# Application of Six Sigma level in delivery schedules in high volatile production environments


## Agganoor Prabhu Kumar*

Department of Mechanical Engineering,
JNTU College of Engineering,
Hyderabad, India
E-mail: prabsjntu@yahoo.co.in
*Corresponding author

## Patel Ram Reddy

JNT University,
Hyderabad, India
E-mail: ramreddydr@yahoo.co.in

## Ravindranath Tagore

Department of Mechanical Engineering,
NIT Warangal, India
E-mail: kovida47@yahoo.co.in


**Abstract:** In the present competitive market conditions, in addition to quality and lower cost of products, sticking to delivery schedules is also a very important factor. This is difficult in a production environment where human skill and psychological factors still play a vital role. Permutations and combinations are used in the PERT method for solving such problems in different conditions through a developed software. The $T_{sch}$ obtained in random and extreme conditions are studied with the actual project completion time over a period of time in the printing organisations to achieve Six Sigma levels in delivery schedules.

**Keywords:** delivery schedules; permutations and combinations; PERT; random and extreme.

**Reference** to this paper should be made as follows: Prabhu Kumar, A., Ram Reddy, P. and Tagore, R. (2009) 'Application of Six Sigma level in delivery schedules in high volatile production environments', *Int. J. Applied Systemic Studies*, Vol. 2, No. 3, pp.218–225.

**Biographical notes:** Agganoor Prabhu Kumar worked in various capacities in the printing industry in India and UK Later he joined in the Mechanical Engineering Department at JNTU College of Engineering, Hyderabad as Associate Professor. He has 16 years of teaching and research experience.

Patel Ram Reddy worked at various capacities viz, Assistant Lecturer, Lecturer, Assistant Professor, Professor, Head of the department, Vice Principal, Principal of JNTU College of Engineering. He was also Registrar of JNT University, Hyderabad. He has about 40 years of teaching and research experience and guided eight PhDs. He has published more than 40 papers in various International and national journals.

Ravindranath Tagore worked for defence organisation, Government of India for ten years. Then, he joined in the teaching profession at National Institute of Technology, Warangal as Assistant Professor and rose to a level of Professor Head of the Department, Mechanical Engineering. He has about 25 years of Teaching and research experience and guided seven PhDs. He has published more than 70 papers in various International and national journals.

## 1 Introduction

The model is a technique for finding project duration time in the atomicity market where a large number of producers and customers are in the market. The market sets the price that the producer chooses. In these conditions, the customer delight plays a vital role. The in-built quality in production and prompt delivery at reasonable costs will deliver an edge over the competitors. The quality and costs can be controlled by adopting various methods. The estimating delivery schedules and delivery on time have become important causes for customer delight, especially where the properties of materials used in production are influenced by the existing environmental factors. Conditioning the environment has been expensive and influences the cost of production. To achieve Six Sigma level in maintaining delivery schedules, on time, the permutation and combinations, along with programme evaluation and review techniques proved to be the best.

## 2 Six Sigma level

Six Sigma level is a rigorous, focused and highly effective implementation of proven principles and techniques. Six Sigma level aims for virtually error free business performance. Sigma, $\sigma$, is a measure of variability in any process.

The sigma level of their business processes measures a company's performance. Traditionally, companies accepted three or four sigma performance levels as the norm, despite the fact that these processes created between 6,200 and 67,000 times not meeting delivery schedules in a million orders. This means 6,200–67,000 unsatisfied customers. This is a large number in the present competitive business environment, which has to be controlled. The Six Sigma standard of 3.4 not meeting delivery schedules in a million orders is a response to the increasing expectations of customers and the increased complexity of modern production and processes.

## 3 PERT

Program Evaluation and Review Technique (PERT) is used to find the probable project completion time. The technique is probabilistic in nature; it concerns the time required to complete an activity as stochastic (Benati, 2006). This stochastic time is based on expert estimates of the minimum (optimistic) $T_{\min}$, mode (most likely) $T_{\text{mean}}$ and maximum (pessimistic) $T_{max}$, time required to complete each activity. The expected time, $T_{exp}$, is an approximate mean calculated as:

$$T_{exp} = \left[ \frac{T_{min} + 4T_{mean} + T_{max}}{6} \right]. \tag{1}$$

These transformed estimates are assumed to be mean values. The $T_{exp}$ represents the approximate average value, while the $T_{mean}$ represents the mean of the beta distribution (Punmia and Khandelwal, 2007). The expected time $T_{exp}$ represents a particular value on the distribution curve that has a 50–50 chance of exceeding, and 50–50 chance of being met. It means there is equal chance for an activity to take more time or less time than the calculated $T_{exp}$. According to the central limit theorem where there are n activities having beta distribution with mean and standard deviation, the distribution of time for the project as a whole will be an approximately normal distribution (Punmia and Khandelwal, 2007). Therefore, the curve will be symmetrical and approach a normal curve with standard deviation.

