses substructures, such as dilute polymer liquids, liquid crystals, animal blood, etc. The aim of this work is to examine the effect of magnetic field on the steady flow of an incompressible electrically conducting micropolar fluid through a diverging channel. Perturbation solutions have been obtained for the velocity and microrotation in terms of effective Reynolds number. The effects of different fluid parameters and magnetic parameter on velocity and microrotation components are shown graphically. The effects of magnetic parameter on the axial pressure gradient for different flow geometries are examined.

2 Formulation of the problem

Consider an incompressible electrically conducting micropolar fluid flow through a slowly varying diverging symmetrical channel. Choose the Cartesian coordinate system such that the x-axis lies on the axis of the symmetry of the channel, y-axis is perpendicular to the channel and the channel is of infinite length in x-direction. A uniform magnetic field $\overline{B}_0$ is applied to the fluid normal to the walls of the channel. The boundary of the channel is assumed as $-b(x) < y < b(x)$, where $b(x)$ defines the wall diverging geometry.

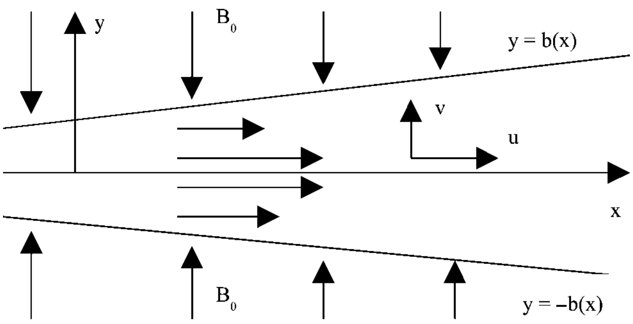

Fig. 1 Geometry of the problem

The equations governing the flow of micropolar fluid under the usual MHD approximations, in the absence of body forces and body couple are

$$\operatorname{div} \overline{q} = 0 \,, \tag{1}$$

$$\rho(\overline{q} \cdot \nabla)\overline{q} = -\operatorname{grad} p + \kappa \operatorname{curl} \overline{v} - (\mu + \kappa) \operatorname{curl}\operatorname{curl} \overline{q} + \overline{J} \times \overline{B} \,, \tag{2}$$

$$\rho j(\overline{q}\cdot\nabla)\overline{\upsilon} = -2\kappa\overline{\upsilon} + \kappa\,\mathrm{curl}\,\overline{q} - \gamma\,\mathrm{curl}\,\mathrm{curl}\,\overline{\upsilon} + (\alpha_1+\beta_1+\gamma_1)\,\mathrm{grad}\;\mathrm{div}\,\overline{\upsilon}\,, \tag{3}$$

where $\overline{q}$ is the velocity vector, $\overline{\upsilon}$ is the micro rotation vector, p fluid pressure, ρ and j are the fluid density and micro gyration parameter, $\overline{B}$ the total magnetic field, ($\overline{B} = \overline{B}_0 + \overline{b}$, $\overline{b}$ is induced magnetic field), $\overline{J}$ is the current density. μ, κ, α_1, β_1, and γ_1 are the material constants (viscosity coefficients) which satisfy the following inequalities.

$$\kappa \geq 0, \quad 2\mu+\kappa \geq 0, \quad 3\alpha_1+\beta_1+\gamma_1 \geq 0, \quad \gamma_1 \geq |\beta_1|\,. \tag{4}$$

Neglecting displacement currents, the Maxwell equations and the generalized Ohm's law are:

$$\mathrm{div}\,\overline{B} = 0, \quad \mathrm{curl}\,\overline{B} = \mu_m\overline{J}, \quad \mathrm{curl}\,\overline{E} = -\frac{\partial\overline{B}}{\partial t}, \quad \text{and} \quad \overline{J} = \sigma(\overline{E} + \overline{q}\times\overline{B})\,, \tag{5}$$

where μ_m is the magnetic permeability, $\overline{E}$ is the electric field and σ is the electrical conductivity of the fluid.

