Consignment stock policy using genetic algorithm for effective inventory management in supply chains

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Abstract: In this paper, we consider single-vendor–multi-buyer Consignment Stock Policy (CSP) inventory model which is a distinctive flavour of Vendor Managed Inventory (VMI). Four different models have been formulated using Genetic Algorithm (GA) to minimise joint total expected cost of vendor and buyer and simultaneously optimise other decision variables such as quantity transported, number of transport operations, delay deliveries and buyer maximum and minimum stocks under stochastic environment. Numerical examples are presented to illustrate the proposed models, and the effects of changes on the cost and system parameters on the inventory are studied by using sensitivity analysis. To solve the iterative procedure involved, the GA is coded in VC++.

Keywords: Consignment Stock Policy; CSP; Supply Chain; SC; Genetic Algorithm; GA; delay delivery; information sharing; crashing cost.

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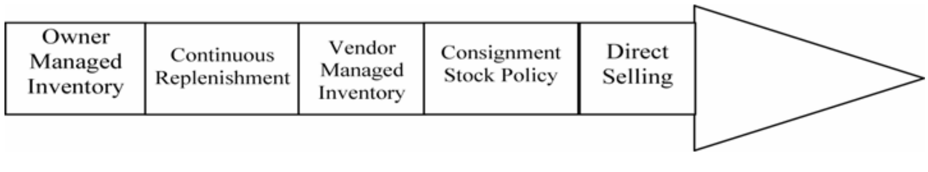
1 Introduction

In today's globalise economy, business is looking for ways to optimise the Supply Chain (SC) network by means of integration and cooperation of network echelons (Drucker, 1998; Douglas and Cooper, 2000). Inventory is one of the most widely discussed areas for improving SC efficiency. Wal-Mart and Procter & Gamble popularised it in the late 1980s. Since the holding of inventories in an SC can cost anywhere between 20% and 40% of product value, the effective management of inventory is critical in SC operations (Ballou, 1992). In this environment, Supply Chain Management (SCM) has become an effective business tool to reduce SC network inventory cost. Houlihan (1985) is credited to be the first person for coining the term SC with insight concepts with a strong case for viewing the SC as a strategy for global business decisions. The SCM is generally viewed as a strategy for integrated network business that work together to source, produce and ultimately distribute products and services to the customer with right quantities, right place and right time. Each echelon of SC performs independent business with integrated information sharing among all the echelons and it holds some inventories which may be unavoidable due to existing uncertainty in the business.

Organisations have followed different strategies and models for optimising inventory levels. Some significant strategies or practices for streamlining inventory along the SC include Consignment Stock models. This paper describes the benefits of Consignment Stock Policy (CSP) inventory models of a single-vendor–multi-buyer model which is viewed as a classification of divergent SC with end-to-multi-end case which is a distinctive flavour of Vendor Managed Inventory (VMI). In CSP vendor stocks his finished products at buyer warehouse, hence it leads to suppression of vendor inventory. The vendor will guarantee for the quantity stored in the buyer warehouse that will be kept between minimum(s) and maximum(s) stock levels with supporting normally distributed stochastic customer demand. In CSP vendor still owns the stock held at buyer warehouse; the change of ownership will occur only when the payment is made to vendor.

With reference to Figure 1, we can predict VMI is transforming into CSP approach and then elimination of intermediary channels transforms this into Direct Selling. The basic fundamentals of CSP is explained in Braglia and Zavanella (2003), Valentini and Zavanella (2003) and Srinivas and Rao (2007). The CSP is conveniently adopted in small size and less cost items. Typically, it is best suitable for Fast Moving Consumer Goods (FMCG), retail items of super and hypermarkets.

Figure 1 Conceptual evaluation of consignment stock policy



This paper is structured in six sections: Section 1 is introduction; Section 2 describes the literature review and the work done in the area of Joint Total Expected Cost (JTEC). Section 3 presents the single-vendor–multi-buyer CSP inventory models and in Section 4 Genetic Algorithm (GA) technique used for solving all the four CSP models is described. Section 5 gives the illustrative examples and results and Section 6 conclusions and future studies.

2 Literature review

Croom et al. (2000) reviewed the SCM literature to identify the nature of research and provided taxonomy as an aid to classify the research in this field as a means of providing a framework for identification of the key contents. The general buying and payment mode includes various strategies (Frazella, 2001), among them Electronic Fund Transfer (EFT) and CSP inventory are important. It facilitates consignment inventory programmes with electronic payment on consumption initiated at the Point of Sale (POS).

Corbett (2001) is credited to be the first person to give about the fundamentals of CSP, whereas Valentini and Zavanella (2003) presented an industrial case and performance analysis of CSP for a single-vendor–single-buyer. Braglia and Zavanella (2003) presented an analytical modelling approach which concerns the deterministic single-vendor–single-buyer, allowing the analyst to identify the optimal inventory level and shipment policy for minimising joint total costs. Piplani and Viswanathan (2003) discussed supplier-owned inventory which posses similar concepts of CSP. They concluded it is always beneficial for the SC as a whole. Zanoni and Grubbström (2004) extended Braglia and Zavanella (2003) with explicit analytical expression of ordering quantity, number of shipments and delay deliveries. Pan and Yang (2002) credited for minimising joint total economic cost of vendor–buyer inventory model with controllable lead-time (LT) which is a decision variables. Ryu and Lee (2003) analysed the effect of investment strategies to control lead-times. Liao and Shyu (1991) decomposed lead-time into '*n*' components, each having a different crashing cost for reduced lead-time. Ben-Daya and Raouf (1994) considered both lead-time and order quantity as decision variables. Their model uses different representatives of the relationship between lead-time crashing cost and lead-time. Ouyang et al. (2004) discussed integrated vendor–buyer model with stochastic demand to integrate production inventory model. The shortages are permitted and it is assumed the lead-time is controllable with added cost so as to optimise ordering quantity. The lead-time crashing components can be more than three and depend on the interest of both vendor and buyer involved to reduce the lead-time and its crashing cost component as much as economically possible, by a technique such as work study (Goyal, 2003). Persona et al. (2005) proposed an analytical model able to take into account the effects of obsolescence in an SC-based consignment stock model. They used deterministic demand and showed the results with presence of obsolescence for short-life components.

Recently, Srinivas and Rao (2007) extended and analysed the models of Braglia and Zavanella (2003). Their results reveal that the cost reduces with information sharing compare to with delay delivery model, and where as in lead-time crashing cost model, it all depends on the type of lead-time crashing values are chosen which involves complexity. The CSP of four models for solving single-vendor–multi-buyer is difficult to solve using enumeration technique and hence GA is used which gives results for all models within 10 seconds. The running time of enumeration technique grows exponentially while increasing the number of variables (Goyal, 1974). The GA technique for more than three buyers with information sharing with five process variables results in less than 10 seconds, whereas enumeration technique could not give results even after 24 hours.

The literature review papers of Aytug et al. (2003) and Chaudhury and Luo (2005) reveal that no approach attempt has been made to develop a heuristic method such as GA to determine inventory levels in SC echelons. Daniel and Rajendran (2005) studied GA, enumeration and random search procedure methods to single product serial SC operating with a base stock periodic review system and to optimise the base stock inventory levels in SC so as to minimise total SC cost, comprising holding and shortage costs at all installations in SC. They found the solution generated by proposed GA is not significantly different from the optimal solution yielded by complete enumeration, but it is good for deterministic replenishments. They did not check for multi-buyer stochastic demand. Li et al. (2001) studied effect of information-sharing strategies on the performance of SC. Their results indicate that information sharing improved SC performance of overall inventory cost and fillrate.