$$\sigma = \left[ \frac{T_{max} - T_{min}}{6} \right] \tag{2}$$

$$\text{Variance } (\sigma^2) = \left[ \frac{T_{max} - T_{min}}{6} \right]^2 \tag{3}$$

$$\sigma = \sqrt{ó2}. \tag{4}$$

$Z$ is the number of standard deviations by which $T_{sch}$ exceeds $T_{exp}$. The scheduled project completion times $T_{sch}$ of both the methods are compared.

$$Z = \left[ \frac{T_{sch} - T_{exp}}{ó} \right] \tag{5}$$

$$T_{sch} = \left[ óZ + T_{exp} \right]. \tag{6}$$

## 4 Permutation and combinations in pert

An activity, when influenced by several conditions, will not give uniform results, but it may result in any one of several possible outcomes (Waheedullah, 2006). The probability of expected time $T_{exp}$ depends on the occurrence of one of the three estimates. The three time estimates are considered by giving equal chances. The permutation and combination method gives the complete stochastic treatment in estimating $T_{exp}$ of an activity.

In PERT, the most likely estimated activity time $T_{mean}$ and approximate variance are insufficient basis for stochastic calculations. However, this can be used as a basis for calculating the critical path. Using the permutations and combinations in the PERT method the three time estimates of an activity i.e., minimum (optimistic) $T_{min}$, mean (most likely) $T_{mean}$ and maximum (pessimistic) $T_{max}$ time were given equal chances of occurrence, with unrestricted repetition. Activity time estimates of ‘*n*’ kinds, and unlimited numbers of each kind after being chosen are replaced in the data. Hence, ‘*r*’ permutations will be filled in ‘*n*’ different ways, with repetition. By the product rule, the numbers of permutations are '$r$' permutations with repetition of '$n$' things is (Riordan, 2002).

$$U = (n, r) = n^r. \tag{7}$$

A software program is developed to calculate the critical path of a network. It generates permutations and combinations of critical activities $T_{sch}$ using the three estimated times.

### *4.1 Chi-square test*

To find the significance or the discrepancy between PERT and permutations and combinations in PERT, a Chi-square test was conducted. The scheduled estimated time $T_{sch}$ calculated by the two methods was tested. The following Chi-square formula was used:

$$\text{Chi-square} = \sum_{i=1}^{n} \left[ \frac{O_i - E_i}{E_i} \right]^2 \tag{8}$$

$O_i$ ($i$ = 1, 2, …, $n$) is a set of observed results and $E_i$ ($i$ = 1, 2, …, $n$) is the corresponding set of expected hypothetical results (Gupta and Kapoor, 1996).

## 5 Methodology

In probabilistic PERT the optimistic time estimate $T_{min}$, pessimistic time estimate $T_{max}$ and most likely time estimate $T_{mean}$ are not given equal chances of occurrence in the calculation of the activity estimated time $T_{exp}$. The weighted average factors considered for the time estimates in calculating $T_{exp}$ are 0.17, 0.17 and 0.66, respectively. In this weighted method the $T_{mean}$ plays a predominant role. Therefore, in most cases, $T_{mean}$ approaches the activity estimated time $T_{exp}$. In this kind of situation where the skill of the personnel and human psychology also plays a vital role, as in the printing industry, this method of calculating $T_{exp}$ based on wrong assumptions. Standard deviation is the most common measure of statistical dispersion, measuring how widely spread the values are in a data set. The standard deviation of each activity duration time is calculated by the PERT method by considering each activity's optimistic and pessimistic time estimates $T_{min}$ and $T_{max}$ only. This is an approximate method of calculation of standard deviation. This is compared and analysed by introducing permutation and combinations in the PERT method under different conditions.