Since the flow is along x-direction, the flow variables are assumed to be independent of the co-ordinate z and we choose the velocity vector as $\overline{q} = u(x,y)\hat{i} + v(x,y)\hat{j}$ and the microrotation vector as $\overline{\upsilon} = \upsilon(x,y)\hat{k}$. Assuming that the induced magnetic field is negligible compared to the applied magnetic field so that magnetic Reynolds number is small and the electric field is zero and μ_m is constant throughout the flow field, the basic field Eqs. (1)–(5) can be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \tag{6}$$

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \kappa\frac{\partial \upsilon}{\partial y} + (\mu+\kappa)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \sigma B_0^2 u\,, \tag{7}$$

$$\rho\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} - \kappa\frac{\partial \upsilon}{\partial x} + (\mu+\kappa)\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)\,, \tag{8}$$

$$\rho\, j\left[u\frac{\partial \upsilon}{\partial x} + v\frac{\partial \upsilon}{\partial y}\right] = -2\kappa\upsilon + \kappa\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + \gamma\left(\frac{\partial^2 \upsilon}{\partial x^2} + \frac{\partial^2 \upsilon}{\partial y^2}\right)\,. \tag{9}$$

The appropriate boundary conditions are

$$\frac{\partial u}{\partial y} = 0, \quad v = 0, \quad \upsilon = 0 \quad \text{at} \quad y = 0 \tag{10a}$$

and

$$u + v\frac{\partial b}{\partial x} = 0, \quad v = 0, \quad \upsilon = 0 \quad \text{at} \quad y = b(x)\,. \tag{10b}$$

Introducing the stream function ψ through

$$u = \frac{\partial\psi}{\partial y}, \quad v = -\frac{\partial\psi}{\partial x} \tag{11}$$

into the Eqs. (6)–(9) and eliminating pressure from the resulting equations, we get

$$\rho\frac{\partial(\psi, -\nabla^2\psi)}{\partial(y,x)} = -\kappa\nabla^2\upsilon - (\mu+\kappa)\nabla^4\psi + \sigma B_0^2\frac{\partial^2\psi}{\partial y^2}\,, \tag{12}$$

$$\rho j\frac{\partial(\psi,\upsilon)}{\partial(y,x)} = -2\kappa\upsilon - \kappa\nabla^2\psi + \gamma\nabla^2\upsilon\,. \tag{13}$$

The boundary conditions Eq. (10) in terms of stream function becomes

$$\begin{aligned} &\frac{\partial^2\psi}{\partial y^2} = 0, \quad \psi = 0, \quad \upsilon = 0 \quad \text{at} \quad y = 0\,,\\ &\frac{\partial\psi}{\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial b}{\partial x} = 0, \quad \upsilon = 0 \quad \text{at} \quad y = b(x)\,. \end{aligned} \tag{14}$$

In addition, we prescribe a constant flux (Q) at the boundary of the channel i.e.

$$\psi = Q \quad \text{at} \quad y = b(x)\,. \tag{15}$$

The function $b(x)$ is assumed to depend upon a small parameter ε such that

$$b(x) = S(\varepsilon x/a_0), \quad (0 < \varepsilon = a_0/L \ll 1)\,,$$

where a_0 is the constant characteristic half width of the channel, L is the constant characteristic length of the channel and S is the function describing the channel wall divergence geometry. In the limit $\varepsilon \to 0$ the channel is of constant width.