2.1 The necessary notations used in this paper are summarised as follows:

s	Batch set-up cost ($) (vendor)	L	Length of the lead-time for the buyer
A_{ti}	Order emission cost ($) (buyer)	C_L	Lead-time crashing cost ($) per cycle
h_v	Vendor stock holding cost ($)/unit/unit time	k^1	Delay deliveries ($\leq n$)
h_b	Buyer stock holding cost ($)/unit/unit time	ϕ	Normal probability density function
P	Vendor production rate	$\Phi_{(z)}$	Cumulative distribution function
d_i	Demand/unit time seen by the buyer	n	Number of transport operations/production batch
σ	Standard deviation of demand	m_i	Delayed deliveries shifted to another buyer ($\leq k$)
π	Unit backorder cost ($) for the buyer	j_{ij}	Delivery shifted from ith buyer to jth buyer

3 Vendor–buyer inventory models

3.1 Consignment stock model

In this model, vendor uses buyer warehouse for keeping the goods produced by the vendor without changing the ownership. This creates a condition of shared benefit; neither the vendor nor the buyer will benefit until the product is sold to an end-user. This shared risk–benefit condition will often be enough to convince the buyer to stock the products. The key benefit to the buyer should be obvious, so that he does not have to tie up his capital $h_{b,\,finance}$. This does not mean that there is no inventory-carrying costs for the buyer; he does still incur costs $h_{b,\,stock}$ related to storing and managing the inventory, i.e. both parties incur holding cost, depending on different rates and the length of time for which materials have been stocked in SC. Finally, the buyer sees a lower inventory cost per unit, i.e. only $h_{\text{b, stock}}$ instead of the entire $h_{\text{b, stock}} + h_{\text{b, finance}}$. The vendor will have set-up cost and holding cost, whereas the buyer will have order emission cost and holding cost.

The average total cost for this model is calculated as follows:

$T_C^{CSP} =$ vendor set-up cost + average vendor holding cost + buyer ordering cost + average buyer holding cost + safety stock cost + shortage cost. (1)

$$T_C^{CSP} = \frac{s}{c} + h_v \frac{c}{2p}\left(\sum_{i=1}^{y}\frac{D_i^2}{n_i}\right) + \frac{1}{c}\left(\sum_{i=1}^{y} A_i + \sum_{i=1}^{y} n_i A_{ti}\right)$$
$$+\sum_{i=1}^{y}\left(\frac{h_{bi}}{2}\left\{D_i c - (n_i - 1)D_i\left[\frac{D_i c}{n_i p} + \sum_{i\neq j}\frac{D_j c}{n_j p}\left(\frac{n_j}{n_i}\right)\right]\right\}\right) \tag{2}$$
$$+\sum_{i=1}^{y}\left(h_{bi} z\sigma_i\sqrt{L_i}\right) + \frac{1}{c}\sum_{i=1}^{y}\pi_i\sigma_i\sqrt{L_i}\Psi(z).$$

Equation (2) is modified as

$$T_C^{CSP} = \frac{1}{c}\left(s + \sum_{i=1}^{y} A_i + \sum_{i=1}^{y} n_i A_{ti}\right) + h_v \frac{c}{2p}\left(\sum_{i=1}^{y}\frac{D_i^2}{n_i}\right)$$
$$+\sum_{i=1}^{y}\left(\frac{h_{bi}}{2}\left\{D_i c - (n_i - 1)D_i\left[\frac{D_i c}{n_i p} + \sum_{i\neq j}\frac{D_j c}{n_j p}\left(\frac{n_j}{n_i}\right)\right]\right\}\right) \tag{3}$$
$$+\sum_{i=1}^{y}\left(h_{bi} z\sigma_i\sqrt{L_i}\right) + \frac{1}{c}\sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_i}\Psi(z)\right).$$

Equation (3) is further modified as

$$T_C^{CSP} = \frac{1}{c}G(n) + H(n)c + \sum_{i=1}^{y}\left(h_{bi} z\sigma_i\sqrt{L_i}\right) + \frac{1}{c}\sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_i}\Psi(z)\right). \tag{4}$$

Here, $G(n) = s + \sum_{i=1}^{y}\left(A_i + n_i A_{ti}\right),$

$$H(n) = h_v \frac{1}{2p}\left[\sum_{i=1}^{y}\frac{D_i^2}{n_i}\right] + \sum_{i=1}^{y}\left(\frac{h_{bi}}{2}\left\{D_i - (n_i - 1)D_i\left[\frac{D_i}{n_i p} + \sum_{i\neq j}\frac{D_j}{n_j p}\left(\frac{n_j}{n_i}\right)\right]\right\}\right),$$

$$\Psi(z) = \phi(z) - z[1 - \Phi(z)]$$

where $\phi(z), \Phi(z)$ are normal probability and cumulative probability density functions. The minimum cost for optimum values of (c, n and z) will be

$$T_C^{CSP} = 2\sqrt{\left(G(n) + \sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_i}\Psi(z)\right)\right)H(n)} + \sum_{i=1}^{y}\left(h_{bi} z\sigma_i\sqrt{L_i}\right). \tag{5}$$

The maximum level of inventory for buyer i

$$= \left\{D_i c - (n_i - 1)D_i\left[\frac{D_i c}{n_i p} + \sum_{j\neq i}\frac{D_j c}{n_j p}\left(\frac{n_j}{n_i}\right)\right]\right\} + z\sigma_i\sqrt{L_i}. \tag{6}$$

3.2 CSP-k^l model (number of delayed deliveries, $k^l < n$)

The CSP is not suitable for limited/small periods because maximum level of buyer's inventory may reach even for limited periods. Hence the CSP model with delayed delivery period (CSP-k^1) is preferred for limited periods. In CSP-k^1 model, the last delivery is delayed until it reaches a state where there is no longer an increase in the maximum level already reached. That means, we have to delay the delivery always whenever maximum level inventory stock is reached.

The average joint total cost in this model is as follows:

$$T_C^{CSP-k^1} = \frac{1}{c}G(n) + E(n)c + \sum_{i=1}^{y}\left(h_{bi} z\sigma_i\sqrt{L_i}\right) + \frac{1}{c}\sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_i}\Psi(z)\right). \tag{7}$$

Here

$$E(n) = h_v\left\{\frac{1}{2p}\left[\sum_{i=1}^{y}\frac{D_i^2}{n_i}\right] + \sum_{i=1}^{y}\left(\frac{D_i}{n_i}\frac{(p-D_i)}{n_i p}\frac{\left(k_i^1+1\right)}{2}k_i^1\right)\right\}$$
$$+\sum_{i=1}^{y}\left\{\frac{h_{bi}}{2}\left\{(n_i-k_i^1)\frac{D_i}{n_i} - \left(n_i-k_i^1-1\right)D_i\left[\frac{D_i}{n_i p} + \sum_{j\neq i}\left(\frac{D_j}{n_j p}\left(\frac{n_j}{n_i}\right)\right)\right]\right\}\right\}.$$