Under random conditions the most likely time is the average observed time of an activity for the project volume $T_{mean}$. The optimistic time $T_{min}$ and, pessimistic time $T_{max}$ of an activity are estimated within the three standard deviations from $T_{mean}$. The expected time $T_{exp}$ is calculated using equation (1). The probability of activity occurrence at $T_{mean}$ is considered as 0.66 and the probability of activity occurrence at $T_{min}$ and $T_{max}$ are considered as 0.17 each. In this condition $T_{exp}$ approaches $T_{mean}$. Therefore, the distribution curve will be symmetrical (Punmia and Khandelwal, 2007). The optimistic $T_{min}$ and pessimistic $T_{max}$ were randomly generated in MAT Lab within ±3 standard deviations to $T_{exp}$. The sum of all activities $T_{exp}$ on the critical path is the $T_{sch}$. On Web, Miller, HMT and Eagle machines, five different projects with five different volumes are considered for the analysis. $T_{mean}$ of each activity is calculated by gathering data of five similar jobs in all aspects. The project duration time for different $T_{exp}$ values and $Z$ value are calculated. $Z$ is the number of standard deviations by which $T_{sch}$ exceeds $T_{exp}$. The scheduled time of project completion time $T_{sch}$ of both the methods with the two different standard deviations is compared. This technique is most helpful in situations where the production cycle time is large and the operational times are very volatile.

In second condition extreme values are given to the optimistic $T_{min}$ and pessimistic $T_{max}$. The projects duration time $T_{sch}$ is calculated by PERT method.

In permutations and combinations in the PERT method, $T_{exp}$ of the project is calculated using the developed software. The time estimates $T_{min}$, $T_{mean}$ and $T_{max}$ of each activity are selected randomly and by permutations and combinations technique available in the program. The $T_{exp}$ of the project is calculated. The scheduled time $T_{sch}$ is calculated by the number of combinations produced within different percentages of probable completion time of the project. The results obtained under random and extreme conditions are studied in the real life situation.

A significance test is conducted for PERT and permutations and combinations by the Chi-square test.

## 6 Results and discussions

In probabilistic PERTs, the optimistic time estimate $T_{min}$, pessimistic time estimate $T_{max}$ and most likely time estimate $T_{mean}$ are not given equal chances of occurrence in calculation of the activity estimated time $T_{exp}$. The weighted average factors considered for the time estimates in calculating $T_{exp}$ are 0.17, 0.17 and 0.66, respectively. In this the weighted factor, $T_{mean}$, plays a predominant role. Therefore, in most cases $T_{mean}$ approaches the activity estimated time, $T_{exp}$. In this kind of situation, the skill of personnel and human dependence also play a vital role, along with environmental effects on raw materials, as in the printing industry. This method of calculating $T_{exp}$ based on wrong assumptions. Standard deviation is the most common measure of statistical dispersion, measuring how widely spread the values in a data set are. The standard deviation of each activity duration time is calculated by the PERT method by considering each activity's two time estimates, $T_{min}$ and $T_{max}$, only. This is an approximate method of calculating standard deviation. The project duration times of printing projects of different volumes are calculated and the results are noted. The results obtained by using equation (2) show lower project completion time, $T_{sch}$, for all projects, with 100, 200, 300, 400 and 500 page volumes on Web, Miller, HMT and Eagle machines than permutations and combinations applied to PERT. Therefore, the project schedule times are different for the same project.

In situations where the prediction of activity time depends on human factors like skill and psychology, the activity duration time may conform to any of the three time estimates. Hence, the permutations and combinations method used, in which all the activity time estimates are considered to have equal chances of an event happening. This method is more realistic than PERT in such situations. The method is applied to similar printing projects. The results obtained show almost equal PERT approximations as permutations and combinations PERT under random conditions.

In extreme conditions, the project completion $T_{sch}$ calculated by PERT, and permutations and combinations by the PERT method show different scheduled times. The values obtained by the PERT method are lower. The permutations and combinations by the PERT method show higher values because the variations in completion of activities of the projects are extreme.

The Chi-square test showed that there is no significant difference in the methods. Over a period of two years the methods are practically evaluated in the printing organisations and showed that the permutation and combination method results are closer to real life situations, and 100% scheduled times are met. Therefore to achieve Six Sigma level in delivery schedules the permutation and combinations in PERT method proved to be appropriate.

The results obtained for various projects duration times with PERT and PC PERT are given in Tables 1–4.