Using the following non dimensional variables

$$x = \frac{a_0 x'}{\varepsilon}, \quad y = a_0 y', \quad \psi = Q\psi', \quad \upsilon = \frac{Q}{a_0^2}\upsilon' \tag{16}$$

in the Eqs. (12)–(15) and neglecting the terms of ε^2 and higher order as well as the primes we get

$$\mathrm{Re}\frac{\partial(\psi, -\nabla^2\psi)}{\partial(y,x)} = -\frac{N}{1-N}\frac{\partial^2\upsilon}{\partial y^2} - \frac{1}{1-N}\frac{\partial^4\psi}{\partial y^4} + \mathrm{Ha}^2\frac{\partial^2\psi}{\partial y^2}\,, \tag{17}$$

$$\mathrm{Re}\;aj\,\frac{\partial(\psi,\upsilon)}{\partial(y,x)} = --\frac{2N}{1-N}\upsilon - \frac{N}{1-N}\frac{\partial^2\psi}{\partial y^2} + \frac{(2-N)N}{m^2(1-N)}\frac{\partial^2\upsilon}{\partial y^2}\,, \tag{18}$$

$$\begin{aligned}
&\frac{\partial^2\psi}{\partial y^2} = 0, \quad \psi = \quad 0, \quad \upsilon = 0 \quad \text{on} \quad y = 0\,,\\
&\frac{\partial\psi}{\partial y} = 0, \quad \psi = \quad 1, \quad \upsilon = 0 \quad \text{on} \quad y = S(x)\,,
\end{aligned} \tag{19}$$

where $\mathrm{Ha}^2 = \frac{\sigma B_0^2 a_0^2}{\mu}$ is Hartmann number, $aj = \frac{j}{a_0^2}$ is the micro inertia parameter, $N = \frac{\kappa}{\mu+\kappa}$ is a coupling number $(0 \le N \le 1)$, $m^2 = \frac{a_0^2\kappa(2\mu+\kappa)}{\gamma(\mu+\kappa)}$ is the micropolar parameter and $\mathrm{Re} = \frac{\rho\varepsilon Q}{\mu}$ is the effective flow Reynolds number.

3 Solution of the problem

Assuming the effective flow Reynolds number (Re) to be very small, the stream function ψ and microrotation component (υ) may be expanded in the power series of Re as

$$\begin{aligned}
&\psi = \psi_0 + \mathrm{Re}\;\psi_1 + \mathrm{Re}^2\psi_2 + \ldots\\
&\upsilon = \upsilon_0 + \mathrm{Re}\;\upsilon_1 + \mathrm{Re}^2\upsilon_2 + \ldots\,,
\end{aligned} \tag{20}$$

where ψ_j and υ_j are functions of $s(x)$ and y.

Substituting (20) in (17)–(19) and collecting the coefficients of various powers of Re on both the sides, we obtain the following set of coupled linear differential equations for ψ_0, υ_0, and ψ_1, υ_1.

Zeroth order in Re:

$$-\frac{N}{1-N}\frac{\partial^2\upsilon_0}{\partial y^2} - \frac{1}{1-N}\frac{\partial^4\psi_0}{\partial y^4} + \mathrm{Ha}^2\frac{\partial^2\psi_0}{\partial y^2} = 0 \tag{21}$$

$$-\frac{2N}{1-N}\upsilon_0 - \frac{N}{1-N}\frac{\partial^2\psi_0}{\partial y^2} + \frac{(2-N)N}{m^2(1-N)}\frac{\partial^2\upsilon_0}{\partial y^2} = 0\,. \tag{22}$$

The corresponding boundary conditions are

$$\begin{aligned}
&\frac{\partial^2\psi_0}{\partial y^2} = 0, \quad \psi_0 = 0, \quad \upsilon_0 = 0\text{on} \quad y = 0\,,\\
&\frac{\partial\psi_0}{\partial y} = 0, \quad \psi_0 = 1, \quad \upsilon_0 = 0 \quad \text{on} \quad y = S(x)\,.
\end{aligned} \tag{23}$$

From (21) and (22), we get

$$\upsilon_0 = -\frac{(2-N)}{2m^2N}\frac{\partial^4\psi_0}{\partial y^4} + \left(\frac{2\mathrm{Ha}^2(2-N)(1-N)}{2m^2N} - \frac{1}{2}\right)\frac{\partial^2\psi_0}{\partial y^2}\,. \tag{24}$$