The minimum cost for optimum values of (c, n, k^l and z) will be

$$T_C^{CSP-k^1} = 2\sqrt{\left(G(n) + \sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_i}\Psi(z)\right)\right)E(n)} + \sum_{i=1}^{y}\left(h_{bi}z\sigma_i\sqrt{L_i}\right). \tag{8}$$

The maximum inventory level for buyer will be as follows:

$$= \left\{\left(n_i - k_i^1\right)\frac{D_i c}{n_i} - \left(n_i - k_i^1 - 1\right)D_i\left[\frac{D_i c}{n_i p} + \sum_{j\neq i}\left(\frac{D_j c}{n_j p}\left(\frac{n_j}{n_i}\right)\right)\right]\right\} + z\sigma_i\sqrt{L_i}. \tag{9}$$

Equation (9) ensures that not less than a single delay has been delayed. When k^l=0, equation (8) becomes the maximum level of buyer's stock in basic CS model equation (6), and when $k^l = (n - 1)$, equation (9) matches with maximum level of buyer's stock of Hill (1999) model in which maximum buyer inventory is equal to '*nq*', where q is the quantity transported per delivery. The delay deliveries strategy is much explained in Zanoni and Grubbström (2004). They also provided a quick method for calculating the optimal total number of deliveries and the number of deliveries to be delayed.

3.3 CSP with information sharing and with delay delivery

Goyal (1976) is credited to be the first person to describe integrated models of single-vendor–single-buyer. Goyal (1977) proposed a Joint Expected Lot Size (JELS) model to minimise total relevant costs and is compared with total costs incurred if vendor and buyer act independently. Banerjee (1986) generalised Goyal (1977) model by assuming vendor with finite rate produces for a buyer on a lot-for-lot basis under deterministic conditions. Goyal (1988) generalised the Banerjee (1986) model by relaxing the assumption of the lot-for-lot policy of the vendor. In an integrated inventory model, one partner's gain exceeds the other partner's loss. Therefore, the net benefit can be shared in some equitable fashion (Goyal and Gupta, 1989). They also summarised the literature on integrated vendor–buyer models up to 1989. In consignment stock with partial information sharing, models include information of demand, shipments and inventory. It is known that information sharing benefits the vendor more compare with buyer due to reduction in vendor inventory and also due to adjusted shipments between buyers, otherwise the vendor may have to keep. In this view, SC is constructed in such a way that if buyer does not need a particular scheduled delivery lot, the vendor finds an alternate buyer in the SC network. To fulfil this, the vendor adjusts exact delivery quantity required to the alternate buyer, i.e. the shifted quantity should be equal to scheduled quantity of alternate buyer.

The average total cost in this model is calculated as follows:

$$T_C^{CSP-IS-k^1} = \frac{1}{c}G(n) + U(n)c + \sum_{i=1}^{y}\left(h_{bi} z \sigma_i \sqrt{L_i}\right) + \frac{1}{c}\sum_{i=1}^{y}\left(\pi_i \sigma_i \sqrt{L_i}\Psi(z)\right). \tag{10}$$

Here

$$U(n) = h_v\left\{\frac{1}{2p}\left[\sum_{i=1}^{y}\frac{D_i^2}{n_i}\right] + \sum_{i=1}^{y}\left(\frac{D_i}{n_i}\frac{(p-D_i)}{n_i p}\frac{\left(k_i^1 - m_i + 1\right)}{2}(k_i^1 - m_i)\right)\right\}$$
$$+\sum_{i=1}^{y}\left\{\frac{h_{bi}}{2}\left\{(n_i - k_i^1 + \sum_{j\neq i} j_{ij})\frac{D_i}{n_i} - \left(n_i - k_i^1 - 1 + \sum_{j\neq i} j_{ij}\right)D_i\left[\frac{D_i}{n_i p} + \sum_{j\neq i}\left(\frac{D_j}{n_j p}\left(\frac{n_j}{n_i}\right)\right)\right]\right\}\right\}.$$

From equation (10) the minimum total cost for optimum values (c, n, k^1, m) is calculated as follows:

$$T_C^{CSP-IS-k^1} = 2\sqrt{\left(G(n) + \sum_{i=1}^{y}\left(\pi_i \sigma_i \sqrt{L_i}\Psi(z)\right)\right)U(n)} + \sum_{i=1}^{y}\left(h_{bi} z \sigma_i \sqrt{L_i}\right). \tag{11}$$

The maximum level of inventory for buyer i

$$= \left\{\left(n_i - k_i^1 + \sum_{j\neq i} j_{ij}\right)\frac{D_i c}{n_i} - \left(n_i - k_i^1 - 1 + \sum_{j\neq i} j_{ij}\right)D_i\left[\frac{D_i c}{n_i p} + \sum_{j\neq i}\left(\frac{D_j c}{n_j p}\left(\frac{n_j}{n_i}\right)\right)\right]\right\} + z\sigma_i\sqrt{L_i}. \tag{12}$$

3.4 CSP-LT model

In this model, the vendor negotiates with a buyer closely to reduce lead-time as much as possible down to a point where it is acceptable to the buyer with his stable production and delivery schedule. The inventory is reviewed continuously and shortages are allowed with fully backordered. It should be noted that the delivery lead-time is null; however, the batch is to be produced, so that there exists a system lead-time other than zero. Adding an additional cost, the system lead-time can be controlled. Thus the system lead-time is crashed one at a time starting from first independent component because it has minimum unit crashing cost per unit time and then the second independent component, and so on. It is clear that when lead-time is reduced, its corresponding handling cost for that time is reduced. The length of lead-time ensures the order transit arrival even though lead-time is crashed and shortages if any are permitted and backordered. Since lead-time is a decision variable in this model, the extra costs incurred by the vendor will be fully transferred to the buyer if shortened lead-time that is requested can be viewed as an investment. The total lead-time crashing cost per cycle is calculated as follows:

$$C_L = C_i(L_{i-1} - L) + \sum_{j=1}^{i-1} C_j(b_j - a_j). \tag{13}$$

$$L_i = L_0 - \sum_{j=1}^{i-1} C_j(b_j - a_j). \tag{14}$$

L_i is the length of the lead-time with components *1*,*2*…, *i* which are to be crashed to minimum duration and $L \in [L_i, L_{i-1}]$ for *i*th component has a normal duration 'b_i', minimum duration 'a_i' and crashing cost per unit time 'c_i', such that $c_1 \le c_2 \le \cdots \le c_n$.

$$T_C^{CSP-LT} = \frac{1}{c}G(n) + V(n)c + \sum_{i=1}^{y}\left(h_{bi} z\sigma_i\sqrt{L_i}\right) + \frac{1}{c}\sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_i}\Psi(z)\right) + \frac{1}{c}\sum_{i=1}^{y} C_{L_i}. \tag{15}$$

Here

$$V(n) = h_v \frac{1}{2p}\left[\sum_{i=1}^{y}\frac{D_i^2}{n_i}\right] + \sum_{i=1}^{y}\left(\frac{h_{bi}}{2}\left\{D_i - (n_i - 1)D_i\left[\frac{D_i}{n_i p} + \sum_{i\ne j}\frac{D_j}{n_j p}\left(\frac{n_j}{n_i}\right)\right]\right\}\right).$$