**Table 1** $T_{sch}$ calculated in PERT and permutation and combinations PERT under random and extreme conditions on Web

| *Web* | *Random* | | *Extreme* | |
|---|---|---|---|---|
| *Volume* | *PERT* | *PC PERT* | *PERT* | *PC PERT* |
| 100 | 52.77 | 53.25 | 94.24 | 105 |
| 200 | 68.36 | 67.73 | 121.6 | 129 |
| 300 | 97.05 | 96.73 | 169.1 | 184 |
| 400 | 114.88 | 116.89 | 158.1 | 197 |
| 500 | 138.30 | 142.47 | 200.3 | 308 |

**Table 2** $T_{sch}$ calculated in PERT and permutation and combinations PERT under random and extreme conditions on Miller

| *Miller* | *Random* | | *Extreme* | |
|---|---|---|---|---|
| *Volume* | *PERT* | *PC PERT* | *PERT* | *PC PERT* |
| 100 | 65.52 | 68.71 | 93.35 | 141 |
| 200 | 92.97 | 95.01 | 135.1 | 200 |
| 300 | 127.83 | 130.92 | 195.8 | 285 |
| 400 | 157.29 | 160.87 | 238.6 | 341 |
| 500 | 200.28 | 204.87 | 337.3 | 500 |

**Table 3** $T_{sch}$ calculated in PERT and permutation and combinations PERT under random and extreme conditions on HMT

| *HMT* | *Random* | | *Extreme* | |
|---|---|---|---|---|
| *Volume* | *PERT* | *PC PERT* | *PERT* | *PC PERT* |
| 100 | 73.8 | 76.1 | 114.3 | 178 |
| 200 | 114.9 | 117.14 | 201.4 | 303 |
| 300 | 161.76 | 164.67 | 338.9 | 500 |
| 400 | 206.06 | 208.9 | 376.9 | 582 |
| 500 | 255.25 | 258.8 | 424.9 | 624 |

**Table 4** $T_{sch}$ calculated in PERT and permutation and combinations PERT under random and extreme conditions on Eagle

| *Eagle* | *Random* | | *Extreme* | |
|---|---|---|---|---|
| *Volume* | *PERT* | *PC PERT* | *PERT* | *PC PERT* |
| 100 | 121.37 | 123.54 | 217.2 | 328 |
| 200 | 205.6 | 208.39 | 361.9 | 495 |
| 300 | 293.39 | 296.13 | 541.4 | 722 |
| 400 | 383.59 | 388.2 | 662.4 | 846 |
| 500 | 423.2 | 428.08 | 737.9 | 968 |

*Chi-square Test results*

Chi-square calculated values for PERT and permutation and combinations PERT

| *Machine* | *Chi-square calculated value (random values PERT and PC PERT)* | *Chi-square calculated value (extreme values PERT and PC PERT)* |
|---|---|---|
| Web | 0.062 | 2.585 |
| Miller | 0.018 | 2.989 |
| HMT | 0.010 | 0.336 |
| Eagle | 0.003 | 1.034 |

## 7 Conclusions

This paper describes the meeting of delivery schedules by adopting permutations and combinations using the PERT method. The permutation and combination method considers all the three time estimates and gives complete stochastic treatment to the three time estimates. The project duration times obtained by this method are nearer to real life situations where the activities involve human skill, and psychological factors play a predominant role. The new approach is simple, and key to keep printing firms competitive in the competitive environment by supporting better decisions in the production schedule that help to achieve Six Sigma level status.

## Acknowledgements

The author thanks Dr. Markandeya for providing valuable information and helping in writing this paper. The author also appreciates the help given by Mr. Venkateswarlu, Deputy Manager, ICFAI Press and Mr. K. Sampath, Assistant Professor, Department of Mathematics, TKR College of Engineering and Technology, Hyderabad.

## References

Benati, S. (2006) 'An optimization model forstochastic project networks with cash flows', *CMS*, Springer-Verlag, Vol. 3, 13 June, pp.271–284.

Gupta, S.C. and Kapoor, V.K. (1996) *Fundamentals of Mathematical Statistics*, Sultan Chand & Sons, New Delhi.

Punmia, B.C. and Khandelwal, K.K. (2007) *Project Planning and Control with PERT and CPM*, Laxmi Publications, New Delhi.

Riordan, J. (2002) *Introduction to Combinatorial Analysis*, Rover Publications Inc., New York.

Waheedullah, A. (2006) *Probability and Statistics*, SMS Education, New Delhi.