Substituting (24) in (21), we get

$$\frac{\partial^6\psi_0}{\partial y^6} - (\alpha^2+\beta^2)\frac{\partial^4\psi_0}{\partial y^4} + \alpha^2\beta^2\frac{\partial^2\psi_0}{\partial y^2} = 0\,, \tag{25}$$

where

$$\alpha^2+\beta^2 = (\mathrm{Ha}^2(1-N) - m^2) \quad \text{and} \quad \alpha^2\beta^2 = \frac{2\mathrm{Ha}^2 m^2(1-N)}{(2-N)}\,.$$

The general solution of Eq. (25) is

$$\psi_0(x,y) = C_1(x) + C_2(x)y + C_3(x)\mathrm{e}^{-\alpha y} + C_4(x)\mathrm{e}^{\alpha y} + C_5(x)\mathrm{e}^{-\beta y} + C_6(x)\mathrm{e}^{\beta y}\,, \tag{26}$$

where $C_1(x)$, $C_2(x)$, $C_3(x)$, $C_4(x)$, $C_5(x)$, and $C_6(x)$ are arbitrary functions of x. Substituting (26) into (24), we get the micro rotation component as

$$\upsilon_0 = A_\alpha(C_3(x)\mathrm{e}^{-\alpha y} + C_4(x)\mathrm{e}^{\alpha y}) + A_\beta(C_5(x)\mathrm{e}^{-\beta y} + C_6(x)\mathrm{e}^{\beta y})\,, \tag{27}$$

where

$$A_\alpha = \frac{(2-N)\alpha^4 + \left[\mathrm{Ha}^2(2-N)(1-N) - Nm^2\right]\alpha^2}{2m^2N}\,,$$

$$A_\beta = \frac{(2-N)\beta^4 + \left[\mathrm{Ha}^2(2-N)(1-N) - Nm^2\right]\beta^2}{2m^2N}\,.$$

Using the boundary conditions (23), we get the expressions for $C_1(x)$, $C_2(x)$, $C_3(x)$, $C_4(x)$, $C_5(x)$, and $C_6(x)$.

The zeroth order pressure gradient is given by

$$\frac{\partial p_0}{\partial x} = \frac{N}{1-N}\frac{\partial^2\upsilon_0}{\partial y^2} + \frac{1}{1-N}\frac{\partial^3\psi_0}{\partial y^3} - \mathrm{Ha}^2\frac{\partial\psi_0}{\partial y}\,. \tag{28}$$

Substituting the expressions for ψ_0 and υ_0, we can get the expression for pressure gradient $\frac{\partial p_0}{\partial x}$.

First order in Re:

$$\frac{\partial(\psi_0, -\nabla^2\psi_0)}{\partial(y,x)} = -\frac{N}{1-N}\frac{\partial^2\upsilon_1}{\partial y^2} - \frac{1}{1-N}\frac{\partial^4\psi_1}{\partial y^4} + \mathrm{Ha}^2\frac{\partial^2\psi_1}{\partial y^2}\,, \tag{29}$$

$$aj\frac{\partial(\psi_0,\upsilon_0)}{\partial(y,x)} = -\frac{2N}{1-N}\upsilon_1 - \frac{N}{1-N}\frac{\partial^2\psi_1}{\partial y^2} + \frac{(2-N)N}{m^2(1-N)}\frac{\partial^2\upsilon_1}{\partial y^2}\,. \tag{30}$$

From (29) and (30), we get

$$\begin{aligned}\upsilon_1 = {}& \frac{(2-N)(1-N)}{2m^2N}\frac{\partial(\psi_0,-\nabla^2\psi_0)}{\partial(y,x)} - \frac{aj\,(1-N)}{2N}\frac{\partial(\psi_0,\upsilon_0)}{\partial(y,x)} \\ & - \frac{(2-N)}{2m^2N}\frac{\partial^4\psi_1}{\partial y^4} + \left(\frac{2\mathrm{Ha}^2(2-N)(1-N)}{2m^2N} - \frac{1}{2}\right)\frac{\partial^2\psi_1}{\partial y^2}\,.\end{aligned} \tag{31}$$