The minimum cost for optimum values of (*c*,*n*,*L*,*z*)

$$T_C^{CSP-LT} = 2\sqrt{\left(G(n) + \sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_i}\Psi(z)\right) + \sum_{i=1}^{y} C_{L_i}\right)V(n)} + \sum_{i=1}^{y}\left(h_{b_i} z\sigma_i\sqrt{L_i}\right). \tag{16}$$

The maximum level of inventory for buyer *i*

$$= \left\{D_i c - (n_i - 1)D_i\left[\frac{D_i c}{n_i p} + \sum_{j\ne i}\frac{D_j c}{n_j p}\left(\frac{n_j}{n_i}\right)\right]\right\} + z\sigma_i\sqrt{L_i}. \tag{17}$$

3.5 Algorithm to CSP-IS-k^1 model

In this section, an iterative algorithm (single vendor–two buyers) including the crashing expenses is presented to find minimum JTEC with optimal decision variables.

j_{ij} = delivery shifted from *i*th buyer to *j*th buyer

j_{12} = delayed deliveries shifted from buyer 1 to buyer 2

J_{21} = delayed deliveries shifted from buyer 2 to buyer 1

$k_1^1 - j_{12}$ = deliveries retained by vendor

$k_2^1 - j_{21}$ = deliveries retained by vendor

m_i = delayed deliveries shifted to another buyer

m_1 = transferred deliveries to buyer 2 from buyer 1

m_2 = transferred deliveries to buyer 1 from buyer 2.

Iterative procedure used in this model:

Step 1: set $n_1 = 1$

Step 2: set $n_2 = 1$

Step 3: set $k_1^1 = 1$

Step 4: set $k_2^1 = 1$

Step 5: set $j_{12} = 1$

Step 6: for each j_{21} perform (i) to (v)

(i) start with $z_{i1} = 0 \Rightarrow \Psi(z) = 0.39894$

(ii) substitute $\Psi(z)$ to evaluate '*c*'

(iii) utilise '*c*' to determine $\Phi(z)$ and then find *z* for next iteration by checking the standard normal table, and hence $\Psi(z)$ for next iteration

(iv) repeat (ii)–(iii) until no change occurs in the values of '*c*' and '*z*'

(v) compute the corresponding $JTEC\left(c, n_2, n_1, k_1^1, k_2^1, j_{12}, j_{21}, z\right)$

Step 7: find $\min JTEC\left(c, n_2, n_1, k_1^1, k_2^1, j_{12}, j_{21}, z\right)$; if $JTEC\left(c^*, n_2, n_1, k_1^1, k_2^1, j_{12}, j_{21}^*, z^*\right)$ is minimum $JTEC\left(c, n_2, n_1, k_1^1, k_2^1, j_{12}, j_{21}, z\right)$, then $\left(c^*, n_2, n_1, k_1^1, k_2^1, j_{12}, j_{21}^*, z^*\right)$ are the optimal solution for fixed $n_2, n_1, k_1^1, k_2^1, j_{12}$

Step 8: set $j_{12} = j_{12} + 1$; repeat Steps 6 and 7 to get $JTEC\left(c^*, n_2, n_1, k_1^1, k_2^1, j_{12}, j_{21}^*, z^*\right)$

Step 9: if $JTEC\left(c^*, n_2, n_1, k_1^1, k_2^1, j_{12}, j_{21}^*, z^*\right) \leq$ $JTEC\left(c^*_{j_{12}-1}, n_{2(j_{12}-1)}, n_{1(j_{12}-1)}, k^1_{1(j_{12}-1)}, k^1_{2(j_{12}-1)}, j_{12(j_{12}-1)}, j^*_{21(j_{12}-1)}, z^*_{(j_{12}-1)}\right)$, then go to Step 8, otherwise go to Step 10

Step 10: set $JTEC\left(c^*, n_2, n_1, k_1^1, k_2^1, j_{12}^*, j_{21}^*, z^*\right) =$ $JTEC\left(c^*_{j_{12}-1}, n_{2(j_{12}-1)}, n_{1(j_{12}-1)}, k^1_{1(j_{12}-1)}, k^1_{2(j_{12}-1)}, j_{12(j_{12}-1)}, j^*_{21(j_{12}-1)}, z^*_{(j_{12}-1)}\right)$

Step 11: set $k_2^1 = k_2^1 + 1$; repeat Steps 6–10 to get $JTEC\left(c^*, n_2, n_1, k_1^1, k_2^1, j_{12}^*, j_{21}^*, z^*\right)$

Step 12: if $JTEC\left(c^*, n_2, n_1, k_1^1, k_2^1, j_{12}^*, j_{21}^*, z^*\right) \leq$ $JTEC\left(c^*_{k_2^1-1}, n_{2(k_2^1-1)}, n_{1(k_2^1-1)}, k^1_{1(k_2^1-1)}, k^1_{2(k_2-1)}, j^*_{12(k_2^1-1)}, j^*_{12(k_2^1-1)}, z^*_{(k_2^1-1)}\right)$ then go to Step 11, otherwise go to Step 13

Step 13: set $JTEC\left(c^*, n_2, n_1, k_1^1, k_2^{1*}, j_{12}^*, j_{21}^*, z^*\right) =$ $JTEC\left(c^*_{k_2^1-1}, n_{2(k_2^1-1)}, n_{1(k_2^1-1)}, k^1_{1(k_2^1-1)}, k^1_{2(k_2^1-1)}, j^*_{12(k_2^1-1)}, j^*_{21(k_2^1-1)}, z^*_{(k_2^1-1)}\right)$

Step 14: set $k_1^1 = k_1^1 + 1$; repeat Steps 6–13 to get $JTEC\left(c^*, n_2, n_1, k_1^1, k_2^{1*}, j_{12}^*, j_{21}^*, z^*\right)$

Step 15: if $JTEC\left(c^*, n_2, n_1, k_1^1, k_2^{1*}, j_{12}^*, j_{21}^*, z^*\right) \leq$ $JTEC\left(c^*_{k_1^1-1}, n_{2(k_1^1-1)}, n_{1(k_1^1-1)}, k^1_{1(k_1^1-1)}, k^1_{2(k_1^1-1)}, j^*_{12(k_1^1-1)}, j^*_{21(k_1^1-1)}, z^*_{(k_1^1-1)}\right)$ then go to Step 14, otherwise go to Step 16.

Step 16: set $JTEC\left(c^*, n_2, n_1, k_1^1, k_2^{1*}, j_{12}^*, j_{21}^*, z^*\right) =$

$JTEC\left(c_{k_1^1-1}^*, n_{2(k_1^1-1)}, n_{1(k_1^1-1)}, k_{1(k_1^1-1)}^1, k_{2(k_1^1-1)}^{1*}, j_{12(k_1^1-1)}^*, j_{21(k_1^1-1)}^*, z_{(k_1^1-1)}^*\right)$

Step 17: set $n_2 = n_2 + 1$; repeat Steps 6–10 to get $JTEC\left(c^*, n_2, n_1, k_1^1, k_2^{1*}, j_{12}^*, j_{21}^*, z^*\right)$

Step 18: if $JTEC\left(c^*, n_2, n_1, k_1^{1*}, k_2^{1*}, j_{12}^*, j_{21}^*, z^*\right) \leq$

$JTEC\left(c_{n_2-1}^*, n_{2(n_2-1)}, n_{1(n_2-1)}, k_{1(n_2-1)}^{1*}, k_{2(n_2-1)}^{1*}, j_{12(n_2-1)}^*, j_{21(n_2-1)}^*, z_{(n_2-1)}^*\right)$ then go to Step 17, otherwise go to Step 19

Step 19: set $JTEC\left(c^*, n_2^*, n_1, k_1^{1*}, k_2^{1*}, j_{12}^*, j_{21}^*, z^*\right) =$

$JTEC\left(c_{n_2-1}^*, n_{2(n_2-1)}, n_{1(n_2-1)}, k_{1(n_2-1)}^{1*}, k_{2(n_2-1)}^{1*}, j_{12(n_2-1)}^*, j_{21(n_2-1)}^*, z_{(n_2-1)}^*\right)$

Step 20: set $n_1 = n_1 + 1$; repeat Steps 4 and 19 to get $JTEC\left(c^*, n_2, n_1, k_1^{1*}, k_2^{1*}, j_{12}^*, j_{21}^*, z^*\right)$

Step 21: if $JTEC\left(c^*, n_2^*, n_1, k_1^{1*}, k_2^{1*}, j_{12}^*, j_{21}^*, z^*\right) \leq$

$JTEC\left(c_{n_1-1}^*, n_{1(n_1-1)}, n_{1(n_1-1)}, k_{1(n_1-1)}^{1*}, k_{2(n_1-1)}^{1*}, j_{12(n_1-1)}^*, j_{21(n_1-1)}^*, z_{(n_1-1)}^*\right)$ then, go to Step 20, otherwise go to Step 22

Step 22: set $JTEC\left(c_{n_1-1}^*, n_{2(n_1-1)}, n_{1(n_1-1)}, k_{1(n_1-1)}^{1*}, k_{2(n_1-1)}^{1*}, j_{12(n_1-1)}^*, j_{21(n_1-1)}^*, z_{(n_1-1)}^*\right) =$

$JTEC\left(c^*, n_2^*, n_1^*, k_1^{1*}, k_2^{1*}, j_{12}^*, j_{21}^*, z^*\right)$

Step 23: $\left(c^*, n_2^*, n_1^*, k_1^{1*}, k_2^{1*}, j_{12}^*, j_{21}^*, z^*\right)$ are the optimal variables and the minimum joint total expected cost is $JTEC\left(c^*, n_1^*, n_2^*, k_1^{1*}, k_2^{1*}, j_{12}^*, j_{21}^*, z^*\right)$

4 Genetic algorithm

We propose a Genetic Algorithm (GA) approach to optimise the *CSP*-based inventory models' joint total expected cost in SC. This study attempts to perform both performance analysis and optimisation of various inventory policy settings. GA is a class of evolutionary algorithms that utilise the theories of evolution and natural selection. GA begins with a population of randomly generated strings that represent the problems' possible solutions. Thereafter, each of these strings is evaluated to find its fitness. The initial population is subjected to genetic evolution to procreate the next generation of candidate solutions (Goldberg, 1989). The members of the population are processed by GA operators such as reproduction, crossover and mutation to create the progenies for the next generation of candidate solutions. The progenies are then evaluated and tested for termination; until a satisfactory solution (based on the acceptability or search stoppage criterion) already at hand is found, the search is stopped.

4.1 Working mechanism of GA

i. encode the initial chromosome using binary and integer

ii. initialise a set of feasible solutions randomly (i.e. initialise a population of chromosomes)

iii. compute fitness value $f_i = \frac{1}{1 + JTEC_{(n,k^1,m,j,c,L,\Phi(z))}} \in \text{chromosome},$ in the entire population

iv. if termination is satisfied, it gives best solution, otherwise go to step 'v'

v. select chromosomes for reproduction by making use of the roulette wheel selection procedure and fitness function value

vi. apply crossover and mutation on the selected chromosomes to produce new chromosomes

vii. if the stopping condition is reached, return the best solution; if not, go to step (ii)

GA works on a population or collection of solutions to the given problem. Each individual in the population is called chromosome. Designing chromosome is a very important step in GA, which contains decision variables that are to be optimised.

The chromosome structures for various models are summarised below:

Basic *CSP* model	$(c, n_i, \Phi_{(z)})$
CSP with delay delivery model	$(c, n_i, k_i^1, \Phi_{(z)})$
CSP with information sharing and delay delivery	$(c, n_i, k_i^1, m_i, j_{ij}, \Phi_{(z)})$
CSP-LT model	$(c, n_i, \Phi_{(z)})$

Integer encoding is used for $(n_i, k_i^1, m_i, j_{ij})$, whereas for '$c$' and '$\Phi(z)$' binary coding is used because decimal range is large. For converting into binary coding, this is first multiplied with 1000 to remove decimal point and then converted to binary. Population Size (ps), number of generations, Probability of Crossover (pc) and Probability of Mutation (pm) are the GA parameters. A large population size means a better exploration of the search space, while a large number of generations allow for better exploitation of the promising solutions found. Generally, the larger these parameters are, the better the algorithm will perform, but at the expenses of longer run-times because more fitness evaluations will be involved. Population size is fixed as 170 and the number of generations is fixed as 500 after experimentation. Probability of crossover varied from 0.5 to 1 with step of 0.1 and optimum value found at 0.7. Probability of mutation is varied from 0.05 to 0.15 in steps of 0.05 and finally fixed at 0.05 as it is giving minimum total cost. In crossover, two strings are picked from the mating pool and some portions of these strings are exchanged between them, attempting to produce new strings of superior fitness by effecting large changes in a string to jump in search of the optimum in the solution space.

An example of chromosome for one-vendor–three-buyers consignment stock with information sharing and with delay delivery model is given below.

Chromosome encoding:

	c	n_1	n_2	n_3	k_1^1	k_2^1	k_3^1	m_1	m_2	m_3	j_{12}	j_{21}	J_{31}	$\Phi(z)$
1st parent chromosome	0.205	3	7	4	2	5	3	1	2	1	1	1	1	0.915
2nd parent chromosome	0.163	7	4	5	5	2	3	3	2	1	2	1	1	0.834

Crossover:

Parent strings after encoding and before crossover:

Binary coding	*Integer coding*												*Binary coding*
c	n_1	n_2	n_3	k_1^1	k_2^1	k_3^1	m_1	m_2	m_3	j_{12}	j_{21}	j_{31}	$\Phi(z)$
1 1 0 0 1 1 0 1	3	7	4	2	5	3	1	2	1	1	1	1	1 1 1 0 0 1 0 0 1 1
1 0 1 0 0 0 1 1	7	4	5	5	2	3	3	2	1	2	1	1	1 1 0 1 0 0 0 0 1 0

Binary coding *Integer coding* *Binary coding*

Offspring after two point crossover:

c	n_1	n_2	n_3	k_1^1	k_2^1	k_3^1	m_1	m_2	m_3	j_{12}	j_{21}	j_{31}	$\Phi(z)$
1 1 1 0 0 0 0 1	3	7	4	**5**	2	3	1	2	1	1	1	1	1 1 1 0 0 0 0 0 1 1
1 0 0 0 1 1 1 1	7	4	5	2	**5**	3	**3**	2	1	2	1	1	1 1 0 1 0 1 0 0 1 1

After crossover, the variables in the Offsprings 1 and 2 are crossed the permissible independent boundary range (see the rounded value), i.e. in Offspring 1, $k_1^1 = 5,$ and in Offspring 2, $k_2^1 = 5,$ but the constraint is $k_i^1 < n_i$. And in Offspring 2, m_1 = 3 which violates the constraint, $m_i \le k_i^1$. As the constraints are violated in both the offsprings, the repair function is used to correct these defective chromosomes.