Substituting (31) in (29), we get

$$\begin{aligned}&\frac{\partial^6\psi_1}{\partial y^6} - (\alpha^2+\beta^2)\frac{\partial^4\psi_1}{\partial y^4} + \alpha^2\beta^2\frac{\partial^2\psi_1}{\partial y^2} \\ &\quad = \frac{2m^2(1-N)}{(2-N)}\left\{\frac{\partial(\psi_0,-\nabla^2\psi_0)}{\partial(y,x)} - \frac{2-N}{2m^2}\frac{\partial^2}{\partial y^2}\left(\frac{\partial(\psi_0,-\nabla^2\psi_0)}{\partial(y,x)}\right) - \frac{a\,j}{2}\frac{\partial^2}{\partial y^2}\left(\frac{\partial(\psi_0,\upsilon_0)}{\partial(y,x)}\right)\right\}.\end{aligned} \tag{32}$$

The corresponding boundary conditions are given by

$$\frac{\partial^2 \psi_1}{\partial y^2} = 0, \quad \psi_1 = 0, \quad \upsilon_1 = 0 \quad \text{on} \quad y = 0\,,$$
$$\frac{\partial \psi_1}{\partial y} = 0, \quad \psi_1 = 0, \quad \upsilon_1 = 0 \quad \text{on} \quad y = S(x)\,. \tag{33}$$

The first order pressure gradient is given by

$$\frac{\partial p_1}{\partial x} = \frac{\partial \psi_0}{\partial y}\frac{\partial^2 \psi_0}{\partial x \partial y} - \frac{\partial \psi_0}{\partial x}\frac{\partial^2 \psi_0}{\partial y^2} + \frac{N}{1-N}\frac{\partial \upsilon_1}{\partial y} + \frac{1}{1-N}\frac{\partial^3 \psi_1}{\partial y^3} - \mathrm{Ha}^2 \frac{\partial \psi_1}{\partial y}\,. \tag{34}$$

Since it is cumbersome to solve Eq. (32), we have used MATHEMATICA to obtain the solution of (32) subject to the boundary conditions (33). As the expressions for the stream function ψ_1 and microrotation component υ_1 are lengthy we have not presented them explicitly.

4 Results and discussion

The numerical values of the arbitrary functions $C_1(x)$, $C_2(x)$, $C_3(x)$, $C_4(x)$, $C_5(x)$, and $C_6(x)$ are obtained by solving the system of equations using the boundary conditions (23) using MATHEMATICA and hence the expressions for stream function and microrotation components of zeroth order i.e. ψ_0 and υ_0. Similarly, the solution of (32) subject to the boundary conditions (33) i.e. the stream function of first order (ψ_1) is evaluated numerically by MATHEMATICA for various values of geometric and fluid parameters. Numerical calculation of the stream function ψ and microrotation component υ (up to first order in Re) is performed and representative set of results are presented graphically in Figs. 2–4 for exponentially diverging geometry $S(x) = \mathrm{e}^x$ and linearly diverging geometry $S(x) = 1 + x$. The effect of micropolar parameter (m) and micro-inertia parameter (aj) on the velocity and microrotation is not significant, hence we set $m = 10$ and $aj = 0.001$.