Repair function: From Offspring 1, k_1^1 value is replaced by the corresponding k_1^1 value of Offspring 2. If k_1^1 value in Offspring 2 is also greater than 'n_1', then randomly substitute k_1^1 with a value less than 'n_1'. Similarly, in Offspring 2, k_2^1 value is replaced by the corresponding k_2^1 value of Offspring 1; if not suitable then substitute k_2^1 value with a value less than 'n_2'. The 'm_1' in Offspring 2 is also replaced by corresponding 'm_1' value of Offspring 1.

Offspring after repair:

c	n_1	n_2	n_3	k_1^1	k_2^1	k_3^1	m_1	m_2	m_3	j_{12}	j_{21}	j_{31}	$\Phi(z)$
1 1 1 0 0 0 0 1	3	7	4	2	2	3	1	2	1	1	1	1	1 1 1 0 0 0 0 0 1 1
1 0 0 0 1 1 1 1	7	4	5	2	2	3	1	2	1	2	1	1	1 1 0 1 0 1 0 0 1 1

Mutation operator:

The need for local search around a current solution also exists and is accomplished by mutation. Mutation is additionally aimed to maintain diversity in the population. Mutation creates a new solution in the neighbourhood of a current solution by introducing a small change in some aspects of the current solution and helps to ensure that no point in the search space has a zero probability of being examined. For binary coding normal swap mutation operator is used. All bits in binary number is mutated with pm = 0.05; for this purpose a uniform random number is generated between 0 to 1; if number is less than probability of mutation, then that bit is changed from 0 to 1 or vice versa. For integer, coding genes in parent population are mutated with pm = 0.05, with sampling a uniform random number, u. If $u \le$ pm, then the value of the corresponding gene is altered as given below:

$$S_{\text{new}} = S_{\text{old}}(1-x) + 2xuS_{\text{old}}. \tag{18}$$

Here S_{new} is new gene after mutation, S_{old} is gene before mutation, u is a uniform random number between 0 and 1 and 'x' denotes the fraction of S_{old}. It is to be noted that if the computed S_{new} takes a non-integer value, then it is rounded off to the nearest integer. In this study 'x' is set to 0.2.

Offspring before mutation:

c	n_1	n_2	n_3	k_1^1	k_2^1	k_3^1	m_1	m_2	m_3	j_{12}	j_{21}	j_{31}	$\Phi(z)$
1 1 1 0 0 **<u>0</u>** 0 1	3	7	4	2	2	3	1	2	1	1	1	1	1 1 1 **<u>0</u>** 0 0 0 0 1 1
1 0 0 0 1 **<u>1</u>** 1 1	7	4	5	2	2	3	**<u>1</u>**	2	**<u>1</u>**	2	1	1	1 1 0 1 0 1 0 **<u>0</u>** 1 1

Offspring after mutation:

c	n_1	n_2	n_3	k_1^1	k_2^1	k_3^1	m_1	m_2	m_3	j_{12}	j_{21}	j_{31}	$\Phi(z)$
1 1 1 0 0 1 0 1	4	7	4	2	2	3	1	2	1	1	1	1	1 1 1 1 0 0 0 0 1 1
1 0 0 0 1 0 1 1	7	4	5	2	2	3	1	2	1	2	1	1	1 1 0 1 0 1 0 1 1 1

After mutation if any damaged genes exist, same repair function as discussed in crossover is used to repair the damaged genes. After crossover and mutation, the new population is called as child population. We have now *N* chromosomes in the initial population and *N* chromosomes in parent population. The best *N* chromosomes, among 2*N* chromosomes in the initial and parent population put together, with respect to *JTEC* are chosen for entry into parent population as the surviving chromosomes for the next generation.

Major savings in CPU time for *CSP-*k^1 and *CSP-IS-*k^1 is almost 99% compare to single-vendor–four-buyers and single-vendor–five-buyers analytical model. The *CSP-IS-*k^1 for single-vendor–four-buyers could give results even after running the model for 24 hours using enumeration technique, whereas GA model yields results in less than 20 seconds for all models up to ten buyers with single vendor.

5 Illustrative examples and results

The closer the total demand rate to the production rate, the greater saving can be obtained. In other words, by gradually declining the ratio of production rate to demand rate, percentage of *JTEC* saving is increased (Figure 2). In contrast, by increasing p/D ratio, the *JTEC* saving decreases. However, it does not mean that the saving diminishes to zero, but it is nearer to zero as (p/D) becomes significantly high as indicated in Figure 2. Our analysis considers the tradeoff of percentage of savings in *JTEC* versus (p/D) ratio from 1 to 5. It is concluded that the production to demand ratio (p/D) of 3.2 is suitable for the set of given data after considering fine iterative tuning analysis between 3 and 3.5. The curved line called efficient frontier shown in Figure 2 is generated iteratively that minimises the SC cost subject to constraints on the maximal p/D ratio. The point 'A' corresponds to a least cost-savings strategy with a maximal p/D ratio of 3.2 at which steady-state vendor and buyer cost yields for all models with stabilisation (Figure 3). Any SC strategy on the efficient frontier is undominated in the sense that no achievable strategy exists that is at least as good as with respect to *JTEC*. Moreover, it is found from the research articles of Goyal (1995, 2000), Ben-Daya and Hariga (2004), Braglia and

Zavanella (2003) and Valentini and Zavanella (2003) that they considered production to demand ratio as 3.2. At (*p/D*) ratio 3.2, the *JTEC* is getting steady state.

Figure 2 Efficient frontier effect of (*p/D*) ratio on the % of savings in JTEC

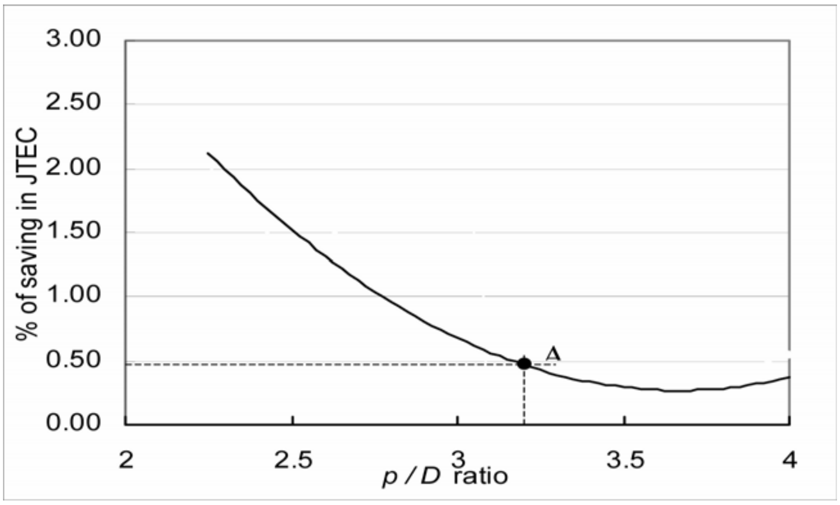
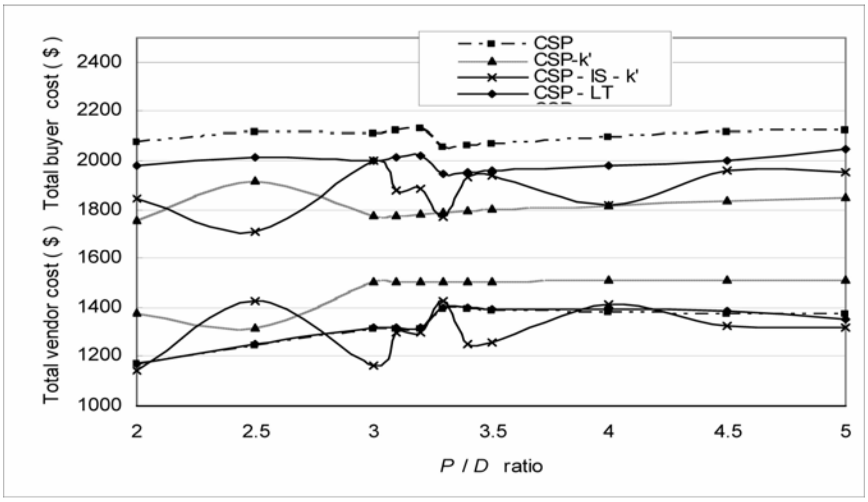
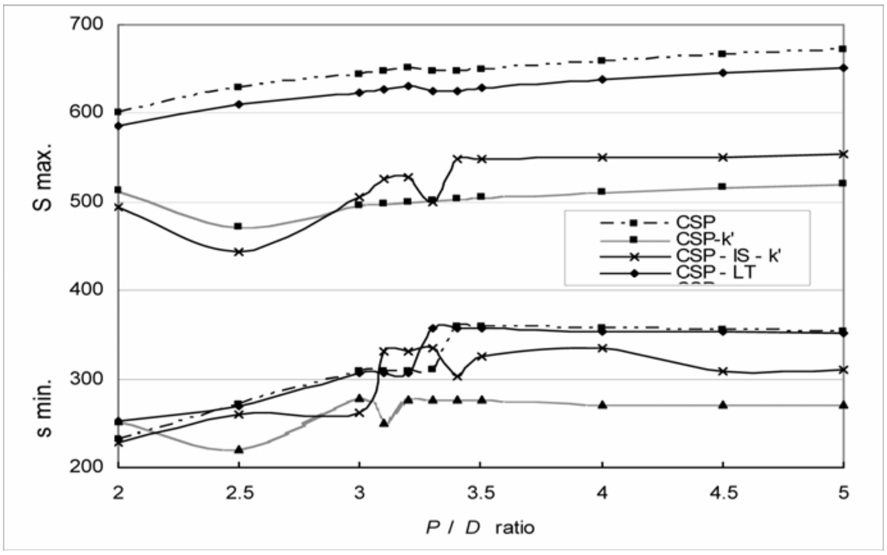


Figure 3 Variation of JTEC of two buyers with (*p/D*) ratio for different strategies



The results refer to input values of Ben-Daya and Raouf (1994), Braglia and Zavanella (2003), Ouyang et al. (2004) and Srinivas and Rao (2007) where h_v = \$4 per unit/year, h_{bi} = \$5 per unit/year, $(p/\sum D_i)$ ratio = 3.2, s = \$400/set-up, A_{ti} = \$25/order, π = \$50/unit. This data is taken up to ten buyers with demand $D_{1,2}$ = 1000, 1300, 800, 1000, 1500, 600, 1200, 1500, 1000, 800 units/year and the corresponding standard deviation are 44.72, 50, 35.7, 30, 30, 20, 30, 30, 30, 20. It is found that the number of shipments in *CSP-IS-*k^1 for single-vendor–two-buyer model is 10 whereas in *CSP* and *CSP-*k^1 it is 5 and 6, respectively – the increase in shipment size is due to information sharing. The buyer maximum stock level and minimum stock level difference in the case of *CSP*-k^1 and *CSP*-*IS*-k^1 is less due to delay delivery and information sharing (Figure 4).

Figure 4 Maximum and minimum stock of two buyers with varying shipments



The *JTEC* for *CSP*-k^1 model with k^1 = 0–7, as shown in Figure 5. In *CSP*-k^1 model, at k^1 = 2 and n = 6 it gives the lowest *JTEC* \$3294 and its corresponding buyers average maximum stock is 500 units for two buyers (Figure 6), whereas *CSP*-k^1 model at k^1 = 0 produces always maximum cost because it adopts basic *CSP* strategy (Figure 5). In single-vendor–two-buyers model, it is assumed that each buyer should receive one shipment and in the sequence order of buyers. The second shipment to any buyer is made only after shipping at least one shipment to each buyer in the SC network.