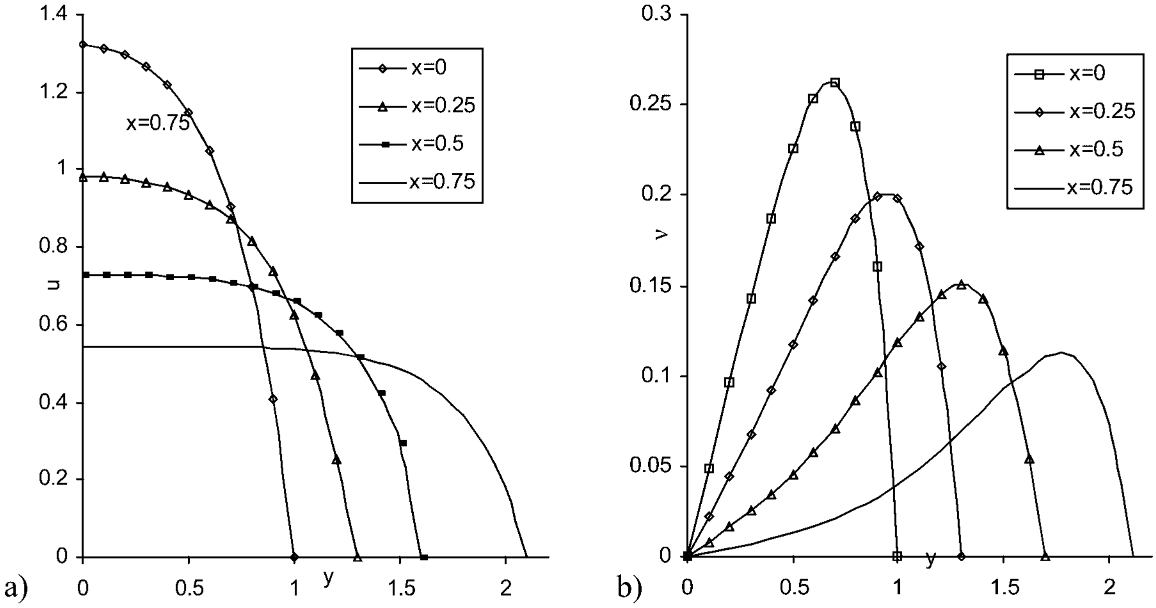

Fig. 2 The profile of (a) velocity and (b) microrotation for $N = 0.5$, $\mathrm{Ha} = 5$, $\mathrm{Re} = 0.1$.

Fig. 2a shows the profile of velocity for the diverging channel geometry $S(x) = \mathrm{e}^x$. As it is expected velocity is maximum at the center of the channel for $x = 0$ and as the channel diverges i.e. the value of x increases, the velocity becomes flattened. Fig. 2b shows the variation of microrotation component with x for the same flow geometry. As x increases, the microrotation component is also decreasing.

The effect of Hartmann number on the velocity is presented in Fig. 3a for fixed value of x. It can be seen from this figure that as Hartmann number increases, the velocity at the centre of the channel decreases and the velocity gradient near the channel wall becomes steeper. This indicates that the fluid velocity can be reduced by an increase in the magnetic fields. Imposing a magnetic field gives rise to a resistive force and slows down the movement of the fluid. The effect of Hartmann

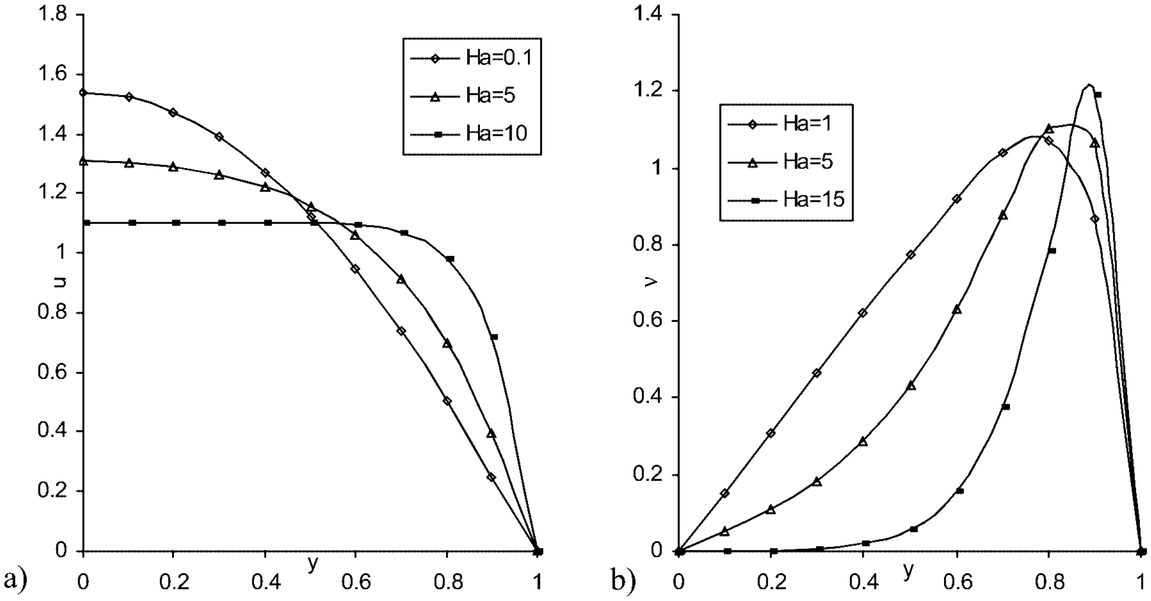

Fig. 3 The effect of Ha on (a) velocity and (b) microrotation for $N = 0.5$, $\mathrm{Re} = 0.1$, $x = 0$.

number Ha, on microrotation is shown in Fig. 3b. Increasing the magnetic field leads to decrease in the microrotation in the domain except near the boundary in which a reverse phenomenon is seen. The intensity of the magnetic field thus can be used for decreasing the angular rotation.

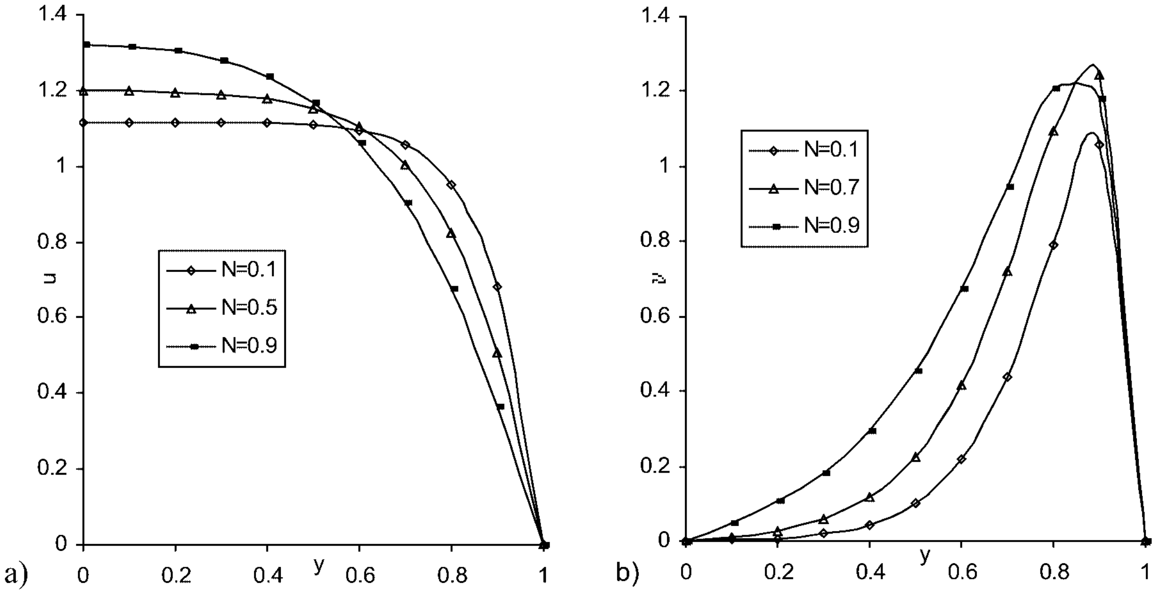

Fig. 4 The effect of N on (a) velocity and (b) microrotation for $\mathrm{Ha} = 10$, $\mathrm{Re} = 0.1$, $x = 0$.