Figure 5 *CSP*-k^1 model JTEC variation for two buyers with delay deliveries

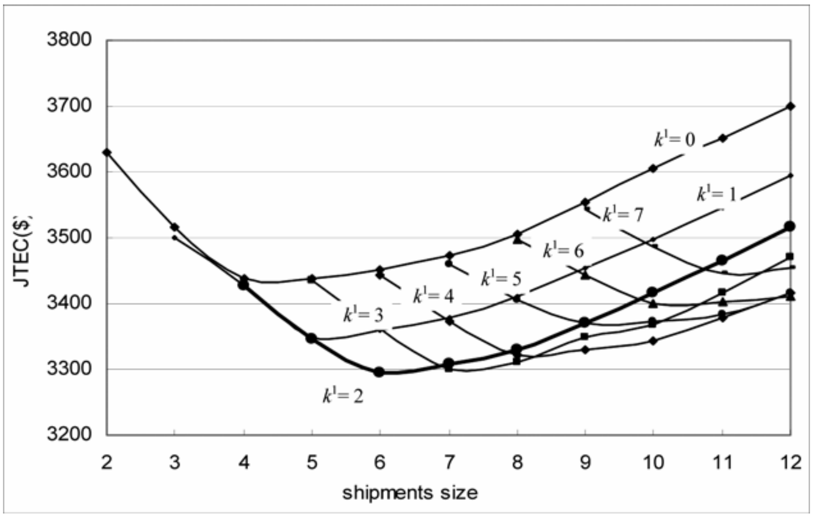


Figure 6 Variation of buyer delay deliveries in *CSP-k*[1] for two buyers

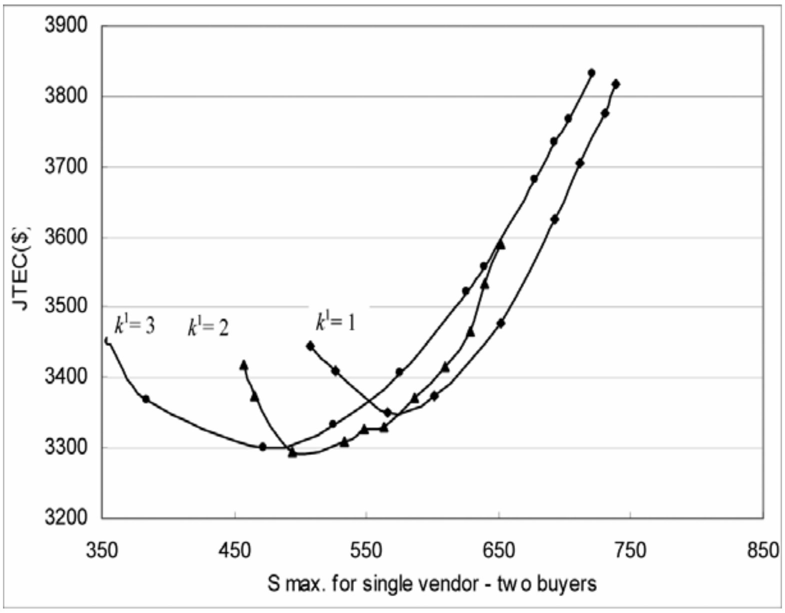


Figure 7 Delayed deliveries and shifted deliveries due to information

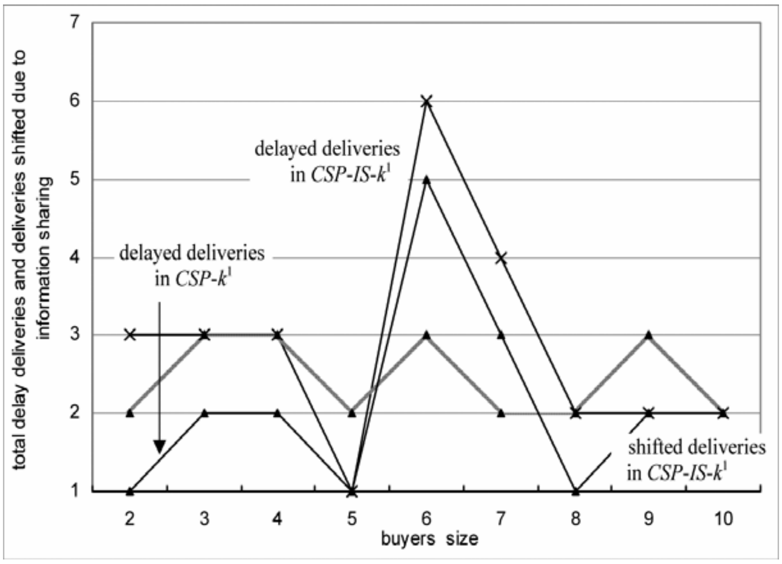


Figure 8 JTEC of single vendor–two buyer for different strategies

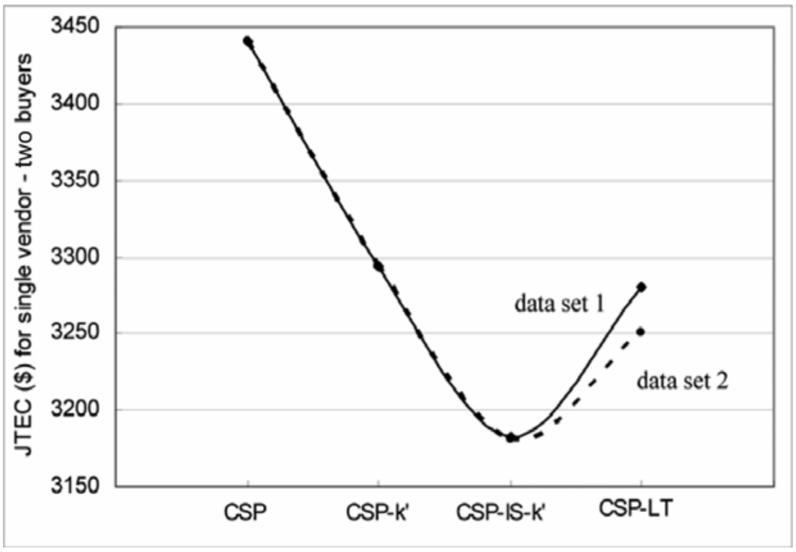


Figure 7 reveals for *CSP-IS-k*^{1} model that the shifted deliveries range from one buyer to another buyer is 1–5 shipments and its corresponding delayed deliveries varies between 1 and 6. Whereas for *CSP-k*^{1}, the delay delivery varies between 1 and 6; it decreases when buyer size increases. The lead-time components taken for *CSP-LT* model in dataset 1 are 14, 10.5, 7 and 5.25 days with unit crashing cost $/day of 0, 0.4, 1.2 and 5.0, and in dataset 2 with unit crashing cost $/day of 0, 0.1, 0.4 and 1.2. As given by Goyal (2003) in his paper that the crashing lead-time components have its impact on the holding cost is misleading and may result in haphazard lead-time. It is to be noted that we do not have to compute the total cost for obtaining the optimal policy like in other models because the lead-time reduction is a joint compromise component of vendor and buyer interest. Perhaps a better approach may be to attribute a benefit of $*x* per day reduction in the lead-time either per cycle or per year. It is to be noted that for single vendor–single buyer, the optimal *JTEC* will occur for second lead-time component, but in the case of single vendor–multi buyer the lead-time may reduce up to minimum component.

Table 1 Comparison of different strategies

Variable	*Hill (1999)*	*Braglia and Zavanella (2003) single vendor–single buyer*			*Proposed models (single vendor–two buyers)*					
		CSP-k^1	CSP-k^1	CSP	CSP-k^1	CSP-k^1	CSP	CSP	CSP-IS-k^1	CSP-LT
Max. buyer stock	110	164	267	376	500	363	651	642	527	610
No. of shipments	5	3	3	4	6	4	5	4	10	4
Delay deliveries	–	2	1	–	2	2	–	–	3	
Total cost ($)	1903	1929	2003	2035	3294	3426	3438	3441	3181	3280

The *JTEC* is decreasing from *CSP* to *CSP-k*[1] and further to *CSP-IS-k*[1]. For the dataset 1, the *JTEC* of *CSP-LT* is more comparing to dataset 2. In brief, the *CSP-IS-k*[1] gives lowest *JTEC* cost compare to *CSP* and *CSP-k*[1]. Whereas for *CSP-LT* it depends on the number and type of lead-time crashing cost reduction components to which the *JTEC* is sensitive in *CSP-LT* model (Figure 8). As described earlier, it is in the interest of vendor–buyer for choosing the structure of lead-time reduction and its crashing cost components. Hence it is very difficult to choose the crashing cost components as well as its lead-time components.

A detailed comparison of the proposed single-vendor–two-buyers models with Hill (1999) and Braglia and Zavanella's (2003) single-vendor–single-buyer model has been given in Table 1. The proposed models basic *CSP* if projected its *JTEC* to single vendor–single buyer gives $111.5 less compare to Braglia and Zavanella (2003) *CSP* model, without considering transportation cost in both the models. It is found that the overall cost in proposed models reduces when chosen to multi-buyers which is an industrial interest. It is to be noted that both Hill (1999) and Braglia and Zavanella (2003) models are based on deterministic demand, whereas the proposed models are based on stochastic demand. In *CSP-LT* the *JTEC* cost increases compare to *CSP-IS-k*[1] due to increase in the buyer stock, and in *CSP* the buyer stock is always more and hence the *JTEC* is more.

6 Conclusions

This paper provides concepts of CSP of four different strategies. The models are developed and illustrated with numerical examples of stochastic customer demand for single vendor–multi-buyer. It is found the *JTEC* of *CSP-IS-k*[1]is less due to information sharing compare to *CSP*, *CSP-k*[1] and *CSP-LT*. But for controllable lead-time model, *JTEC* depends on the lead-time components and its crashing cost components. Future studies have to be made in the area of *CSP* for single vendor–multi-buyer with multi-products.

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