Fig. 4a depicts the effect of the coupling number N on the velocity. It can be seen that the velocity increases at the center with increase of the coupling number. Also, the velocity in case of micropolar fluids is greater than that of viscous fluids (since the limit $N \to 0$ corresponds to viscous fluid). However, the reverse trend is observed near the boundary. Fig. 4b illustrates that the microrotation increases with N.

Fig. 5a shows the variation of axial pressure gradient with the Hartmann number for the channel geometry $S(x) = \mathrm{e}^x$ at the center of the channel ($y = 0$). As the Hartmann number increases, the pressure gradient decreases. The effect of the Hartmann number on the pressure gradient for the diverging geometry $S(x) = 1 + x$ is shown in Fig. 5b. For this geometry also, the pressure gradient decreases as Hartmann number increases.

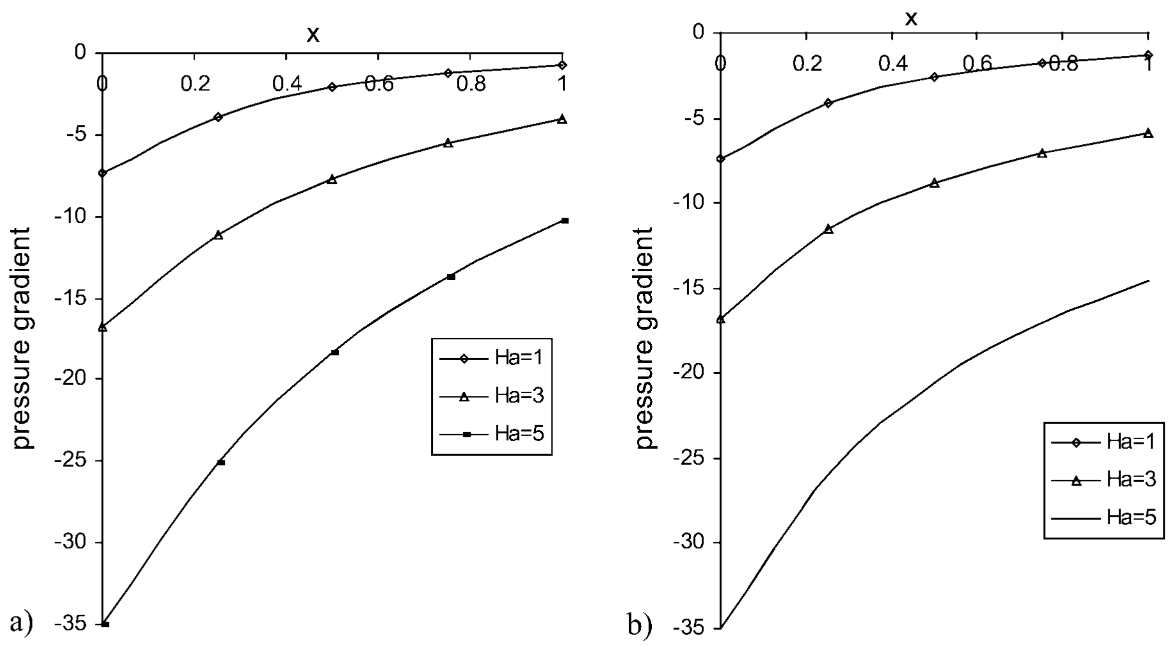

Fig. 5 The effect of Ha on axial pressure a) $S = e^x$ b) $S = 1 + x$ for $N = 0.5$, $\mathrm{Re} = 0.1$, $y = 0$.

5 Conclusion

The steady conducting micro polar fluid through a diverging channel under the influence of an applied uniform magnetic field has been studied. Perturbation solutions have been obtained for the stream function and microrotation components in terms of effective Reynolds number. The profiles of velocity and microrotation components are presented for different micropolar fluid parameters and magnetic parameter. As the micropolar nature of the fluid increases, the velocity also increases. Intensity of the magnetic field reduces the velocity at the center of the channel and the microrotation. It is also shown that increasing the magnetic field decreases the pressure gradient.

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